



FEDERAL RESERVE BANK
OF MINNEAPOLIS

Research
Division

STAFF REPORT
No. 613

Jacks of All Trades and Masters of One: Declining Search Frictions and Unequal Growth

September 2020

Paolo Martellini

Federal Reserve Bank of Minneapolis

Guido Menzio

New York University and NBER

DOI: <https://doi.org/10.21034/sr.613>

Keywords: Search frictions; Biased technical change; Growth; Inequality

JEL classification: E24, J24, J31, J64, O47

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Jacks of All Trades and Masters of One: Declining Search Frictions and Unequal Growth*

PAOLO MARTELLINI
FRB Minneapolis

GUIDO MENZIO
NYU and NBER

August 2020

Abstract

Declining search frictions generate productivity growth by allowing workers to find jobs for which they are better suited. The return of declining search frictions on productivity varies across different types of workers. For workers who are “jacks of all trades”—in the sense that their productivity is nearly independent from the distance between their skills and the requirements of their job—declining search frictions lead to minimal productivity growth. For workers who are “masters of one trade”—in the sense that their productivity is very sensitive to the gap between their individual skills and the requirements of their job—declining search frictions lead to fast productivity growth. As predicted by this view, we find that workers in routine occupations have low wage dispersion and growth, while workers in non-routine occupations have high wage dispersion and growth.

JEL Codes: E24, J24, J31, J64, O47.

Keywords: Search Frictions, Biased Technical Change, Growth, Inequality.

*Martellini: Federal Reserve Bank of Minneapolis, 90 Hennepin Avenue, Minneapolis, MN 55401, USA (email: pa.martellini@gmail.com); Menzio: Department of Economics, New York University, 19 West 4th Street, New York, NY 10012 (email: gm1310@nyu.edu). We are grateful to Pieter Gautier, Victoria Gregory, Chris Moser and Robert Ulbricht for their insightful comments.

1 Introduction

Search frictions cause labor misallocation, in the sense that workers are not necessarily employed in the jobs in which they are most productive. As search frictions decline—because of, say, advances in information and communication technologies—the allocation of labor improves and average labor productivity increases.¹ The return of declining search frictions to productivity, though, is unequal for different groups of workers. For workers who are specialists—in the sense that their productivity varies a great deal across different jobs in their labor market—the decline in search frictions leads to high productivity and wage growth. For workers who are generalists—in the sense that their productivity is similar across different jobs—the decline in search frictions leads to minimal productivity and wage growth. Thus, declining search frictions lead to lower growth for “jacks of all trades” (the generalist workers) and higher growth for “masters of one trade” (the specialized workers).

We formalize the above argument using a version of the search-theoretic model by Martellini and Menzio (forthcoming) with multiple types of workers. Different types of workers populate different labor markets. In each labor market, workers and firms are located along a circle of unit circumference and meet randomly. The distance between a particular worker and a particular firm determines the productivity of their match and, in turn, the decision of whether the firm and the worker consummate their match. The environment is non-stationary, as both the production and the search technologies improve over time. As in Martellini and Menzio (forthcoming), we restrict attention to a Balanced Growth Path (BGP), which is an equilibrium path consistent with the empirical observation that the unemployment rate, the vacancy rate, the rate at which unemployed workers become employed (UE rate), and the rate at which employed workers become unemployed (EU rate) display no secular trend. The existence of a BGP implies some restrictions on the fundamentals of the model. In particular, it implies that productivity must be isoelastic with respect to the distance between the worker and the firm, which we interpret as the distance between the skills of the worker and the skill requirements of the firm’s job.

The main theoretical finding of the paper is that for workers of a particular type, the growth rate of productivity and wages has two components. The first component is the growth rate of the production technology used by that type of worker. The second component is the growth rate of the efficiency of the search technology multiplied by the type-specific elasticity of productivity with respect to the firm-worker skill distance. Therefore, the growth rates for two groups of workers may differ not only because of biased technical change, but also because the return to declining search frictions is higher for one group—the group with a higher elasticity of productivity with respect to the firm-worker skill distance—and lower for the other group—the group with a lower elasticity of productivity with respect to the

¹Bhuller, Kostol and Vigtel (2019) provide direct evidence that the introduction of broadband Internet reduced the extent of search frictions in Norway. They find that when broadband Internet becomes available, firms report fewer problems in finding workers, workers find employment more quickly, and the wage of newly hired workers increases. These findings are consistent with the predictions of our model in response to a lumpy fall in search frictions.

skill distance.

The paper’s main empirical finding is a relationship between wage dispersion and wage growth across different occupations (which we take to be a proxy for workers’ types). Specifically, we find that occupations featuring more wage dispersion in 1980 tend to have higher wage growth between 1980 and 2015. The finding is consistent with the hypothesis that unequal growth is caused partly by differential returns to declining search frictions. Indeed, if a group of workers is more specialized—in the sense that its productivity is more sensitive to the firm-worker skill distance—the cross-sectional wage distribution is more dispersed, the return to declining search frictions is higher, and the growth rate of wages is higher.

We then group occupations into equally-sized bins according to their degree of routineness. We find that routine occupations have lower wage dispersion and lower wage growth than non-routine occupations. These findings have a natural interpretation in light of our theory of unequal growth. Routine workers have a low effective elasticity of productivity with respect to the firm-worker skill distance because, almost by definition, routine jobs are all similar. Non-routine workers have a high effective elasticity of productivity with respect to the firm-worker skill distance because, again by definition, non-routine jobs are dissimilar. It then follows that declining search frictions lead to lower growth for routine than for non-routine workers. A back of the envelope calibration of the model suggests that unequal returns to declining search frictions account for about 25% of the growth differential between routine and non-routine occupations.

Our paper contributes to the literature on rising wage inequality between different groups of workers. The literature started by documenting the rise in the college premium (Katz and Murphy 1992). The canonical explanation for this phenomenon is that technological progress is biased in favor of skilled workers either by fiat (see, e.g., Katz and Murphy 1992 and Card and DiNardo 2002) or because capital equipment is complementary to skilled labor (Krusell et al. 2000). More recently, the literature has switched its focus from the college premium to the rise in wage inequality between routine and non-routine workers (Autor and Dorn 2013). The standard explanation for this phenomenon is again a version of the skilled-biased technical change hypothesis, in which automation erodes the wages of routine workers (see, e.g., Autor and Dorn 2013, Acemoglu and Restrepo 2018, and Ray and Mookherjee 2020). Our paper provides a complementary explanation, based on the view that improvements in the search technology have different effects on routine and non-routine workers.

