

Federal Reserve Bank of Minneapolis
Research Department

Historical

ADVERSE SELECTION IN A MODEL OF
REAL ESTATE LENDING

V. V. Chari and Ravi Jagannathan*

Working Paper 321

Revised May 1988

NOT FOR DISTRIBUTION
WITHOUT AUTHOR APPROVAL

*Chari, Federal Reserve Bank of Minneapolis; Jagannathan, Department of Finance, University of Minnesota, and J. L. Kellogg Graduate School of Management, Northwestern University. We would like to thank Anjan Takor for useful comments. All errors are our own. This research was supported in part by the Banking Research Center, Northwestern University. Ravi Jagannathan gratefully acknowledges support from a grant by the McKnight foundation to the University of Minnesota.

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. This paper is preliminary and is circulated to stimulate discussion. It is not to be quoted without the authors' permission.

Conventional home mortgage contracts contain a vast array of contingent clauses. The most controversial of these is the due-on-sale clause that gives the lender the contractual right to declare the loan fully payable if all or part of the mortgaged property is sold. Dunn and Spatt (1985) argue that the due-on-sale clause can provide insurance to the borrower. They consider an environment where individuals face uninsurable risk in their income. If incomes are high when borrowers move, they will prefer a contract which imposes a prepayment penalty when they sell their property in return for a subsidy, perhaps in the form of lower interest rates, when they stay. Dunn and Spatt characterize the optimal contract when the income of borrowers is unobservable to the lender. However, all borrowers are ex ante identical.

In this paper we examine a market where individuals differ in the riskiness of their potential gains to selling their property. If this information is private to the borrower, the kind of insurance arrangement described above must be modified to deal with the obvious selection problems that arise. Any lender must therefore ensure that any insurance contract offered to individuals with riskier income streams does not offer so much insurance at such a favorable price that individuals with less risky income also accept the same contract. The classic analysis of this type of problem is by Rothschild and Stiglitz (1976) and our analysis follows theirs. In order to bring attention to bear on the nature of the adverse selection problem, we assume two types of individuals. Both types have the same expected income stream. For simplicity we assume that type I individuals do not move. Type II individuals must move with some probability. Future income is random for all individuals but those type II individuals who move receive on average a higher income than those who do not move. Clearly not all moves are accompanied by a rise in income and our model reflects this fact. We show that when the types of the

agents are known to the bank, the optimal contract provides the same initial debt level to both types accompanied by a prepayment penalty for type II individuals who move and a bonus for type II individuals who do not move. If types are not known to the bank, then the type I agents would purchase this contract and the bank would suffer a loss. We model the nature of the optimal contract with adverse selection and show that the optimal contract provides partial insurance.

The motivation behind this analysis is explaining the existence of "points" on mortgage loans. Banks and savings and loan associations charge fees which depend upon the size of the debt incurred and generally form a sizable portion of the face value of the debt. We show that when prepayment penalties are prohibited, a combination of points and a lower interest rate on the loan can provide insurance even in the presence of adverse selection. We also show that points will be used if banks cannot make interest rates contingent on the amount borrowed.

Traditional explanations for points rely upon the option value of fixed interest rate loans. If interest rates rise, then the bank suffers a capital loss, while if interest rates fall, the borrower refinances the loan and the bank does not receive a corresponding capital gain. By charging a fee, the bank recovers this option value. In our view, this analysis is deficient because an alternate means of capturing this option value to the borrower is through a higher interest rate. A higher interest rate is likely to be preferred for two reasons. First, the value of the prepayment option is more for those borrowers who are more likely to stay in one place and hence prepay only when interest rates fall. Those who are more likely to move will value the prepayment option relatively less. Capturing the option value by a higher interest rate will, to some extent, take this into account because the

higher interest rate will be paid only so long as the loan is outstanding. Second, for a large class of borrowers, income levels will rise over time and the marginal utility of consumption will decline. One would expect that such individuals would prefer to see their gains through fixed rate contracts captured through higher interest rates than through points.

2. An Adverse Selection Model

The economy consists of a large number of consumers and a bank. Consumers are infinitely lived and have von-Neumann-Morgenstern utility functions defined over an infinite-dimensional commodity space:

$$U(c_0, c_1, \dots) = \sum_{t=0}^{\infty} \beta^t U(c_t), \quad 0 < \beta < 1.$$

where U is strictly increasing, strictly concave, bounded and twice continuously differentiable.

