Constructing the Capital Stock and Calibrating Parameters

Capital Stock and Delta

Standard national accounts, such as those of Finland, do not report a series for the capital stock, so we have to construct such a series using the data on investment. We construct this series using the law of motion for capital in the model,

$$(1) \qquad K_{t+1} = (1-\delta)K_t + I_t$$

This commonly used procedure for calculating a capital stock is referred to as the perpetual inventory method. The inputs necessary to construct the capital stock series are a capital stock at the beginning of the investment series and a value for the constant depreciation rate, δ. The value of δ is chosen to be consistent with the average ratio of depreciation to GDP observed in the data over the data period used for calibration purposes. For Finland, we find that the ratio of depreciation to GDP over the period 1980–2005 is

$$(2) \qquad \frac{1}{26}\sum_{t=1980}^{2005} \frac{\delta K_t}{Y_t} = 0.1693$$

Without explicit data on the capital stock at the beginning of the investment series, we have to adopt a more or less arbitrary rule. The rule that we use in this example is that the capital-output ratio of the initial period should match the average capital-output ratio over some reference period. Here we choose the capital stock so that the capital-output ratio in 1960 matches its average over 1961–70:

$$(3) \qquad \frac{K_{1960}}{Y_{1960}} = \frac{1}{10}\sum_{t=1961}^{1970} \frac{K_t}{Y_t}$$

The system of equations (1)–(3) allows us to use data on investment, I_t to solve for the sequence of capital stocks and for the depreciation rate, δ. There are 27 unknowns: K_{1980}, δ, and K_{1981}, K_{1982}, ..., K_{2005}, in 27 equations: 25 equations (1), where t =1980, 1981, ..., 2004, (2), and (3). Solving this system of equations, we obtain the sequence of capital stocks and a calibrated value for depreciation, δ = 0.0556.

Alpha

We can directly measure α from the data, but we need to make some adjustments. If we are using national accounting data under SNA93, as we are in the case of Finland, the income definition of GDP is the sum of three categories: Compensation of Employees, Net Taxes on Production and Imports, and Gross Operating Surplus and Mixed Income.

We define the labor income share as unambiguous labor income divided by GDP net of the ambiguous categories (household net mixed income and indirect taxes):

$$(4) \qquad \text{Labor Share} = \frac{\text{Compensation of Employees}}{\text{GDP - Household Net Mixed Income - Net Indirect Taxes}}$$

This procedure is equivalent to splitting the ambiguous categories between labor income and capital income in the same proportions as in the rest of the economy. The capital share, α, is then 1 – Labor Share. Our calculations produce an average value of the labor income share of 0.6410 over the period 1980–2005, so that α = 0.3590.

Beta and Gamma

To calibrate a value for β, we use the first order condition for intertemporal substitution

$$(5) \qquad \frac{C_{t+1}}{C_t} = \beta(1-\delta+\alpha A_{t+1}K_{t+1}^{\alpha-1}L_{t+1}^{1-\alpha})$$

to write

$$(6) \qquad \beta = \frac{C_{t+1}}{C_t(1-\delta+\alpha Y_{t+1}/K_{t+1})}$$

We compute β using our values for δ, α, and our constructed capital stock in every period. We take the average over 1970–80. That is, we calibrate household behavior to a period outside that in which we are interested. We find β = 0.9752.

The procedure for calibrating γ is similar. We use the first order condition for intratemporal substitution

$$(7) \qquad (1-\alpha)A_tK_t^{\alpha}L_t^{-\alpha}(\bar{h}N_t - L_t) = \frac{1-\gamma}{\gamma}C_t$$

to write

$$(8) \qquad \gamma = \frac{C_tL_t}{Y_t(\bar{h}N_t - L_t)(1-\alpha)+C_tL_t}$$

Using data on consumption, hours worked, population, and output and the value for α, we find that the average value over 1970–80 is γ = 0.2846.

TFP Values

Finally, we construct the series of TFP values that we use in the MATLAB program by rearranging the production function and plugging in our calibrated values for α, and the capital stock.

$$(9)\ A_t = \frac{Y_t}{K_t^{\alpha}L_t^{1-\alpha}}$$