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SOME UNSOLVED PROBLEMS FOR MONETARY THEORY

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Some Unsolved Problems for Monetary Theory

In this paper, I discuss two policy questions which I regard as unresolved: should interest be paid on money and should currency provision be in the hands of the government? An affirmative answer to the first question stands as one of the few widely accepted general results of monetary theory. My discussion is intended to cast doubt on it. There is no widely accepted answer to the second question; some have asserted that currency provision is a public good while others have asserted that currency provision should be left to the market. My discussion of currency provision will not provide a resolution. Instead, I will discuss a way of formulating the question that seems to offer some hope for resolving it.

I. Payment of Interest on Money

The casual statement of the case for paying interest on money is familiar. Real balances are produced at zero social cost. In an equilibrium in which the real yield on other assets exceeds that on money--or, more generally, in which the marginal rate of substitution between future consumption and present consumption exceeds the real return on money--individuals face a positive alternative cost of holding money. Given the zero social cost, this positive alternative cost implies that too little money is being held. Payment of interest on money removes the positive alternative cost. The aspect of this casual statement on which I focus is how an equilibrium with a real return on money less than the relevant intertemporal marginal rate of substitution arises.

In order for a model to have such an equilibrium, it must contain something that prevents individuals from borrowing at the real return on money. This is accomplished in money-in-the-utility-function models by the assumption that the money in the utility function is outside money, not the sum of outside and inside money with individuals free to issue inside money. It is accomplished in cash-in-advance (Clower-constaint) models by a similar identification of the objects that qualify as cash. It is accomplished in a more straightforward manner in a model like Bewley's (1983) in which it is simply assumed that there are no private credit instruments.

These kinds of models leave implicit what prevents individuals from borrowing at the real return on money. Thus, for example, Samuelson says about his model of money in the utility function,

> "This is not the only way of introducing the real convenience of cash balances. An even better way would be to let utility depend only on the time stream of consumptions, and then to show that holding an inventory of money does contribute to a more stable and greatly preferable stream of consumptions." [1968, page 9].

Samuelson does not say why the alternative would be better. I think it would be better because explicit assumptions about what prevents individuals from borrowing at the real return on money are likely to have implications for the feasibilty of tax-financed payment of interest on money and for the feasibility of other policies for dealing with the seeming nonoptimality. I will

illustrate this by describing a formulation that is more explicit than the models just mentioned about what prevents individuals from borrowing at the real return on outside money. In this formulation, the feature that prevents private borrowing makes feasibility of tax-financed interest on money questionable. And, in it, if tax-financed interest on money is feasible, then so are other policies that seem at least as desirable.

An Overlapping Generations Model Without Private Borrowing

This is a stationary, one good per date, pure exchange model of two-period lived overlapping generations. There is diversity in tastes and endowments within generations but no diversity across generations. Each person cares only about his own lifetime pattern of consumption according to a twice differentiable strictly quasi concave utility function. Finally, suppose that the people who are in the second period of their lives at the initial date, the initial old, own in the aggregate one unit of fiat outside money.

Such settings are usually analyzed under the assumption that nothing prevents individuals when young from borrowing at the real return on outside money. I want to make a different assumption, namely that individuals cannot borrow at any return because they cannot credibly commit themselves to repay when old. Although young people know they will receive an endowment of the good when they are old, I assume that they cannot commit this as collateral on a loan and that nothing prevents old people, whether or not they have borrowed, from consuming their endowment, and, hence, from reneging on any loan.

Under very weak additional assumptions to be described momentarily, the following propositions hold for this economy.

1. No nonintervention equilibrium is Pareto optimal.
2. There exists a nonintervention equilibrium with a positive and constant value of fiat money, one with a gross real return on money equal to unity.
3. There exists an interest-on-money scheme financed by lump sum taxes payable when old which gives rise to a Pareto optimal equilibrium.
4. At least some people in each generation are worse off under any stationary optimal interest on money equilibrium than under the nonintervention equilibrium.

I will describe both the assumptions under which these propositions hold and how they are proved with the aid of some simple diagrams.

Figure 1 shows endowments and (common) preferences of two members of generation t. It also shows the boundaries of their budget sets when these people face no taxes and the option of acquiring money with a real gross return of unity.

For proposition 1, it is sufficient that (a) marginal rates of substitution at the endowment are not equal for all members of generation t, and (b) the marginal rate of substitution of some member of generation t exceeds unity at the endowment. (Both conditions are displayed in Figure 1.)

