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GRESHAM'S LAW IN A LEMONS MARKET

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ABSTRACT

We describe a simple environment in which assets of varying qualities may be used for transactions and consumption. The quality of an asset is known to the seller but not the buyer. We show that this feature can generate a negative relationship between the transactions velocities of assets and their rates of return. We also discuss several versions of Gresham's Law which hold in this environment.

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In this note we describe a simple environment in which assets of varying qualities may be used for transactions and consumption. The quality of an asset is known to the seller but not the buyer. We show that this feature can generate a negative relationship between the transactions velocities of assets and their rates of return. We also discuss several versions of Gresham's Law which hold in this environment.

The model is one of stationary pure exchange with overlapping generations of three period lived agents and a single consumption good at each date. Throughout, the analysis will be confined to steady states and hence, time subscripts will be omitted.

There is a continuum of infinitely lived assets indexed by q where q is the per period dividend. We assume that the support of the distribution of assets according to dividends is given by the interval $[\underline{q},\bar{q}]$ where $0 < \underline{q} < \bar{q} < \infty$. We let $G(\cdot)$ and $g(\cdot)$ denote the cumulative distribution function and the probability density function, respectively, of assets according to dividends. The quality of an asset is here identified with its constant dividend payoff in the obvious manner and the total quantity of assets is constant over time and equal to $\hat{z}$.

Let w_s and c_s be the endowment and the consumption, respectively, of an agent in the sth period of life where s is either 1, 2, or 3. The preferences of an agent are given by the utility function $U(c_1,c_2,c_3)$ which is assumed to be twice continuously differentiable, strictly increasing and strictly concave.

Lastly, agents may borrow and lend in a consumption loans market and may also buy and sell the real assets in the economy. The quality of an asset (its dividend yield) is known to a potential seller but is not known to a potential buyer until after he/she buys it. We also assume that sellers are anonymous, that is, from the point of view of a buyer all sellers look the same. As will become clear in a moment, this is done to prevent the identity of a seller (whether middle aged or old) from revealing some information about the average quality of assets that he/she is offering for sale.[1] Buyers of assets will, of course, base their decisions on the average quality of assets offered for sale on the market.

It is easy to describe verbally the pattern of asset holdings and transactions in a steady state equilibrium. Let $\hat{q}$ be some cut off value of quality, to be determined in equilibrium. The old (s=3) will be holding one half of each asset with quality higher than $\hat{q}$ and none of the assets with any lower quality. The middle aged (s=2) will be holding the remaining one half of each asset with quality higher than $\hat{q}$ and all of the assets with quality lower than $\hat{q}$. At each date, the young (s=1) will purchase assets from both the old and middle aged. These purchases will consist of one half of each asset with quality higher than $\hat{q}$ (from the old) and all the assets with quality lower than $\hat{q}$ (from the middle aged). As is obvious, this pattern will then be repeated indefinitely.

Let p be the price of assets, R the gross rate of return on consumption loans, (ℓ_1, ℓ_2) the quantity of consumption loans

made (borrowed, if negative) when young and when middle aged, respectively, and z be the quantity of total assets purchased by the young. Let $\tilde{G}$ be the distribution of assets purchased by the young. This is unknown to the young but will become known one period later. The budget constraints can now be written as follows.

$$(1) \qquad c_1 = w_1 - pz - \ell_1$$

$$(2) \qquad c_2 = w_2 + z[p\tilde{G}(\hat{q})+E(q|\tilde{G})] + R\ell_1 - \ell_2$$

$$(3) \qquad c_3 = w_3 + z\left[p\left(1-\tilde{G}(\hat{q})\right)+ \int_{\hat{q}}^{\bar{q}} q \, d\tilde{G}\right] + R\ell_2.$$

In equation (2) and elsewhere, $E(\cdot)$ is the expectation operator. The consumer chooses $\hat{q}$ and ℓ_2 when middle aged and z and ℓ_1 when young to maximize utility. This leads to the following necessary and sufficient first order conditions for an interior maximum.

$$(4) \qquad R = U_1/U_2 = U_2/U_3 = 1 + \hat{q}/p$$

$$(5) \qquad pU_1 = U_2\left[p\tilde{G}(\hat{q})+E(q|\tilde{G})\right] + U_3\left[p\left(1-\tilde{G}(\hat{q})\right)+ \int_{\hat{q}}^{\bar{q}} q \, d\tilde{G}\right].$$

In equilibrium the following conditions must hold.

$$(6) \qquad \tilde{G}(q) = \begin{cases} 2G(q)/[1+G(\hat{q})] & \text{for } q \in [\underline{q},\hat{q}] \\ [G(q)+G(\hat{q})]/[1+G(\hat{q})] & \text{for } q \in [\hat{q},\bar{q}] \end{cases}$$

$$(7) \qquad z = \hat{z}[1+G(\hat{q})]/2$$

$$(8) \qquad \ell_1 + \ell_2 = 0.$$

It is easy to verify that conditions (6)-(8) imply the following goods market clearing condition.

