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Present Value Procedures for Capital Investment Decision Making at Federal Reserve Banks

Introduction

In the course of daily business the Minneapolis Fed and its counterparts, like hundreds of other institutions, are continuously called upon to make capital spending decisions. Many of the questions involve relatively small amounts of money, some fairly large amounts and a few involve substantial sums. Some expenditure questions can be framed fairly simply. For example, the choice of a typewriter from among several competing models, all judged to be equal in quality and service, can usually be made by simply selecting the model with the lowest price. Other spending decisions are, of course, much more complex.

And it is not simply that difficult questions may involve more money. Major problems are encountered when spending and the related benefits are spread over time. We must then find an explicit way to measure the value of dollars to be received or spent at future dates, compared with dollars received or spent today. Questions involving costs and benefits extending further into the future also usually involve greater uncertainty than those with cash flows concentrated in the present or near future.

These investment decision problems are well known and the professional literature pronouncing various approaches is enormous. Economists, accountants, buiness finance experts, operations research specialists and others have addressed themselves to the issue. It would be a rare individual who knew nothing of interest rates and notions of the time value of money which underlay them. But in spite of this general familiarity with financial concepts (or perhaps because of it) great mystery and confusion surrounds investment decision-making procedures. Capital spending decisions at the Minneapolis Fed in the past have not, in general, involved

explicit procedures which account for time.

The purpose of this report is to examine these issues using everyday language; to explain some of the rules commonly used; to point out some of the limitations; to establish a framework within which valid procedures can be selected and applied at Federal Reserve Banks; and perhaps most important, to begin to develop a set of practical case examples through which the various interested Federal Reserve staffs can effectively accumulate and extend their experiences in systematic application of present value procedures.

The present report is preliminary and it is being offered for critical review and constructive improvement around the Federal Reserve System. As currently conceived the report is divided into four parts. Part I discusses the rationale for using present-value procedures in government agency capital spending decisions and explores the tricky problem of selecting an appropriate discount rate for government agency decisions. Part II describes in general terms a set of recommended procedures for application of present-value concepts to Federal Reserve capital spending decisions. Part III attempts to illustrate the general procedures by discussing in detail a real case application. Part IV is intended to provide for a continuing collection of case applications from throughout the Federal Reserve that will serve to illustrate successes and limitations of present-value procedures applied to a widening spectrum of typical Federal Reserve situations. One case has been provided as a starter for Part IV.

The basic concept we have for this report, therefore, is that it constitutes the beginning of a proposed "loose leaf" manual, all of it subject to change and improvement, to be developed by and for the Federal Reserve Banks and the Board.

PART I. RATIONALE FOR PRESENT-VALUE PROCEDURES IN CAPITAL INVESTMENT

Before setting down a rationale for the use of cash flow analysis we will explain some of the terms and concepts to be used. At this point discussion will be limited to basic ideas. These concepts will be explained and illustrated in more detail in Part II.

Basic Terminology and Time Conventions

In this paper we speak of capital investment or capital spending decisions as any decision that entails flows of costs (expenditures) and benefits (receipts) taking place over some span of time. Decisions to buy a coin sorting machine, to lease computer equipment, or to sponsor an employee training program, are examples of capital investment decisions.

An investment project involving spending and benefits can be characterized by a sequence of numerical net cash flows over time. The relevant time period, called the 'project life' or project horizon,' is divided into convenient subperiods, e.g., months, quarters, years. Generally then, expenditures and receipts will take place during each subperiod. From these cash flows a net cash flow stream can be computed for the entire project horizon. We will adopt the convention here that receipts or inflows be designated as positive (+) numbers and expenditures or outflows as negative (-) numbers. If for the first subperiod receipts are ten dollars (+$10) and expenditures two hundred dollars (-$200), then the NET CASH FLOW for that subperiod would be a negative one hundred and ninety dollars (+10) + (-$200) = (-$190). Whether in practice many Federal Reserve investment projects can adequately be reduced to measurement by numerical cash flows is an important question -- one that will be avoided in these introductory remarks, but dealt with in a later section.

At its simplest, the premise supporting use of some form of time-weighted decision procedure is that "future money" is less valuable in some sense than "current money." The reason, which is essentially that physical capital is productive,

will be discussed in more detail subsequently. On these grounds, future flows of money should be given correspondingly less weight in any decision than flows closer to the present date. More specifically, we will argue that future cash flows ought to be discounted at some interest rate back to the present date before summing up the net cash flows for use as an indicator of worth of the project. Rationale for these kinds of procedures has nothing to do with 'risk' or 'uncertainty'; that's a separate issue to be tackled later. The procedures would apply equally well to projects for which future cash flows were known with certainty.

To speak in terms of projected "cash flows" suggests projects more in the realm of private profit-seeking businesses than activities of the Federal Reserve Banks or other public agencies. Yet the considerable body of literature that has by now emerged on optimal methods for public investment decision-making by public institutions agrees that, here too, correct procedures require discounting future costs and benefits at some appropriate rate of interest. That in turn suggests the need to place dollar valuations on the expected benefits to flow from Federal Reserve Bank activities. Accurate measurement of benefits would provide very valuable information for decision-making; however, the basic procedures can be modified to suit circumstances in which only cost projections are available.

Considerable difference of view exists on the question of <u>what</u> interest rate (conceptually) is appropriate for discounting, as well as how in practice one would determine the correct numerical <u>value</u> of that interest rate. We will deal here only summarily with the theoretical rationale for use of discounting procedures and the choice of discount rates. It seems important to spend most of our time in this report examining the ways in which existing procedures can be used in practical applications at the Federal Reserve Bank. A bit of justification, however, is important. Because the literature is rather complex -- often confounding -- we do not attempt a balanced survey.

A rationale for using a present-value procedure can be constructed at two different levels. Which is more appropriate depends essentially on the constituency to whom our investment actions are to be justified. The one we shall consider first estabishes simply that, as long as there is some safe alternative earning rate at which we could place our funds, then to fail to use present-value procedures (with that safe rate of discount) is to risk a poorer showing on the Bank's earnings and expense statement over time than would be possible had we used present-value procedures.

The second rationale, the cumbersome details of which we'll take up a little later, establishes that something higher by way of a discount rate is required (higher than the safe earning rate, that is) if we wish to do what is more "socially optimal."

FINANCIAL ARGUMENTS FOR PRESENT-VALUING AT SOME SAFE ALTERNATIVE INTEREST RATE

We'll proceed with this first-level rationale by illustrative example, though it should be clear that the argument can be generalized. Consider the following annual cash flow series,

-100, 30, 80

which is intended to represent an initial outlay of 100 (say the units are thousands of dollars) followed at the end of the first year by receipt of 30 and at the end of the second year by receipt of 80. If we ignore the time pattern of flows, that particular project promises to return over its life $10,000 more than we put into it, and that might seem to make it an attractive prospect for investment. But it's not attractive if an alternative safe earning rate of, say, 6% is available as can be seen if we apply the present-value procedure as follows:

(1) COMPUTE THE PRESENT VALUE OF THE CASH FLOW SERIES AT 6% INTEREST.

First, discount to the present time each of the flows in the series and sum them algebraically to obtain the present value of the series. We take the initial outflow as the "present," so discounting doesn't affect it. The flow at the end of the first year is divided by 1.06,

that at the end of the second year by $(1.06)^2$.

$$PV@6\% = -100{,}000 + \frac{30{,}000}{1.06} + \frac{80{,}000}{1.06^2}$$

$$= -100{,}000 + 28{,}300 + 71{,}200$$

$$= -500$$

(2) EXAMINE THE PRESENT VALUE OF THE PROJECT; IF IT IS POSITIVE, GO AHEAD WITH THE INVESTMENT, IT IT IS NEGATIVE REJECT THE INVESTMENT.

With that quick preview of the text-book present-value procedure (which will be discussed in more detail in Part II of this report) our example project "X", with flows,

-100, 30, 80

has been rejected.

The correctness of the decision from a purely financial point of view can also be demonstrated by computing the amount of cash each alternative would have accumulated at the end of the two-year horizon. We assumed an alternative safe earning rate of 6%. Compare the two options of investing in the project and placing our funds at the safe rate for the same period of time:

	Cash flows at end of year...		
Option I: Invest in Project "X"	0	1	2
An initial outlay of $100,000	-100,000		
will return $30,000 at end of year 1...		30,000	
and $80,000 at end of year 2...			80,000
while the $30,000 received at end of year 1 can be placed at safe 6% and convert to $31,800 at the end of year 2...			31,800
which gives us a total cash on hand at end of year 2 of...			$111,800

Option II: Place funds at safe 6%	0	1	2
$100,000 placed at simple 6% will show	-100,000		
... a balance at end of year 1 of...		106,000	
and a balance at end of year 2 of...			$112,360

Comparison shows our financial statement would be $560 poorer at the end of year 2 by going ahead with the project than by placing the funds at a safe 6% -- even if we allow our first-year cash inflow to be placed and earn at that rate during the second year. The message is fairly clear on the face of that evidence that project "X" is earning a return less than 6%.

Our final example is project "Y," with the following pattern of cash flows: -100, 95, 12.

Ignoring the time pattern of flows (which is what failing to use present-value procedures essentially achieves) project "Y" seems inferior even to project "X," since "Y"'s inflows exceed outflows by only 7 compared with net inflow of 10 for "X."