Endowments

All consumers have a fixed endowment e in period 0. Future income is random. Consumers are of two types. Type I agents receive a random endowment in period 1 which remains constant at that level thereafter. Let z denote the present value of income from period 1 onwards. The cumulative distribution function from which the present value of type I individuals' income is drawn is denoted by $F(\cdot)$. Type II agents first receive a signal $\theta \in \{1, 2\}$. The probability that $\theta = 1$ is π , and the probability that $\theta = 2$ is $(1-\pi)$. If $\theta = 1$, then the type II agent must move and if $\theta = 2$ the agent stays. Conditional on the signal θ , the present value of type II agents' income is drawn from a distribution function $G(\cdot|\theta)$. Income in periods 2, 3, ..., is the same as in period 1. We assume that if type II agents move their income is on average higher than the income of type I agents. However, if

type II agents stay, their income is on average lower than type I agents' income. We assume that type I agents do not move. Formally, we assume that

$$(2.1) \quad G(z|i) = F(z-\delta_i), \quad i = 1,2, \quad \text{and} \quad \pi\delta_1 = (1-\pi)\delta_2.$$

It follows from (2.1) that the income distribution of type II agents is a mean preserving spread of the income distribution of type I agents. This way of modeling random future income recognizes that some individuals move when their incomes are low (say due to divorce) and some when their incomes are high (say for a better job). However, we assume that on average those who move have higher incomes than those who stay. All our results go through if we allow type I agents to move as long as the incomes of type I agents is uncorrelated with the moving decision and the probability of a move is small.

Banks and Markets

The bank is risk-neutral. All individuals and the bank can borrow or lend at a given, constant interest rate r . There is no interest rate uncertainty in the economy under consideration.

Information

Technology, preferences and the interest rate are common knowledge. However, only individuals know their own type as of period 0. The bank does not have access to this information. In period 1, the endowments are realized. We assume that these endowments are private information. What is observed is whether a type II individual moves or not. The bank can make contracts contingent on whether the individual moves or not but not on the realized value of the endowment.

Restrictions on Contracts

In this section we consider contracts where prepayment penalties are prohibited by law. In section 3 we consider an environment where the interest rate cannot depend upon the amount borrowed but there are no restrictions on prepayment penalties.

It is useful at this stage to define the indirect utility functions over wealth from period 1 onwards which we will use to characterize the nature of the optimal contract. Let y be the wealth level (present discounted value of income) of an individual who chooses an infinite consumption stream to maximize the sum of discounted utility. Let $V(y,r)$ be the associated indirect utility function. Then, we have

$$V(y,r) = \max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} \leq y.$$

We assume that the indirect utility function displays decreasing absolute risk aversion. Formally,

$$(2.2) \quad V_1(y,r) \text{ is convex in } y.$$

Characterization of the Optimal Contract

Consider first the problem of a type I agent. Let the present value of the income from period 1 onwards be denoted by z . The type I agent decides the amount to be borrowed, B_1 , from the bank by solving

$$\max_{B_1} U(B_1) + \beta \int V(z - (1+r)B_1) dF(z)$$

where we have suppressed the period zero endowment, e , for convenience.

The first order condition is

$$(2.3) \quad U'(B_1) = \beta(1+r) \int V'(z - (1+r)B_1) dF(z).$$

The problem faced by the type II agent is more complicated. We start by assuming that the type of the agent is known to the bank. Furthermore, the bank can observe whether the type II agent moves or not. However, the contract cannot be made contingent upon the realized value of the income. Thus, the problem is to provide insurance to the type II agent given that only the signal θ is observed. Let x denote the amount borrowed by the type II agent in period 0. Let y denote the present value in period 1 of the repayments to the bank in the event that $\theta = 1$ and w the present value of repayments in the event that $\theta = 2$.

The problem of designing the optimal contract is then

$$(2.4) \quad \max U(x) + \beta \left[\pi \int V(z-y) dG(z|1) + (1-\pi) \int V(z-w) dG(z|2) \right]$$

subject to

$$(2.5) \quad x \leq \frac{\pi y}{(1+r)} + \frac{(1-\pi)w}{(1+r)}.$$

Equation (2.9) is the bank's break-even constraint. The right side is the expected return from the loan and the left side is the amount of the loan.

The first order conditions are

$$(2.6) \quad U'(x) = \beta(1+r) \int V'(z-y) dG(z|1)$$

$$(2.7) \quad U'(x) = \beta(1+r) \int V'(z-w)dG(z|2).$$

We now establish that under the optimal contract, the type II agent borrows the same amount as the type I agent.