(9) $\sum_s (c_s - w_s) = \hat{z}E(q|G)$.

One can also see that by virtue of (4) and (5), the budget constraints (1)-(3) can be collapsed into the following single lifetime budget constraint.

(10) $\sum_s (c_s - w_s)/R^{s-1} = 0$.

This enables us to solve for equilibrium values of R and the $\{c_s\}$. Note that the conditions (4) are equivalent to maximizing utility subject to (10). We then impose the market clearing condition (9).

Proposition 1

There exists an R greater than one and $\{c_s\}$ such that $\{c_s\}$ maximize utility subject to (10) and also satisfy (9).

Proof

At R = 1, the utility maximizing $\{c_s\}$ lead to excess supply in (9). However, as R tends to infinity the utility maximizing $\{c_s\}$ must be such that $(\sum_s c_s)$ also tends to infinity leading to excess demand in (9). The proposition follows.□

We will now establish a solution for $\hat{q}$ and p, given R. To do this, divide (5) by U_1 and substitute 1/R for U_2/U_1 and $1/R^2$ for U_3/U_1 from (4). Similarly, substitute $\hat{q}/(R-1)$ for p, also from (4) and, finally for $\tilde{G}(\cdot)$ in terms of $G(\cdot)$ from (6). We thus obtain a single equation in the one unknown $\hat{q}$, which may be simplified as follows.

$$(11) \qquad \phi(\hat{q}) \equiv (1+1/R)\left(\hat{q}-E(q|G)\right) + (1-1/R)\int_{\underline{q}}^{\hat{q}} G\, dq = 0.$$

Proposition 2

For each equilibrium R, there exists a unique $\hat{q}$ and p that solve (4) and (5).

Proof

From (11) we see that $\phi(\underline{q})$ is negative whereas $\phi(\bar{q})$ is positive. Uniqueness follows from the fact that $\phi'(\hat{q})$ is everywhere positive.□

It is now easy to see the sense in which the above model produces a negative relationship between transactions velocities and rates of return of assets. Let v(q) and r(q) be the transactions velocity and the net rate of return, respectively on an asset of quality q. The transactions velocity of an asset is defined as the value of that asset that is turned over each period divided by the value of the total stock of that asset. It is now obvious that v(q) and r(q) are given by the following.

$$(12) \qquad v(q) = \begin{cases} 1/2 & \text{for } q > \hat{q} \\ 1 & \text{for } q < \hat{q} \end{cases}$$

$$(13) \qquad r(q) = q/p.$$

The higher quality assets are hoarded for future consumption whereas the lower quality assets are exchanged for current consumption. This results in bad assets circulating more rapidly than good assets. It is perhaps worth pointing out that this result does hinge on the private information regarding asset quality. Without private information, prices of different assets

would adjust until each of them yielded exactly the same return and the pattern of transactions velocities would be totally random.

Next, we describe several versions of Gresham's famous law for the above environment. That is, that "bad money drives out good money." This may be interpreted in various ways in the context of the above environment and we will consider some of these that we regard as plausible.

First, consider the following notion of good and bad money. Assets with $q > E(q|G)$ are good and the rest are bad. It is easy to show that there is always some bad money masquerading as good, in the sense of having the same low transactions velocity.

Proposition 3

$\hat{q}$ is less than $E(q|G)$.

Proof

Obvious because $\phi(\cdot)$ is increasing and $\phi(E(q|G))$ is positive.□

The above proposition suggests the following experiment. Suppose that we increase the amount of bad money in the system (that is, of quality levels closer to $\underline{q}$) without changing the quantity of good money (that is, of quality levels closer to $\bar{q}$). Such an experiment may be conceptually thought to consist of several separate steps. It consists of: (i) an increase in the total quantity of money ($\hat{z}$) without a change in the distribution, (ii) a decrease in the average quality of money without a change

in the quantity of good money or the total flow of goods, and (iii) an increase in the relative amount of bad versus good money without a change either in the total quantity of money or its average quality (and, hence, no change in the total flow of goods). We consider the results of each of these steps, one by one.

Proposition 4 (Gresham's Law 1)

Suppose that excess demand $\sum_s(c_s-w_s)$ is an increasing function of R for $R \geq 1$, where $\{c_s\}$ maximize utility subject to (10).[2] Then, there is a unique equilibrium value of R and an increase in $\hat{z}$ increases R, lowers $\hat{q}$ and lowers p.

