COMMUNICATION BARRIERS IN
THE DIAMOND-DYBVIG MODEL:
THE SEQUENTIAL-SERVICE CONSTRAINT
AND OTHER IMPLICATIONS

Neil Wallace*

Working Paper 370

September 1987

NOT FOR DISTRIBUTION
WITHOUT AUTHOR APPROVAL

ABSTRACT

The Diamond-Dybvig model of banking (Journal of Political Economy, 1983) is amended by introducing communication barriers—these being implicit in their model and in most explanations of why people hold so-called liquid assets. These barriers imply the sequential-service constraint that Diamond and Dybvig imposed on private intermediation and have other implications: infeasibility of the policy that Diamond and Dybvig identify with deposit insurance and desirability of dependence of the realized return on deposits on the random order of withdrawals.

*Federal Reserve Bank of Minneapolis and University of Minnesota.

The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. This paper is preliminary and is circulated to stimulate discussion. It is not to be quoted without the author's permission.
In their 1983 paper, "Bank Runs, Deposit Insurance, and Liquidity," Diamond and Dybvig provided for the first time an almost complete description of an environment or model in which beneficial trading arrangements require the existence of an intermediary with an illiquid portfolio. I say "almost complete" because Diamond and Dybvig left vague and implicit the features of the environment that imply their critical sequential-service constraint on intermediaries. They offer hints about such an environment when they say that imposing their sequential-service constraint allows them "to capture the flavor of continuous time (in which depositors deposit and withdraw at different random times) in a discrete model" (p. 408). However, in attempting to reconcile the feasibility of their deposit insurance policy with the sequential-service constraint, they say only that "the realistic sequential-service constraint represents some services that a bank provides but which we do not explicitly model" (p. 414). Here, I give an explicit description of an environment consistent with the one they hinted at, an environment with communication barriers. These barriers, in addition to giving rise to their sequential-service constraint, are shown to have two other consequences. First, the policy that Diamond and Dybvig identify with deposit insurance is not feasible. Second, desirable arrangements have realized returns on deposits that depend on the random order in which people withdraw.

These results, which hold for the version of the model with aggregate randomness in the intertemporal pattern of demand, are presented in section 4. They are preceded by descriptions and
brief discussions of the physical environment (section 1), the equilibrium concept (section 2), and the special case of no aggregate randomness (section 3).

1. The Physical Environment

Diamond and Dybvig consider a three-date economy with dates labeled \( t = 0, 1, \) and 2. At \( t = 0 \), there are a large number of identical people, each of whom is uncertain about whether at \( t = 1 \) he will want to consume then or will be patient enough to wait until \( t = 2 \) to consume. Some people, in fact, turn out to be impatient and others patient. At \( t = 0 \), each person has an endowment of one unit of a good which can serve as an input into a constant returns-to-scale intertemporal technology with gross rate-of-return \( R_1 \) between \( t = 0 \) and \( t = 1 \), and \( R_1 R_2 \) between \( t = 0 \) and \( t = 2 \). Thus, under autarky, an agent's maximum date 1 consumption is \( R_1 \) and his maximum date 2 consumption is \( R_1 R_2 \). If the input is pooled, then these bounds do not apply. Whether pooling arrangements can and do take a form that resembles bank deposits—and in particular, demand deposits—depends on what is assumed about communication possibilities at \( t = 1 \), a matter which Diamond and Dybvig left implicit. I follow their hints and assume that \( t = 1 \) is an interval of time, that people are isolated from each other during that interval, and that they in a random fashion contact a central location during that interval.

The assumptions that determine what is feasible—those about the \( t = 1 \) interval, the possibilities for the pooling of inputs, and the assumption that whether a person turns out to be impatient or patient is private information—can all be described
using the image of a vending machine—a machine, however, that is productive and that is as smart as a cash machine. I suppose that pooled input is deposited into a vending machine in which it and aggregate withdrawals at \( t = 1 \) and \( t = 2 \), which must be consumed when withdrawn, are subject to the productive technology. The machine operates like a cash machine in that it is able to check a person's account in order to determine whether the person is entitled to make a withdrawal. Moreover, at \( t = 0 \), the machine can be programmed to compute functions of withdrawals as they occur and to make subsequent withdrawals depend on previous withdrawals. The machine cannot, however (say, by way of a lie detector), determine whether a person is impatient or patient. The machine has a location and people contact it once and at random during the \( t = 1 \) interval.