Our paper also contributes to the literature on search frictions and wage inequality. Generally, this literature is focused on measuring the contribution of search frictions to the dispersion of wages among observationally similar workers (see, e.g., Albrecht and Axel 1984, Burdett and Mortensen 1998, Bontemps, Robin and Van Den Berg 2000, Postel-Vinay and Robin 2002, Bagger et al. 2014, Lise and Postel-Vinay 2018, Gregory 2020, Morchio and Moser 2020). In contrast, our paper is focused on understanding the effect of declining search frictions on wage inequality between different groups of workers.

Our paper is part of a broader research effort aimed at understanding the macro conse-

quences of declining search frictions. Martellini and Menzio (forthcoming) identify conditions under which there exists a BGP in which unemployment, vacancies and workers' transition rates remain constant over time in the face of declining search frictions. They show that in such a BGP, declining search frictions contribute to economic growth by allowing workers to find more jobs that suit them better. Brugemann, Gautier and Tyros (2020) study the effect of a one-time decline in search frictions on the firm's choice between technologies requiring general or specialized skills. In the context of the product market, Menzio (2020) identifies conditions under which there exists a BGP where price dispersion for similar goods does not vanish as search frictions decline. He shows that in such a BGP, declining search frictions lead to economic growth by allowing firms to design products that are more specialized. Our paper contributes to this line of research by showing that declining search frictions may have different rates of return for different agents in the economy, depending on their degree of specialization.

2 Environment

The economy is populated by a measure μ_i of workers of type $i = 1, 2, \dots, N$, with $\mu_i \in [0, 1]$ and $\sum_{i=1}^N \mu_i = 1$. Different types of workers belong to different labor markets. Within their own labor market, workers of type i are uniformly distributed along a circle with unit circumference. The worker's type captures characteristics (e.g., educational attainment, preferences over broad lines of work, etc.) that determine the labor market in which the worker sells his labor. The worker's location along the circle captures the type of tasks for which the worker is best suited. The objective of a worker of type i is to maximize the present value of flow income discounted at the rate $r > 0$, where flow income is given by some wage $w_{i,t}$ when the worker is employed and by some unemployment benefit $b_{i,t}$ when the worker is unemployed.

The economy is also populated by a positive measure of firms. The objective of a firm is to maximize the present value of profits discounted at the rate r . The firm hires workers by opening, maintaining, and filling vacancies. The firm opens a vacancy in labor market i and locates it randomly along the circle of unit circumference. The firm pays a flow cost $k_{i,t} > 0$ to maintain the vacancy. The location of the vacancy along the circle captures the type of tasks required by the firm. Once the vacancy is filled, the firm and the hired worker produce an income flow of $y_{i,t}z_i(\epsilon)$, where $y_{i,t}$ is a component of productivity that is common to all matches between a firm and a worker of type i and $z_i(\epsilon)$ is a component of productivity that is specific to a particular match between a firm and a worker of type i . Specifically, $z_i(\epsilon)$ is a strictly positive and strictly decreasing function of the distance $\epsilon \in [0, 1/2]$ along the circle between the location of the worker and the location of the firm's job. We refer to ϵ as the firm-worker skill distance.

Labor markets are subject to search frictions. Unemployed workers and vacancies need

to search the market to locate each other.² The outcome of the search process is a flow $A_t M(u_{i,t}, v_{i,t})$ of random, bilateral meetings between unemployed workers and vacant jobs, where $u_{i,t}$ and $v_{i,t}$ are the measures of unemployed workers and vacant jobs in labor market i , M is a constant return to scale function,³ and A_t is the efficiency of the search process. The rate at which an unemployed worker meets a vacancy is $A_t p(\theta_{i,t})$, where $\theta_{i,t} = v_{i,t}/u_{i,t}$ is the tightness of labor market i and $p(\theta) \equiv M(1, \theta)$ is a strictly increasing and concave function such that $p(0) = 0$ and $p(\infty) = \infty$. The rate at which a vacancy meets an unemployed worker is $A_t q(\theta_{i,t})$, where $q(\theta) \equiv p(\theta)/\theta$ is a strictly decreasing function such that $q(0) = \infty$ and $q(\infty) = 0$.

Upon meeting, a firm and a worker observe the distance ϵ between them. After observing ϵ , the firm and the worker decide whether to match.⁴ If they do, the firm and the worker bargain over the terms of an employment contract and start producing an income flow of $y_{i,t} z_i(\epsilon)$. Production continues until the match is broken off. If they do not match, the worker remains unemployed and the firm's job remains vacant.

The terms of the employment contract are determined by the axiomatic Nash bargaining solution. That is, the terms of the employment contract maximize the product between the worker's gains from trade taken to the power of γ and the firm's gains from trade taken to the power of $1 - \gamma$, with $\gamma \in (0, 1)$. The worker's gains from trade are defined as the difference between the value of the employment relationship to the worker and his disagreement point, which we take to be the value of unemployment. Similarly, the firm's gains from trade are defined as the difference between the value of the employment relationship to the firm and its disagreement point, which we take to be the value of an unfilled vacancy. The contract specifies a path for the worker's wage and, directly or indirectly, a break-up date. We assume that the contract has enough contingencies to guarantee that the break-up date maximizes the joint value of the match. Since the wage transfers income from the firm to the worker at the rate of 1 to 1, the Nash bargaining solution is such that the worker captures a fraction γ and the firm captures a fraction $1 - \gamma$ of the surplus, which is defined as the difference between the joint value of the match and the sum of the disagreement points.

As mentioned earlier, the economic environment is non-stationary, as both the production and the search technologies improve over time. Specifically, the common component of productivity $y_{i,t}$ grows at some constant rate $g_{y,i} \geq 0$, and the efficiency of the search process A_t grows at some constant rate $g_A > 0$. Additionally, the unemployment income $b_{i,t}$

²We assume that workers can search the labor market only when unemployed. As shown in Martellini and Menzio (forthcoming, Appendix C), the conditions and properties of a BGP are essentially the same in a version of the model where workers are allowed to search on the job. The same is true for the results on unequal growth, which are the focus of this paper.

³Without population growth, the assumption of constant returns to scale in M is essentially without loss in generality. With population growth, the returns to scale in M provide an additional source of declining search frictions (see Martellini and Menzio forthcoming).

⁴Since a firm and a worker perfectly observe the distance ϵ before deciding whether to start an employment relationship, we say that firm-worker matches are pure inspection goods (as in, say, Pissarides 1984). As discussed in Martellini and Menzio (forthcoming), a BGP does not exist if firm-worker matches are pure experience goods.

and the vacancy cost $k_{i,t}$ grow at the rates $g_{b,i}$ and $g_{k,i}$, respectively.

