THE WORKWEEK OF CAPITAL
AND ITS CYCLICAL IMPLICATIONS

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Abstract

The neoclassical growth model studied in Kydland and Prescott [1982] is modified to permit the capital utilization rate to vary. The effect of this modification is to increase the amplitude of the aggregate fluctuations predicted by theory as the equilibrium response to technological shocks. If following Solow [1957], the changes in output not accounted for by changes in the labor and tangible capital inputs are interpreted as being the technology shocks, the statistical properties of the fluctuations in the post-war United States economy are close in magnitude and nature to those predicted by theory.

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I. Introduction

In Kydland and Prescott [1982], the competitive neo-classical growth model with shocks to technology is studied. Leisure was introduced as an additional argument affecting preferences and labor services as an input to the aggregate production function. This artificial economy was calibrated so that its equilibrium would display the growth observations of Kuznets [1966]. In particular, its average consumption and investment shares of output match those of the post-war U.S. economy as do the capital-output ratios and factor shares. Further, the allocation of time between market and nonmarket production activities is consistent with household studies and does not change secularly as technology advances.

Services from inventory stocks were introduced as a factor of production and a production period for new capital consistent with micro observations assumed. We found that the competitive equilibrium for this economy displays fluctuation in output having amplitude and serial correlation properties close to those of the U.S. economy. The relative variability of both consumption and investment matched the data. Further, the majority of the variability in output was accounted for by variability in the labor input. This exercise lead us to the conclusion that if, following Solow [1957], residuals in the aggregate production function are interpreted as shocks to technology, there would be a puzzle if industrial market economies did not display business cycle fluctuations.
For both the neoclassical growth model and post-war American economies capital stocks do not fluctuate with output. At the business cycle frequencies they are nearly orthogonal. But, the services of capital need not be equal to the capital stock as for the model that we studied. The utilization of capital can and surely does fluctuate systematically with output and employment. A factory can be operated two shifts rather than one or operated six rather than five days a week. If hours of labor services are proportionate to the workweek of capital, increases in labor services do not reduce the ratio of capital services to labor services.

The question that is addressed in this paper is how does the nature of the fluctuations predicted by the stochastic growth model change if the workweek of capital is permitted to vary? The principal finding is that fluctuations are larger and the stochastic growth model better mimics the behavior of the American economy.

In Section II, the economy that we study is specified. In Section III, we show that by defining workdays of different lengths to be different commodities, the economy can be put into the Debreu [1954] general equilibrium framework. The competitive equilibrium is found by supporting the Pareto optimum. The model is calibrated in Section IV and results reported in Section V. Section VI contains some concluding comments.
II. The Economy

Preferences

The stand-in household's preferences are ordered by

$$E \sum_{t=0}^{\infty} \beta^t u(c_t, \sum_{i=0}^{\infty} a_i \ell_{t-1})$$

where $c_t > 0$ is date $t$ consumption, $\ell_t > 0$ date $t$ leisure, $0 < \beta < 1$ is the discount factor, and $E$ the expectation operator. The parameters $a_i$ are constrained to sum to 1 and $a_{i+1}/a_i = (1-\eta)$ for $i = 1, 2, \ldots$. Thus, $0 < a_0 < 1$ and $0 < \eta < 1$ determine the values of the $a_i$. The case of $a_0 = 1$ corresponds to a standard time-separable utility function and is used as a benchmark. Theoretically, of course, $a_0$ could be greater than one, implying that leisure in neighboring periods are complements rather than substitutes. Justifications for so restricting it and for this distributed lag of leisure based upon household production theory with household capital can be found in Kydland [1983] and evidence based on panel data in Hotz, Kydland and Sedlacek [1985].

The period utility function is assumed to have the form

$$u(c_t, a(L)\ell_t) = [c_t^\sigma a(L)\ell_t^{1-\mu}]^{1/\gamma}$$

where $\gamma < 1$ and $\gamma \neq 0$. To summarize, $\beta$, $a_0$, $\eta$, $\gamma$ and $\mu$ are the parameters of preferences. The unit elasticity of substitution between $c_t$ and $a(L)\ell_t$ was dictated by the requirement that there be no secular change in $a(L)\ell_t$ associated with secular increase in productivity. The household endowment of time per period is normalized to be one.
Let \( h_t(\tau), 0 < \tau < 1, \) be the workday at instant \( \tau \) of period \( t \). Total hours worked during period \( t \) is

\[
h_t = \int_0^1 h_t(\tau) d\tau
\]

and total leisure \( f_t = 1 - h_t \).