Proposition 1

If types of agents are known, then the amount borrowed by the type II agent, x , is equal to the amount borrowed by the type I agent, B_1 .

Proof: From (2.6) and (2.7) using (2.1) we get

$$\int V'(z-y)dF(z-\delta_1) = \int V'(z-w)dF(z+\delta_2).$$

It immediately follows by a change of variables that

$$\int V'(z-(y-\delta_1))dF(z) = \int V'(z-(w+\delta_2))dF(z).$$

Hence, we have

$$(2.8) \quad y - \delta_1 = w + \delta_2.$$

Substituting (2.8) in (2.6) and using (2.1) we get

$$(2.9) \quad x(1+r) = w + \delta_2 = y - \delta_1.$$

Substituting (2.9) into (2.6) and changing variables again we get that the amount borrowed by the Type II agent satisfies

$$U'(x) = \beta(1+r) \int V'(z-x(1+r))dF(z).$$

This is identical to the first order condition for the type I agent given in (2.7). \diamond

We now show that the optimal contract for the type II agent can be supported by observed contractual arrangements such as points and prepayment penalties together with a "due-on-sale" clause. Let the prepayment penalty per unit of debt be denoted by p , the "points" per unit borrowed by D , the amount borrowed by B and the interest rate on the loan by R . With a due-on-sale clause, in the event that the borrower moves he must pay off the loan together with any prepayment penalty. The amount paid is then given by $B(1+R+p)$. In the event the borrower does not move, the present value of the payments to the bank is $BR(1+r)/r$. The net amount borrowed is $B(1-D)$. Thus, we set $x = B(1-D)$, $y = B(1+R+p)$, $w = BR(1+r)/r$.

We can solve for B , D , R , and p given x , y , and w . Obviously there is some indeterminacy here because we have four variables and three equations. There is no compelling reason to argue for points as opposed to a prepayment penalty. Of course, if prepayment penalties are prohibited by law, then points must be positive.

We now examine the nature of the optimal contract when the bank cannot determine the type of the individual. In such a case, the optimal contract must also satisfy a self-selection constraint given by

$$(2.10) \quad U(B_1) + \beta \int V(z - B_1(1+r))dF(z) \geq U(x) + \beta \int V(z - w)dF(z).$$

The left side of (2.10) is the utility of the type I agent when he purchases a contract with no points or prepayment penalties. The right side is the utility of the type I agent when he purchases the contract offered to the type II agent.

An optimal contract with adverse selection solves (2.4) subject to (2.5) and (2.10) where B_1 solves (2.3).

We now show that (2.10) must be binding in an optimal contract.

Proposition 2

If banks are unable to discern the type of the consumer-borrower, then constraint (2.10) must be binding in any solution to the optimal contract problem.

Proof: Suppose (2.10) is not binding. The solution to the optimal contract must then be the same as when types are known. From Proposition 1 we then have that $x = B_1$. Furthermore, from (2.9) we have that $w < x(1+r)$. Hence, (2.10) is violated. \diamond

We have assumed that type I agents do not move. This assumption can be relaxed without great difficulty. Proposition 2 continues to hold as long as the income of type I agents is uncorrelated with a decision to sell their property and the probability of a move is sufficiently small.

We now show that the optimal contract with adverse selection provides insurance though less than when types are known.

Proposition 3

The optimal contract with adverse selection is characterized by $y > x(1+r)$ and $w < x(1+r)$.

Proof: We first show that $w < y$. From the bank's break-even constraint it is clear that this implies the proposition. The proof is by contradiction. Consider first the case where $w = y$. Consider the problem of maximizing (2.8) given $w = y$. Denote the solution to this problem by \hat{x} . This solution must satisfy

$$U'(\hat{x}) = \beta(1+r) \left[\pi \int V'(z - \hat{x}(1+r)) dG(z|1) + (1-\pi) \int V'(z - \hat{x}(1+r)) dG(z|2) \right].$$

From our assumption that $V'(\cdot)$ is convex (see (2.5)) and that the income distribution of the type II agent is a mean preserving spread of the type I agent's distribution (see (2.1 and (2.2)) we have that

$$U'(\hat{x}) > \beta(1+r) \int V'(z-(\hat{x})(1+r))dF(z).$$

Because B_1 solves (2.3), we have that $\hat{x} < B_1$. Hence, (2.10) holds with strict inequality. Hence the solution \hat{x} is feasible for the optimal contracting problem. But this contradicts Proposition 2. Therefore $w \neq y$.

