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A MULTIPLE MEANS-OF-PAYMENT MODEL

Edward C. Prescott\*

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## Introduction

In this paper an economy is studied in which there are two technologies for making payments. The first is currency and the second is bank drafts drawn on interest-bearing demand deposits. The interest-bearing asset does not dominate the noninterest-bearing currency because there is a fixed recordkeeping cost incurred whenever a bank draft is used as the means-of-payment. The steady state equilibrium is characterized. It is found that the value of the good, or more precisely, package of goods purchased at a given location determine which means-of-payment is used. Bank drafts are used for large purchases and currency for small purchases.

In the environment studied, the highly centralized Arrow-Debreu competitive equilibrium is impractical as the number of date-event-location contingent commodities is so large that the resources required for information collection and processing would be prohibitive. In this sense we follow Brunner and Meltzer (1971) in assigning the chief role of "money" as an economizer on costly information collection and processing.

The approach is close in spirit to that of Townsend (1980) who views the payment system as a communication system.<sup>1/</sup> It differs in that no effort is made to find the best arrangement. The arrangement studied, however, is sufficiently explicit that one can calibrate the model and then examine the costs and benefits associated with modifying the scheme--say by imposing reserve requirements or interest rate ceilings. Upper bounds for

the gains that can be realized from alternative systems can be computed. A system that is simple, implementable, and nearly optimal--independent of the exact specification of the environment--is the most economic theory can provide.

The scheme studied requires collective actions, which are virtually necessary for any payment system that uses fiat money. The question of what would develop, absent any collective action save for the enforcement of contracts, is not addressed.<sup>2/</sup> In this environment there are no gains from credit arrangement because the interest-bearing debit accounts dominate. In the real world credit is used, particularly when there are ongoing relations. An important extension of this research would be to introduce some feature into the environment that would give rise to the use of credit as well as currency and debit accounts.

The model is close to the growth model, a structure which has proven so useful in public finance and macroeconomics. This, I think, is desirable for the closer a specification is to the ones used in other economic applications, the greater the prior knowledge that can be used to restrict the parameters of the model. A second desirable feature of the construct is that the recordkeeping costs associated with using a bank draft as the means-of-payment can be, and have been, measured. The number obtained is not small, being nearly a half-dollar per draft, and would be larger if the value of the transactor's time associated with writing a check and verifying the identity of the payee were

taken into account. The number of transactions also can be measured and then used to restrict the theory. To summarize, the hope is that this line of research will lead to the development of a theory that can be used to quantitatively evaluate alternative payment systems and lead to the design of better payment arrangements.

The paper is organized as follows. In Section 1 the environment is specified. The means-of-payment decisions of an agent, given the market interest rate and that agent's total expenditures, is solved in Section 2. In spite of the fixed cost per bank draft, standard convex analysis is applicable. This is possible because each purchase is a negligible fraction of total expenditures. Previously, others have assumed a finite number of transactions and the resulting nonconvexity held back the development of a transactions-based, general equilibrium theory of money. In Section 3 the saving-consumption expenditure decision is considered. It is found that the resulting behavior is essentially the same as for the neoclassical growth model. The steady state capital stock equates the marginal product of capital to the sum of the depreciation rate of capital and the subjective time discount rate. This steady state capital stock is invariant to the inflation rate for this model. The steady state equilibrium is determined in Section 4 and the final section contains some illustrative uses of the construct.

## 1. The Economy

There are a continuum of agents and products. Time is discrete with half the agents allocating their time to production in odd periods and to shopping-consumption in even periods. The other half of the population produce goods in even periods and shop in odd periods. The shoppers purchase many different goods at different locations--indeed a continuum of them. For an individual, the goods can be categorized into a finite number of equivalence classes, which are indexed by  $\theta$  belonging to a finite set of positive reals  $\Theta = \{\theta_1, \dots, \theta_n\}$ . The classes of goods are ordered so that  $\theta_i < \theta_{i+1}$ . Element  $\pi_\theta$  is the fraction of goods of type  $\theta$ . Within an equivalence class of goods, all goods enter the individual's utility function symmetrically. Letting  $e$  name the goods within an equivalence class consumed and  $\varepsilon_\theta$  their total measures, the utility function used has the form

$$(1.1) \quad \sum_{t, \theta} \beta^t \int_0^{\varepsilon_\theta} u[\theta x_{\theta t}(e)] de$$

where  $x_{\theta t}(e)$  is the quantity of good  $e$  in class  $\theta$  in period  $t$  consumed and  $0 < \beta < 1$  is the subjective time discount factor. The function  $u$  is continuously differentiable, strictly increasing and strictly concave and is defined on the nonnegative reals. Further,  $u(0) = 0$ . Consequently, I need not, and do not, keep track of goods not consumed.