**Technology**

Aggregate output is constrained by

\[
f(z_t, h_t, k_t, y_t) = [(z_t h_t^{\psi} k_t^\theta) - \omega y_t^{1 - \psi}]^{-1/\nu}
\]

where \( k_t \) is the productive stock of physical capital other than inventories, \( y_t \) the stock of inventories, and \( z_t \) the date \( t \) technology shock. Parameters are \( \psi, \theta, \sigma \) and \( \nu \). Restrictions on the parameters are \( 0 < \theta < 1, \sigma > 0, \nu > 0, \) and \( 1 - \theta < \psi < 1 \). Because \( \psi + \theta > 1 \), there are apparent increasing returns. With \( \psi = 1 - \theta \) the only margin over which hours can be varied is the number of workers operating a given quantity of capital. With \( \psi = 1 \), the only margin is the length of the workday of capital and labor. Since over the cycle both elements are surely present, we restrict \( \psi \) to lie between these values.

The production function differs from the one we used previously in another important respect. For the above CES production function, capital and labor are combined to form a composite input and that composite input combined with inventories to produce goods and services. Previously, \( k_t \) and \( y_t \) were nested together to form a composite capital good which in turn was combined with the labor input to produce output. We think these as-
sumptions made here match better with micro observations. They result in inventories being a procyclical variable, rather than moving countercyclically, in conformity with observations.

Total output is allocated to noninventory investment $i_t$, inventory investment $y_{t+1} - y_t$, and consumption $c_t$:

$$i_t + c_t + y_{t+1} - y_t < f(z_t, h_t, k_t, y_t).$$

New productive capacity is not built in a period, which we take to be a quarter of a year. The construction period is $J$ quarters, with $\phi_{j+1-j}$ being the fraction of the value put in place in the $j^{th}$ period of construction for $j = 1, \ldots, J$. Let $s_j$ for $j = 1, \ldots, J$ be the total value of projects (that is, the sum of value put into place at each of the $j$ stages) that are $j$ periods from completion. Then,

$$i_t = \sum_{j=1}^{J} \phi_j s_j.$$

Note that the $\phi_j$ sum to one and are positive as they are fractions.

The capital stocks that index the effect of past production decisions upon subsequent production possibilities are $s_1, \ldots, s_{J-1}, k_t$ and $y_t$. Productive capital is assumed to depreciate geometrically at rate $0 < \delta < 1$ so

$$k_{t+1} = (1-\delta)k_t + s_1t.$$

Further,

$$s_{j, t+1} = s_{j+1, t}.$$
for $j = 1, \ldots, J-1$. The latter set of constraints is that projects $j + 1$ stages from completion are $j$ stages from completion next period.

The shock to technology $z_t$ is the sum of a persistent component $z_{1t}$ and a transitory component $z_{2t}$

$$z_t = z_{1t} + z_{2t} + \varepsilon_t$$

The process on $z_{1t}$ is first-order autoregressive with

$$z_{1,t+1} = \rho z_{1t} + \xi_{1,t+1} \quad 0 < \rho < 1$$

while

$$z_{2,t+1} = \xi_{2,t+1}$$

Further $z_t$ is observed only with error at the beginning of the period. Observed is

$$\tilde{z}_t = z_t + \xi_{3t}.$$

Decisions on employment $h_t$ and new capital projects $s_{jt}$ are made contingent on $\tilde{z}_t$ and the previous history of the $z_t$. Output is then observed and $z_t$ deduced. Then the consumption-inventory carry-over decision is made contingent upon $z_t$ as well as the history of shocks. The vectors $\xi_t$ are iid normal with mean zero and diagonal covariance. With this assumption, the conditional expectations of $z_{1t}$ and $z_{2t}$ based upon available information are sufficient, relative to the history, for decisionmaking. Further, the conditional expectations are linear functions of their previous value and the latest observation whether it be $\tilde{z}_t$ or $z_t$. For further details, see Kydland and Prescott [1982].
III. Supporting the Pareto Optimum

The technology appears to display increasing returns to scale and does, if the obvious commodity point is employed—namely, the one for which hours and capital are inputs. Because competitive equilibria fail to exist with increasing returns, an alternative commodity point is needed if we are to use competitive analysis. Our approach is to assume that capital is specific to the household, that is, only the worker can effectively operate his equipment. Then we treat hours/capital pairs as being different commodities. A commodity of type \((k,h)\) provides

\[ zh^k h^l \]

units of productive services. As there are a continuum of commodities, measures are needed to specify the quantities of all these different commodities. Let \(I = [0,1] \times \mathbb{R}_+\). The input vector, then, is an element of the space \( M_1 \), the signed measures on the Borel sigma algebra of \( I \).

Besides the commodities \( m_1 \), also traded are inventory services \( m_2 \in \mathbb{R} \) and the composite output commodities \( m_3 \in L_\omega(z) \). The composite commodity is indexed by shock \( z \) because the quantity obtained depends upon its realization.

Commodity Point and Technology

With these definitions, our period commodity space is

\[ M = M_1 \times \mathbb{R} \times L_\omega(z) \].