But if we compute the present value of the flows at 6% for project "Y," we get

$$PV@6\% = -100{,}000 + \frac{95{,}000}{1.06} + \frac{12{,}000}{1.06^2}$$

$$= -100{,}000 + 89{,}623 + 10{,}680$$

$$= +330$$

The decision rule mentioned earlier tells us to proceed with project "Y." We can also calculate our second-year financial statement as we did for the two earlier options:

	end of year		
Option III: Invest in Project "Y"	0	1	2
An initial outlay of $100,000...	-100,000		
will return $95,000 at end of year 1...		95,000	
and $12,000 at end of year 2...			12,000
while the $95,000 received at the end of year 1 can be placed at 6% and convert to $100,700 at the end of year 2...			100,700
resulting in total cash on hand end of year 2 of...			$112,700

The final year cash total, $112,700, is greater than the $112,360 we would have accumulated placing our initial funds at 6%. The outcome of the present-value decision rule likewise signals that project "Y" promises to earn at a greater-than-6-percent rate.

In summary then, one reason for using present-value procedures in choosing among investment alternatives is that you may be throwing away money if you don't. In Part II we discuss cases where no revenue estimate is possible and the choice among alternative ways to accomplish the same project is made on the basis of minimum outlays. The time pattern of outlays is crucial there, too, and the use of present-value procedures can help you avoid the prospect of selecting an alternative on the basis of minimum overall outlay while you are in fact penalizing your ultimate financial position because of differences you did not take into account in the pattern and timing in outlays. But the case for government agencies using present-value procedures is generally made and discussed at a different level; we turn there next.

THE OPTIMAL SOCIAL RATE OF DISCOUNT

What rate of interest ought a Federal Reserve Bank, or any other government agency, use for present-value computations? Most recent literature on the subject is in agreement that we ought to be using a rate that represents what society gives up when we command resources into a investment project -- a rate representing the "opportunity cost" to society of our decision. If we do not, then we would be drawing resources into government sector activities that return less to society than those resources would yield by investment in the private sector. That's simple enough in principle, but the problem of determining the appropriate rate of "opportunity cost" is formidable, and no unambigous estimate is available.

The practical upshot, though, whether we can pin down "the" optimal rate or not, is that the rate we should be using is higher than the rate on safe, liquid earning assets discussed in the preceding section. And in general for any given agenda of projects the higher the discount rate, the greater the number of projects that will be rejected under a present-value decision rule.

We accept in principle the social opportunity cost rationale for using present-value procedures at the Federal Reserve Bank and we turn now to sketching out an approach to identifiying a rate, or range of rates, to be considered "best" for our present value computations.

1. A SIMPLIFIED MARKETS MODEL OF THE ECONOMY

At an abstract level we can consider an economy with three sectors: government, business, households (the public-at-large). From conventional theory two basic determinants of interest rate levels in the ideal markets are posited.

The first is a "rate of time preference" that summarizes the public's willingness, in aggregate, to trade income today for income in the future. Specifically it may be assumed that for each individual there is some amount of dollars, R, such that he or she will give up 100 dollars today in return for the payment of 100 + R dollars with certainty at a specific future date. If the date is one year from today, then R reprsents an annual interest rate. Higher rates of

2. RISK, CORPORATE INCOME TAX, AND OTHER REAL LIFE "IMPEDIMENTS."

A slightly more realistic model emerges from attempts to accommodate some real life institutional features that, when introduced into the above simple model, act to "drive a wedge" between the public's marginal time preference rate and the business sector's marginal productivity of capital.

So, if we were to start with an estimate of the public's marginal time preference rate and were able to construct reasonable approximations of the rate impact of the major institional "wedges," then we would end up with an estimate of marginal productivity of business sector capital. That may seem rather roundabout, but the marginal productivity of capital, which is key in determining the appropriate rate of opportunity cost of public investment, is not observable directly in the markets. That in brief is the track pursued by a number of recent articles in the relevant literature, and is the structure of argument to be sketched out next.

PURE TIME PREFERENCE. Our starting point, then, is the premise that the public's pure time preference rate is fairly well reflected in the rate on safe, liquid financial assets, e.g., U.S. government securities. We'll take the 10-year bond rate as an appropriate basis for comparison (to give us a term more nearly like that of durable capital commitments) and adjust the rate to approximately eliminate any apparent "inflation premium" built into our observable market data. Basically the reason we want to eliminate inflation effects is that marginal productivity of capital is a rate measured in "real" terms, so our preferred starting point is the public's marginal time preference in real terms (what might be called the marginal time preference for goods as distinguished from marginal time preference for dollars).

We can observe that long-term U.S. government bond yields ranged over the past five years between 5 and 7 percent. How much of that was inflation premium we cannot say with any precision, but we think it reasonable to argue that most of the increase that occurred since 1966 was inflation premium. In consequence we think that, in round numbers, 4% to 5% per annum reasonably brackets the marginal

time preference rate of the public. For purposes of computation we'll start with 4 1/2%.

RISK. By that token 4 1/2% return would be what the public would require in order to give up the use of its money to a business corporation with certainty of full return. But since real life investment payoff isn't certain, there's always the risk that a project will fail to deliver as expected and that a company will default or go bankrupt. Therefore the public requires a "risk premium" over and above their pure time preference rate to induce them to provide equity funds to individual firms in the business sector.

While our model of the economy may crisply posit a single risk premium, the task of measuring what that risk premium might be in our observation of real life market rates is something else again. Differences between corporate bond rates and government bond rates might give a clue, though other market characteristics would enter into any observed differences. Reasonable guesses would seem to place such a risk premium in the 2% and 3% per annum range. We shall take 2 1/2% for computation purposes.

Next we proceed to add our so-determined risk premium rate to our estimate of marginal rate of time preference. Then 7% (=4 1/2% + 2 1/2%) would be the necessary minimum return on investment an individual corporation could set out to earn and still expect to obtain funds from the public -- and that would mean a marginal productivity of capital of about 7%.

The notion of "risk" as we've used it here has a fairly simple statistical probability model underlying it. It says that even if a firm's decisions, accurately calculated, lead it to expect a rate of return of, say, 7%, that figure is somehow only the most probable or average return it can expect. What the firm really faces is a distribution of various outcomes as in the following diagram where the height of the curve at a particular value for rate of return is proportional to the probability of getting that rate of return.

While the expected rate of return is 7%, the firm could earn less or more. For example, the probability of earning less than 5% is represented by the shaded area under the curve.

"Uncertainty" about the future is measured by the variability of the curve -- the more uncertainty, the more spread out it is. "Risk" is measured by the probability that the rate of return will fall below some arbitrary cutoff value (generally a value representing loss or failure to an investor). The "risk premium" we conjectured earlier can be thought of in terms of the above diagram as the excess of a firm's expected rate of return over an available safe rate, say 4 1/2%, that is required to reduce the shaded area to an acceptably small proportion.

CORPORATE INCOME TAX. The effect of a corporate income tax when introduced into our prior model is to require any corporation to earn a higher marginal return on its investments than it must yield after taxes to its stockholders. At a 50% marginal tax rate, for example, in order to return an after-tax 7% to its stockholders, the corporations' investments must earn 14%. In general, where t is the marginal tax rate:

$$\text{\% return before taxes} = \frac{\text{\% return after taxes}}{1-t}$$

In that context, then, if it were true in the real world that an effective marginal tax rate of 50% applied across the business sector, then 14% return on investment would indeed mark the point in the business sector's schedule of

potential projects where a cutoff must be made -- no project with a lower expected return would be undertaken. Then 14% is a reasonable estimate of the marginal productivity of capital and can be taken as the opportunity cost of diverting resources away from corporate business.

However, the effective marginal tax rate on corporate income is not 50%, but something lower. Rapid amortization provisions, percentage depletion allowances and other special tax provisions for corporations all operate to reduce it. Our "horseback" figure for the effective rate is 40%. Then an after-tax return to stockholders of 7% would require

$$\begin{array}{l}\text{\% return}\\ \text{before taxes}\end{array} = \frac{7\%}{1-0.4} = 12\%.$$

But not all capital funds that corporations invest are equity funds. Some are borrowed funds and, since interest costs are deductible for corporate income tax computation purposes, the marginal tax rate doesn't induce any need for higher pretax return in support of those funds. If we assume the corporate sector to be 80% equity financed and 20% debt financed then our estimate of the required marginal productivity of capital for the corporate sector would be reduced to

$$\left(\frac{20}{100}\right) 7\% + \left(\frac{80}{100}\right) 12\% = 11\%$$

As an overall judgment, then, we would place marginal productivity for the (corporate) business sector at about 11%.

<u>SECTOR EFFECTS</u>. The actual opportunity cost of a particular government investment decision depends on where specifically resources are drawn from. Resources commanded by government at full employment could in principle, be drawn from

corporate producers, noncorporate producers, or from the consumer sector.* Suppose all the resources are drawn from the corporate part of the business sector. Then the 11% figure for marginal productivity of capital suggested in the preceding section would seem a reasonable measure of the opportunity cost of government sector investment. But suppose all of the resources were drawn instead from the noncorporate part of the business sector. With no distorting effect from the corporate income tax to interfere, marginal productivity of capital may be taken to be close to the 7% "risk rate" estimate we made earlier. Finally, to the extent that government sector decision to invest diverts resources from consumption then the appropriate opportunity would seem to be the 4% to 5% "safe rate" -- the public's time preference rate for goods.