Although the assumptions concerning the \( t = 1 \) interval, which together will be referred to as the isolation assumption, may seem extreme, they should be regarded as a simple version of a widely accepted notion about what lies behind the demand for "liquid" assets—namely, that people hold such assets because they may find themselves impatient to spend when and where they are not connected to asset markets in which they can sell any asset at its "market" price. Since demand deposits have usually been described as providing the holder with the possibility of spending at any time, if not also at any place, it is not surprising that some version of people being isolated plays an important role in a model that implies the existence of assets that resemble demand deposits.
The specific assumptions are as follows.

There is a continuum, the unit interval, of agents and three units of times--instants \( t = 0 \) and \( t = 2 \), and an interval around and labeled \( t = 1 \). At \( t = 0 \) and \( t = 2 \), agents are together. During interval \( t = 1 \), they are isolated, although each one contacts a central location at some instant during the \( t = 1 \) interval. The order in which agents contact the central location is random (and viewed by them as random) and uniformly distributed: the probability that an agent is among the proportion \( v \) who first make contact is \( v \).

Each agent is endowed at \( t = 0 \) with one unit of a good and with access to the following technology set. A triplet \((a^1, a_2, a_3)\), with \( a_i \) being output at time \( i \), is in the set if \((a^1, a_2, a_3) \in [-a, \lambda R_1, a, (1-\lambda)R_2^1 R_2 a] \) for some \( a > 0 \) and some \( \lambda \in [0,1] \) where the \( R_i \) are positive scalars. Note that if \( C_{-1} \) is total consumption at date \( i \), then \((C_{-1}, C_{-2})\) is consistent with the endowment, the technology, and free disposal if and only if

\[(1) \quad 1 \geq C_{-1}/R_{1} + C_{-2}/(R_1 R_2).\]

Each agent at \( t = 0 \) maximizes the expected value of

\[U(c) = \sum_{h=1}^{2} \alpha_h u^h(c^h, c_i^h)\]

where \( c = (c^1, c^2, c_1^1, c_1^2) \) and \( c_i^h \) is consumption at time \( i \) of an agent who learns that he is type \( h \) at the start of interval \( t = 1 \). It is assumed that \( \alpha_h \) is both the probability that the agent will be type \( h \) (i.e., will have the ex post utility function \( u^h \)) and the proportion of agents who will turn out to be type \( h \). Each
agent learns his type at the beginning of interval $t = 1$. Although $U$ is itself an expected utility, I say that agents maximize the expected value of $U$ because there are sources of uncertainty in addition to type—$\alpha_h$ and the ordering of agents during $t = 1$.

I assume that $\alpha_1$ is a discrete random variable with finite support: $\alpha_1 = a_{1}^{k}$ with probability $p_k$, $k = 1, 2, \ldots, K$ and $a_1^k + a_2^k = 1$. Following Diamond and Dybvig, $u^h(c_h^1,c_h^2) = g(c_h^1 + \delta_h^1c_h^2)$, where $0 < \delta_1 < R_2^{-1} < \delta_2$, $g'>0$, $g'' < 0$, and $g'(R_1)/g'(\delta_2R_2R_1) > \delta_2R_2$. (Note that these conditions on $g$ are satisfied by $g(x) = -bx^{-b}$, $b > 0$.) These ex post utility functions are such that if faced with the ex post return $R_2$, type $h$ agents wish to consume only at time $h$.

2. An Equilibrium Concept

The equilibrium concept I use has both cooperative and noncooperative elements and is closely related to that of Harris and Townsend [1981]. It is consistent with people getting together at $t = 0$ and doing the best they can for the representative person by, for example, setting up and determining the policy of an intermediary organized as a mutual (the cooperative element), subject to not being able to coerce people (the noncooperative element)—for example, either to join at $t = 0$ or to truthfully reveal type at $t = 1$.