Rather than keeping track of the quantities of every good consumed by every person, I need, and will, only keep track of the distribution of the quantities of goods consumed in the

various equivalence class of goods. The quantity of a good consumed is indexed by  $x$  belonging to some finite set of positive real numbers. Indirect utility functions will be defined over arrays  $z_{\theta xt}$  for  $\theta \in \Theta$ ,  $x \in X$ , and  $t \in \{0,1,2,\dots\}$ . Element  $z_{\theta xt}$  is the number (or more precisely the measure) of good of type  $\theta$  consumed in quantity  $x$  in period  $t$ . The utility function defined on  $z$  has the form

$$U(z) = \sum_{t, \theta, x} \beta^t u(\theta x) z_{\theta xt}.$$

The sets  $\Theta$  and  $X$  are assumed to have a finite number of elements, for expository, not technical, reasons. In the following arguments the only property needed is that the cross product of  $\Theta$  and  $X$  be a compact separable metric space. Since  $\Theta$  and  $X$  are subsets of the real line, their cross product is compact if the sets are closed and bounded. Sets of measures that are defined on the Borel sigma algebra of a separable compact metric space and that are closed and bounded are compact with respect to the weak-\* topology.<sup>3/</sup> At points in the analysis we will proceed as if  $X$  were a continuum and differentiate with respect to  $x$ . Implicitly we are assuming the points in  $X$  are so closely spaced that the derivative and the finite difference are for all practical purposes the same.

Producers are located at spatially distinct points, or islands, and shoppers visit a random sample of islands. At each island there is precisely one type (not class) of goods sold by a number of producer-sellers and as a result, prices are determined

competitively. The sample of purchasing opportunities is large. Consequently, a shopper receives a representative sample of purchase opportunities. Letting  $\lambda$  denote the number (measure) of purchase opportunities,  $\pi_\theta \lambda$  is the number of opportunities for the purchase of different goods in class  $\theta$ . A given good is of type  $\theta$  for fraction  $\pi_\theta$  of the population. Thus some goods are more highly valued by a given individual--that is, have higher  $\theta$ --but the fraction of the population which values a given good at a given level is the same for all goods. This introduces symmetry in both the goods and agent space, which simplifies the subsequent analysis. In equilibrium all goods will have the same price and the same distribution of purchase quantities because of this symmetry.

There is the additional restriction that a given good is purchased once, using either currency or a draft, or not at all. This constraint greatly simplifies the formulation and is nonbinding. There is no gain from making five one-dollar purchases of a given good using currency rather than one five-dollar purchase. If drafts are used, it is wasteful to make multiple purchases of the same good because the fixed cost, unnecessarily, would be incurred more than once.

## 2. The Means-of-Payment and Purchase Decision

The means-of-payment and quantity-purchased decision is first considered. By symmetry the price of all goods must be the same in equilibrium. Units are selected so that this price is one unit of currency. Let  $M$  be currency holdings at the beginning of

a shopping period and B be bank deposits. Letting  $m = (m_{\theta x})$  be the measure of cash purchases and  $d = (d_{\theta x})$  be the measure of debit purchases, wealth at the end of the period is

$$(2.1) \quad W' = (1+r)B + M - \sum_{\theta, x} xm_{\theta x} - \sum_{\theta, x} xd_{\theta x} - \gamma \sum_{\theta, x} d_{\theta x}.$$

As the first summation is the value of all cash purchases, the second summation is the value of all credit transactions, and the third is the sum of the fixed costs of credit transactions ( $\gamma$  is the cost per transaction and  $\sum d_{\theta x}$  is the number of transactions).

Let

$$(2.2) \quad M = \sum_{\theta, x} xm_{\theta x}$$

$$(2.3) \quad D = \sum_{\theta, x} xd_{\theta x}$$

$$(2.4) \quad S = \gamma \sum_{\theta, x} d_{\theta x}.$$

As  $W = M + B$ , it follows that

$$W' = (1+r)W - Y$$

where

$$(2.5) \quad Y = (1+r)M + D + S.$$

Variable Y is total expenditures on goods and bank services plus foregone interest earnings on currency holdings.

The program facing the individual with these definitions is

$$(2.6) \quad U(Y, r) = \max_{m, d > 0} \sum_{\theta, x} u(\theta x)(m_{\theta x} + d_{\theta x})$$

subject to

$$(2.7) \quad \sum (m_{\theta x} + d_{\theta x}) \leq \lambda \pi_{\theta} \quad \text{for all } \theta$$

(purchases of a given class of good are constrained by the number of goods in that class found while searching) and to

$$(2.8) \quad (1+r) \sum_{\theta, x} x m_{\theta x} + \sum_{\theta, x} x d_{\theta x} + \gamma \sum_{\theta, x} d_{\theta x} \leq Y.$$

This is a linear program. The constraint set is closed and bounded and is nonempty. Consequently, an optimum exists.