Suppose now that $w > y$. Consider an alternative solution given by $\hat{w} = \hat{y} = x(1+r)$. We claim that (2.10) is not violated. This follows because the left side of (2.10) maximizes type I agent's utility when the interest rate is r . Note that this solution transfers income actuarially fairly (recall the bank's break-even constraint) from high income to low income states. Hence, utility is strictly higher. \diamond

It is useful again to represent the optimal contract in terms of observed institutional arrangements such as points and prepayment penalties. We use Proposition 3 to show that if there is a legal constraint prohibiting prepayment penalties then points are necessarily positive. From (2.18) using Proposition 3 we have that

$$w = \frac{BR(1+r)}{r} < B(1-D)(1+r), \text{ and } y = B(1+R+p) > B(1-D)(1+r).$$

Thus we have that

$$(2.11) \quad \frac{R}{r} < (1-D) < \frac{(1+R+p)}{1+r}.$$

Suppose now that prepayment penalties are forbidden by law. Then $p \leq 0$. From (2.11) it follows that $R(1+r) < (1+R)r$. Hence, $R < r$ and $D > 0$. We have proved

Proposition 4

If prepayment penalties are constrained to be nonpositive, then in an optimal contract, points are positive and the interest rate on the loan is less than the market rate.

3. The Optimal Contract With No Restrictions on Prepayment Penalties

A central assumption of the model described in section 2 is that the bank can observe the amount borrowed and make the interest rate a function of the amount borrowed. In fact, some restriction on the face value of the debt seems essential to ensure that adverse selection does not occur. The scheme described in the previous section cannot function at all if there is no constraint on the amount that can be borrowed by Type I agents. They will then borrow the amount required for the points from some other source and repay the creditor at the market interest rate. It is easy to see that this would make them better off.

We now characterize the optimal contract assuming that the interest rate charged by the bank, R , cannot depend on the amount borrowed. We assume that the bank can impose a constraint on the total amount borrowed, which we will call \bar{B} . Presumably the maximum amount of the face value of the debt, \bar{B} , is determined by considerations of bankruptcy. For our purposes, we will regard \bar{B} as determined outside the model. For convenience we assume that \bar{B} is equal to the amount borrowed by the type I individual. Recall that this equals B_1 . Our argument goes through unchanged if $\bar{B} > B_1$ as long as it is not too much larger. Now the programming problem solved by the bank can be rewritten as follows.

$$(3.1) \quad \max U(B(1-D)) + \beta \left[\pi \int V(z-B(1+R+p))dG(z|1) \right. \\ \left. + (1-\pi) \int V(z-BR(1+r)/r)dG(z|2) \right]$$

subject to

$$(3.2) \quad (1-D) \leq \pi \left(\frac{1+R+p}{1+r} \right) + (1-\pi)/R/r$$

$$(3.3) \quad U(B_1) + \beta \int V(z - B_1(1+r)) dF(z) \geq \max_{x \leq B_1} \left[U(x(1-D)) + \beta \int V\left(z - \frac{xR(1+r)}{r}\right) dF(z) \right]$$

$$(3.4) \quad B \leq B_1.$$

Note that in (3.3) we have incorporated the idea that the interest rate R charged by the bank is independent of the amount borrowed B as long as $B \leq B_1$. The value of B_1 is determined by (2.3).

We now prove that the optimal contract which solves the programming problem (3.1) above will necessarily involve positive points.

Proposition 5

The solution to the programming problem (3.1) is characterized by $D > 0$, and $R < r$.

Proof: The objective function (3.1) is continuous and the constraint set is compact. We show below that the constraint set is nonempty. Hence a solution exists. The simplest feasible solution is given by $D = 0$, $p = 0$, and $R = r$. We will call this contract the simple contract. Let the solution to the maximization problem (3.1) given the simple contract be denoted by B^* , where B^* solves

$$(3.5) \quad U'(B^*) = \beta(1+r) \left[\pi \int V'(z - B^*(1+r)) dG(z|1) + (1-\pi) \int V'(z - B^*(1+r)) dG(z|2) \right].$$

Using (2.2), and the assumption that $V'(\cdot)$ is convex, (3.5) implies

$$(3.6) \quad U'(B^*) > \beta(1+r) \int V'(z - B^*(1+r)) dF(z).$$

Using (2.3) it follows that $B^* < B_1$. Hence this contract is feasible.

