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STOCHASTIC COMPARATIVE STATICS:  
A COMMENT ON STOCKMAN

Patrick J. Kehoe\*

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\*Federal Reserve Bank of Minneapolis and University of Minnesota.

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The purpose of Stockman's paper is twofold: First, it promotes a type of stochastic comparative statics (SCS) as an alternative to standard deterministic comparative statics (DCS). Second, it shows how the degree of completeness of markets, here international financial markets, may be crucial for the SCS results. Stockman accomplishes this through a series of examples that clearly illustrate his points, and I have nothing to add to them. Instead I will expand on his two main themes.

#### Comparative Statics: Deterministic Versus Stochastic

When we study a dynamic economy, we are often concerned with how the economy will respond to various types of shocks. We would like to perform experiments in which these shocks can be interpreted as taking place in a single economy in real time, that is, calendar time. I argue that there is no logically consistent way to carry out such experiments in a deterministic setting. However, in a stochastic setting such experiments are straightforward.

In a deterministic setting, shocks are classified as either "unanticipated" or "anticipated." With unanticipated shocks we consider an economy in which agents are assumed to know the future with certainty and then we ask what happens if some event unexpectedly occurs. This question is ill-posed since we solve for an equilibrium conditional on certain assumptions which we then violate in our thought experiment. This makes our experiment internally inconsistent and, hence, nonsensical.

A simple example should make this clear. Consider a world in which Stockman knows that his house will never burn

down. Now suppose that it does. What happens? This question is ill-posed because if Stockman knew his house could burn down, he would have had enough sense to have already bought some insurance--or at least a fire extinguisher.

More generally, the logical problem is the following. We start by assuming there is, using Arrow's terminology, a single possible state of the world. Part of this state includes Stockman's house sitting there in fine condition. Given this state we define the natural commodity space and we define preferences over this space. We can then compute equilibrium allocations and welfare for this economy. However, serious problems arise if we attempt to evaluate equilibrium allocations and welfare in some "unexpected" state of the world in which Stockman's house has burned down. Since these allocations are not contained in our original commodity space and our preference order is not even defined over such a point, I have no idea what the word welfare means in this context.

In order to avoid a possible misunderstanding, I should expand on one small point. If we simply ignore these logical difficulties and interpret such experiments as if they were conducted in a truly stochastic world, then there are special cases in which we may get the "right" answer. In particular, if in the stochastic world either there is no possibility for sharing risk--because of the market structure or the physical environment--or there is no value in sharing risk--because agents are risk neutral--then we will get the right answer in the analogue deterministic environment in the sense that we will obtain the same numerical values for the consumption allocations either way.

An example of such a stochastic environment is a representative agent Lucas-tree economy, that is, a pure exchange economy populated by agents who have identical preferences and who, for every possible realization of uncertainty, have identical endowments. In such a model no matter how we conduct our experiments and no matter what we assume about markets, the equilibrium will always be "don't trade and eat your own fruit." This may be a useful model for studying asset prices, but it is not very useful for studying trade. As soon as we add a little heterogeneity to this environment--either in preferences or in endowments--how we conduct our thought experiments and what we assume about market structure become crucial. [For an analysis of how such experiments work in a Lucas model with heterogenous agents, see Backus and Kehoe (1987).]

Finally, some may attempt to salvage DCS experiments as reasonable approximations to SCS experiments in which the shocks under consideration occur "rarely." However, they cannot be salvaged. For example, an SCS experiment in an economy where there is a small positive probability of a house burning down will typically be vastly different from a DCS experiment in an economy where there is a zero probability of a house burning down.

With the other type of deterministic shocks--the anticipated shocks--we consider two distinct settings for some economically exogenous variable. For each setting, we solve for a separate perfect foresight equilibrium and then compare the endogenous variables, prices, and allocations across the equilibria. From this description it is clear that these comparisons cannot be

interpreted as taking place in a single economy in real time. Rather, for economies specified at the country level, they should be interpreted as cross-country experiments. Thus, even though they are internally consistent, they are useless for many of the thought experiments we want to consider.

In a stochastic setting, it is straightforward to model shocks that take place in real time. The basic algorithm for conducting consistent experiments involving a shock to an exogenous variable is the following:

- Consider an economy in which agents place a positive probability on at least two values of this variable.
- Compute one equilibrium in which agents engage in all mutually beneficial trades and in which their expectations are confirmed.
- Draw different time paths of realizations of the exogenous stochastic variables.
- Compare equilibrium prices and allocations across these realizations.

In this setup we can compute cross-moments between any variables of interest. [For interesting examples of this algorithm, see Svensson (1985) and Stockman and Svensson (1985).]

For some variables, such as endowments or productivity, these experiments have a straightforward interpretation. However, for government policy variables the interpretation is less clear. In his paper, Stockman models government policies as exogenous stochastic variables. The policy experiments he consid-

ers are comparisons across realizations of these processes. How should we interpret such experiments? That depends on the underlying model of government behavior.