The aggregate production possibility set is simply
The constraints defining the technology are concave so \( Y \) is convex. Using the weak* topology for \( m_1 \) and the product topology for \( m = (m_1, m_2, m_3) \), \( Y \) has an interior. There is no economic meaning for measures \( m_1 \) which have negative part. But if we restricted \( m_1 \) to have positive part only, \( Y \) would not have an interior, and the Hahn-Banach theorem could not be used to support the Pareto optimum allocation. The subset of \( Y \) for which \( m_1 > 0 \) is also supported at the optimum \( m^* \) (note \( m^* > 0 \)) by the supporting hyperplane which supports \( Y \) at \( m^* \). Thus, if a supporting hyperplane exists for \( Y \) at an \( m^0 \) with \( m_1^0 > 0 \), then it also is a supporting hyperplane for the set \( Y^+ = \{m \in Y : m_1 > 0\} \) at \( m^0 \).

Consumption Possibility Set and Preferences

The household chooses a point (primes denotes end of period or beginning of next period values of a variable)

\[(k, s, y, a, k', s', y', a', c, n, m)\]

which satisfies the following constraints

(i) \( k' = (1-\delta)k + s_1 \)

(ii) \( s_j' = s_{j+1} \) \( j = 1, \ldots, J-1 \)

(iii) \( y'(z) - y + \phi_1 s_1 + \cdots + \phi_{J-1} s_{J-1} + \phi J s_J + c(z) < m_2(z) \)

(iv) \( m_2 < y \)

(v) \( \int \int m_1(du, dh) < 1 \)
(vi) \[ \int \int_{k \leq 0} m_1(du, dh) = 0 \]

(vii) \[ n = \bar{h} + \int \int_0^\infty (h-\bar{h})^2 m_1(du, dh) \]

where \[ \bar{h} = \int \int_0^\infty h m_1(du, dh) \]

(viii) \[ a^* = (1-\eta) a + n \]

and the nonnegativity constraints.

Constraints (i) and (ii) are statements concerning how capital is transformed. Constraint (iii) is that inventory investment plus investment in other capital plus consumption is constrained by output. Constraint (iv) is that the supply of inventory service is less than or equal to inventory stocks.

Constraints (v) and (vi) need further explanation. Constraint (v) is that the fraction of instances an individual works \( h \) hours during the period using \( u \) units of capital summed over all instances is less than or equal to one. One cannot work more than 100 percent of the time. Further, one cannot use more than \( k \) units of capital if one has only \( k \) units. We assume agents can use less even though in equilibrium they do not. Giving the agent this option convexifies the consumption possibility set.

Constraints (vii) and (viii) reflect the fact that preferences depend upon the total time worked during the period \( \bar{h} \) and also the variability of the work week. The latter was introduced so that the household would strictly prefer a constant workweek over the period to a varying one. The variable \( a^* \) is the
effect of past labor allocations upon current utility flows. The household maximizes

$$J = \int [u(c(z), 1 - \alpha) - (1 - \alpha) na] + \beta v(a', k', s', y', z)]dF(z|\pi)$$

where $v$ is the value function from dynamic programming. Its current period budget constraint is

$$w_3(m_3) < w_1(m_1) + p_2 \cdot m_2$$

where $w_1$, $p_2$ and $w_3$ are the prices. It can be shown that $w_1(m_1)$ has the representation

$$w_1(m_1) = \int p_1(h, k)m_1(dh, dk)$$

where $p_1(h, k)$ is a continuous bounded function. The argument mimics the ones used by Mas-Colell [1975] and Jones [1984], who have used signed measures in general equilibrium analyses.

Given that agents discount, an equilibrium $w_3$ exists with the representation

$$w_3(m_3) = \int p_3(z)m_3(z)dF(z|\pi)$$

with $p_3 \in L_1(z)$.

The existence of a continuous equilibrium pricing function supporting the Pareto optimum is guaranteed by Theorem 2 of Debreu [1954], as the consumption possibility set is convex, preferences are convex, the aggregate production possibility set is convex, preferences are continuous, and the production possibility set has an interior. Given a separating hyperplane exists,
Theorem 2 of Prescott and Lucas [1972] guarantees one exists with \( w_2 \) belonging to \( \mathbb{L}^1(z) \). The key conditions are that the consumer discounts highly improbable outcomes.

Given there are no utility gains to having a variable workweek over the period and there are costs in terms of foregone output, in equilibrium the measure \( \mu \) puts mass one on some point \((k, h)\) and zero mass elsewhere. Consequently, when solving the social planner's problem, one can solve the problem for which the workday is restricted to be of the same length at every instance of each time period, though free to be of different lengths in different time periods. With this restriction, finding the Pareto optimum is a standard discounted dynamic programming problem (see, for example, Majumdar and Radner [1983]).

IV. Steady State and Calibration

To determine, for each set of parameter values, the set of stochastic difference equations describing the equilibrium for this model, we exploit the result that the equilibrium can be found by solving the stand-in utility maximization problem described in the previous sections. The aim is to determine the variance-covariance properties of the model and to contrast them with data for the post-war U.S. economy. Due to the form of the utility function and the nonlinearity of the resource constraint, this is infeasible for the original formulation of the model. Instead, we shall study the approximate economy which is found by making a quadratic approximation around the steady state of the exact model. The steady state will also be used for the purpose
of calibrating the model in the sense of making it consistent with
average relations from growth observations as well as from micro-
data. For those reasons, we now turn to the determination of the
steady state.