I suppose that each individual's strategy consists of an investment decision at $t = 0$ and an announcement of type at $t = 1$. An equilibrium, then, consists of strategies for each agent and an allocation rule, giving each individual's consumption as a function of all the strategies and possibly the realizations of
random variables like the order in which a person contacts the
central location at \( t = 1 \), that maximizes \( t = 0 \) expected utility
of the representative agent subject to the following conditions:
(i) the strategies and rule are feasible; (ii) the strategies
include truthful revelation of type at \( t = 1 \); and (iii) given the
rule, the strategies are such that: (a) they constitute a Nash
equilibrium at \( t = 0 \), (b) truthful revelation of type at \( t = 1 \) is
a Nash equilibrium in the subgame that begins after individuals
learn their type, and (c) truthful revelation of type at \( t = 1 \) is
a best response for each agent for any beliefs about whether other
agents truthfully reveal their types.

Note that (i) requires that the allocation rule be
consistent with the description of the physical environment in­
cluding the assumption that people are isolated at \( t = 1 \) and
contact the central location at random. Condition (iii) describes
the noncooperative features of the equilibrium concept and in­
cludes incentive compatibility conditions. Condition (iii.c)
rules out bank runs. Although it implies (iii.b), I list (iii.b)
separately because I will at times want to note that some strate­
gies and rules satisfy (iii.b), but not (iii.c).

Diamond and Dybvig seemed to use an equilibrium concept
somewhat like this one, but possibly one that does not include
(iii.c). I impose (iii.c) as part of the equilibrium concept,
because without it, people at \( t = 0 \) must be assigned beliefs about
whether other agents will truthfully reveal type at \( t = 1 \). (For a
more extended discussion of the Diamond-Dybvig equilibrium con­
cept, see Postlewaite and Vives [1987].)
3. The Diamond-Dybvig Model With No Aggregate Risk

Here the proportion of each type is nonrandom \((K = 1)\). Diamond and Dybvig considered allocations that maximize \(U(c)\) subject only to (1). The assumptions imply that the solution, denoted \(\hat{c}\), is unique and given by \(\hat{c}_2^1 = \hat{c}_2^2 = 0\), (1) at equality, and \(g'(\hat{c}_1^1)/g'(\hat{c}_2^2) = \delta_2 R_2\). It follows that \(\delta_2 R_2 R_1 > \delta_2 \hat{c}_2^2 > \hat{c}_1^1 > R_1\), where the middle inequality is a consequence of \(\delta_2 R_2 > 1\) and the other inequalities follow from that and the conditions on \(g\).

Diamond and Dybvig showed that the following arrangement is an equilibrium that supports \(\hat{c}\). The strategy for each agent is to deposit his entire endowment at \(t = 0\) and to truthfully reveal type at \(t = 1\) when he contacts the machine. The machine offers a gross return of \(\hat{c}_1^1\) on \(t = 1\) withdrawals until total time 1 disbursements cumulate to \(\hat{c}_1^1\). At that point, the machine shuts down ("suspends") until \(t = 2\), when all deposits not withdrawn at \(t = 1\) share equally on a pro rata basis everything that remains.

In terms of my definition of an equilibrium, it is easily seen that this arrangement satisfies all the restrictions, including (iii.c). In particular, it is consistent with the assumption that people are isolated at \(t = 1\). Moreover, that assumption is necessary for claiming that \(\hat{c}\) is supported by, and only by, a scheme that resembles a demand deposit arrangement.

Without the isolation assumption, the above arrangement violates (iii.a) in the following way (see Jacklin [1987]). Consider the situation of an agent who deviates by not depositing his endowment at \(t = 0\), but instead engages in autarkic investment. If he turns out to be type 2, he does nothing at \(t = 1\) and has
higher realized utility than if he deposited. If he turns out to be type 1, then he offers at \( t = 1 \) to sell his autarkic investment to a person who deposited and who turns out to be type 2. A beneficial trade is possible with any such person who can succeed in withdrawing---one possibility being that the autarkic investment is sold for \( \hat{c}_1^1 \). This gives the type 2 person \( R_2 \) at \( t = 2 \) which exceeds \( \hat{c}_2^2 \) and gives the \( t = 0 \) deviant \( (c_1^1, c_2^2) = (\hat{c}_1^1, R_2) \) ex ante, which dominates \( (c_1^1, c_2^2) \). And, without isolation, arrangements that do not resemble demand deposit arrangements can be used to support \( \hat{c} \). For example, let everyone who pooled endowments meet at \( t = 1 \) and let people be asked to identify themselves as to type. If the proportion claiming to be type 1 exceeds \( \alpha_1 \), let no one get anything or let those claiming to be type 1 get \( R_1 \) and the rest \( R_2 \); otherwise, let returns per unit be \( (c_1^1, c_2^2) \). It is easy to see that this scheme supports \( \hat{c} \) except, of course, for violating isolation at \( t = 1 \).