The model is a version of Martellini and Menzio (forthcoming) in which the economy is populated by N types of workers. Different types of workers operate in distinct labor markets, and they differ along two crucial dimensions. First, different types of workers use different production technologies, each associated with its own level of productivity $y_{i,t}$ and its own growth rate $g_{y,i}$.⁵ For example, one type of worker may use a production technology that grows at a high rate. Another type of worker may use a production technology that grows at a lower rate. Second, different types of workers face a different relationship $z_i(\epsilon)$ between the component of productivity that is specific to a particular match and the distance ϵ between the worker’s individual skill and the job’s skill requirements. For example, one type of worker may face a function $z_i(\epsilon)$ with a very low elasticity of z with respect to ϵ . This type of worker is a “jack of all trades,” as he is essentially equally productive doing any kind of job (within his labor market). Another type of worker may face a function $z_i(\epsilon)$ with a very high elasticity of z with respect to ϵ . This type of worker is a “master of one trade,” as he is much more productive doing a job that requires skills similar to his own, rather than doing a job that requires different skills.

3 Balanced Growth Path

We focus on a Balanced Growth Path, BGP, an initial condition and an associated equilibrium path along which—for each type of worker—the unemployment rate, vacancy rate, and UE and EU rates are all constant over time. In a model with only one type of worker, the focus on a BGP is natural, given the lack of any clear secular trend during the last 100 years of US history in the aggregate unemployment, vacancy, UE and EU rates (see Martellini and Menzio forthcoming). In a model with multiple types of workers, the focus on a BGP seems like a natural starting point, as the stationarity of the unemployment, vacancy, UE and EU rates for each type of worker guarantees the stationarity of these objects at the aggregate level.⁶ Moreover, the stationarity of the aggregate rates requires the type-specific rates to eventually become constant—as long as the type-specific rates do not feature offsetting cycles.

To formally define a Balanced Growth Path (BGP), we need some notation. Let $V_{i,t}(z)$ denote the joint value of an employment relationship of quality z between a firm and a worker of type i . Let $U_{i,t}$ denote the value of unemployment for a worker of type i . Let $S_{i,t}(z) \equiv V_{i,t}(z) - U_{i,t}$ denote the surplus of a match of quality z between a firm and a worker of type i . Further, let $\theta_{i,t}$ denote the tightness of labor market i , $u_{i,t}$ the measure of unemployed

⁵Throughout the paper, we will refer to $y_{i,t}$ as productivity and $g_{y,i}$ as the growth rate of the production technology. More generally, though, $y_{i,t}$ represents revenues and depends on both technology (which affects the quantity of output produced) and preferences (which affect the price of the output). Similarly, $g_{y,i}$ is the growth rate of revenues, which depends not only technological progress but also on the shape of preferences.

⁶In this paper, we keep things simple and we do not consider the possibility of trends in the measure of different types of workers. We do not expect such trends to generate quantitatively significant departures from aggregate stationarity of unemployment, vacancy, UE and EU rates.

workers of type i , and $G_{i,t}(z)$ the c.d.f. of employed workers of type i across match qualities. The initial state of the economy is the distribution of workers across employment states at date $t = 0$, that is, $\{u_{i,0}, G_{i,0}\}$. A rational expectation equilibrium is a path for $V_{i,t}$, $U_{i,t}$, $\theta_{i,t}$, $u_{i,t}$, and $G_{i,t}$ such that the agents' decisions are optimal, markets clear, and the evolution of $u_{i,t}$ and $G_{i,t}$ is consistent with the agents' decisions and the initial state. A BGP is an initial state and an associated rational expectation equilibrium such that the unemployment rate, the vacancy rate, and the UE and the EU rates are constant over time for all types of workers, while the distribution of employed workers, $G_{i,t}$, grows at some constant, endogenous rate $g_{z,i}$ —in the sense that every quantile of the distribution grows at the rate $g_{z,i}$.⁷

The value $V_{i,t}(z)$ of a match of quality z between a firm and a worker of type i is such that

$$V_{i,t}(z) = \max_{T \geq 0} \int_t^{t+T} e^{-r(\tau-t)} y_{i,\tau} z d\tau + e^{-rT} U_{i,t+T}. \quad (3.1)$$

At date $\tau \in [t, t+T]$, the firm and the worker produce a flow $y_{i,\tau} z$ of income. At date $t+T$, the firm and the worker break up. After the break up, the worker's present value of income is $U_{i,t+T}$ and the firm's present value of income is the value of an unfilled vacancy, which must be zero because firms are free to open and close vacancies at will.

The optimal break-up date T is such that

$$z y_{i,t+T} + \dot{U}_{i,t+T} \leq r U_{i,t+T}, \text{ and } T \geq 0, \quad (3.2)$$

where the two inequalities hold with complementary slackness.⁸ The left-hand side of (3.2) is the marginal benefit of delaying the break-up, which is given by the flow income of the match plus the time-derivative of the value of unemployment to the worker. The right-hand side of (3.2) is the marginal cost of delaying the break-up, which is given by the annuitized value of unemployment to the worker. Condition (3.2) states that either $T = 0$ and the marginal cost of delaying the break-up exceeds the marginal benefit or $T > 0$ and the marginal cost equals the marginal benefit.

The reservation quality $R_{i,t}$ is defined as

$$y_{i,t} R_{i,t} = r U_{i,t} - \dot{U}_{i,t}. \quad (3.3)$$

It follows from (3.2) that an existing match between a firm and a worker of type i is maintained at date t iff its quality z exceeds $R_{i,t}$. Similarly, a meeting between a firm and a worker of type i is consummated iff its quality z exceeds $R_{i,t}$. That is, $R_{i,t}$ is the lowest quality for which existing matches are maintained and new matches are consummated. From (3.1) and (3.2), it also follows that $S_{i,t} = V_{i,t}(z) - U_{i,t}$ is strictly positive iff z exceeds $R_{i,t}$.

⁷This is a standard condition for a BGP in which one of the equilibrium objects is a distribution of productivities or some other variable that grows over time (see, e.g., Lucas and Moll 2014 or Perla and Tonetti 2014).

⁸Condition (3.2) is obviously necessary. As explained in Martellini and Menzio (forthcoming), condition (3.2) is also sufficient.

The value $U_{i,t}$ of unemployment for a worker of type i is such that

$$rU_{i,t} = b_{i,t} + A_{i,t}p(\theta_i)\gamma \int_0^{z_i^{-1}(R_{i,t})} S_{i,t}(z_i(\epsilon))2d\epsilon + \dot{U}_{i,t}. \quad (3.4)$$

The left-hand side of (3.4) is the annuitized value of unemployment to the worker. The first term on the right-hand side of (3.4) is the worker's flow income when unemployed. The second term is the worker's option value of searching, which is the rate at which the worker meets a firm times a fraction γ of the expected surplus of the meeting between the worker and the firm. The last term is the time-derivative of the value of unemployment to the worker. Note that the expression for the expected surplus makes use of the fact that the distance ϵ between the worker and the firm is a random variable that is uniformly distributed over the interval $[0, 1/2]$ and that only matches with a distance $\epsilon \leq z^{-1}(R_{i,t})$ are consummated.