We shall let variables without subscripts denote the
steady state. It is found by setting \( z_t \) equal to its uncondi-
tional mean of \( z \) in every period.

The parameters that play a role for the steady state are
\( \beta, \gamma, \nu, \omega, \alpha, \phi_1, \ldots, \phi_j, \rho, \) and \( \theta \), from technology. The variables of the
model are consumption \( c \), investment plans \( s \) which are equal to
actual investment in the steady state, inventories \( y \), capital
stock \( k \), workweek \( h \), and the real interest rate \( r \). For each set
of parameter values, a steady state results. Thus, if we denote
by \( \omega \) the vector of parameters and let \( S \) be the vector of state
variables, then one can determine a mapping \( S(\omega) \).

For our purpose, the inverse mapping is of particular
interest, that is, the mappings from part of the state vector \( S \) or
functions thereof to a subset of the parameters. The state vector
typically consists of variables for which there are long-run
relations in the data that have been essentially constant through-
out the sample period. Examples are consumption-output, inven-
tory-output, and capital-output ratios, and households' average
allocation of time to market activity.

The parameters can be divided into two groups, those
that affect the steady state vector a great deal, and those which
do not. It is natural, then, to use the steady state observations to narrowly restrict the first set of parameters. As will be seen shortly, this is the strategy that we adopt. Some of the remaining parameters are determined from microstudies of individual and household behavior. For example, previous studies imply a narrow range of possible values for the curvature or risk-aversion parameter $\gamma$ in the utility function. The magnitude of the variance of the exogenous technology shocks and their serial correlation properties are measured by examining the statistical properties of changes in aggregate output not accounted for by changes in the inputs. Only the parameter $\nu$, which determines the elasticity of substitution between inventories and the composite capital-labor input, was free. We did not find good measures of this parameter in the literature. Observations at the firm technology level suggest limited substitution opportunities and consequently we restrict this parameter to be large. Admittedly if good measurement found that this parameter was in fact smaller, the prediction of theory would change as it would change if better measurement of the technology change shock variance found that parameter to significantly differ from the values we used.

In our model, the quarterly depreciation rate $\delta$ is set equal to 0.025 on the basis of microstudies. The steady state interest rate will, in general, have a close relationship to $\beta$ and the rate of growth of consumption. Abstracting from growth, however, we have $r = (1 - \delta)/\beta$. With a quarterly real interest rate of one percent, a value for $\beta$ of 0.99 is implied. The resource
allocation to new capital is assumed to be uniform over the construction period, and so, all the \( q_i \)'s are assumed to be equal to \( 1/J \). The value of \( J \) was set equal to three.

The parameter \( z \) can be viewed as a scale parameter. Without loss of generality, we can choose \( z \) so that steady state output equals one. The autoregressive coefficient \( \rho \) in the law of motion for the permanent part of \( z_t \) is fixed at 0.95. This leaves eight parameters to be determined.

Once the depreciation rate is given, we obviously have \( s = \delta k \). In the steady state, there is no inventory accumulation. Thus, when \( s \) is determined, steady state consumption is then total output minus this quantity. We can therefore, without loss of generality, choose the variables \( y, k, \) and \( h \) as the ones whose steady states remain to be determined. Out of these, average \( y \) (interpreted as inventory-output ratio) and \( h \) have been stable in the post-war period and will be determined by the data, and so will \( k \) from the capital's share of output, which has also changed very little.

The parameters that affect the steady state a lot are \( \mu \), \( \theta \), and \( \sigma \). In particular, there are close relationships between the three parameter-variable pairs \( \mu \) and \( h \), \( \theta \) and \( k \), and \( \sigma \) and \( y \).

From the condition \( f_{y} = r \) and the fact that steady state output is one, it follows that

\[
\sigma = ry^{\nu+1}.
\]
Letting consumption be the numeraire, the steady state relative price of services from capital is 
\[ q = \frac{(r+\delta)}{\sum_{j=1}^{J} (1+r)^{j-1} \phi_j}. \]  
Steady state capital stock then follows from the relation 
\[ \theta = \frac{qk}{1 - \sigma^{-1}}, \]  
where the ratio is determined as an average relationship in the data. We can now also determine from the production function the value of \( z \) that will make steady state output equal one as follows 
\[ z = \frac{(1-\sigma^{-1})^{\nu}}{h \psi k^{\theta}}. \]

Turning to the consumer side, notice that we already have \( c \), and all that is left to be determined from utility maximization is \( h \). Without the dependence of current utility on past leisure choices, this would have followed from the condition \( u_{ht}/u_c = f_h \). In the present case, however, the relevant first-order condition can be written as 
\[ \frac{u}{c_t} f_{ht} = (1-u) \sum_{j=0}^{\infty} \frac{\beta^j a_j}{\alpha_{(L)} t_{t+j}} \]  
for all \( t \). 