Note that having agents deposit their entire endowment in deposits with gross returns \( \hat{c}_1^1 \) at \( t = 1 \) and \( \hat{c}_2^2 \) at \( t = 2 \) is not the only way to support the \( \hat{c} \) allocation. It can also be supported using diversified portfolios for individuals. One possibility is as follows. If \( (x_1, x_2, \delta) \) is given by the unique solution to

\[(2) \quad 1 = a_1 x_1 / R_1 + a_2 x_2 / R_1 R_2 \]

\[(3) \quad x_1 = \delta x_2 \]

\[(4) \quad \hat{c}_1^1 = (1-\delta)x_1 + 8R_1 \]
then a strategy for each agent of investing \( \beta \) autarkically and
\( (1-\beta) \) in pooled input that pays \( x_1 \) per unit withdrawn at date 1
until date 1 withdrawals total \( (1-\beta)x_1 \) and pays out what remains
at \( t = 2 \) on a pro rata basis to remaining deposit claims is an
equilibrium that supports \( \hat{c} \). The proof of this assertion—which
involves showing that \( x_2 = \frac{(1-\beta)x_2 + \beta R_2}{\beta} \) is
straightforward and left to the reader. Note that by (3), type
1's want to withdraw at \( t = 1 \), while type 2's are willing to wait
until \( t = 2 \), which is what they must be assumed to do. Given such
truthful revelation, (2) is implied by the technology. Note that
subject to incentive compatibility, (3) maximizes the date 1
return and, hence, the degree of illiquidity of the pooled portfo-
lio. Another way to support \( \hat{c} \) is to have individuals diversify
between deposits with the above returns and deposits with returns
given by (2) and \( x_1 = \delta_1 x_2 \), returns that maximize the date 2
return subject to incentive compatibility.

These arrangements suggest that \( \hat{c} \) can be supported by
schemes that are decentralized in the following way: firms offer
deposits with payoff streams that depend only on the parameters in
(2) and the \( \delta_1 \), while individuals allocate their endowments among
such deposits and autarkic investment. In fact, such schemes,
which more closely resemble actual banking arrangements, could
support allocations that are optimal subject only to (1) in ver-
sions of the model with some diversity; individuals could have
different endowments and different preferences in the form of
different \( g \) functions.
Finally, note that any of the ways of supporting consistent with the isolation assumption nicely depict the situation of holders of ostensibly low-return, liquid forms of assets in actual economies. A plausible description of the situation facing holders of such assets is that they face a distribution of possible circumstances: in some the pecuniary return on the liquid assets will exceed that on other assets and in some the reverse will occur. The former exist even though they are not systematically recorded and even though recorded "market rates of return" are such that liquid assets are dominated in rate of return. In the model, the time 0 situation of agents is similar with the high time 1 return deposits playing the role of liquid assets. If an agent turns out to be type 1, he converts all his assets into consumption and realizes a higher return on liquid assets than on other assets. If, instead, he turns out to be type 2, the reverse occurs. In both the model and the actual economy, the circumstance in which the liquid asset has a higher return occurs outside of organized asset markets.

4. Random Aggregate Proportions of Types

Diamond and Dybvig analyzed this version by first considering allocations that depend only on the realization \( \alpha^k = (\alpha^k_1, \alpha^k_2) \) and that solve the following problem: choose \( c^h_i(\alpha^k) \) for \( h, i = 1, 2, \) and \( k = 1, 2, \ldots, K \) (a 4K element vector) to maximize

\[
\text{EU}(c) = \sum_k \rho_k \left[ \sum_h \alpha^k_h u^h(c^1_h(\alpha^k), c^2_h(\alpha^k)) \right]
\]

subject to
(6) \[ 1 \geq \sum_{h} a^{k} c_{1}(a^{k})/R_{1} + \sum_{h} a^{k} c_{2}(a^{k})/R_{1}R_{2}; \quad k = 1, 2, \ldots, K \]

where (6) is simply (1) applied to such allocations.