The tightness of labor marker i is such that

$$k_{i,t} = A_{i,t}q(\theta_i)(1 - \gamma) \int_0^{z_i^{-1}(R_{i,t})} S_{i,t}(z_i(\epsilon))2d\epsilon. \quad (3.5)$$

The left-hand side of (3.5) is the cost to the firm from maintaining a vacancy in labor market i . The right-hand side is the benefit, which is given by the rate at which the vacancy meets a worker times a fraction $1 - \gamma$ of the expected surplus of the meeting between the vacancy and the worker. Condition (3.5) states that in equilibrium, the tightness θ_i equates the cost and the benefit of maintaining an additional vacancy in labor market i .

In a BGP, the unemployment rate, the market tightness, and the UE and the EU rates are required to be constant for each worker type. These requirements are fulfilled iff

$$A_t p(\theta_{i,t}) 2 z_i^{-1}(R_{i,t}) = h_{ue}^i, \quad (3.6)$$

$$G'_{i,t}(R_{i,t}) \dot{R}_{i,t} = h_{eu}^i, \quad (3.7)$$

$$u_i h_{ue}^i = (\mu_i - u_i) h_{eu}^i. \quad (3.8)$$

The UE rate at date t is the product between the rate at which an unemployed worker meets a firm and the probability that the distance between the worker and the firm is below the reservation distance $z^{-1}(R_{i,t})$. Condition (1) states that the UE rate is constant over time. The EU rate at date t is the product between the density of the $G_{i,t}$ distribution at the reservation quality $R_{i,t}$ and the time-derivative of $R_{i,t}$. Condition (1) states that the EU rate is constant over time. The condition for the stationarity of the unemployment rate is (1), which states that the flow of workers out of unemployment equals the flow of workers into unemployment. The condition for the stationarity of the market tightness is implicit in (3.5), which requires the market-clearing tightness to be constant.

In a BGP, the distribution $G_{i,t}(z)$ is required to grow at some endogenous constant rate $g_{z,i}$. That is, $z_{i,t}(x) = z_{i,0} \exp(g_{z,i}t)$ for all $x \in [0, 1]$ and all $t \geq 0$, where $z_{i,t}(x)$ is the x -th

quantile of $G_{i,t}$. The condition is satisfied iff

$$\begin{aligned} & (\mu_i - u_{i,t})G'_{i,t}(z_{i,t}(x))z_{i,t}(x)g_{z,i} + u_i A_t p(\theta_i) 2 [z^{-1}(R_{i,t}) - z^{-1}(z_{i,t}(x))] \\ &= (\mu_i - u_{i,t})G'_{i,t}(R_{i,t})\dot{R}_{i,t}. \end{aligned} \quad (3.9)$$

The left-hand side of (1) is the flow of workers into matches with quality lower than an x -th quantile growing at the rate $g_{z,i}$. The right-hand side is the flow of workers out of matches with quality lower than an x -th quantile growing at the rate $g_{z,i}$. Condition (1) thus guarantees that the measure of workers in matches below an x -th quantile growing at the rate $g_{z,i}$ remains constant over time.

4 Unequal Growth

For each worker type, the definition of a BGP is the same as in Martellini and Menzio (forthcoming). Therefore, the necessary and sufficient conditions for the existence of a BGP are exactly the same as those in Martellini and Menzio (forthcoming, Theorems 1 and 2) applied to each different type of worker. The necessary and sufficient conditions are reported below, together with the characterization of the properties of a BGP.

Theorem 1. (*Existence and Properties of a BGP*).

1. A BGP exists iff for each worker type $i = 1, 2, \dots, N$, (a) the function $z_i(\epsilon)$ has the isoelastic form $z_{i,\ell} \epsilon^{-1/\alpha_i}$ for some $z_{i,\ell} > 0$ and some $\alpha_i > 1$; (b) the growth rate $g_{k,i}$ of the vacancy cost and the growth rate $g_{b,i}$ of the unemployment income are both equal to $g_{y,i} + g_A/\alpha_i$; and (c) the discount rate r is such that $r > g_{y,i} + g_A/\alpha_i$.
2. If a BGP exists, it is unique and for each worker type $i = 1, 2, \dots, N$, it is such that (i) unemployment, vacancy, and UE and EU rates are constant; (ii) $G_{i,t}$ is a Pareto distribution with coefficient α_i truncated at $R_{i,t}$ and grows at the constant rate $g_{z,i} = g_A/\alpha_i$; and (iii) average labor productivity grows at the rate $g_{y,i} + g_A/\alpha_i$.

The first part of Theorem 1 lists the necessary and sufficient conditions for the existence of a BGP. The intuition is simple. On the one hand, improvements in the search technology lead to an increase in the rate at which unemployed workers meet vacancies. On the other hand, improvements in the search technology increase the reservation quality for workers and firms, thus reducing the probability that a meeting turns into a match. The two effects offset each other, and thus the UE rate remains constant iff the function $z_i(\epsilon)$ has constant elasticity $1/\alpha_i$ and the vacancy cost and the unemployment income grow at the rate $g_{y,i} + g_A/\alpha_i$. The EU rate remains constant as long as the distribution $G_{i,t}$ is Pareto, which is the case as long as the initial distribution $G_{i,0}$ is Pareto. The unemployment rate is stationary because the UE and EU rates are constant.

The conditions on the growth rate of the vacancy cost and the unemployment income may appear to be knife-edge, but they are not. To understand why this is the case, note that

$g_{y,i} + g_A/\alpha_i$ is the growth rate of the average productivity of workers of type i . Thus, the vacancy cost grows endogenously at the rate $g_{y,i} + g_A/\alpha_i$ as long as the input in maintaining a vacancy in labor market i is some constant amount of labor supplied by workers of type i . For the same reason, the unemployment income grows endogenously at the rate $g_{y,i} + g_A/\alpha_i$ as long as $b_{i,t}$ is some constant fraction of the average output of workers of type i . These observations are formalized in Martellini and Menzio (forthcoming, Appendix B).