Each specific government sector investment could in fact affect sector resource use differently -- particular industries with marginal capital productivities either very much higher or very much lower than the corporate aggregate figure could be differentially hit. Regional differences in impact, too, would occur. (That exercise really brings us into a more complicated model than we have been describing.) As a consequence, some authors point out, the "true" opportunity cost probably varies with each government investment decision. Furthermore, if we abandon our full-employment premise, the true rate of opportunity cost would tend to vary with the state of the economy -- with the rate effectively lower during times of greater slack and underutilized labor and plant.

Our leanings are toward use of an economy-wide rate for opportunity cost. The suggested refinement of tracing subsector, industry, or regional impact seems beyond any foreseeable technical competence and may perhaps even be conceptually in error. An economy wide rate would presumably be some weighted average of the

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*Where the resources come from will depend to some extent on how government finances its expenditures; financing can be done either by taxing or by inflating.

sector marginal rates developed above. Without wishing to defend rigorously the artibtary weights chosen, we generate the following average rate for the record:

70/100	corporate sector	@	11%
20/100	noncorporate sector	@	7%
10/100	consumption	@	4.5%
overall economy average		=	9.5%

Furthermore we would make no direct adjustment for "slack" in the economy. While some indirect adjustment would tend to occur with cyclical movements in general interest rate levels, we don't feel that deliberately introducing an extra push in the Bank's capital spending propensities is an appropriate way to abet national economic stabilization policy.

Finally, it will turn out later (although we don't describe the process in detail until Part II) that our recommended procedure involves no single discount rate, but rather examination of "what goes on" with present values over a range of discount rates. Given all the preceding rate discussion, to be sure, our major attention will focus on what happens over the 7% to 11% range.

3. SOCIAL RISK, PRIVATE RISK AND THE ISSUE OF "RISK ADJUSTMENT."

One last set of matters we wish to take up in this opening part of the report. We want to deal with a couple of contentions frequently appearing in the literature on the matter of adjusting the data or the rates for "risk." One contention is that government investment decisions ought to use a safe rate (or the Treasury borrowing rate) for discounting purposes. The other is that prospective investments ought to be stored into "risk classes" and higher discount rates be used for riskier classes of investment.

SOCIAL RISK, PRIVATE RISK. A case for government use of a safe (risk-free) rate in present-value calculations is based ont he argument that because of the very large number of projects undertaken by government, the "risk pooling" effect makes achievement of the aggregate expected value of all its returns taken together virtually certain, or "riskless." The argument is made that the social risk in any particular government investment undertaking is less than the private risk to an

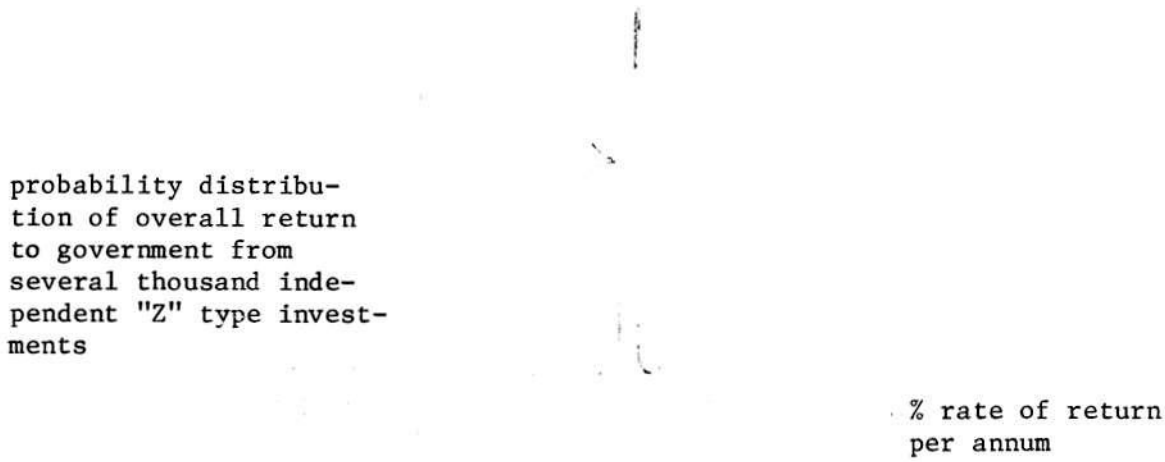
individual firm undertaking a comparable project. The model that generates this conclusion can be illustrated fairly simply. It assumes that uncertainty about any individual investment project confronts an investor with a distribution of possible rate-of-return outcomes. For example:

Project "Z" is expected to earn 7%, but it could do very much better or very much worse.

Now consider corporation "A," organized to undertake a dozen different and independent projects, all of which have an identical distribution of outcomes to that of Project "Z" (the mean or expected value of outcomes is the same and the spread of outcomes around the mean is the same). The statistical consequence of combining the several projects in one firm is to reduce the spread of combined outcomes leaving the mean value unchanged at 7% as follows:

Corporation "A," by pooling the risks over several projects has cut the risk to the investor from what it would have been in any of the projects taken individually. "Risk" here is defined by implication as the probability of earning less than some specified rate of return -- for example the shaded areas in the above diagrams represent outcomes earning less than 5%; say that funds "cost" that much and so to earn less would be to go broke as a corporation.

The statistical "law of large numbers" guarantees that if government were to undertake many thousands of independent projects, each with a spread of rate of return outcomes identical to that of project "Z," the combined outcome with virtual certainty would be very close to the expected value of 7%.



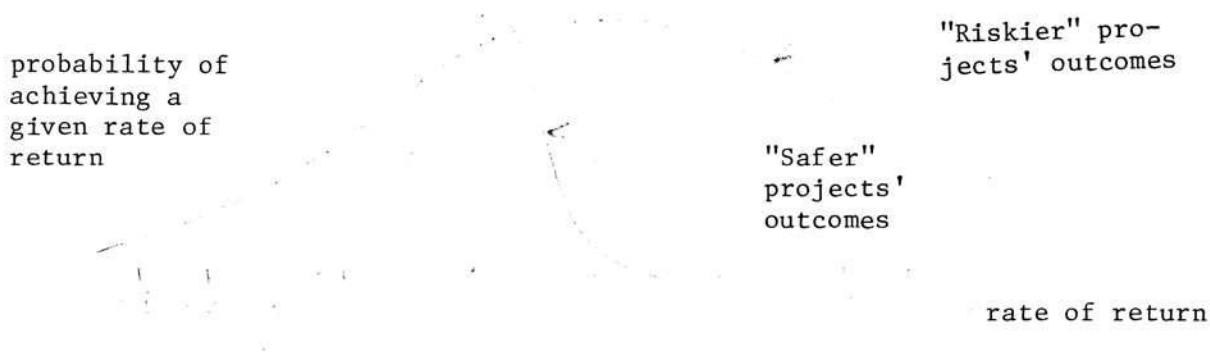
"Social risk," the risk of loss to society from the undertaking of individually "risky" projects, thus can be very low for government investment because large numbers of projects are involved -- and large numbers are involved in government sector decision because of the size of that sector. The critical point here, however, is that while the "private risk" of an individual firm's decisions may be appreciable (as illustrated in the second diagram above), the "social risk" of private decisions about "Z" type projects is as low as that for government decisions -- again because from society's point of view very large numbers of projects are involved. Thus there are no grounds from a "social risk" standpoint for treating government

decisions on a different risk footing from that applied to individual private firm decisions.

And the point was established earlier that the proper "footing" to be used in social decisions is the opportunity cost to society of those decisions -- and in the choice of interest rate measures that clearly is not the riskless, safe rate.

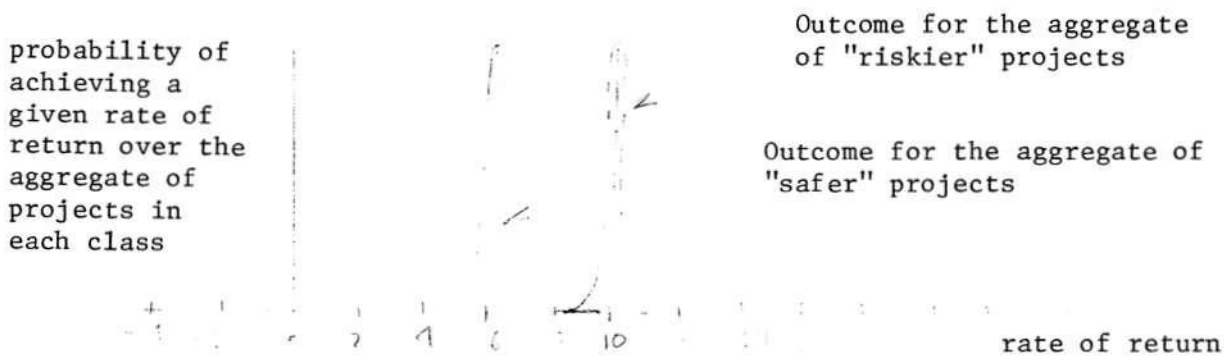
RISK CLASSES AND DIFFERING PROJECT RISKINESS. Should investment projects be classified according to inherent "riskness" and higher discount rates be applied in computing present value for "riskier" classes of projects? Our answer here is an unequivocal, "no." The function of the discount rate is in effect to measure the opportunity cost of an investment and the opportunity cost is not affected nor altered by the particular properties of an individual investment project that may be under consideration.