This problem splits into \( K \) separate problems, one for each \( k \). Moreover, for each \( k \) the problem is exactly like that which determined \( \hat{c} \) above. Thus, the solution, denoted \( \hat{c}(a^{k}) \), \( k = 1, 2, \ldots, K \), is for each \( k \) given by

\[ \hat{c}_{1}(a^{k}) = \hat{c}_{2}(a^{k}) = 0, \quad (6) \]

at equality, and \( g'(\hat{c}_{1}(a^{k}))/g'(\delta_{2}\hat{c}_{2}(a^{k})) = \delta_{2}R_{2} \). It follows that \( \hat{c}_{1}(a^{k}) \) and \( \hat{c}_{2}(a^{k}) \) vary with \( k \), a fact used below.

Diamond and Dybvig showed that no pooling arrangement that satisfies their sequential-service constraint can support \( \hat{c}(a^{k}) \). Their argument, which shows that \( \hat{c}(a^{k}) \) is not feasible given the isolation assumption, considers two exhaustive possibilities for time 1 returns on pooled investment consistent with the sequential service constraint. If such returns depend on the order in which agents contact the machine, then for some \( k \) two different type 1 agents will receive different consumptions. If such returns do not depend on the order, then the return does not vary with \( k \). Put differently, if the vending machine must make payments during the \( t = 1 \) interval before it can cumulate all requests, then it cannot achieve payments that depend on \( k \) and that for a given \( k \) do not depend on the order in which agents appear.

4.1 Diamond-Dybvig Deposit Insurance

Despite the above result, Diamond and Dybvig claimed that there is a feasible policy, one which they identify with deposit insurance, that permits \( \hat{c}(a^{k}) \) to be supported as an equi-
librium. The crucial feature of this policy is that the deposit insurance agency is allowed to tax and make transfers to everyone at \( t = 1 \) that depend on the realization of \( a^1 \). That is, their scheme permits the deposit insurance agency to cumulate all requests for \( t = 1 \) withdrawals and to make net disbursements to everyone that depends on the total of such requests. It is as if the deposit insurance agency holds a meeting at \( t = 1 \) at which it announces \( \hat{c}(a^K) \) as a schedule of returns that is conditional on the proportion who when asked announce themselves to be type 1. With a suitable proviso about what happens if more than \( a^K \) respond that they are type 1—for example, no one gets anything or those announcing themselves to be type 1 get \( R_1 \)—such a scheme satisfies equilibrium condition (iii.c).

In defense of allowing their deposit insurance agency to violate the sequential service constraint imposed on intermediaries, Diamond and Dybvig said only the following:

As the government can impose a tax on an agent after he or she has withdrawn, the government can base its tax on \( f \), the realized total value of \( T = 1 \) withdrawals. This is in marked contrast to a bank, which must provide sequential service and cannot reduce the amount of a withdrawal after it has been made. This asymmetry allows a potential benefit from government intervention. The realistic sequential-service constraint represents some services that a bank provides but which we do not explicitly model (p. 414).

This is not convincing. If there is a way to reconcile the sequential-service constraint with \( \hat{c}(a^K) \) and their deposit
insurance scheme, Diamond and Dybvig have not provided it. As noted above, $\hat{c}(a^k)$ and the Diamond-Dybvig deposit insurance are not compatible with the isolation assumption. And, of course, if that assumption is invoked, there is no need to appeal to some unmodeled service of banks in order to justify the sequential service constraint; the risk-sharing services of banks must be provided subject to the sequential service constraint.

4.2 Nontrivial Suspension

The Diamond-Dybvig argument given above shows that $\hat{c}(a^k)$ is not feasible given the isolation assumption. That being so, it is far from obvious what is an equilibrium arrangement. I now show that there are economies in which any equilibrium arrangement will have realized date 1 consumption dependent on the order in which type 1 agents contact the central location. I do this by showing that an arrangement in which realized date 1 consumption depends in a simple way on that ordering satisfies equilibrium conditions (i)-(iii) and gives higher expected utility than any feasible arrangement which does not display any such dependence.