In the steady state, \( c_t = c \), \( L_t = L \), and \( f_{ht} = f_h \) for all \( t \). Using also the facts that \( a_j \) sum to one, that \( \sum_{j=0}^{\infty} \beta^j a_j = (\alpha_0 r + \eta)/(r + \eta) \), and that \( L = 1 - h \), this expression can be written as 
\[ (1-u) \frac{\alpha_0 r + \eta}{r + \eta} c = (1-h)f_h, \]
where 
\[ f_h = \psi(zk^{\theta})^{-\nu} h^{-\psi-1}. \]
If average h is given, then the above relation can be solved to determine the parameter μ.

Given the steady state, the remaining steps in the computation of equilibrium are similar to those described in Kydland and Prescott [1982]. To deal with nonlinearity in the resource constraint, we substitute for \( c_t \) from the resource constraint in the utility function and then make a quadratic approximation around the model's rest point. As the remaining constraints are linear, it is now fairly straightforward to determine recursively the decision rules that describe the equilibrium behavior of the approximate economy. The decision variables are \( h_t, s_t, y_{t+1}, \) and \( c_t \), of which \( c_t \) is determined from the resource constraint. The equilibrium can be represented by a linear stochastic system of difference equations consisting of constraints determining next period's capital stocks, the decision rules which are functions of a state vector that includes conditional expectations of the unobserved permanent and transitory technology shocks, and the linear relations for updating these conditional expectations from the most recent observations.

Given sets of parameter values and variances of the three shocks, sample series can be generated from the model. These series are decomposed into a cyclical and a growth component using the same method as for the data. In both cases, several statistics are computed and used for comparison. They are autocorrelations of output of order up to six, standard deviations of the cyclical components of all the variables, and their correla-
tions with cyclical output. The U.S. statistics, using quarterly data, for the 116 quarter period 1954-82 are reported in Table 1. Using the model, 20 repeated samples of length 116 quarters are drawn, and averages and standard deviations are computed for each of the statistics. This approach provides a measure of the sampling error.

The average value for $h$ is 0.3 based on the microfinding that households allocate about 30 percent of their nonsleeping time to market production. The implied value of the parameter $\mu$ varies somewhat depending on model version. For the case of intertemporally nonseparable utility in leisure, for example, the value of $\mu$ is 0.325 when the production function parameter $\psi$ equals $1 - \delta = 0.64$, while its value becomes 0.274 in the example with $\psi = 0.82$. The average inventory-output ratio is approximately one when output is measured quarterly. This yields a value for $\sigma$ of .01.

To determine capital's factor share, we added depreciation of capital to rental, corporate, and interest income. These national income account numbers were augmented by estimates of depreciation and implicit rental income on durables owned by the household. The results of these calculations for 1977, which is a typical year, was a factor share for capital of 36 percent. Besides implying $\theta = .36$, this share implies 10 for the capital-output ratio, with output still being measured on a quarterly basis.
V. Results

Given that the present model shares many of the features of the one presented in Kydland and Prescott [1982], it is natural to use that model as a benchmark and to concentrate on the differences in results due to the introduction of variable workweek. We shall begin by setting the parameters that are common to the two models equal to the values that were used in the earlier paper. Thus, we set $\alpha_0 = 0.5$, $\eta = 0.1$, $\gamma = -0.5$, and use the same relative variances for the shocks in both models.

The parameter $v$ determines the elasticity of substitution between inventories and the other inputs. Due to the change in the form of the production function, this parameter is not quite comparable to the old $v$. Changes in $v$ affect most statistics very little, with the exception of the correlation between inventories and output, and to a lesser extent the volatility in inventories. The elasticity of substitution between inventories and other inputs should be very small, so we use 3 as the value for $v$, yielding a correlation between inventories and output of about 0.60 for the version with variable workweek.

Another small change is in the time $J$ it takes to build new capital. Previously, we used four quarters. In this study three quarters are assumed. The reason for this change was that durable good and housing construction periods are typically less than a year. The principal effect of this modification is the prediction that over the cycle all the capital stock should be nearly orthogonal to output rather than negatively correlated.
With this change the model matches a little better the post Korean War U.S. observations.

The Kydland and Prescott [1982] model was intended to have the technology shock be the sum of a permanent and transitory component. For computational reasons and so as to keep \( z_t \) from wandering away from where the approximation is good, a value of 0.95 was used throughout. The values for the standard deviations of the permanent, transitory, and indicator shocks, respectively, are 0.0077, 0.0016, and 0.0077. The implied variance of the technology stock is approximately equal to that obtained by Prescott [1986] using Solow growth accounting to obtain a \( z_t \) sequence. 2/

Statistics describing some of the cyclical properties of the model are presented in Table 2. The value of the production function parameter \( \psi \) is 0.82, which is between the two extremes of \( 1 - \theta = 0.64 \) and one; that is, both margins are operative. 4/

These statistics can be compared with the case of not allowing the workweek to vary, which corresponds to letting \( \psi \) equal 0.64. Results for this case, with otherwise exactly the same parameter values, including the variances of the shocks, are presented in Table 2. This is the version of the model that is closest to the model in Kydland and Prescott [1982], the difference being the way in which inventories enter the production technology.