The second part of Theorem 1 lists some of the key properties of a BGP. First, in a BGP, the unemployment, vacancy, UE and EU rates are constant for each type of worker, and hence they are constant at the aggregate level. Second, in a BGP, the distribution $G_{i,t}$ of workers of type i across match qualities is a Pareto distribution with coefficient α_i truncated at the reservation quality $R_{i,t}$ and grows at the rate $g_{z,i} = g_A/\alpha_i$. To understand this second property, note that when a firm and a worker of type i meet, the distance ϵ between the worker's skills and the firm's job requirements is a random draw from a uniform distribution with support $[0, 1/2]$. The match between the firm and the worker is consummated iff $\epsilon \leq z^{-1}(R_{i,t})$ and if this is the case, the quality of the match is given by $z_i(\epsilon) = z_{i,\ell}\epsilon^{-1/\alpha_i}$. Therefore, the distribution of quality between newly-created matches is

$$\begin{aligned} \Pr(\tilde{z} \leq z) &= \Pr(\epsilon \geq z^{-1}(z) | \epsilon \leq z^{-1}(R_{i,t})) \\ &= 1 - (R_{i,t}/z)^{\alpha_i}. \end{aligned} \tag{4.1}$$

The above expression shows that the distribution of new matches is a Pareto with coefficient α_i truncated at $R_{i,t}$. Since previously created matches are also distributed as a Pareto with coefficient α_i and survive iff their quality exceeds $R_{i,t}$, the overall distribution $G_{i,t}$ is also given by (4.1). The distribution $G_{i,t}$ grows at the rate $g_{z,i} = g_A/\alpha_i$, as this is the growth rate of the reservation quality $R_{i,t}$.

The second part of Theorem 1 also states that the growth rate of the average productivity of labor for workers of type i is given by $g_{y,i} + g_A/\alpha_i$. To see why this is the case, note that $G_{i,t}$ is given by (4.1), and hence

$$\int y_{i,t} z dG_{i,t}(z) = \frac{\alpha_i}{\alpha_i - 1} y_{i,t} R_{i,t}. \tag{4.2}$$

The above expression grows at the rate $g_{y,i} + g_A/\alpha_i$, as $y_{i,t}$ grows at the rate $g_{y,i}$ and $R_{i,t}$ grows at the rate $g_{z,i} = g_A/\alpha_i$. The growth rate of average productivity for workers of type i has two distinct sources. The first source is $g_{y,i}$, progress in the production technology used by workers of type i . The second source is g_A/α_i , progress in the search technology which has a return on the productivity of workers of type i that is equal to the elasticity $1/\alpha_i$ of $z_i(\epsilon)$ with respect to ϵ . That is, the return of declining search frictions on the productivity of workers of type i is equal to the elasticity of the idiosyncratic component of productivity of a match with respect to the distance between the skills of a worker of type i and the requirements of a firm's job.

The growth rate of productivity translates into wage growth. Following Pissarides (1985)

and Mortensen and Pissarides (1994), let us assume that a firm and a worker bargain over the wage at time intervals of length $dt \rightarrow 0$. Then, it is easy to show that the wage for a worker of type i in a match of quality z is given by

$$w_{i,t}(z) = \gamma y_{i,t} z + (1 - \gamma) y_{i,t} R_{i,t}. \quad (4.3)$$

From (4.1) and (4.3), it follows that the wage distribution $L_{i,t}(w)$ for workers of type i is

$$L_{i,t}(w) = 1 - \left(\frac{\gamma y_{i,t} R_{i,t}}{w - (1 - \gamma) y_{i,t} R_{i,t}} \right)^{\alpha_i}, \quad (4.4)$$

and the average wage for workers of type i is

$$\int w dL_{it}(w) = \left[\gamma \frac{\alpha_i}{\alpha_i - 1} + (1 - \gamma) \right] y_{i,t} R_{i,t}. \quad (4.5)$$

Clearly, the average wage for workers of type i grows at the rate $g_{y,i} + g_A/\alpha_i$, which is the same as the growth rate of average productivity.

We are now in the position to state the main theorem of the paper.

Theorem 2. (*Unequal Growth*) *Consider two worker types i and j , $i \neq j$. Wage growth for workers of type i may be higher than for workers of type j because (a) technological progress in production is biased toward workers of type i : $g_{y,i} > g_{y,j}$; and (b) technological progress in search has a higher rate of return for workers of type i : $1/\alpha_i > 1/\alpha_j$.*

Theorem 2 identifies two sources of unequal wage growth for different groups of workers. The first source of unequal wage growth between workers of type i and workers of type j is that progress in the production technology is biased in favor of workers of type i . This is the canonical explanation for the rise in the college premium (i.e., progress in the production technology is biased in favor of college graduates) or for the decline in the wages of routine workers relative to non-routine workers (i.e., progress in automation erodes the surplus generated by routine workers).

The second source of unequal wage growth is novel. Specifically, the second source of unequal wage growth between workers of type i and workers of type j is that progress in the search technology has a higher rate of return for workers of type i . The logic is simple. Suppose that workers of type i are specialists, in the sense that their productivity is more elastic to the distance between their idiosyncratic skills and the skill requirements of their job. In contrast, workers of type j are generalists, in the sense that their productivity is less elastic to the distance between their idiosyncratic skills and the skill requirements of their job. Declining search frictions allow both workers of type i and workers of type j to become more selective with respect to the jobs they accept. The increase in selectivity, however, is going to have a higher rate of return in terms of productivity—and, hence, wages—for workers of type i than for workers of type j .

As a matter of interpretation, it is useful to point out that the notions of specialists and

generalists are relative to jobs. That is, workers of type i may be more specialized than workers of type j because they are more productive at jobs that suit them well and less productive at jobs that do not. Equivalently, workers of type i may be more specialized than workers of type j because the jobs that are available to them are more heterogeneous in terms of skill requirements. In either case, workers of type i end up facing more heterogeneity in the component of their productivity that is match specific.

Declining search frictions cause unequal growth iff different types of workers have a different elasticity of productivity with respect to the firm-worker skill distance. In turn, differences in the elasticity of productivity with respect to the firm-worker skill distance manifest themselves as differences in wage dispersion. Hence, declining search frictions cause unequal growth iff there are differences in the extent of wage dispersion across different types of workers. Specifically, declining search frictions cause higher growth for the types of workers displaying more wage dispersion.

It is easy to formalize the above argument. From (4.4), it follows that the ratio between the x_1 -th and the x_0 -th quantiles of the wage distribution for workers of type i is given by

$$\frac{w_i(x_1)}{w_i(x_0)} = \frac{\gamma(1-x_1)^{-1/\alpha_i} + (1-\gamma)}{\gamma(1-x_0)^{-1/\alpha_i} + (1-\gamma)}. \quad (4.6)$$

Unless there are differences in bargaining power, the wage quantile ratio in (4.6) is different for two types of workers i and j iff the elasticity of productivity with respect to the firm-worker skill distance is different between the two types. In turn, the growth rate induced by declining search friction is different for workers of type i and workers of type j iff the elasticity is different. Moreover, since (4.6) is strictly increasing in $1/\alpha$, the growth rate g_A/α induced by declining search frictions is higher for workers of type i than for workers of type j iff the wage quantile ratio (4.6) is higher for workers of type i .