To prove proposition 1, one simply notes that optimality requires equality of marginal rates of substitution for all members of generation t; that under assumption (a) this requires a real gross return on money no smaller than the maximum marginal rate of substitution at the endowment; and, that under assumption (b) this, in turn, requires a return that exceeds and is bounded away from the unity for all t, which is equivalent to a minimum positive rate of appreciation of money. However, since the real value of money is bounded by the sum of first period endowments for a generation, a constant in this model, such appreciation cannot occur as an equilibrium.

For proposition 2, it is sufficient that (c) the marginal rate of substitution of some person be less than unity at the endowment (e.g., person h in Figure 1). The relevant equilibrium condition can be expressed as follows. First, for each h in generation t, let $s^h(r,v)$ be the utility maximizing choice of $w_1^h - c_1^h$ (saving) subject to

$$(1) \qquad c_1^h + c_2^h/r \leq (1-v)(w_1^h + w_2^h/r)$$

$$(2) \qquad c_1^h - w_1^h \leq 0$$

where (c_1^h, c_2^h), the vector of arguments of h's utility function, is h's consumption when young and when old respectively, (w_1^h, w_2^h) is the corresponding vector of pretax endowments of h, v is a tax rate levied on wealth, and r is the gross real return faced by h. Note that constraint (2) is the no-borrowing restriction implied by the assumption that taxes are payable when

old. The positive and constant equilibrium value of money is simply $\sum s^h(1,0)$, where the summation (and all those below) is over the members of a single generation. Assumption (c) guarantees that the sum is positive.

In order to prove proposition 3, I make two other assumptions: (d) there exists some scalar r^* that is at least as large as the marginal rate of substitution at the pre-tax endowment of any member of generation t; (e) for each person, first and second period consumption are normal goods. Assumption (e) implies that with positive taxation payable when old, the marginal rate of substitution at the after-tax endowment is smaller than at the pre-tax endowment. Thus, it implies that at any return $r > r^*$ and nonnegative tax rate, everyone wants to save so that constraint (2) is not binding.

Given these assumptions, proposition 3 is established by showing that there exists a triplet (r,v,p) that satisfies $r > r^*$, $v \in (0,1)$, $p > 0$ and

$$(3) \qquad \sum s^h(r,v) = p$$

$$(4) \qquad (r-1)p = v(rW_1 + W_2)$$

where W_1 is the sum of endowments when young in a generation and W_2 is the corresponding sum when old. Equation (4) is the condition that interest payments (the left-side) be equal to tax revenue (the right side). One way to prove that such a triplet exists is to show that for any $r > r^*$, there exists a $v \in (0,1)$ that satisfies

$$(5) \qquad (r-1)\sum s^h(r,v) = v(rW_1+W_2)$$

To see this, note that with $r > 1$, the left side of (5) exceeds the right side at $v = 0$; that there exists a v large enough but less than unity for which the right side exceeds the left side (essentially because $\sum s^h(r,v) < W_1$); and that both sides of (5) are continuous functions of v. That any such $(r,v(r))$ implies a positive p from (3) follows from $r > r^*$ and assumption (e), which together imply $s^h(r,v(r)) > 0$ for all h. This last fact implies equality of marginal rates of substitution and Pareto optimality because $r^* > 1$.

Proposition 4 follows from the fact that the interest-on-money equilibrium satisfies feasibility with equality,

$$(6) \qquad \sum [c_1^h+c_2^h-(w_1^h+w_2^h)] = 0$$

and is such that everyone is saving. It follows that if someone in generation t ends up outside the budget set faced in the non-intervention equilibrium, then someone else ends up inside it and, hence, is worse off. The sense in which the "average" person in generation t is worse off under the interest payment scheme is depicted in Figure 2. It shows the situation of someone who ends up on the boundary of the non-intervention budget set, or, equivalently, of someone who ends up paying taxes equal to the interest on his money holdings. This worse off person is the "average" person because (6) holds. It is, of course, the initial old who benefit from the interest on money scheme.

The above shows that with lump-sum taxes payable when old, it is easy to support on optimal equilibrium. However, such taxing seems to contradict the assumption that people cannot commit themselves to repay debts. If the government can enforce taxes payable when old, why can't it enforce debt repayment? Taxes payable when young would not seem to suffer from the same enforcement problem, but as I now indicate, such tax schemes are subject to other difficulties.

Proposition 4 was proved by showing that there are high enough returns and corresponding supporting taxes so that constraint (2) is not binding for any person. With taxes payable when young, each person faces constraint (1), but a more stringent version of (2), namely

$$(2') \qquad c_1^h - w_1^h \leq -v(w_1^h + w_2^h/r)$$

which is equivalent to $c_2^h \geq w_2^h$. In terms of Figure 2, for a given return and tax rate, constraint (2') requires that each person choose to be northwest of point s', while constraint (2) only requires the person to choose to be northwest of point s. It is easy to produce innocent examples for which there does not exist a steady state with a uniform tax rate, with taxes payable when young, and with (2') nonbinding.[1/] Although there exist vectors of tax rates which support a steady state with (2') not binding, the tax rates must vary across people and must be such that relatively high saving people pay relatively high taxes, while still regarding those taxes as lump-sum. Obviously, the informational requirements for such taxing are substantial.