Suppose that we assume a government is a single administration that chooses a policy function to maximize its objectives. This function will have as arguments the state variables of the economy which include, among other things, all exogenous stochastic variables. As in Stockman's model, government policy will follow a stochastic process. However, there are some differences. Basically, we have pushed the exogenous uncertainty back to a deeper level--back from the level of an institution called the government to the more primitive level of agents' tastes and technology. As a result, policy introduces no new randomness into the economy. Of course, if we introduce shocks into the government objective function, government policy will add to the randomness. However, if we start building a model of these shocks, we will end up with them being functions of the original primitive shocks. Government policy will again introduce no new randomness. I will discuss the implication of this in a moment. For now, simply realize that in this interpretation we are investigating the operating characteristics of the economy under a single policy regime, where I define a regime to be a particular policy function of the government. Note that although we can give the word regime many reasonable definitions, I will use it in the concrete sense just described.

Suppose now, however, that we are interested in comparing outcomes across regimes. One way to do this is to specify two

different objective functions for the government and then solve for two equilibria. In the first equilibrium, agents correctly believe that with probability one the government maximizes the first objective function; in the second equilibrium, agents correctly believe that with probability one the government maximizes the second objective function. Although we can compare the operating characteristics of these two regimes, we cannot interpret comparisons across these equilibria as real-time experiments for the same reasons as before.

There is, however, an alternative to this type of experiment. Suppose that the government is composed of a sequence of administrations with possibly differing objective functions. Suppose, for simplicity, that there are only two possible administrations and that for some as-yet-to-be-specified process the government switches randomly between them. Then, for each administration we can solve for a policy function, and we can solve for a single equilibrium and consistently compare across these regimes. As far as I know, this is the only way to consistently compare regimes in a way that can be interpreted as taking place in a single economy in real time. [For a good exposition of these ideas, see Cooley, LeRoy, and Raymon (1984).]

Since the main point of Stockman's paper is to show how the degree of completeness of markets can affect SCS results, he does not need to develop a deep model of government behavior. However, the nature of the underlying model is important for two reasons: it clarifies the possible interpretations of Stockman's experiments, and it helps us think about what financial markets we need in order to have complete markets.

In Stockman's model the fundamental uncertainty is in government policy itself. In this setup, to have complete markets Stockman needs securities that pay off as functions of government policy. With casual reading, we may leave Stockman's paper with the mistaken impression that if we do not see securities that explicitly depend on government policies, then we necessarily have incomplete markets. With more careful reading, however, we realize that this is simply because Stockman took a useful shortcut in modeling government behavior. With a deeper model of government behavior, government policy will itself be a function of other stochastic variables, such as productivity. In this case, to have complete markets we do not need securities that depend on government policy directly; we only need to have enough securities that are correlated with the primitive stochastic elements.

#### Market Completeness and Stochastic Comparative Statics

Stockman's second purpose is to investigate how the degree of completeness of international financial markets affects the results of SCS. To show this, Stockman conducts experiments in two polar regimes: one with complete international financial markets and another with no international financial markets. The punchline of these examples is that the results may differ widely across the regimes.

Loosely speaking, the intuition for these examples is as follows. With complete markets, optimal behavior by agents involves eliminating all diversifiable income effects, while with incomplete markets, agents are artificially constrained so that they cannot eliminate all of these effects. In both cases, how-

ever, substitution effects remain. Then for a given SCS experiment, if the substitution effects go in the opposite direction of the income effects, it is possible to have experiments that have opposite signs in the two cases. Basically, the income effects due to incomplete markets need to swamp the substitution effects.

Since Stockman's examples illustrate these points clearly, I will concentrate on answering this question: Why should we be interested in knowing how the completeness of markets affects SCS results?

A reason Stockman seems to favor is that the increasing sophistication of financial markets in countries like the United States means that we are moving from a regime of less complete markets to one of more complete markets. Thus, wisdom gleaned from the earlier stages of market development may soon prove faulty. I am not that comfortable with this motivation.

Another reason, which I find more appealing, is that this analysis may give us insight into which traditional trade theory results obtained using deterministic models will be overturned once we switch to stochastic models. This is because DCS results often are very similar to SCS results with incomplete markets. Basically, both get the income effects wrong in the same direction. Thus, if the completeness of markets overturns an SCS experiment under incomplete markets, it may also overturn the analogous DCS experiment.

A final reason is that Stockman's paper is the beginning of a research project that investigates the effects of incomplete markets more broadly. If this is true, then I would like to add a

word of caution. We have learned from Harris and Townsend (1981) that in terms of thinking about what it means for government policy to be optimal, there is a world of difference between an environment in which incomplete markets are simply imposed and one in which markets are as complete as they can be, given the informational-spatial-communication structure.

If we are not careful, we may end up analyzing what Ed Prescott calls a "chicken model." The analysis of such a model goes something like this: First, assume that the private sector wants chickens but can't make them. Next, assume that governments can make chickens. The amazing policy result is that in equilibrium the government should make chickens and supply them to the private sector. I hope we have more exciting things to work on than this.

Of course, Stockman has not fallen into the chicken coop. Rather, he has provided us with a series of thought-provoking examples.

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