Presumably what is meant by "riskier" class of projects is a set of projects for which the range or spread of outcomes is larger than that of some standard set of projects, as in the following diagram:



The mean, or "expected value," of the "riskier" class is shown here to be larger than that of the safer class, although there is no reason that need be assumed (though in general that's likely to be true for those projects seriously considered by businessmen). The point is that a class of projects may be riskier (viewed individually as having a greater variance in outcomes), yet from a social point of view, when a large number of such projects is undertaken, uncertainty about aggregate outcome drops toward zero and with it, risk. The "spread" of expected outcomes collapses around the mean value as the pooling of a large number of projects occurs. All that is required of that model of "riskness" is that the projects be independent and that the distribution of outcomes and the expected mean value be accurately known.

For the case illustrated on the above diagram, once the "pooling" effect of large numbers has whittled away the uncertainty (spread) of outcome, society would be better off from investment in the "riskier" class of projects because they will return 10% per year with virtual certainty whereas the "safer" projects will return only 6% per year as in the following:



Thus it could prove perverse from a social point of view to insist on application of a higher rate to classes of "riskier" projects.

The final thing we want to do in this section of the paper is to briefly describe how uncertainty over future conditions can enter legitimately into present-value computations.

Uncertainty about the future clearly enters any effort to make projections of cash flows for any investment. And the more distant the time period, the more uncertain the projections are likely to be. A projected cash flow series presumably will be built up from projections of the various cost and revenue (or benefit) elements entering total costs and revenues. Examples are: the expected volume of output of an operation, unit labor requirements, wage rate, prices, maintenance costs, power requirements, and scrap, trade-in, or resale values. Each of the elements could trace a variety of alternative paths over the future and we do not know in advance which of the many possible paths the values for the variable will be taken in the final outcome.

In principle we can accommodate some measure of our uncertainty by properly structuring our projections. Future cash flows, or the elements that comprise them, can be thought of in terms of a distribution of possible outcomes to which we can assign probabilities of occurrence. These procedures, while possibly tedious, are variously described in standard references. We'll simply illustrate the idea by considering our earlier simple example that had the following cash flows:

-100, 30, 80.

To single out just one quantity, the projected cash inflow of 80 for the end of year 2 is in any practical case uncertain. One possible model of our uncertainty would be that 80 merely represents the expected value (the mean) of a distribution of possible outcomes:

Cash flows projected for other time periods are also uncertain (though presumably those nearer the present have less variance). Discounting the expected values for the cash flows, which is what we did earlier, will generate a single present value at any given discount rate. If we could take fully into account the probabilities of getting cash flows different from the expected values (and that's a tricky matter since they are not in general independent) then we could generate a distribution of present values that would reflect our uncertainty over cash flow projections.

A mechanically simpler approach is to express our uncertainty by a small number of discrete outcomes, to which we assign odds in such a way that the total of probabilities adds up to 1. For an example of this, take the second-year inflow of $80,000. Suppose that value to be derived from (a) a volume estimate and (b) an estimated net per unit. We judge we are most likely to produce 21,000 units, but there is some chance output could be lower. We may choose to express our uncertainty about volume of output in the following way:

Production Volume	Probability of that Outcome
21,000	3/4
17,000	1/4

Similarly, say we are uncertain about cost levels (gross per-unit value assumed unvarying) and we express our uncertainty about the net value per unit as follows:

Net Value Per Unit	Probability of that Outcome
$4.20	1/2
3.80	1/2

If we assume the volume and unit value outcomes to be independent, then the following probabilities for the various unit net value and volume combinations are generated:

If volume is...	and unit net value is...	then cash flow is...	with probability...
21,000	$4.20	$88,200	(3/4)(1/2)=3/8
21,000	3.80	79,800	(3/4)(1/2)=3/8
17,000	4.20	71,400	(1/4)(1/2)=1/8
17,000	3.80	64,600	(1/4)(1/2)=1/8

Assuming at this point that the first two cash flow figures in our example are known with certainty, we have now available a probability distribution of cash flow series embodying our expressed uncertainties over the final cash flow:

Cash flow Outcome	Probability of that Outcome	Present value @ 6%
-100, 30, 88.2	3 out of 8	6.8
-100, 30, 79.8	3 out of 8	-0.7
-100, 30, 71.4	1 out of 8	-8.2
-100, 30, 64.6	1 out of 8	-14.2

We can of course then compute a "discrete" probability distribution of present values at any given discount rate as in the third column above. While an <u>expected</u> present value (the weighted average) can still be computed, the <u>distribution</u> of present values conveys useful information about uncertainty and risk. The general effect, though, of introducing into the cash flow projection procedures these notions of alternative outcomes and their associated probabilities, is to reduce the projected flows as risk increases -- giving greater weight to the pro-

spect of less favorable production, price and cost showings. In sum, through the approach we've outlined, "risk" is given appropriate consideration, and no "risk adjustment" of discount rates is required.

Part II. General Description of Present-Value Procedures in Capital Investment Decision Making

The objective of Part II is to explain present-value procedures and how they can be used to provide information useful in investment decision making. It will be clear from the examples that these procedures are merely tools of analysis, and they may prove to be more or less useful in a particular application depending on the nature of the decision-making problem. We will take up first what we call the standard case involving problems for which dollar values can be assigned both to the stream of future benefits expected to flow from the project and also to expected cost flows. The second class of cases to be considered will be those for which dollar valuation of the expected benefits is not possible.

A. The Standard Case with Both Receipts and Expenditures Known

We start with a hypothetical example to illustrate the steps involved in organizing and analyzing data in the standard case. Suppose our bank is considering establishment of a new banking service for which a charge will be made. It has determined two possible levels of activity. The first level, call it alternative A, will produce estimable revenues over a period of years and will generate particular costs for acquisition of equipment, additional labor and so on. The second (higher) level of activity, alternative B, would generate greater revenue but would also require greater outlays for equipment, labor and supplies to handle the greater activity. The question is which, if any, alternative should the bank choose?

1. Constructing the Net Cash Flow Streams

Four general steps are involved in developing the necessary net cash flow data: (1) define the project horizon and time units to be used, (2) estimate expenditure flows, (3) estimate receipts flows and (4) compute the resulting net cash flow streams. All four steps require judgments as to what facets of the problem can (can't) be adequately quantified and incorporated into net cash flow data. These decisions largely determine whether useful information will come out of the present-value analysis.

Project Horizon and Time Units

A number of factors usually influence choice of the project horizon. The expected life of key processing equipment or conventional replacement practices might indicate an appropriate planning cycle. Equipment lease periods may play a part. For our example we will assume the analysts have decided, after considering the factors above and other relevant information, that revenue and cost streams throughout the foreseeable future will follow a reasonably predictable three-year cycle. It may be adequate in any particular application to simply examine one three-year cycle. That can be established in any event by extending the analysis to cover additional three-year cycles to determine whether the conclusions would be altered. There are few if any general rules to guide the analyst in the choice of project planning horizon.

Having chosen a three-year horizon it remains to select an appropriate time unit. Sometimes the nature of the project

will suggest an obvious choice, sometimes not. Suppose in our example that elements of cash flows (billings, equipment rental payments, wage payments, etc.) occur variously at quarterly, monthly, semimonthly, biweekly and weekly. Some considerations in choice of a time unit:

1. If we choose the week as the time unit, the important monthly and quarterly data must arbitrarily be interpolated to derive weekly figures. Then too, three years (the project horizon) of weekly data involves over 150 time observation intervals, and in general it is more costly to handle large amounts of data.
2. If we choose the month as the time unit, semimonthly data could easily be aggregated to provide monthly figures though weekly and biweekly would cause some problems. Quarterly data must still be interpolated to derive equivalent monthly figures. With a monthly time unit the total number of data intervals is reduced to 36 (three years of monthly data), not an unwieldly number.
3. A time unit of one quarter eliminates the need to interpolate data. Twelve intervals (three years of quarterly data) is certainly manageable. The question becomes, is this too few observations? Will quarterly data mask important features of the problem or would all the essential information still be revealed? Again, there are few general rules to guide the analyst. From our experience 12 observations is not obviously too few. Doubt can be resolved

in any particular case by extending the analysis to the next finer time unit to determine whether the conclusions differ. For our example we assume quarterly data are acceptable.

4. From the standpoint of computational requirements, time units greater than one quarter also could be used; half years, annual data, even a unit measuring the full three years is not mechanically impossible. The main point of a cash flow analysis, however, is to properly account for the importance of differential timing of cash flows. To choose a time unit so large that all flows are clustered at a few points widely spaced in time conceals the very information we are trying to exploit.

One final point on the matter of time scale. Our example will involve 12 quarterly time intervals. We will adopt the convention of numbering points in time (rather than intervals) as illustrated below.

Let the letter t be our index of points in time so that $t = 0$ indicates the beginning of the first quarter of the project horizon, $t = 1$ denotes the end of the first quarter and the beginning of the second, and so on through $t = 12$ which is the end of the twelfth quarter and also the end of the project horizon for this problem.

Expenditure Flows

Our investment decision will involve accepting one or the other of two mutually exclusive courses of action, or rejecting both alternatives. One alternative is to establish a certain (lower) level of bank service (alternative A). The second is to establish a higher level of bank service (alternative B). Analysis of both alternatives must be based on the time scale described above. The task before us now is to estimate the sequence of quarterly cash outflows (expenditures) for each alternative.