For several reasons, then, the above model makes tax-financed interest on money a questionable policy. A tax scheme with taxes payable when old seems to contradict the assumption that private debt agreements cannot be enforced. A tax scheme with taxes payable when young must have tax rates depend on individual characteristics. And, finally, any such tax schemes seem to have strange distributional consequences.

Of course, the above is simply one particular model--and, perhaps a bizarre one. However, there are few, if any, other models which both account for the inability of individuals to borrow at the real return on outside money and imply that tax supported interest on money is a feasible and good policy. The one model I know of which comes closest to displaying both properties is Townsend's "Turnpike model of exchange" (1980, pages 267-275). But even in his set-up, as he notes (1980, page 297), the feature that rules out private borrowing, an extreme form of spatial and informational separation, makes one uneasy about the feasibility of tax schemes for supporting interest on money.

II. Currency Provision

Here, I will discuss an approach to studying currency provision. The goal of such study is ultimately to answer questions of the following sort. Should currency provision be a government activity? And, if it should be, how should it be managed? Armed with a theory that address such questions, we might hope to appraise the United States coinage act of 1792 which, among other things, created a government mint, set up a denomina-

tional structure, directed the mint to issue coins to those depositing gold and silver bullion at the mint with no expense to the depositor, and made no provision for replacing worn coins with new coins. We also might hope to appraise the current system of currency provision under which, for example, ten $1 bills exchange for one $10 bill despite the obvious production cost discrepancies, and under which worn units of currency are exchanged for new units despite the cost of making the replacement. (The latter policy is currently coming under pressure because of the large demand for new or almost new units of currency arising from the use of automatic teller machines.) We might also be able to better interpret the fairly general view that some economies have suffered from an inadequate currency. It is claimed, for example, that during the Colonial period and during periods in the 19th century, the United States suffered a shortage of small denomination currency (see Hanson 1979 and Timberlake 1978, Chapter 9). It is also claimed that England during the 19th Century suffered from a currency that was badly worn (see Jevons 1918).

As these comments suggest, issues about currency provision arise in both what we may loosely call fiat monetary systems and in commodity monetary systems. That being so, it may be helpful to interpret the following discussion in the context of commodity money systems and, in so doing, avoid the additional complications that must be faced in studying fiat monetary systems.

I want to approach the study of currency provision in commodity money systems by using suggestions, made repeatedly, about the properties of objects that render them suitable as commodity monies. Among the suggested properties are: divisibility, durability, portability, and recognizability. However, I want to use this kind of list in what may be a new way. I want to treat the listed properties both as conjectures about properties that permit an object to play a prominent role in exchange and as conjectures about properties that a decentralized system, a laissez-faire system, will in some sense underproduce. Naturally, to even discuss the possibility of underproduction of these properties, the environments or models we use better not have these properties be too freely available. Thus, for example, if we want to study divisibility, then we had better start out with a model in which not everything is divisible to start with and in which in making things more divisible is costly. Also, in order to give the conjectures a chance, we want to have an environment in which there is conceivably some nontrivial role for media of exchange. Obviously, study of Robinson Crusoe alone on his island will not do. Before I discuss some features of a candidate model, I want to express in a loose way why one might be willing to entertain the possibility that a decentralized system underproduces some of the listed properties.

Consider divisibility and imagine a world in which the production of anything that is durable--think, of a coin--costs more in resources than one slice of bread, bread being nondur-

able. It would seem, then, that if there is only private coinage production, one would never observe a coin that exchanges for as little as one slice of bread. Could this also be a world in which it would be convenient to have coins that exchange for as little as one slice of bread, convenient in that people would in some sense be willing to be taxed to provide a subsidy to be used to support the production of a stock of such coins? If it could be, then we would have an instance of underproduction of small denomination currency. Notice that in this loose kind of story, coins differ from other durable goods because it is posited that the services people get from coins depend upon what they exchange for. One modeling challenge is to determine whether something like this story can be made to emerge as an implication of a coherent environment.