Proposition. There exist economies for which any equilibrium has date 1 consumption dependent on the order in which agents contact the central location during the $t=1$ interval.

Proof. An upper bound on expected utility achievable under schemes without dependence on order is given by the solution to maximizing (5) subject to (6) and $c^h_1(a^k) = c^h_1$, a constant not dependent on $k$. Denote this allocation $\tilde{c}(a^k)$ and note that it
satisfies $c_1^2 = c_2^1(a^k) = 0$ and (6) at equality for each $k$. Now let $K = 2$ so that the nonzero part of $\bar{c}(a^k)$ is a triplet $(\bar{c}_1^1, \bar{c}_2^1(a^1), \bar{c}_2^2(a^2))$. Moreover, let the function $g$ be such that this solution satisfies $\bar{c}_1^1 < \delta_2\bar{c}_2^2(a^k)$ for $k = 1, 2$. If $g(x) = -bx^2$, $b > 0$, then, since the solution to maximizing (5) subject to (6) approaches $c_1^1 = R_1$, $c_2^2 = R_1 R_2$ as $b$ approaches 0, for sufficiently small $b$, $\bar{c}_1^1 < \delta_2\bar{c}_2^2(a^k)$.

Now consider the following strategies and rule. The strategies are to pool the entire endowment and to truthfully reveal type. The rule is as follows: for some positive $\varepsilon$ still to be determined, the return per unit of withdrawal is $(\bar{c}_1^1 + \varepsilon)$ until withdrawals total $\alpha_1^1(\bar{c}_1^1 + \varepsilon)$, the return per unit of subsequent withdrawals is $(\bar{c}_1^1 - \varepsilon)$ until withdrawals total $\alpha_1^1(\bar{c}_1^1 + \varepsilon) + (\alpha_2^1 - \alpha_1^1)(\bar{c}_1^1 - \varepsilon)$, then no further withdrawals are allowed. Finally, at $t = 2$, all resources are divided on a pro rata basis among remaining deposit claims. Note that this scheme satisfies equilibrium conditions (i)-(iii) for a sufficiently small $\varepsilon$. In particular, since $\bar{c}_1^1 < \delta_2\bar{c}_2^2(a^k)$, (iii.c) is satisfied.

In order to express expected utility as a function of $\varepsilon$, I first express $C_j$ as a function of $\alpha_k$ and then use (1) at equality to express $c_2^2(a^k)$ as a function of $\varepsilon$. If $\alpha = \alpha_1$, then $C_1 = \alpha_1^1(\bar{c}_1^1 + \varepsilon)$, while if $\alpha = \alpha_2$, then $C_1 = \alpha_1^1(\bar{c}_1^1 + \varepsilon) + (\alpha_2^1 - \alpha_1^1)(\bar{c}_1^1 - \varepsilon)$. In each case, $c_2^2(a^k) = (R_1 R_2 - R_2 C_1)/(1-\alpha_1^k)$. Then expected utility is

\[
G(\varepsilon) = p_1[\alpha_1^1 g(\bar{c}_1^1 + \varepsilon) + (1-\alpha_1^1)g(\delta_2\bar{c}_2^2(a^1))] \\
+ p_2[\alpha_2^1(\alpha_1^1/a_1^1)g(\bar{c}_1^1 + \varepsilon) + \alpha_1^1(1-\alpha_1^1/a_1^1)g(\bar{c}_1^1 - \varepsilon) \\
+ (1-\alpha_1^2)g(\delta_2\bar{c}_2^2(a^2))]
\]
where \( a_1^1/a_1^2 \) is the probability conditional on \( \alpha = a_1^2 \) that a type 1 agent contacts the machine early enough to get the return \((c_1^1+\epsilon)\), while \((1-a_1^1/a_1^2)\) is the probability that he is too late and gets only \((c_1^1-\epsilon)\).