Proof

The first part is obvious from (9). The second part follows because from (11), $\phi(\cdot)$ is increasing in R and in $\hat{q}$. The last part follows from (4).□

We now consider the experiment in which the average quality of assets is lowered without affecting the quantity of good assets or the total flow of goods. Consider a change in the distribution of asset returns from $G(\cdot)$ to $G^*(\cdot)$ such that,

$$(14) \qquad G^*(q) > G(q) \text{ for } q \in (\underline{q},\hat{q})$$
$$= G(q) \text{ for } q \in [\hat{q},\bar{q}].$$

The above change lowers the average quality of assets and hence reduces the total flow of goods. Therefore, to maintain the same total quantity of goods, we increase $\hat{z}$ to $(\hat{z})^*$ as follows.

$$(15) \qquad (\hat{z})^* E(q|G^*) = \hat{z}E(q|G).$$

We then have

Proposition 5 (Gresham's Law 2)

A change in the distribution of assets from $G(\cdot)$ to $G^*(\cdot)$ and the quantity of total assets from $\hat{z}$ to $(\hat{z})^*$ as given by (14) and (15) lowers $\hat{q}$ as well as p.

Proof

First, note that the above change has no effect on R. Second, a slight manipulation of (11) yields,

$$(16) \qquad \phi(\hat{q}) \equiv (1+1/R)[\hat{q}-\bar{q}+ \int_{\hat{q}}^{\bar{q}} G\, dq] + 2 \int_{\underline{q}}^{\hat{q}} G\, dq$$

which shows that at the previous equilibrium value of $\hat{q}$, $\phi(\cdot)$ increases. Therefore, the new equilibrium value of $\hat{q}$ must be lower. Consequently, p must also be lower.□

Next, we consider a mean preserving increase in the spread of the distribution of asset returns. Let $G(\cdot,\sigma)$ be a one parameter family of distributions such that changes in σ are mean preserving and an increase in σ corresponds to an increase in risk. That is,

$$(17) \qquad \int_{\underline{q}}^{q} G_2(q,\sigma)dq \geq 0 \text{ for } q \in [\underline{q},\bar{q}]$$

$$> 0 \text{ for some } q \in (\underline{q},\bar{q})$$

$$= 0 \text{ for } q = \bar{q}.$$

Again, it is obvious that because changes in σ are mean preserving, they have no effect on the equilibrium values of R. Therefore, all we have to do is to show that the solution for $\hat{q}$ from (11) decreases when σ increases.

Proposition 6 (Gresham's Law 3)

A mean preserving increase in the riskiness of the distribution of assets lowers $\hat{q}$.

Proof

It is straightforward to verify that under the given conditions, the function $\phi(\cdot)$ in (11) is strictly increasing in σ for all $\hat{q} \in (\underline{q},\bar{q})$. Since $\phi(\cdot)$ is also increasing in $\hat{q}$, the result follows.□

A different way to approach the analysis of good versus bad money is as follows. Suppose that there is another infinitely lived asset in amount z' and known quality q' selling at price p'. This leads to the following additional arbitrage condition and a modified market clearing condition.

(18) $R = 1 + q'/p'$

(19) $\sum_s (c_s - w_s) = \hat{z}E(q|G) + z'q'$.

The determination of R, $\hat{q}$, and p proceeds as before and from (18) we can then determine p'. This new asset may be considered "bad" relative to the previous "good" asset on the following grounds. From (4), (18), and proposition (3), it follows that $q'/p' < E(q|G)/p$. That is, the average rate of return on this

asset is less than that on the previous asset with unknown quality. We can now show that an increase in z' lowers $\hat{q}$, thereby lowering the average transactions velocity of the good money.

<u>Proposition 7</u> (Gresham's Law 4)

Under the assumptions of proposition (4), an increase in z' lowers $\hat{q}$.

<u>Proof</u>

Similar to proposition (4).□

Footnotes

[1]Eliminating this restriction has the following interesting implication. It turns out that, in equilibrium, the old and the middle aged will be holding an identical portfolio of assets but will face different prices. In particular, the price faced by the middle aged will be sufficiently lower so that they will choose not to sell any part of their portfolio. Thus, we can endogenously generate a situation in which consumption loans are used to finance short term consumption whereas real assets are used solely to finance long term consumption. The reason for this is that at the same price, the old will be supplying their portfolio of known average quality inelastically, whereas the middle aged will be selling assets from the "bottom of the barrel," and hence of much lower average quality. Consequently, in equilibrium, the price faced by the middle aged is sufficiently lower so that they simply choose to retain their asset holdings.

[2]This would be true, for instance, if the utility function is log-linear and agents receive a positive endowment only in their first period.

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