We have thus established the following result.

Theorem 3. (*Wage Dispersion and Growth*) *Consider two groups of workers, i and j , $i \neq j$, with $g_{y,i} = g_{y,j}$. Wage growth for workers of type i is higher than for workers of type j iff for any x_0, x_1 in $[0, 1]$ with $x_1 > x_0$, the ratio between the x_1 -th and the x_0 -th quantile of the wage distribution is higher for workers of type i than for workers of type j .*

Theorem 3 suggests an empirical test for the hypothesis that differential returns to declining search frictions are a quantitatively important cause of unequal growth across different groups of workers. If the hypothesis is true, we should expect to see a positive relationship between wage dispersion and wage growth across different groups of workers. If the hypothesis is false, there may or may not be a positive relationship between wage dispersion and wage growth.

We implement the empirical test using wage and salary income data from the Decennial Census (1980) and the American Community Service (2015). We construct our sample using workers aged 25 to 55 who are neither in the military nor enrolled in school. We further

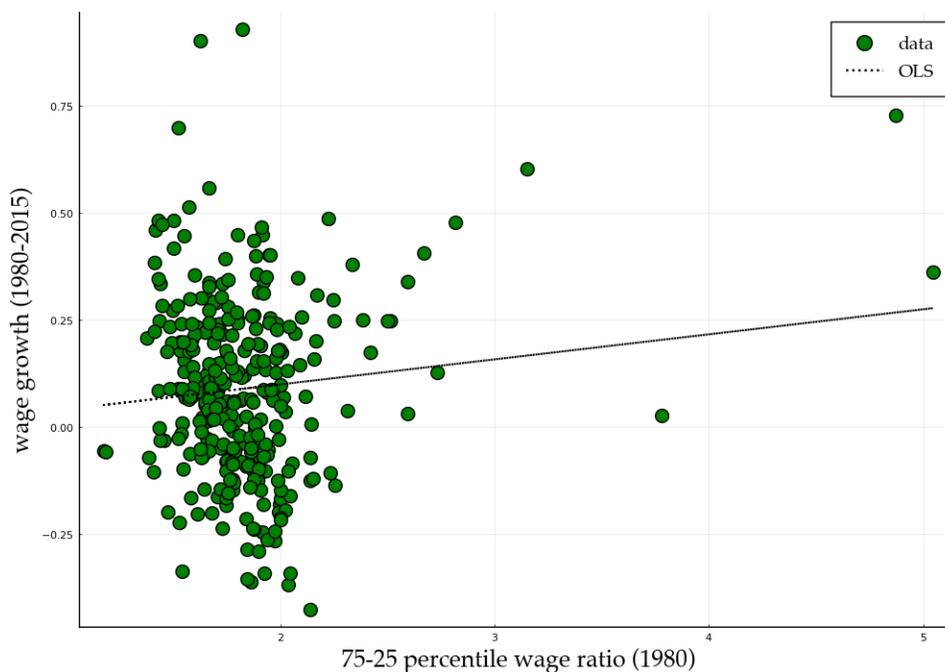


Figure 1: Wage Dispersion and Growth by Occupation

restrict attention to “full-time” workers, that is, individuals who work at least 35 hours per week and 45 weeks per year with an annual income of at least US\$ 5,000, where income is measured in 2014 dollars. We proxy a worker’s type as their 3-digit occupation,⁹ using the crosswalk developed by David Dorn to maintain a consistent definition of occupation over the years.

Figure 1 is a scatter plot of wage dispersion and wage growth. Wage dispersion is measured as the ratio between the 75th and the 25th percentile of the wage distribution in 1980. Wage growth is measured as the ratio between the average wage in 2015 and the average wage in 1980. The relationship between wage dispersion and wage growth is quite noisy but positive, as revealed by a linear regression. On average, occupations with more wage dispersion in 1980 have higher wage growth between 1980 and 2015.

Next, we want to entertain the hypothesis that declining search frictions might be the cause of unequal wage growth between routine and non-routine occupations. To this end, we follow Autor and Dorn (2013) and classify each occupation based on the degree of routineness of the tasks it involves. We then aggregate occupations into 20 equally-sized clusters with increasing degrees of routineness. We define the wage dispersion of a particular cluster as the average of the 75th-to-25th wage percentile ratios in 1980 across all occupations that belong to that cluster.¹⁰ We define the wage growth of a particular cluster as the average of the

⁹The choice of occupation as a proxy for a worker’s type is not without flaws. In particular, while the type is assumed to be a permanent trait of a worker in the model, in the data, workers do switch occupations. This discrepancy between model and data might be mitigated by using clusters of occupations with a similar degree of routineness as our proxy for a worker’s type.

¹⁰Theorem 3 applies to any wage percentile ratios. In the main text, we use the 75th-to-25th percentile