There are two major differences between the results for \( \psi \) equal to 0.82 and 0.64. First, for given variability of the shocks, output is much more variable when the workweek is permitted to vary. Fluctuations with \( \psi = 0.82 \) are very close in magni-
tude to those experienced by the American economy in the post-war period. Second, the statistics for the inventory stock are much closer to those of U.S. data in Table 1, both in terms of variability and lead-lag properties. The U.S. data on inventories clearly include a significant portion of what in our model is physical capital under construction. We have therefore added half of these goods to inventories before computing the statistics in Tables 2 and 3. In steady state, they represent one-eighth of final-good inventories. A smaller fraction than one-half would have reduced the variability of the inventory stock, the standard deviation being 0.68 if no capital goods in process were included.

All the standard deviations in Table 3 are obviously smaller than in Table 2 just as output is much less variable. It is interesting to note, however, that in Table 2 the standard deviation of hours is 38 percent larger than that of productivity, as compared with being 18 percent larger in Table 3. In spite of this improvement, there is still substantial discrepancy left between the hours variability in the model and in the data.

Another way to see this discrepancy is in terms of the output-labor elasticity emphasized by Lucas [1970], who estimated it to be near one. A puzzle at the time was the magnitude of the difference between this empirical elasticity and the one predicted by standard production-function specifications to equal labor's share. In our model, we can obtain the elasticity of cyclical output with respect to cyclical labor input by simply dividing the standard deviation of output by that of hours and multiplying by
the correlation between hours and output. Doing that for each of
the 116-period samples from the model on which Table 2 is based
gives us an average elasticity of 1.51 with a standard deviation
of 0.05. This is of course much larger than the labor share,
which illustrates the importance of technology shocks for our
model.

In terms of the output-labor elasticity, we are faced
with the opposite problem of Lucas, that is, we need to explain
why the empirical elasticity for the U.S. is so low relative to
the model. This issue is of course related to the above-mentioned
discrepancy in hours variability. A possible explanation is
measurement errors. Hansen (1984) divided the population into six
demographic groups according to age and sex indicates that actual
cyclical hours should be multiplied by a factor of 0.94 in order
to obtain hours in efficiency unity. In Kydland [1984], prime-age
males were further divided into skill groups using years of educa-
tion as the criterion. It was found that hours varied substan-
tially more over the cycle for individuals with less education.
Adding this element would further reduce the efficiency factor
below 0.94. Furthermore, aggregate hours are difficult to measure
accurately as evidenced by the discrepancy in figures obtained
from establishment data and household data. (See Prescott
[1986].) These factors reduce the empirical elasticity of output.

One may wonder to what extent larger values of $\psi$ are a
substitute for the intertemporally nonseparable utility function
in terms of increasing the variability of hours and output or
reducing the output-labor elasticity. The time-separable case is the special case of \( \alpha_0 = 1 \). The finding is that as \( \psi \) increases from 0.64 to 0.82 to 1.0, the magnitude of fluctuations increases from 1.30 percent to 1.46 percent to 1.68 percent when in fact they were 1.80 percent. Thus the finding that fluctuations in response to technology shocks are larger if the workweek varies appears robust to the specification of preferences.

One final point is that permitting the workweek of capital to vary systematically with hours of employment affects hardly at all the estimate of the variance of the technology shock. Prescott [1986] finds its value is reduced from 0.763 percent to only 0.759 percent when \( \psi \) is increased from 0.64 to 0.85. With \( \psi = 0.85 \), the estimated variance of the shock is at its smallest.

VI. Concluding Comments

Our analysis should not be interpreted to mean that fluctuations are optimal and that there is no role for stabilization policy. Our view is that public-finance considerations are not the principal factor driving the cycle and that abstracting from them at this stage is warranted. Only when we have considerable confidence in a theory of business cycle fluctuations would the application of public finance theory to the question of stabilization be warranted. Such an extension is conceptually straightforward, though in all likelihood it will be difficult and will require considerable ingenuity.
Footnotes

*We thank Antonio Borges, Kevin Cotter, Rodolfo Manuelli, John Shoven, and Charles Whiteman for helpful comments, and the National Science Foundation and the Minneapolis Federal Reserve Bank for financial support.

¹/See Kydland and Prescott [1982, footnote 15].

²/Subsequently, all statistics reported are for the cyclical components.


⁴/The value for Ψ of 0.82 was close to the largest one for which our computational methods were applicable.