The first order conditions are

$$(2.9) \quad u(\theta x) \leq \mu_{\theta} + (1+r)x\phi \quad \text{with equality if } m_{\theta x} > 0,$$

and

$$(2.10) \quad u(\theta x) \leq \mu_{\theta} + (\gamma+x)\phi \quad \text{with equality if } d_{\theta x} > 0$$

where  $\mu_{\theta}$  are the Lagrange multipliers associated with constraints (2.7) and  $\phi$  is the multiplier associated with constraint (2.8).

If purchases are made using currency, the quantity purchased satisfies

$$(2.11) \quad \theta u'(\theta x) = (1+r)\phi$$

while if a draft is the means of payments, the quantity purchased satisfies

$$(2.12) \quad \theta u'(\theta x) = \phi.$$

Let  $x_m(\theta, \phi)$  and  $x_d(\theta, \phi)$  be the solutions to (2.11) and (2.12), respectively. Currency will be used if

$$(2.13) \quad u[ex_m(\theta, \phi)] - (1+r)\phi x_m(\theta, \phi) > u[\theta x_d(\theta, \phi)] - \theta x_d(\theta, \phi) - \gamma\phi,$$

and drafts if the inequality is in the opposite direction. With equality, it is optimal to use either means of payment for purchase of goods of that marginal type. The fraction of purchases of that type good using the two alternative means of payment, however, is determined so there is a unique solution to the program. For  $\theta$  less than some critical value  $\theta(Y, r)$ , currency is the means of payment. For  $\theta$  greater than  $\theta(Y, r)$ , bank drafts are used.

The larger  $Y$ , the smaller is  $\phi$  which is the marginal utility of additional expenditures. The smaller  $\phi$ , the larger are the purchase quantities  $x_m(\theta, \phi)$  and  $x_d(\theta, \phi)$ . Further, as  $\theta$  increases, drafts are used for the purchase of more goods; that is,  $\theta(Y, r)$  is decreasing in  $Y$ . Consequently, the value of purchases using drafts increases as  $Y$  increases. As

$$(2.14) \quad M = \frac{Y - D - S}{1 + r},$$

a one-unit increase in  $Y$  results in a change in  $M$  that is bounded from above by  $(1+r)^{-1}$ . Thus the optimal currency holding  $M(Y, r)$  has slope less than one with respect to  $Y$ .

It is readily verifiable that increases in  $r$  decrease the use of currency and therefore increase the use of bank drafts.

### 3. The Dynamic Problem

The production functions of an individual is

$$(3.1) \quad f(k)$$

where  $f$  is increasing, strictly concave, and continuously differentiable with  $f'(0) = \infty$  and  $f'(\bar{k}) = 0$ . As all goods enter symmetrically and all goods have the same equilibrium price, which we normalize to be one. Then if  $k$  units of capital are rented and  $W$  is the beginning of production-period wealth, end-of-period wealth is

$$(3.2) \quad W' = (1+r)W + f(k) - rk$$

as sales,  $f(k)$ , are realized and capital rental payments,  $rk$ , made at the end of the period.

Letting  $v_1(W)$  and  $v_2(W)$  be the dynamic programming value functions for beginning of production and purchase periods, respectively, one optimality condition is

$$(3.3) \quad v_1(W) = \max_k [\beta v_2[(1+r)W + f(k) - rk]].$$

Given  $v_2$  is increasing, the first order condition is

$$(3.4) \quad f'(k) = r$$

which is the usual steady state condition for the optimal growth model.

Another result is that if  $v_2$  is concave, then  $v_1$  is concave given  $f$  is concave.

Using the indirect utility function derived in the previous section, the optimality condition requires

$$(3.5) \quad v_2(W) = \max_Y \{U(Y,r) + \beta v_1[(1+r)W-Y]\}.$$

As  $U$  is concave in  $Y$ ,  $v_2$  is concave if  $v_1$  is concave. By standard discounted dynamic programming results, functional equations (3.3) and (3.5) have unique solutions which are concave and continuous. These solutions are the optimal value functions for this discounted dynamic program.

#### 4. Steady State Equilibrium

In order to determine the steady state equilibrium, the interest rate  $r$  must be determined. An individual, a shopper at date  $t$ , can transform  $Y_t$  into  $Y_{t+2}$  via borrowing or lending at rate  $(1+r)^2$ . The marginal rate of substitution relative to the indirect utility function  $U$  between  $Y_t$  and  $Y_{t+2}$  is  $1/\beta^2$ . Consequently, the steady state interest rate is

$$(4.1) \quad r = \beta^{-1} - 1.$$

Given  $r$ , steady state  $k$  is determined by condition (3.4).