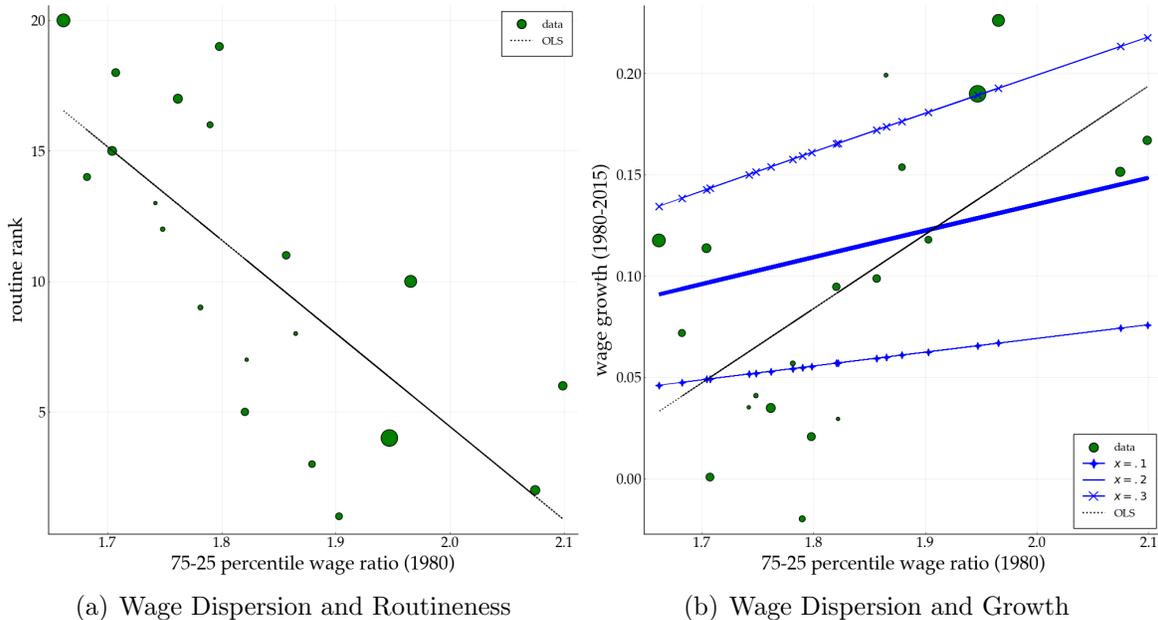


Figure 2: Wage Dispersion and Growth by Routineness

2015-to-1980 wage growths across all occupations that belong to that cluster.

Figure 2(a) shows that occupations that are more routine have a systematically lower wage dispersion than occupations that are less routine. The finding has a natural interpretation in light of our theory. In a routine occupation, jobs are—almost by definition—similar to each other. Hence, the productivity of a particular worker is bound to be similar across different jobs. In contrast, in a non-routine occupation, jobs are—almost by definition—differentiated, and hence the productivity of a particular worker is likely to be quite different in different jobs. In other words, workers in routine occupations are jacks of all trades (because all jobs involve the same tasks and require the same skills) and workers in non-routine occupations are masters of one trade (because different jobs involve different tasks and require different skills).

Figure 2(b) shows that occupations in which wage dispersion is lower—which are the routine occupations—display systematically lower wage growth than occupations in which wage dispersion is higher—which are the non-routine occupations. The relationship between wage dispersion and wage growth is now much clearer than in Figure 1, presumably because we have averaged out some of the occupational-specific noise. The coefficient on wage dispersion in a regression of wage growth is .36, and the R^2 of the regression is 40%.¹¹

We now take a somewhat more structural approach to measure declining search frictions’ contribution to wage growth in routine and non-routine occupations. First, using (4.6), we

ratio. In Appendix A, we use the 90th-to-10th percentile ratio and find the same patterns.

¹¹The reader may be concerned that some of the differences in wage dispersion across occupation clusters may be due to differences in the rate at which workers accumulate human capital over the life-cycle. To address this concern, Appendix B shows that the same relationships between routineness, wage dispersion and wage growth hold if we restrict attention to male workers aged 25 to 35.

can recover the return of declining search frictions, $1/\alpha_i$, from the extent of wage dispersion in the occupation cluster i . Since some wage dispersion may be caused by differences in worker-specific productivity rather than in match-specific productivity, we choose $1/\alpha_i$ so that it generates some fraction $x \in [0, 1]$ of the wage dispersion in cluster i .¹² Second, as explained in Martellini and Menzio (forthcoming), we can recover the rate of decline of search frictions, g_A , from the growth rate of the average number of applications received by a firm before filling its vacancy. We find that g_A averages about 2% per year between 1980 and 2010. Third, we multiply the estimates of g_A and $1/\alpha_i$ to obtain a measure of the contribution of declining search frictions to the growth of wages in cluster i .

Figure 2(b) illustrates our findings. The three lines represent the wage growth caused by declining search frictions for $x = 0.1$ (bottom), 0.2 (middle), and 0.3 (top).¹³ The three lines are computed assuming that the worker's share γ of the surplus is equal to 0.9.¹⁴ For $x = 0.2$, the wage growth due to declining search frictions goes from about 9% for the most routine occupations (i.e., those with the least wage dispersion) to about 15% for the least routine occupations (i.e., those with the most wage dispersion). In the data, wage growth for the most routine occupations is about 0, and that for the least routine occupations is about 20%. Thus, heterogeneous returns on declining search frictions account for about 6 out of 20 percentage points of the growth differential between routine and non-routine occupations. The residual wage growth differential may be due to biased technological change, as suggested by Autor and Dorn (2013). If we assume that x is 0.1 (*0.3*) rather than 0.2, that is, we assume that the fraction of wage dispersion coming from differences in match-specific productivity is lower (*higher*), we find that declining search frictions generate lower (*higher*) growth for all occupations, and they account for about 3 (*7*) percentage points of the growth differential between routine and non-routine occupations. In any case, the message is similar: declining search frictions appear to have benefitted more non-routine workers than routine ones.

¹²To isolate match-specific heterogeneity from worker-specific heterogeneity (due to, say, differences in human capital among young workers and differences in human capital between young and old workers) we would need a life-cycle model estimated using panel data. We plan on carrying out this more ambitious exercise in the future.

¹³For $x = 0.2$, the average value of α is about 7. For $x = 0.1$, the average value of α is 12. For $x = 0.3$, the average value of α is about 4.5. These are conservative values for the thickness $1/\alpha$ of the tail of the distribution of match-specific productivity. For instance, using data on the wage paths of individual workers and a model similar to ours, Martellini (2019) estimates α to be 3.6.

¹⁴For the sake of simplicity, our model abstracts from search on the job. In on-the-job search models, multiple firms compete for the services of a particular worker, and hence the share of the surplus accruing to workers ends up being quite large (see, e.g., Bagger et al. 2014, Menzio, Telyukova and Visschers 2016, Gregory 2019 or Martellini 2019). We set γ equal to 0.9 to capture this important feature of on-the-job search models.