⁵/See Shoven and Whalley [1984] for a review of applications of general equilibrium analysis to issues in public finance and trade.
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Appendix
Equilibrium Process For the Model

Laws of Motion

\[ a_{t+1} = 0.9a_t + r_t \]
\[ k_{t+1} = 0.975k_t + s_{1t} \]
\[ s_{1,t+1} = s_{2t} \]
\[ s_{2,t+1} = s_{3t} \]
\[ z_{1,t+1} = 0.95z_{1t} + 0.05917 + \xi_{1,t+1} \]
\[ z_{2,t+1} = \xi_{2,t+1} \]
\[ z_t = z_{1t} + z_{2t} \]
\[ \pi_t = z_t + \xi_{3t} \]

Forecasting Rules
Let \( m_{0t} \) be the expectation of \( (z_{1t}, z_{2t})' \), conditionally on the previous history of \( z_r \), \( r < t \). Then the expectation of \( (z_{1t}, z_{2t})' \), given \( \pi_t \), is

\[
\begin{bmatrix}
0.50129 \\
-0.49871
\end{bmatrix}
\begin{bmatrix}
0.49871 \\
0.019659
\end{bmatrix}

m_{1t} = m_{0t} + \begin{bmatrix}
0.50129 \\
-0.49871
\end{bmatrix}
\begin{bmatrix}
0.49871 \\
0.019659
\end{bmatrix} \pi_t.
\]
The expectation of \((z_{1t}, z_{2t})\), given \(z_t\), is

\[
m_{2t} = \begin{bmatrix} 0.037924 & -0.96208 \\ -0.037924 & 0.96208 \end{bmatrix} m_{1t} + \begin{bmatrix} 0.96208 \\ 0.037924 \end{bmatrix} z_t.
\]

Decision Rules

<table>
<thead>
<tr>
<th>State Variables</th>
<th>Decision Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_t)</td>
<td>-0.0093319</td>
</tr>
<tr>
<td>(y_t)</td>
<td>-0.0067304</td>
</tr>
<tr>
<td>(a_t)</td>
<td>-0.036587</td>
</tr>
<tr>
<td>(s_{1t})</td>
<td>-0.022726</td>
</tr>
<tr>
<td>(s_{2t})</td>
<td>-0.010831</td>
</tr>
<tr>
<td>const.</td>
<td>0.15672</td>
</tr>
<tr>
<td>(m_{11t})</td>
<td>0.30548</td>
</tr>
<tr>
<td>(m_{12t})</td>
<td>0.58741</td>
</tr>
<tr>
<td>(m_{21t})</td>
<td></td>
</tr>
<tr>
<td>(m_{22t})</td>
<td>0.78731</td>
</tr>
<tr>
<td>(n_t)</td>
<td>2.3943</td>
</tr>
<tr>
<td>(s_{jt})</td>
<td>-0.32354</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>0.00050171</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>0.94847</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>-0.0059959</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>-0.34657</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>-0.33468</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>-1.0833</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>-1.4633</td>
</tr>
<tr>
<td>(y_{t+1})</td>
<td>-1.0833</td>
</tr>
</tbody>
</table>
Parameter Values

Preference
\[ \begin{align*}
\beta &= 0.990 \\
\alpha_0 &= 0.500 \\
\eta &= 0.100 \\
\mu &= 0.274 \\
\gamma &= -0.500
\end{align*} \]

Technology
\[ \begin{align*}
\psi &= 0.274 & \rho &= 0.950 \\
\rho &= 0.360 & \sigma &= 0.010 \\
J &= 3 & \bar{z} &= 1.000 \\
\phi_1 &= 0.333 & \nu &= 3.000 \\
\phi_2 &= 0.333 & \delta &= 0.025 \\
\phi_3 &= 0.333
\end{align*} \]

Covariance Matrix of Shocks\(^8/\)
\[
\Sigma \xi = 1.183^2 \begin{bmatrix}
0.0077^2 & 0 & 0 \\
0 & 0.0016^2 & 0 \\
0 & 0 & 0.0077^2
\end{bmatrix}
\]

\(^8/\text{The 1.183 is the unconditioned mean of the } \pi_t \text{ process.}\)
Table 1  
Cyclical Behavior of the U.S. Economy  
Deviations From Trend of Key Variables  
1954:1-1982:4

<table>
<thead>
<tr>
<th>Variables x</th>
<th>Standard Deviation</th>
<th>Cross Correlation of Output With</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>x(t-2) x(t-1) x(t) x(t+1) x(t+2)</td>
</tr>
<tr>
<td>Gross National Product</td>
<td>1.8%</td>
<td>.57</td>
</tr>
<tr>
<td>Personal Consumption Expenditures</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>.6</td>
<td>.53</td>
</tr>
<tr>
<td>Nondurable Goods</td>
<td>1.2</td>
<td>.54</td>
</tr>
<tr>
<td>Durable Goods</td>
<td>5.0</td>
<td>.60</td>
</tr>
<tr>
<td>Fixed Investment Expenditures</td>
<td>5.3</td>
<td>.59</td>
</tr>
<tr>
<td>Nonresidential Investment</td>
<td>5.2</td>
<td>.28</td>
</tr>
<tr>
<td>Structures</td>
<td>4.6</td>
<td>.23</td>
</tr>
<tr>
<td>Equipment</td>
<td>6.0</td>
<td>.27</td>
</tr>
<tr>
<td>Capital Stocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Nonfarm Inventories</td>
<td>1.7</td>
<td>-.13</td>
</tr>
<tr>
<td>Nonresidential Structures</td>
<td>.4</td>
<td>-.36</td>
</tr>
<tr>
<td>Nonresidential Equipment</td>
<td>1.0</td>
<td>-.26</td>
</tr>
<tr>
<td>Labor Input</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonfarm Hours</td>
<td>1.7</td>
<td>.25</td>
</tr>
<tr>
<td>Average Weekly Hours in Mfg.</td>
<td>1.0</td>
<td>.54</td>
</tr>
<tr>
<td>Productivity (GNP/Hours)</td>
<td>1.0</td>
<td>.60</td>
</tr>
</tbody>
</table>