The interest rate  $r$  does not determine a unique steady state wealth for an individual. Equilibrium in the goods market determines it. In particular, equilibrium in the goods and service markets requires

$$(4.2) \quad f(k) = M(Y,r) + D(Y,r) + S(Y,r).$$

As purchases of goods and services are increasing in  $Y$ , equation (4.2) can be solved for  $Y$  given  $r$  and  $k$ .

Letting  $W_1$  and  $W_2$  be the beginning-of-production and shopping-period wealth,

$$(4.3) \quad k = W_1 + (W_2 - M).$$

Further, from (3.2)

$$(4.4) \quad W_2 = (1+r)W_1 + f(k) - rk.$$

Given  $M$ ,  $k$ , and  $r$ , these equations, linear in  $W_1$  and  $W_2$ , determine  $W_1$  and  $W_2$ .

The one remaining variable to be determined is the price level. The steady state price level  $P$  satisfies

$$(4.5) \quad \frac{M(Y,r)}{2} = \frac{M^S}{P}$$

where  $M^S$  is the money supply for per capita and  $M(Y,r)/2$  is the average real cash balance of agents. Thus, the quantity theory holds for this economy.

## 5. Illustrated Uses of the Construct

The purpose of this discussion is to indicate the type of questions that can be addressed within models of this type. Before a model such as this is confronted with the data, it would be necessary to incorporate many additional features. For example, one reason for using drafts as a means of payment is that then there is a record of payment that is needed for tax purposes. Conversely, currency may be used so that there is no

record of the payment in order to facilitate illegitimate economic activities, to avoid taxation, or not be used as much as it might otherwise be in order to reduce the risk of loss by theft or fire. Still another possibly important feature is that income allocated to bank services is not taxable when it is financed by lower interest payments on deposits. A final caveat before discussing applications is that the model is a steady state model. Such models are not designed for studying fluctuations in interest rates and various monetary aggregates. It is suited only for the study of the smoothed data series for a given economy or for cross-country, time-averaged data. Given these caveats, the illustrative uses are as follows.

Suppose there are two economies, alike in every way except that for one the marginal product of capital is uniformly higher. Steady state capital and output are greater for the more productive economy. Checks will be used for more purchases and these purchases will be larger in the rich country. One implication is that more banking services are used in the rich country. Predictions with respect to the use of currency are ambiguous because they depend upon the distribution of goods by types. In the high income country, goods using currency as the means of payment are purchased in greater quantities leading to a greater use of currency. This effect is offset by the use of checks for a greater fraction of the purchases.

A second application is the question of the optimal growth rate of money. Suppose injections of money are in the form

of lump sum transfers of money to agents at the time that they are selling their goods. Withdrawals of money are accomplished by lump sum taxes also at the time the households-firms sell their products. Assuming the money supply grows (declines) at a constant rate  $\phi$ , prices will grow at rate  $\phi$  and the nominal interest rate paid on demand deposits will be  $i = \phi + r$ . The larger  $\phi$ , the greater the use of checks and the smaller the steady state consumption (output less banking services). For  $\phi = -r$ , the nominal interest rate  $i$  is zero. This minimizes the amount of resources allocated to banking services and maximizes steady state consumption. In this sense this model supports the view of Friedman and Samuelson that since currency is costless to produce, it is optimal to deflate at the real interest rate. This, of course, assumes lump sum taxes and no private information, features which are needed for taxation to have no deadweight loss. Optimal taxation implies a zero tax on liquidity only if other taxes are distortion-free. But having a zero nominal interest rate does eliminate incentives to economize upon currency holdings for this economy.

The economy considered has zero reserve requirements, but reserve requirements are easily introduced. Suppose, instead, there is a reserve requirement, interest is not paid on reserves, and currency is supplied perfectly elastically. This is closer to the American payment system than the model's. The quantity theory would still hold, but for currency plus reserves rather than for currency, as for the model considered. If  $\rho$  is the reserve re-

quirement, the interest on demand deposits would be  $(1-p)r$  rather than  $r$ . Deposits would be just large enough to ensure zero deposits after payments were made. The banks would finance only part of the capital investment and would charge interest rate  $r$ . In summary, the steady state behavior of this economy is very much like the static, textbook models of money and banking.

A final use is to consider what happens as the record-keeping cost goes to zero. In the limit, currency is not used and there is no numeraire. Consequently, the price level is indeterminate. In such an environment, a reserve requirement, along with a fixed supply of reserves, is an arrangement for which the price level is determined.

Footnotes

1/Other related models are those of Lucas (1980) and Hellwig and Gale (1984).

2/See King (1983) for an insightful discussion of the economics of the private provision of money.

3/For the development of general equilibrium theory with signed measures used as the commodity point, see Mas-Colell (1975) and Jones (1984). They exploit them for the case of a continuum of differentiated products.

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