References

- [1] Acemoglu, D., and P. Restrepo. 2018. “The Race between Man and Machine: Implications of Technology for Growth, Factor Shares and Employment.” *American Economic Review*, 108: 1488-1542.
- [2] Albrecht, J., and B. Axell. 1984. “An Equilibrium Model of Search Unemployment.” *Journal of Political Economy*, 92: 824-840.
- [3] Autor, D., and D. Dorn. 2013. “The Growth of Low Skill Service Jobs and the Polarization of the US Labor Market.” *American Economic Review*, 103: 1553-1597.
- [4] Bagger, J., F. Fontaine, F. Postel-Vinay, and J.-M. Robin. 2014. “Tenure, Experience, Human Capital, and Wages: A Tractable Equilibrium Search Model of Wage Dynamics.” *American Economic Review*, 104: 1551-1596.
- [5] Bhuller, M., A. Kostol, and T. Vigtel. 2019. “How Broadband Internet Affects Labor Market Matching.” Working Paper, Norges Bank.
- [6] Bontemps, C., J. Robin and G. Van den Berg. 2000. “Equilibrium Search with Continuous Productivity Dispersion: Theory and Non-Parametric Estimation.” *International Economic Review*, 41: 305-358.
- [7] Brügemann, B., P. Gautier, and S. Tyros. 2020. “Sorting, Technology Choice and Specialization.” Manuscript, Tinbergen Institute.
- [8] Burdett, K., and D. Mortensen. 1998. “Wage Differentials, Employer Size and Unemployment.” *International Economic Review*, 39: 257-273.
- [9] Card, D., and J. DiNardo. 2002. “Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles.” *Journal of Labor Economics*, 20: 733-783.
- [10] Ellison, G., and S. Ellison. 2018. “Match Quality, Search, and the Internet Market for Used Books.” NBER Working Paper 24197.
- [11] Gregory, V. 2020. “Firms as Learning Environments.” Manuscript, NYU.
- [12] Katz, L., and K. Murphy. 1992. “Changes in Relative Wages: 1963-1987: Supply and Demand Factors.” *Quarterly Journal of Economics*, 107: 35-78.
- [13] Krusell, P., L. Ohanian, J. Rios-Rull, and G. Violante. 2000. “Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis.” *Econometrica*, 68: 1029-1053.
- [14] Lise, J., and F. Postel-Vinay. 2020. “Multidimensional Skills, Sorting, and Human Capital Accumulation.” *American Economic Review*, 110: 2328-2376.
- [15] Lucas, R., and B. Moll. 2014. “Knowledge Growth and the Allocation of Time.” *Journal of Political Economy*, 122: 1-51.

- [16] Martellini, P. 2019. “The City-Size Wage Premium: Origins and Aggregate Implications.” Manuscript, University of Pennsylvania.
- [17] Martellini, P. and G. Menzio. Forthcoming. “Declining Search Frictions, Unemployment and Growth.” *Journal of Political Economy*.
- [18] Menzio, G. 2020. “Product Design, Competition and Prices with Declining Search Frictions.” Manuscript, New York University.
- [19] Menzio, G., I. Telyukova and L. Visschers. 2016. “Directed Search over the Life Cycle.” *Review of Economic Dynamics*, 19: 38-62.
- [20] Morchio, I., and C. Moser. 2020. “The Gender Pay Gap: Micro Sources and Macro Consequences.” Manuscript, Columbia University.
- [21] Mortensen, D., and C. Pissarides. 1994. “Job Creation and Job Destruction in the Theory of Unemployment.” *Review of Economic Studies*, 61: 397-415.
- [22] Pissarides, C. 1984. “Efficient Job Rejection.” *Economic Journal*, 94: 97-108.
- [23] Pissarides, C. 1985. “Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages.” *American Economic Review*, 75: 676-690.
- [24] Perla, J., and C. Tonetti. 2014. “Equilibrium Imitation and Growth.” *Journal of Political Economy*, 122: 52-76.
- [25] Postel-Vinay, F., and J. Robin. 2002. “Equilibrium Wage Dispersion with Worker and Employer Heterogeneity.” *Econometrica*, 70: 229
- [26] Ray, D. and D. Mookherjee. 2020. “Growth, Automation, and the Long-Run Share of Labor.” Manuscript, New York University.

Appendix

A 90-10 Percentile Ratio

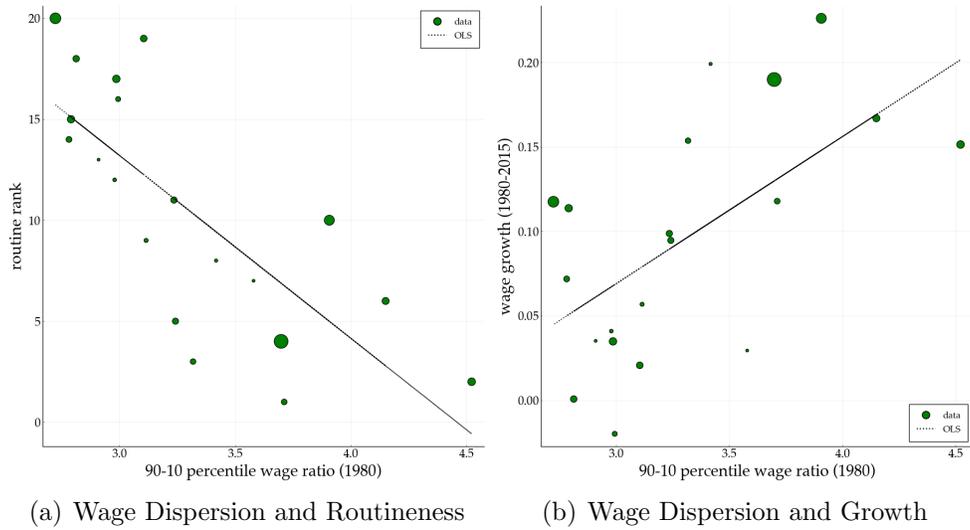


Figure 3: 90-10 Percentile Ratio and Growth by Routineness

B Male Workers Aged 25-35

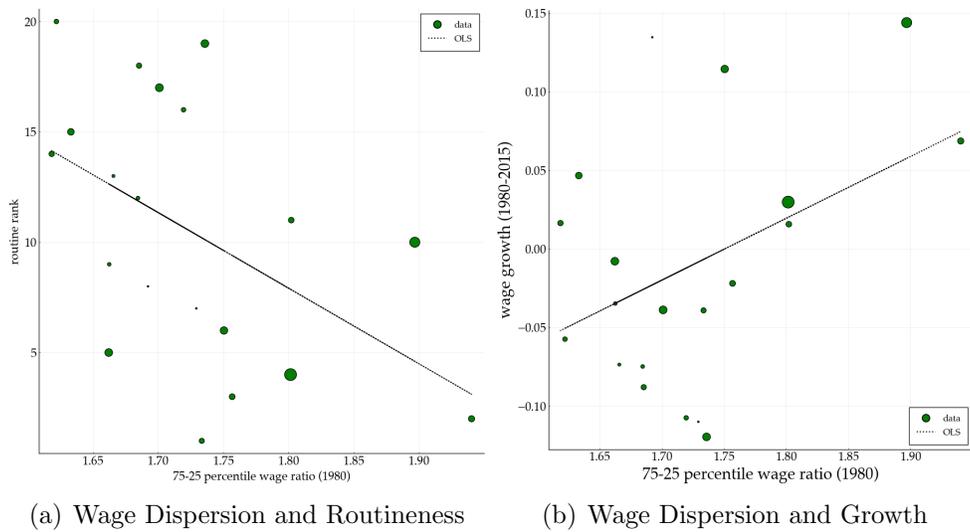


Figure 4: Wage Dispersion and Growth by Routineness, Males 25-35