Cash flow data, as the name suggests, includes only monetary transactions in which the ownership of funds is transfered to or from the individual or corporate entity making the decision. Accounting artifices such as depreciation accounts or reserves for possible future losses have no part in a cash flow analysis.[2/]

Constructing a cash flow stream requires that we forecast future events -- a process involving uncertainty. An exercise frequently useful in forecasting is to estimate the lowest, most likely, and the highest outcome. And, of course, more sophisticated statistical procedures exist. The procedure mentioned at the end of Part I could be helpful. Basically that involves estimating two or more realistic alternative levels for expenditure outcomes, assigning probabilities on the

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[2/] Except insofar as they may affect cash flows, as depreciation reduces income tax payments. We assume here no such situation exists and in fact that either there is no income tax or that we have decided a before-tax analysis is appropriate to the present problem.

assumption one or another alternative will occur, and calculating weighted averages ("expected values") for each period. For present-value analysis it is important that the cash flow estimates represent the most likely (i.e., mean or average) outcomes, not simply an ideal or favorable outcome.

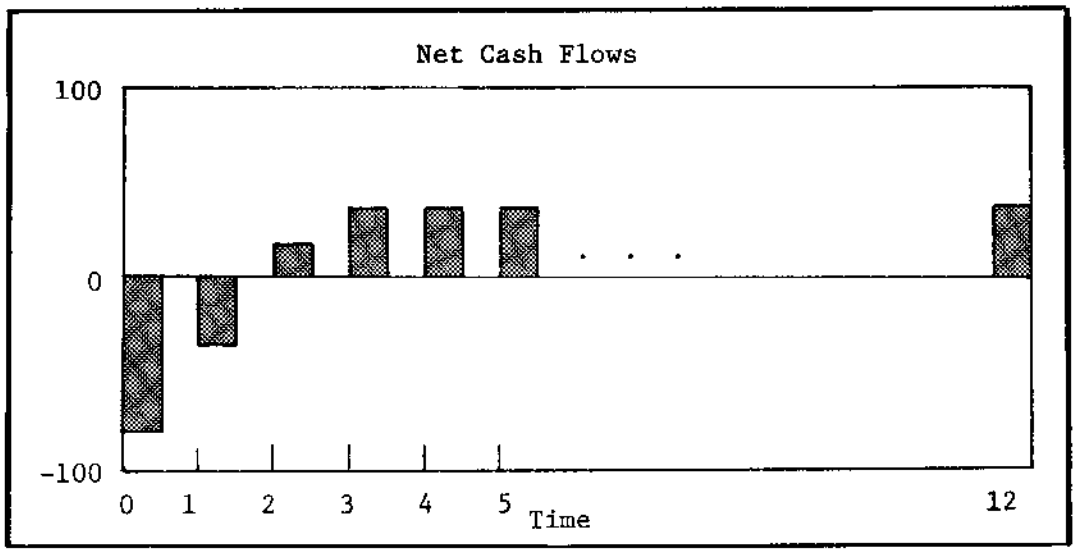
Actual cash flow transactions occur on a particular day. For example, a regular monthly equipment lease payment may be made on, say, the first of February. Our time scale has no first of February. It has a first of January and a first of March (the beginning and end respectively of the quarter containing February 1st). To which of these two points do we attach the lease payment? Again, no hard and fast rules exist. The analyst must decide what procedures are best suited to the problem at hand. If the time unit chosen is sufficiently small the problem will be a minor one. If the time unit is larger (relative to the total project horizon especially) then more attention must be given to this issue.

In our example for both alternatives the major large outlays for equipment and other factors are assumed to be assignable to the point $t = 0$. By $t = 2$ and beyond the outflows have settled down to reflect recurring operating expenses and regular cash payments for lease contracts. The estimated expenditure stream includes a small, steady increase due to anticipated increases in labor and other costs. Table 1 gives assumed cash outflow data for each alternative. The numbers may be thought of as units of $1000. The notation E^A

as negative numbers. If for a particular value of t we have receipts of \$300 and expenditures of \$250, the net cash flow for that value of t is...

$$(+\$300) + (-\$250) = (+\$50)$$

Problems amenable to cash flow analysis frequently involve large expenditures (outflows) with little or no offsetting revenues in the early periods. The start-up costs may reflect large initial outlays for plant and equipment, intensive early planning and development requirements, low initial productivity and so on. Later periods in the horizon typically produce a more or less steady stream of positive net cash flows (inflows) as income earned on the larger initial investment. In many cases, including the present example, much of the large initial investment outlay can be bunched at t = 0, the very beginning of the project horizon. There may or may not be additional net outflows in other early periods. Heavy equipment installation or plant construction typically extends over several early periods. The net cash flow for alternative A is shown below and is typical of the time pattern for B and for many other practical problems.



2. Analysis of the Net Cash Flow Stream

Selecting the Appropriate Interest Rate

The next step in the standard case investment decision process is selection of an appropriate interest rate to be used in computing present values for each net cash flow stream. Two rationales for choice of interest rates were discussed in Part I of this report. Neither generates an unambiguous numerical value. The first rationale suggested the rate should be at least as high as that paid on some relatively safe and liquid financial asset -- an alternative "earning rate" directly and safely available to the institution's decision makers. Such a rate might today fall in the range of 5% to 6% per annum. The second rationale argued that the appropriate rate for discounting is the rate "society" would earn if the funds in question were invested in the most profitable

The discount factor at, say, t = 5 tells us that $1000 to be received at t = 5 is worth only $908 to us at t = 0. Put the other way around: $908 invested at time t = 0 at an 8% annual interest rate will be worth $1000 five quarters later. Discount factors are available in standard financial tables or can easily be calculated on electronic computers.

Computing the Present Value of the Project

In table 4 we've entered the projected net cash flows assumed earlier for hypothetical project alternatives A and B. Using the discount factors from table 3, the present value for each quarterly flow can be computed by simple multiplication

$$PV_t = D_t \times F_t$$

for most cash flow streams.*

Simple Decision Rules Based on Present-Value Computations

We will discuss two fairly standard decision rules by which alternatives can be judged on the basis of their present values at a given interest rate. The first applies to any project, whether or not alternatives are being considered. These are cases of "accept-reject" decisions where the alternatives are either to adopt a contemplated project or not to adopt it.

> Rule I: If the present value of a project is greater than zero, adopt it. If the present value of a project is less than or equal to zero, do not adopt it.

The second rule applies in the case of multiple alternatives which are mutually exclusive, that is, only one of the several alternatives can be selected. The hypothetical example given above is of this type; select either project A or B. If we adopt one we can't adopt the other.

> Rule II: If two or more mutually exclusive alternatives have present values greater than zero, adopt that alternative with the greatest present values.

Applying rule I to our hypothetical problem we would conclude that neither alternative should be selected. Both have negative present values discounted at 8%. We

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*It can happen for rather strange patterns of cash flows that, within certain ranges of the discount rate, the present value will increase if the interest rate is increased a little. A cash flow that experiences a large expenditure late in the time horizon may have this property. We will encounter few if any cash flows of this type.

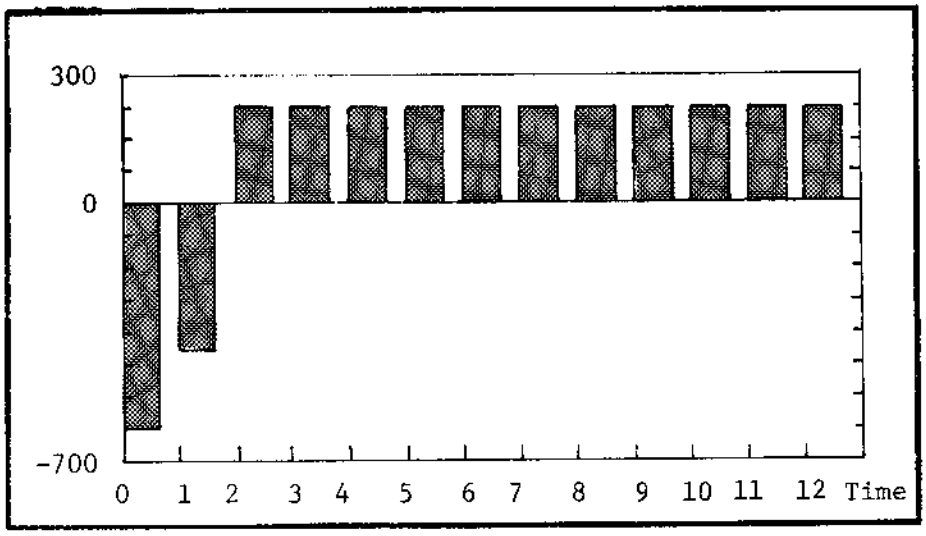
might note that although alternative B appeared superior to A in terms of the simple sums of net flows ($263 for B compared with $195 for A), after discounted at 8% the ordering is reversed (-$105 for B compared with -$32 for A).

3. Multiple Present Value Computations, Internal Rate of Return, and More Generalized Decision Procedures.

There is simply no way in practice to single out one "best" interest rate for investment analysis purposes. We recommend that a range of interest rates -- rather than just one -- be used as a basis for present value calculations. Multiple present value computations, now rendered relatively simple through computer programs, will considerably enhance the information provided to the decision maker. We suggest that present value information for each project be prepared as a continuous graph over a reasonably wide range of interest rates, say zero to 16 or 20 percent, and that the graphs so prepared be the basis for displaying present-value information about investment alternatives. The next subsection explains such a procedure.