We now show that there exist contracts which yield a higher level of utility to the type II individual than the utility obtained under this simple contract.

Suppose the bank offers the following contract in addition to the simple contract. Let $(1-D) = B^*/B_1$, $R = rB^*/B_1$, and $p = B^*/B_1 - 1$.

We have constructed this contract so that the utility level of the type II individual when $B = B_1$ is the same as the utility attainable by choosing the simple contract. It can be verified that the bank's break even constraint (3.2) is satisfied. We now show that the adverse selection constraint (3.3) holds with strict inequality. Consider the right side of (3.3). This is given by

$$\max_{x \leq B_1} U(x(1-D)) + \beta \int V(z-xR(1+r)/r)dF(z).$$

Note that $R < r$ and D is positive. Hence $B = B_1$. Consequently the type I individual's utility if he chooses the proposed contract is given by

$$U(B^*) + \beta \int V(z-B^*(1+r))dF(z).$$

Because $B^* < B_1$, the adverse selection constraint (3.3) holds with strict inequality. Because (3.1) through (3.4) are all continuous in B , D , R , and p , there exists some contract which increases the utility of the type II agent when compared to the simple contract without violating the adverse selection constraint.

We now show that D is strictly positive. Suppose $D = 0$. Then it must be the case that $R \leq r$. For suppose that $R > r$ and $D = 0$. Then from the bank's break even constraint, p is negative. Any such contract can be improved by setting $R = r$ and $p = 0$. Hence $R \leq r$. But any contract with $R < r$ and $D = 0$ must violate the incentive constraint. Hence either $R = r$ and $D = 0$

or $R < r$ and $D > 0$. We have already shown that $R = r$ and $D = 0$ can be improved upon. Hence $D > 0$. \diamond

It is possible that the prepayment penalty which solves the problem above is negative. In such a case, the bank provides a prepayment bonus. The essential point of this analysis, however, is to stress the fact that if the interest rate cannot depend upon the amount borrowed, then points must be positive even if there are no legal constraints on prepayment penalties.

4. Conclusions

We have attempted to explain the puzzling institution of points which are frequently found in home mortgage loans. Our analysis builds on two key features. First, insurance markets are unavailable for labor income. Second, the due on sale clause allows banks to offer loan contracts that provide partial insurance against fluctuations in labor income. In our environment, banks offer loan contracts at the nominal rate below the market rate and points serve the role of prepayment penalties. The optimal contract reflects the probability of moving and the gain to moving. Our analysis therefore suggests that a variety of loan contracts with differing points-interest rate combinations will be observed in the market place. Several states impose restrictions on points as well as prepayment penalties. In our model such restrictions lead to Pareto inferior allocations.

Our results should not be misconstrued to imply that those who plan to stay in a given house only for a short period of time will accept loans with high points. Obviously this is not true. In our model those who anticipate large wage increases in the event of a move are better off by accepting a loan with points. The model can be easily adapted to allow for all agents to move. The essential feature is that those whom we label type II individuals

must be interpreted as those who receive relatively large increases in labor income when they move. Of course, some people suffer a decline in their income when they move. Our model only requires that on average type II agents are better off when they move. A key feature is that contracts cannot be made contingent upon future income. The same considerations that prevent labor markets from providing insurance, such as moral hazard, will prevent loan contracts from providing perfect insurance.

The results in this paper suggest that the value of the option to prepay the loan may not be the sole reason we observe points. We would, for example, predict that even with adjustable rate loans, contracts with points will be offered.

The issues discussed in this paper are closely related to the literature on credit rationing (Stiglitz and Weiss 1981). If individuals could borrow at the market rate to pay the points on the loan contract, then in equilibrium contracts with points will not be offered. An essential component of our analysis is that the debt equity ratio of the borrower be observable to the bank. This observability allows the bank to offer different contracts to different borrowers.

Avenues for further research include attempts to confront the model with observations and extensions of the model to include default risk and the effect of fluctuations of interest rate over time.

References

- Dunn, K. B., and Spatt, Chester. 1985. "Prepayment Penalties and the Due-On-Sale Clause," Journal of Finance 40, pp. 293-308.
- Rothschild, M., and Stiglitz, J. 1976. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," Quarterly Journal of Economics 90, pp. 629-50.
- Stiglitz, J. E., and Weiss, A. 1981. "Credit Rationing in Markets With Imperfect Information," The American Economic Review 71, pp. 393-410.