My own thinking about candidate models takes off from Milton Harris' model in his 1979 paper, "Expectations and Money in a Dynamic Exchange Model." Harris' model is one of pure exchange in which the resources or endowments consist of stocks of non-depreciating durable goods that yield services in proportion to the amounts held. Time is discrete and there are a large number of infinitely lived people, each of whom maximizes the expected value of discounted instantaneous utility, where instantaneous utility at date t depends upon the vector of services from the vector of durable goods carried over from date t - 1 to date t. The special feature of the model--and the one that attracts me to it--is that people meet pairwise and at random at each date. This

pairwise random meeting pattern is treated by Harris as ruling out all trades except spot trades in durables between people meeting at a date. Treated that way, Harris' model is as conducive to a role for media of exchange as any setup I can imagine. However, to use the Harris model for the purposes I have in mind, it has to be amended.[2/]

All of Harris' durable goods are endowed to an infinite degree with the properties I listed above as those that make objects suitable as commodity monies. In particular, they are all perfectly divisible, durable, portable, and recognizable. Thus, for example, if we want to study divisibility, we at least have to have a version where some or all durable goods are initially indivisible to a degree and in which there are costly technologies available for making things more divisible, say by splitting them up in some way.

Unfortunately, I have not gotten beyond this very loose way of formulating questions about currency provision. Nevertheless, even this very preliminary discussion is suggestive both about directions to pursue and about how far we have to go.

Footnotes

1/ One example has a common utility function given by $c_1^h c_2^h$ so that the choice of c_2^h subject only to (1) is $c_2^h/w_2^h = [1-v(r)]r(w_1^h/w_2^h)/2 + [1-v(r)]/2 < [1-v(r)]r(w_1^h/w_2^h)/2 + 1/2$. In this case, the solution for $v(r)$ from (5) is $v(r) = [(r-1)/(r+1)](rW_1-W_2)/(rW_1+W_2)$, a function of r and aggregate endowments only. It follows that $[1-v(r)]r$ is bounded above for all r by $2(W_1+W_2)/W_1$. Therefore, if for some h, $w_1^h/w_2^h < W_1/2(W_1+W_2)$, then for that h, $c_2^h/w_2^h < 1$, which violates (2').

2/ One should also impose some version of rational expectations which Harris failed to do. To accomplish this, one must have subjective views about future trading possibilities be endogenous--not exogenous as in Harris' formulation--and consistent with what agents know about other agents.

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Figure 1

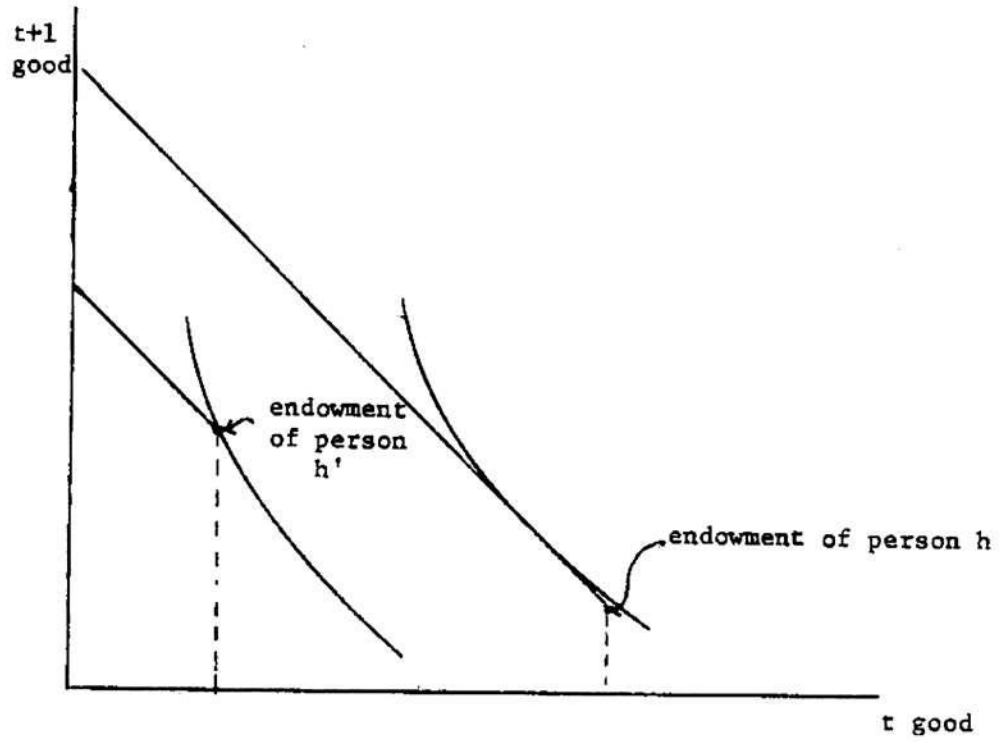

Figure 2