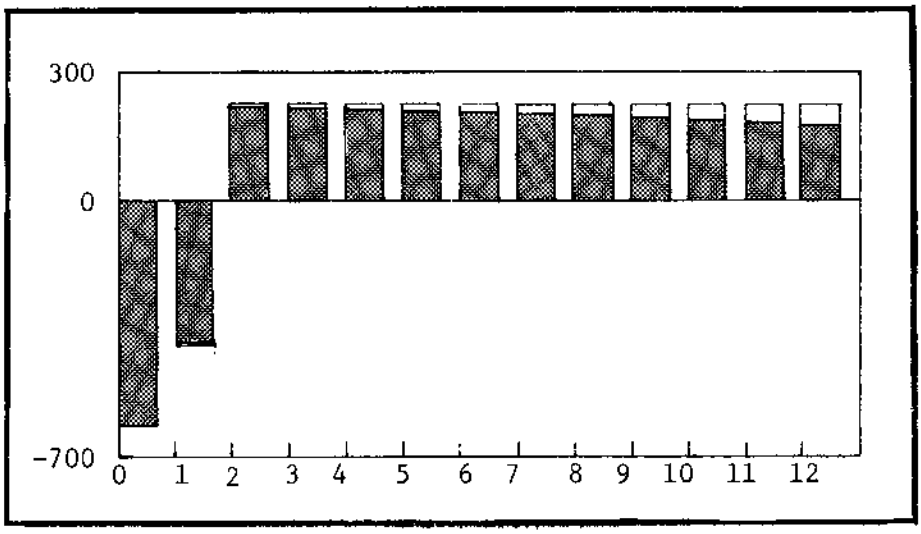
a. Present value properties over a range of interest rates.

In our prior example, alternative A had a series of net cash flows as represented in the following diagram:



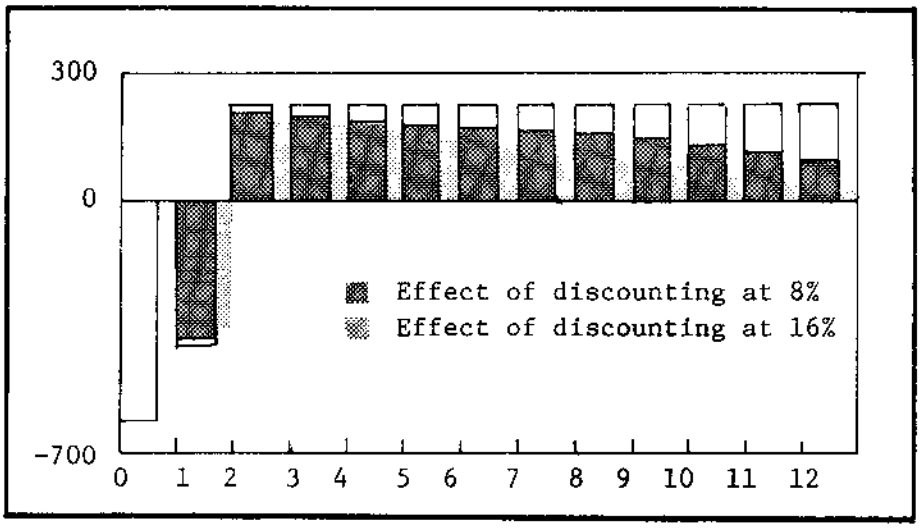
The actual quarterly net cash flows for each value of time t can be thought of as present values for the special case in which the discount rate is equal to zero. To put it another way, not discounting at all is the same thing as discounting with the interest rate set equal to zero. Thus, present value for the project (at a zero rate of discount) can be viewed as the algebraic sum of the heights of the bars in the above diagram -- a sum we calculated in table 5 to be $195.

Discounting the flows to time t = 0 at, say, a 4% rather than 0% annual rate can be viewed as a process that has no effect on the length of the bar at t = 0, reduces the length of the bar at t = 1 by about 1%, the bar at t = 2 by about 2%, and so on. Graphically:



Given that the greatest proportionate reduction takes place for the later points in time where positive flows predominate, the impact of discounting on the project, of course, is to reduce its present value. In contrast to the $195 present value we got at a zero rate of discount, we have only a $74 present value at a 4% rate of discount.

For cash flow patterns of the sort we have been looking at, successively raising the selected discount rate will so shrink the contribution of the later, positive inflows that the overall project present value would eventually be reduced to zero and then become negative. At an 8% annual rate of discount alternative A has a present value of -$32; and higher interest rates would result only in larger negative values.



We can summarize all of this information on one graph by plotting present value on the vertical axis against

estimated directly from the graph, and has a special meaning. The rate at which the present value of net cash flows is zero is the so-called "internal rate of return" (IRR) of a project or investment. The internal rate of return is often used as an index profitability and there are various suggested investment criteria that build their decision rules around the internal rate of return concept. Present-value rules, however, are more generally applicable. We will use present value as the basic criteria in our procedure. The internal rate of return will serve as an additional piece of information, falling out of the calculations.

b. Comparing projects using a range of interest rates.

Now let's return to our earlier example and plot the "present-value curve" for each of the two alternatives on the same graph. This we do in Figure 2.2 (again, in this example the curves were arrived at by computing PV's for even percentage rates and connecting the resultant points by straight lines).

over B. This crossing of present-value curves is not uncommon and in such cases the choice of the most "appropriate" interest rate for discounting purposes may become quite critical in helping to judge which project is superior. If 6% per year is chosen as the discount rate then alternative A is still attractive (has a positive PV) but alternative B is no longer acceptable under the conventional decision rule because its present value is negative (slightly). At an interest rate of 8%, neither project has a positive present value, and neither project would be acceptable as measured by present value criteria.

We recommend the range 8% to 11% as the significant region for present value comparisons. That could mean, in some cases, application of the decision "rules" would be ambiguous. We reemphasize our earlier point that present-value computations provide some, but never all, of the information necessary to sound decision making. So even if the present value comes out negative over most if not all of the 8%-to-11% range, it would not be conclusive that the decision must be to reject the project. There are always intangibles associated with any major decision. In the case of alternative A we might imagine it represents a new line of activity with some significant longer term promise which we have no way of incorporating into the cash flow data. Or perhaps it represents a possible avenue for staff development that we believe to be important but

can't assign specific dollar values to today. So we might still choose to go ahead with alternative A. And our present value calculations may still be useful as measures of the "opportunity cost" that the intangibles in the project must be judged to outweigh.

In another situation we may weigh two alternatives that both happen to have positive present values and decide to go with the lower PV alternative. We may reason, for example, that it represents purchase from a different equipment manufacturer we believe we should encourage in the interests of improving competition in the suppliers' market. And again, we can read from our graphs what this choice is costing us by observing the difference between the two present values in the relevant range of interest rates.

Finally we may note that alternative A has an internal rate of return of just under 7% which is modestly greater than the IRR we've calculated for B (about 5-1/2%). Perhaps that information may give us some additional feel for what's involved in our choice, or rejection, of one or the other alternative.

B. The Case in Which Benefits (Receipts) Cannot be Measured

In the standard case considered in section A both the expenditure (cost) stream and the revenue (benefits) stream were known. For public institutions, including the Federal Reserve, it is difficult or even impossible to accurately measure in monetary

terms the social value of many of the activities undertaken. Some public agencies do attempt to assign benefit values to their projects. This may involve very subjective and arbitrary decisions on values affecting the social worth of say a bridge, a sewer system, a recreational site and so on. Generally, we recommend that an attempt be made to estimate benefits, so that decision-making procedures applicable to the standard case may be used. If, however, it is not possible to make a sufficiently accurate estimate of benefits, cash flow analysis procedures are still available which can provide useful information for decision making. We will discuss two situations of this kind.

1. Choose from Two or More Alternatives by Minimizing Present Value of the Cost Stream (Benefits Assumed to be the Same For Each).

Typical of this kind of problem is the situation in which we have been instructed (say by Congress or the Board of Governors) to engage in an activity. The problem is to select the most efficient (e.g., least cost) means to accomplish the objectives of the activity. Another common situation arises when management has determined, based on analysis at a broader level, that an objective is appropriate; that even though we can't measure the benefit stream, it is surely large enough in their judgment that the net cash flow stream does have a positive present value at relevant discount rates.

More generally, we include here any situation in which it can be assumed that the alternatives all have benefit

streams with the same present value. The time patterns of the benefit streams need not be identical so long as the present values are the same. A common example of this type is the lease-buy decision. A decision may have been made to acquire, say, specific computer equipment. The question is whether to buy or to lease, and if lease, which of several lease options to elect. A rule applicable to these cases is:

> Rule III. For projects assumed to have future benefit streams with the same present value, accept the alternative for which the present value of the cost stream is a minimum, provided that the present value of benefits is judged to be at least as great as the present value of costs.

To illustrate this situation we will consider an example where the problem is to determine on what basis to acquire terminal equipment for access to a commercial computer timesharing service, to be used mainly by the Research Department. All of the basic principals, problems, and procedures regarding time scale, constructing cash flows, and so on carry over here.

The objective of the acquisition is to facilitate economic research involving the use of large econometric models. We assume here that objective has already been judged by management to be worth pursuing, and that various alternative approaches to providing larger and faster computer facilities, including (a) acquisition of a larger Bank computer, (b) using various computers service bureaus on a walk-in basis, and (c) using the Board of Governor's

computer by transmitting jobs and receiving the output by mail have been considered. On the basis of an initial review, which need not have included any present value procedures, the decision has been made that commercial timesharing services offered the most efficient and flexible approach. Computer terminal equipment must now be installed, and the question is, on what basis?