The derivative of \( G \) with respect to \( \epsilon \) evaluated at \( \epsilon = 0 \) is

\[
G'(0) = p_1 a_1^1 H(a_1^1) + p_2 (2a_1^1-a_1^2) H(a_1^2)
\]

where \( H(a_k) = \frac{g'(\bar{\alpha}_k^1)}{\delta_1^2} - \delta_2^2 g''(\bar{\alpha}_k^2(a_k)) \). Since \( \bar{\alpha}_k^1(a_k) \) is the solution to maximizing (5) subject to (6) and \( \bar{\alpha}_1^1(a_k) = \bar{\alpha}_1^1 \), it satisfies

\[
\begin{align*}
p_1 a_1^1 H(a_1^1) + p_2 a_1^2 H(a_1^2) &= 0. \quad (9)
\end{align*}
\]

Substituting from (9) into (8), \( G'(0) = p_2 (2a_1^1-a_1^2) H(a_1^2) \). Since \( a_1^1 < a_1^2 \), \( g'' \) < 0, and \( \bar{\alpha}_k^1(a_k) \) satisfies (6) at equality, it follows from the definition of \( H(a_k) \) that \( H(a_1^2) < H(a_1^1) \). This inequality and (9) imply that \( H(a_1^2) < 0 \), which implies \( G'(0) > 0 \).

Note that this proposition does not describe an equilibrium. That is, even for the simple \( K = 2 \) case, I have not found an arrangement that maximizes expected utility subject to conditions (i)-(iii) that are part of the equilibrium concept. However, since for some economies, I have found an arrangement that satisfies those conditions and gives higher expected utility than any feasible arrangement that does not make consumption depend on the order in which agents contact the central location at \( t = 1 \), I can conclude that for those economies any equilibrium arrangement displays such dependence.
Although this dependence property of an equilibrium has been established only for a very special class of economies, it almost certainly holds quite generally. Arrangements that do not display such dependence necessarily limit randomness to date 2 consumption. In general, higher expected utility should be achievable using arrangements that shift some of the randomness to date 1 consumption even if such shifting is accomplished by introducing a new source of randomness, dependence on ordering—that by itself is utility reducing. Indeed, a plausible conjecture is that aggregate randomness in the desired intertemporal pattern of consumption and some version of the isolation assumption are in general sufficient to imply that equilibrium arrangements involve some dependence of returns on the order in which people withdraw.

Note, in this connection, that events under an equilibrium arrangement in which returns are a decreasing function of earlier withdrawals match quite well the qualitative features of U.S. banking experience during the nineteenth century. In particular, during suspension episodes, those who withdrew late, after suspension occurred, received a lower return—their checks passed at a discount—than those who were able to withdraw early. Under this interpretation, suspension episodes are identified with high realizations of $a_i^k$, not with runs in the sense of type 2 agents claiming to be type 1 agents.

That interpretation can be pursued further to account both for the search for better banking arrangements—in particular, for ones under which suspensions do not occur—and for the banking difficulties of the early 1930s in the U.S. As regards
the former, since some depositors during suspension episodes received lower returns than others simply because they happened to withdraw later, since economies subject to aggregate randomness of withdrawal demand and some version of the isolation assumption are quite complicated, and since the perspective on feasible arrangements provided by the model was not available, it is not at all surprising that there was a continual search for better banking arrangements. 5

As regards the latter, a version of the view set out by Friedman and Schwartz [1963, p. 329] can be adopted. According to them, once the Federal Reserve System was created, suspension by individual banks and groups of banks became a less viable option, even though, contrary to what was widely presumed, no reliable mechanism to replace suspension was in fact put into place. When that presumption was tested and found not to be correct—when, in terms of the model, people learned that the arrangement in place did not satisfy equilibrium condition (iii.c)—then deposits became a less desirable form in which to hold wealth. 6

5. Concluding Remarks

Since the main purpose of a model like that described above is the analysis of alternative banking arrangements or policies, I want to point out that there are versions of regulatory and other policies that, in contrast to the Diamond-Dybvig version of deposit insurance, are feasible in the model. Bank capital can be modeled as the setting aside at t = 0 of some pooled input, the returns from which are paid out to other than shareholders and at other than date 2 only in unusual circum-
stances. A complete description, certainly of capital requirement, calls for spelling out those circumstances. Feasible versions of deposit insurance can also be formulated. For example, a deposit insurance agency could promise to meet date 2 obligations not met by intermediaries, with the promise backed by a contingent tax on nondeposit wealth at \( t = 2 \). However, while formulating such policies is relatively easy, analyzing their consequences is not.