Source: Citibank Data Base
Table 2
Cyclical Behavior of the Modified Kydland-Prescott Economy With Variable Workweeks *

<table>
<thead>
<tr>
<th>Variables x</th>
<th>Standard Deviation</th>
<th>x(t-2)</th>
<th>x(t-1)</th>
<th>x(t)</th>
<th>x(t+1)</th>
<th>x(t+2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.77 (.14)</td>
<td>.38</td>
<td>.64</td>
<td>1.00</td>
<td>0.64</td>
<td>.38</td>
</tr>
<tr>
<td>Consumption</td>
<td>.55 (.07)</td>
<td>.26</td>
<td>.56</td>
<td>.90</td>
<td>.73</td>
<td>.58</td>
</tr>
<tr>
<td>Investment</td>
<td>5.19 (.39)</td>
<td>.40</td>
<td>.55</td>
<td>.88</td>
<td>.80</td>
<td>.42</td>
</tr>
<tr>
<td>Inventory Stock</td>
<td>2.06 (.34)</td>
<td>.25</td>
<td>.16</td>
<td>.59</td>
<td>.51</td>
<td>-.03</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>.57 (.09)</td>
<td>-.32</td>
<td>-.04</td>
<td>-.10</td>
<td>.08</td>
<td>.57</td>
</tr>
<tr>
<td>Hours</td>
<td>1.11 (.08)</td>
<td>.35</td>
<td>.54</td>
<td>.94</td>
<td>.57</td>
<td>.27</td>
</tr>
<tr>
<td>Productivity (Output/Hours)</td>
<td>.80 (.07)</td>
<td>.34</td>
<td>.65</td>
<td>.89</td>
<td>.61</td>
<td>.45</td>
</tr>
<tr>
<td>(Annual)</td>
<td>.25 (.05)</td>
<td>.36</td>
<td>.64</td>
<td>.63</td>
<td>.41</td>
<td>.36</td>
</tr>
</tbody>
</table>

*These are the means of 20 simulations, each of which was 116 periods long.
The numbers in parentheses are standard errors.
Table 3
Cyclical Behavior of the Kydland-Prescott Economy*

<table>
<thead>
<tr>
<th>Variables x</th>
<th>Standard Deviation</th>
<th>x(t-2)</th>
<th>x(t-1)</th>
<th>x(t)</th>
<th>x(t+1)</th>
<th>x(t+2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.43 (.11)</td>
<td>.44</td>
<td>.70</td>
<td>1.00</td>
<td>.70</td>
<td>.44</td>
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<tr>
<td>Consumption</td>
<td>.48 (.05)</td>
<td>.29</td>
<td>.60</td>
<td>.92</td>
<td>.76</td>
<td>.60</td>
</tr>
<tr>
<td>Investment</td>
<td>4.32 (.32)</td>
<td>.39</td>
<td>.58</td>
<td>.88</td>
<td>.82</td>
<td>.51</td>
</tr>
<tr>
<td>Inventory Stock</td>
<td>1.13 (.07)</td>
<td>.31</td>
<td>.49</td>
<td>.78</td>
<td>.66</td>
<td>.21</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>.39 (.06)</td>
<td>.33</td>
<td>-.20</td>
<td>-.06</td>
<td>.17</td>
<td>.54</td>
</tr>
<tr>
<td>Hours</td>
<td>.85 (.07)</td>
<td>.42</td>
<td>.61</td>
<td>.93</td>
<td>.68</td>
<td>.37</td>
</tr>
<tr>
<td>Productivity (Output/Hours)</td>
<td>.72 (.06)</td>
<td>.37</td>
<td>.68</td>
<td>.90</td>
<td>.59</td>
<td>.44</td>
</tr>
<tr>
<td>Real Interest Rate (Annual)</td>
<td>.17 (.02)</td>
<td>.46</td>
<td>.68</td>
<td>.84</td>
<td>.56</td>
<td>.42</td>
</tr>
</tbody>
</table>

*These are the means of 20 simulations, each of which was 116 periods long.

The numbers in parentheses are standard errors.