The inquiry is based on a two-year project horizon and three alternatives open to us through agreements with vendors.

(i) Purchase the terminal, pay cash in the full amount at time $t = 0$.

(ii) Purchase the terminal, pay for it in twenty-four equal monthly installments.

(iii) Lease for twelve months, renew the lease for the second year.

For the two purchases options it is necessary to estimate the "scrap" value of the terminal equipment at the end of the two-year horizon. This is essentially the price we expect the equipment can be sold for on the used market, less all transaction costs at time $t = 24$. This scrap value then appears as an inflow and as such has a positive (+) sign attached to it. Table 2.1 gives the monthly cash flows for each alternative. Notice that there are twenty-four monthly intervals so we have twenty-five time points ($t = 0, 1, \ldots, 24$). All lease or purchase payments are assumed to be made at the beginning of the month. Thus,

cash flow for the lease option is zero at the end of the twenty-fourth month (t = 24) but equal to the scrap value ($3340) for both purchase plans.

Multiple discount rate analysis is now applied to these alternatives. Table 2.2 gives the resulting present values for selected discount rates, the results are graphed on Figure 2.3. The "naive decision rule" that fails to account for the time value of money (i.e., zero discount rate) shows the purchase-with-one-payment option to be superior (cost of $13,360), with purchase-using-monthly-payments as second choice (cost of $15,304), and the lease option as the least desirable choice (cost of $18,000). Over the 8%-to-12% range of discount rates, the same ranking holds but the difference in present value of cost is narrowing. For discount rates greater than about 14%, the option to purchase on time payments becomes more desirable than the one-pay purchase (that ranking holds out to at least a 25% discount rate although the lease option closes rapidly on the one-pay purchase option).

So we decide to buy the machine and pay cash, right? Not necessarily. The three cash flows were constructed by considering only actual known cash payments and an estimate of the scrap value of the terminal if it were purchased. No other aspects of the problem were explicitly incorporated into the cash flows and the

present-value analysis can provide no information on these excluded factors. There could be important additional considerations. One is the desirability of maintaining a flexible position; to keep open the option of disposing of the terminal at low cost in the event our early experience with the facility fails to meet our advance expectations, or our needs change.

We mean to emphasize by this discussion that present-value data represents merely a tool in decision making, not an all-conclusive key. So it is quite possible in the face of the present-value data generated by our example to arrive at the judgment that the benefits of greater flexibility offered by the lease option will be sufficient to offset the cost differential measured by the factors present in the present-value analysis. Still we can get useful quantitative information from the present-value table. At an 8% discount rate the lease option costs $16,737 while the least expensive option, purchase for cash, costs only $13,836. The difference in cost -- $2901 -- represents a minimum value which all factors not explicitly represented in the cash flow streams ought to exceed in order to support our choice of the lease option.

2. Accept or Reject an Investment Opportunity for Which Benefits Are Not Known.

We will illustrate this kind of decision problem by modifying the computer terminal example used above. Suppose

this time we have just one alternative, the one pay purchase option, and the choice is to accept or reject acquisition of a terminal. The benefits available from use of the terminal cannot accurately enough be estimated to apply the standard decision rules. A rule applicable to this situation is:

> Rule IV: If the present value of the future benefit stream is judged to be at least as great as the present value of the cost stream then accept the project; reject the project otherwise.

Again, the same cash flow stream is constructed. The same multidiscount rate present-value computations are performed. The present value of costs at 8% is $13,836. Although we don't know the present value of benefits we may be able to say that, whatever the value, it is at least $13,836. If we can do that the Rule IV says go ahead, accept the project. Not all decisions involving unmeasurable benefits can be resolvable by framing the question in this manner. But it provides an additional piece of information which may be of some value.

There are other ways to manipulate cash flow data that may provide information useful in a given problem. The analyst will need to exercise ingenuity in any practical ease. One computation that may be helpful is to compute an equivalent equal-payment monthly benefit stream which corresponds to the known expense flows. For the present example, this monthly annuity amount for twenty-four months if $626. If management can determine that the value of the benefits is at least equal to $626 per month over the project life, then, again, the decision to accept the project can be made even though the exact value of benefits are not known.

Part III A Case Study

In April 1971 Federal Reserve Banks were instructed by the Board of Governors to provide wrapped coin to all member banks. In the past this service has been offered only to country banks. The volume of wrapped coin is expected to increase from about 5 million rolls per year at present to about 18 million rolls per year when the service is fully established in 1973. Later this year an extensive study will be conducted to choose from among a number of alternative systems of coin wrapping equipment and to decide on the purchase or lease question. The illustration presented here is based on this coin wrapping question. Less alternatives are considered here than will be treated in the subsequent study and a number of minor but bothersome considerations which must be dealt with later are ignored in this illustration. Nevertheless, the general approach developed here is a realistic one in response to a realistic problem. The subsequent study will probably be based on a similar framework, augmented to treat the alternatives ignored here.

Description of the Problem

Although several equipment manufacturers have submitted proposals to provide coin wrapping equipment, this illustration will consider only one brand of equipment. A choice must be made between leasing or purchasing the machine system. Also considered here will be the question of how long to keep the equipment. Generally the longer an investment can be kept earning revenue the more favorable will be the net cash flow stream it generates. Usually offsetting this favorable tendency is the sometimes substantial increase in maintenance cost associated with old equipment, a decrease in productivity due to more frequent equipment failures, and finally obsolescence which results from technological improvements available in new

equipment models. A thorough analysis of optimal equipment replacement cycles is very difficult and won't be attempted here. Some information on the question will be provided simply by running the cash flow analysis for both a three-year life and a five-year life.

Time Units and Horizon Quarterly data will be used for this illustration. Monthly data are available and would add to the precision of the information to be generated. The results of this exercise will show that quarterly data are probably precise enough.

Two time horizons will be examined.

(i) Three years ($t = 0,1,2,\ldots,12$)

(ii) Five years ($t = 0,1,2,\ldots,20$)

Expenditures Flows The following cash outflows must be considered:

(i) Labor cost. Six operators will be assigned full time to the coin wrapping facility along with one-half of one supervisor's time. Total cost for salaries and benefits are estimated to be $50,500 in the first year of operation and are projected to increase by 5% per year in each of the following years.

Year	Labor Cost
1	$50,500
2	53,025
3	55,676
4	58,460
5	61,383

Labor costs are assumed to be paid at the end of each quarter.

(ii) Equipment Maintenance Contract. If the machine systems are purchased, an equipment maintenance contract will be signed with the vendor. The cost for labor is $1,560 per quarter with no charge

for the first quarter. Parts are supplied at no cost to the Bank during the first year. In subsequent years parts costs are estimated to be:

Year	Parts Cost
1	$ 0
2	2,000
3	2,400
4	3,200
5	4,800

Equipment maintenance costs are assumed to be paid at the end of each quarter.

(iii) Building Maintenance. The policy of the Bank regarding building costs for wrapped coin service is to recover building maintenance costs but not the original building investment. A rate of $5.60 per square foot per year is charged for this purpose in the present building and this figure will be used here. Maintenance costs for the new building are not yet determined. The coin wrapping operation will require 2,100 square feet. The cost is $2,940 per quarter and has been held constant over the five-year horizon. Building maintenance costs are assumed to be paid at the end of each quarter.

(iv) Supplies. Supplies include mostly paper and bags. Historical records indicate supplies cost to be about $0.12 per 100 rolls of wrapped coin (12¢ per 100 wraps). This cost has been very stable -- even declining a bit over recent past years -- and is not projected to increase during the coming five years.

Volume of wraps for this example is projected to be 14,000,000 in the first year of operation, and is expected to increase by 5% per year in each of the four subsequent years.

Year	Volume	Supplies Cost
1	14,000,000	$16,800
2	14,700,000	17,640
3	15,434,000	18,524
4	16,207,000	19,448
5	17,017,000	20,420

Supplies costs are assumed to be paid at the end of each quarter.

(v) Purchase Price. Twelve coin wrapping systems are required at a total purchase price of $97,443. In addition three scales are required at a cost of $9,000. The entire purchase amount is payable at t = 0.

(vi) Lease Contract. The quarterly lease cost is $10,866 and is fixed by contract for the first three years. It has been assumed the lease price would be the same in both the fourth and fifth years also. The lease contract includes equipment maintenance -- both parts and labor. Scales, at $9,000, must still be bought if the lease option is elected. Lease payments are assumed to be due at the beginning of each quarter.

Revenue Flows The Minneapolis Federal Reserve Bank has established a price of $.85 per hundred wraps to be charged member banks. With the expected volume, annual revenue will be:

Year	Revenue
1	$119,000
2	124,950
3	131,189
4	137,760
5	144,645

Revenue is assumed to be received at the end of each quarter.

Additional inflows are generated by scrap value of equipment at the end of the time horizon. Scrap values are assumed to be:

	Wrapping Equipment	Scales	Total Scrap Value
Three-Year Horizon	$1,000	$3,000	$4,000
Five-Year Horizon	0	$3,000	$3,000

Net Cash Flows Using this information, and converting to quarterly time units, the basic cash flow data are developed as shown in table 1. Net cash flow data for each of the alternatives are constructed in tables 2 through 5. The four alternatives considered are:

Alternative A: Purchase the coin wrapping equipment with a three-year horizon.