First, such policies have a potential role only in the version of the model with aggregate randomness. To establish an actual role, therefore, it would have to be shown that they improve upon the best nontrivial suspension policy. Note, by the way, that since it was shown above that desirable arrangements have returns dependent on the order in which agents appear, one would not want to preclude such dependence a priori—even though actual deposit insurance systems seem to.

Second, to the extent possible, it is desirable to avoid having conclusions about such policies depend on nonrobust and inessential features of the model. For example, the ex post utility functions, the \( u_h \), used above display constant marginal rates of substitution, the \( \delta_h \). A consequence is that schemes that force agents at \( t = 0 \) to hold any wealth that is convertible into consumption only at \( t = 2 \)—as would happen under a capital requirement—are costly in terms of expected utility. However, such schemes would not necessarily be costly with more plausible "smooth" ex post preferences. Another nonrobust and inessential feature of the model is its single, nonstochastic intertemporal
technology. A richer specification would permit the model to address the moral hazard aspects of some deposit insurance schemes.

A generalized version of the model would, by the way, have implications consistent with some of the main features of the U.S. banking experience under the deposit insurance scheme put into place in 1934. Leaving aside the well-known moral hazard aspects of the scheme, those features are: no suspensions of any kind, no bank runs, and general satisfaction with the scheme. If the model were amended to include sufficient nondeposit wealth at $t = 2$ to make credible the kind of deposit insurance described above, then under a version of such insurance, it could account for those features. The insurance could fully substitute for limiting the total of time 1 withdrawals. And, if it is specified to prohibit returns dependent on ordering, as actual insurance is, then such returns would not be observed. Finally, although dependence on ordering might be desirable, this would not be recognized unless a connection were made between the magnitude of (date 1) returns on deposits and the effective prohibition of nontrivial suspension—a connection that is difficult to make in the absence of the model.

In my view, the essential ingredients of the model, ones that should be maintained in an analysis of banking arrangements, are its specification of preferences that leads people not to want to precommit to specific intertemporal patterns of consumption, aggregate risk concerning that pattern, the goods-in-process aspect of the technology, some version of the isolation assump-
tion, and private information. As the above discussion suggests, much remains to be done in exploring fruitful ways of embedding those features into complete models that allow one to conduct disciplined discussions of alternative banking arrangements. Diamond and Dybvig are, I think, to be credited with being the first to point out a way of doing that.
Footnotes

1 Since the model describes an entire economy, the machine must be regarded as the consolidated banking-business sector of that economy.

2 There is an important irreversibility or goods-in-process aspect of the technology. The marginal return $R_2$, between $t = 1$ and $t = 2$, is available only by adjusting $\lambda$. It is not available de novo as a one-period technology using output of time 1 good as input. I assume that no such one-period technology is available, although that assumption could be relaxed without affecting the results.

3 A more straightforward way to produce aggregate randomness is to have a finite number of agents with type identically and independently distributed among them.

4 It satisfies (iii.a) because given the isolation assumption and the rule, any deviation at $t = 0$ gives the agent some weighted average of $(R_1, R_1 R_2)$ and $(\hat{c}_1, \hat{c}_2)$ with less than full weight on the latter.

5 Note that I am suggesting that banks and depositors behaved as postulated by the model even though no one knew the model. Banks faced with meeting random withdrawals sequentially could easily be led to adopt suspension policies—in part, as some said, to protect bank assets. Depositors would be led to hold deposits even knowing that suspensions might occur for exactly the reasons described by the model; namely, that deposits have a relatively high pecuniary return in some circumstances. Depositors would not need to view themselves as sharing risk with other depositors.
These remarks should not be construed as suggesting that the model be judged by how well it matches historical events. Such matching is relevant in deciding between this and other models that also permit one to analyze alternative policies toward banking.

Jacklin [1987] explores the consequence of such preferences in a version of the model with markets in assets at $t = 1$. 
References