Alternative B: Lease the coin wrapping equipment with a three-year horizon.

Alternative C: Purchase the coin wrapping equipment with a five-year horizon.

Alternative D: Lease the coin wrapping equipment with a five-year horizon.

Interpretation of Cash Flow Data Present values for each alternative are given in table 6. Consider first the three-year horizon. Present value curves for both alternatives are shown in chart 1. At a zero discount rate (in effect ignoring the time distribution of cash flows) it appears clearly better to purchase. The present value of cash flows is +$3,713 compared with -$8,676 for the lease option. As the choice of discount rate increases preference for the purchase option decreases until the rate reaches just under 10% where the present-value criterion shifts in favor of the lease option. At any discount rate greater than about 2% both options produce negative present values.

Another way in which cash flow analysis can be used in this instance is to determine the price we should charge for wrapped coin in order that the present value of cash flows at our selected discount rate is just equal to zero. This is a more general way to define a break-even point than simply assuming a zero discount rate. The problem framed in this way becomes: choose the alternative which minimizes the price per 100 wraps that we must charge in order to yield a zero present value of cash flow discounted at, say, 8%. Using the data in chart 1 and an 8% discount rate, the purchase option is superior but the present value is less than zero. If the cash flow computations were repeated using a higher price per 100 wraps, both of the present-value curves would be raised on the chart but, given the particular data in this example, they would closely maintain their relative positions. It is not difficult in principle to determine the price necessary to produce the desired zero present value.

If the selected discount rate were greater than about 10% the same methods could be employed with the result that the lease option would permit the lowest price necessary to achieve a zero present value.

Present values for the five-year horizon case are shown on chart 2. Here the purchase option clearly dominates leasing for all discount rates and in fact yields very high positive present values over a wide range of rates. The present value reaches zero at a rate of about 25%. If we accept these data as realistic, cash flow analysis shows the importance of extending the revenue-producing life of an asset if it can be done at reasonable increases in maintenance cost, and at small enough sacrifices in productivity so as not to substantially increase labor costs. Data on assumed productivity (not shown) provided for this illustration implied it would not be necessary to rely on overtime or an additional work shift even though productivity of the machines is assumed to be declining while volume is increasing. This is obviously a critical aspect of the study and in an actual application would warrant careful consideration. Fortunately with the ready availability of computers to aid in analyzing problems of this kind it is feasible in many cases to examine a large number of possible outcomes for each basic alternative. It would be possible in this example to investigate the impact not only of price, and machine replacement cycle, but also the effect of differing volume demands, different productivity assumptions, buying less equipment and running two shifts, and so on.

If this were all the information we could incorporate into the cash flow data -- hence all we could expect analysis of the data to tell us -- where would we be? If after examining additional factors we felt it most appropriate to plan on a three-year equipment replacement cycle, and if we chose a discount rate of 8% then present-value analysis would tell us to purchase the equipment. We might also reexamine the $.85 per 100 wraps price. That price has the Bank about breaking even at discount rates near zero percent but produces a fairly large deficit (-$9,430) in present value at an 8% discount rate.

If we decided to go with the five-year horizon we would clearly purchase equipment regardless of the choice of discount rate (up to some high number like about 30 or 40% anyway). Here again we may choose to adjust the price of wrapped coin, lowering it in this case to eliminate the high positive present values. Recall that the discount rate at which the present value curve crosses the axis (the rate at which the present value equals zero) is called the internal rate of return and is a measure of profitability used by many analysts to assess projects. Alternative C has an internal rate of return of about 25% and exposes the Bank to possible criticism for overpricing.

Alternatively we may wish to explore further the question of what is the best equipment replacement cycle. The optimal cycle depends on the equipment chosen and on whether it is leased or bought. Therefore the analysis would have to be repeated for each of the alternatives under consideration. The framework for study would, in the first instance anyway, have to extend the time horizon off into the far distant future, forever in fact. Summing an infinite number of cash flows will in most practical problems still yield a finite present value because for points in time far into the future the discount factors -- $1/(1+r)^t$ -- are so small that the corresponding discounted cash flows in turn become so small as to have almost no effect on the present value of the cash flow stream. Extending the time horizon to infinity obviously creates difficult problems in estimating costs, volume and other stuff necessary to perform the analysis. In practice it is possible, hopefully, to examine some shorter period of time and still get acceptably accurate information. We could, for example, try an 18-year time horizon for the three-year cycle. And for a five-year

cycle we may find it adequate to use a 20-year horizon covering four cycles. Then it may be accurate enough to compare these results to judge which machine replacement cycle offers the better alternative. Hopefully, other cycles could also be examined so that the information obtained would also be comparable.

PART IV Case Applications

Part I of this report discussed the theoretical rationale for use of cash flow analysis procedures in investment decision making. Parts II and III illustrated the basic procedures by applying them to simplified problems. The purpose of Part IV is to present more detailed case applications of decision problems faced by the Bank. A main goal will be to establish to what extent the kinds of decisions confronting the Bank can in fact be subjected to cash flow analysis. It is one thing to outline desirable decision-making techniques. It is another matter to shape the particular problems of a given institution into the form required by the decision technique and still a further problem to collect accurate data required for application of the methods. It is clear at the outset that few Bank decision problems will easily yield accurate data on benefits. It is equally clear that a certain amount of cost data can be determined for many decision problems. Only by experience will we learn the extent to which cash flow analysis can provide useful information for our decision problems. Experience and sound managerial judgment will be required to determine in each case how much of the total information relevant to the problem can accurately be captured in the cash flow data, and how much is outside the ability of mechanical techniques to correctly assess.

Part IV will evolve into a loose-leaf notebook into which new and unique or otherwise noteworthy applications can be added. This first attempt to compile applications is a beginning.

Case I Bank Automobiles; Lease or Buy

Based on experience and studies over past years Bank management has made various decisions relating to automobile transportation for staff while in the conduct of Bank business. It has been decided, for example,

that employees shall be encouraged to use Bank-owned cars when traveling on business. Studies have shown it is preferable to trade cars in when they are two years old or accumulate 30,000 miles.

The Bank maintains 11 cars for general use by staff. Since a good deal of travel involves long distance highway driving, full-size cars are preferred, with features and accessories chosen to promote safety and comfort. It is not the intent here to reexamine these decisions. The purpose rather is to illustrate a decision as to the best (least cost in present value) means to acquire the automobiles -- whether to purchase outright or to lease. Recent past data will be used to avoid the problem of predicting future auto prices and trade-in values.

Fortunately the Bank has kept extensive records on automobile expenses from which cash flow data can be constructed. The problem will be framed in terms of lease or buy one car.

Time Units and Horizon: Monthly data will be used. The time horizon is 24 months ($t = 0,1,\ldots,24$).

Expenditure Flows: Based on 1,250 miles per month (30,000 miles over a 24-month period).

1. Operating Costs and Maintenance

Operating Costs	\$ 73.00 per month
In-House Maintenance	68.00 per month
	\$141.00 per month

2. Purchase Price and Scrap Value

Purchase Price at $t = 0$	\$3,510
Scrap Value at $t = 24$	1,977

3. Lease Price is \$117 per month on a 24-month basis at about 30,000 miles. The Bank must still insure and maintain the car.

Using this information the alternative cash outflows can be constructed. Assume operating costs and maintenance expenses are paid at the end of the month. Lease payments are made at the beginning of the month. Present value of costs are given in table 2 for selected discount rates, and are plotted in figure 1.

Present value of costs for the buy alternative is only slightly affected by changes in the discount rate over the range considered. This is because the great bulk (over 70%) of the total expense is incurred at $t = 0$ which is unaffected by discounting. Lease option present values follow a more typical pattern with declining present value of costs as the discount rate is increased. At 8% it is clearly superior to buy; the present values are $4,941 for purchase and $5,616 for lease. It isn't until the discount rate exceeds 23% that the lease option becomes more desirable according to the present value of costs criterion.

A Computational Shortcut

The decision in this particular problem centers on the difference between the present value of cash outflows between two mutually exclusive alternatives (just another way of saying select the alternative with the lowest present value of cost). In this case it is acceptable to net out cost factors that are common to both alternatives. When originally framing the problem it could have correctly been argued that since the operating costs and maintenance expenses are exactly the same in either alternative they can be ignored. The resulting cash flows would have been as shown in table 3.

The present values for these cash flows (not shown) would each have been less negative than those in table 2 but the two present value lines in the graph would have maintained the same relative positions, still crossing

at just over a 23% discount rate. The decision would have been the same as that based on complete expenditure data.

Netting out common costs in this manner saves some work and is appropriate when only the difference in the present value of cost streams is of interest. There is loss of information in using computational shortcuts, however. The present value of costs for a particular alternative no longer measures total costs. Hence it is not useful as a guide in asking the question: is the present value of benefits (which are assumed not to be measurable) at least as great as the measured present value of costs? In problems presenting more than two alternatives terms netted out must be present in exactly the same amount and time pattern in each alternative cash flow. Attempts at shortcut cash flow construction expose the analysis to additional risk of mistaken judgment in framing the problem and also limit the possibility of augmenting the decision exercise by easily adding newly discovered alternatives which may not share the common term previously netted out.

The best justification for netting out cash flow terms is in cases in which it is not possible to measure the particular costs accurately. Then the cash flow analysis framework may have to be designed to permit use of a decision rule which does not depend on the unknown common cost elements.