



## **International Evidence on Long-Run Money Demand**

Luca Benati

University of Bern

Robert E. Lucas, Jr.

University of Chicago

Juan-Pablo Nicolini

Federal Reserve Bank of Minneapolis and Universidad Di Tella

Warren Weber

University of South Carolina, Bank of Canada,  
and Federal Reserve Bank of Atlanta

**Staff Report 587**

**June 2019**

DOI: <https://doi.org/10.21034/sr.587>

Keywords: Long-run money demand; Cointegration

JEL classification: E41, C32

An earlier version of this staff report circulated as Working Paper 737. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

---

Federal Reserve Bank of Minneapolis • 90 Hennepin Avenue • Minneapolis, MN 55480-0291

<https://www.minneapolisfed.org/research/>

# International Evidence on Long-Run Money Demand\*

Luca Benati

University of Bern

Robert E. Lucas, Jr.

University of Chicago

Juan Pablo Nicolini

Federal Reserve Bank of Minneapolis and Universidad Di Tella

Warren Weber

University of South Carolina, Bank of Canada,

and Federal Reserve Bank of Atlanta

## Abstract

We explore the long-run demand for M1 based on a dataset comprising 38 countries and relatively long sample periods, extending in some cases to over a century. Overall, we find very strong evidence of a long-run relationship between the ratio of M1 to GDP and a short-term interest rate, in spite of a few failures. The standard log-log specification provides a very good characterization of the data, with the exception of periods featuring very low interest rate values. This is because such a specification implies that, as the short rate tends to zero, real money balances become arbitrarily large, which is rejected by the data. A simple extension imposing limits on the amount that households can borrow results in a truncated log-log specification, which is in line with what we observe in the data. We estimate the interest rate elasticity to be between 0.3 and 0.6, which encompasses the well-known squared-root specification of Baumol and Tobin.

---

\*We thank Francisco Buera, Giuseppe Cavaliere, V.V. Chari, Jesus Fernandez-Villaverde, Patrick Kehoe, Tim Kehoe, Helmut Luetkepohl, Albert Marcet, Rodolfo Manuelli, Edward Nelson, B. Ravikumar, Pedro Teles, and Nancy Stokey. Special thanks to Joao Ayres, Nicole Gorton, Ruth Judson, Eva Werner, and Junji Yano for invaluable help with the data. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

# 1 Introduction

This paper describes and analyzes a new dataset containing annual measurements of money supplies, both real and nominal output (GDP) and thus price levels, and short-term nominal interest rates for 38 countries, for periods that go from three decades to over a century. The framework we use for organizing these data is a money demand function that relates the money that the public and private sectors of the economy choose to hold to the rate of production of goods and the short-term interest rate,

$$M_t = P_t y_t \phi(r_t), \tag{1}$$

where  $\phi$  is a decreasing function of  $r_t$ .

The formula (1) contains some strong implications. One is that, if  $r_t$  is stationary,  $M_t$  and  $P_t y_t$  should grow at common rate in the long run, for any continuous function  $\phi$ . If, on the other hand,  $r_t$  has a unit root—possibly because inflation is driven in part by permanent shocks—then  $M_t$  and  $P_t y_t$  should grow at common rate in the long run, once controlling for the impact of permanent shocks to  $r_t$ . Another implication is that it should be possible to use both cross-country and within-country time series to trace out the function  $\phi$ . This is the agenda carried out in this paper.

In recent years, many economists and central bankers have come to doubt the usefulness of measures of the money supply (such as M1) in the conduct of monetary policy. What was thought to be a central pillar of the monetary policies of the newly established European Central Bank in 1999 has come to be seen as too unreliable to be of any use. These concerns were not without empirical basis. Conventional time series models of money demand could be unstable, especially at high frequencies. Seeking “long run” relations seemed to be the only course. Our idea here is to utilize the methods of cointegration to make precise what we mean by “long-run” relations and to apply these methods in a uniform way to a very wide variety of countries.

We take pains to ensure that terms such as “short-term interest rate” and “money” are measures of the same thing (or almost!) in different countries and over time within countries. The set of countries is very heterogeneous in terms of size, income per capita, and world region. More importantly, the countries also differ substantially in terms of their monetary history: our sample includes countries that experienced hyperinflations, together with countries in which inflation has been almost always within a single digit. The periods covered include very different growth experiences, different monetary

arrangements, and different degrees of integration to the world markets. We will explicitly ignore all those differences and will look at this diverse set of countries through the lens of an extremely simple model. The high degree of variation in nominal interest rates across countries and over time within each country is what we will exploit in building our case that the basic features of the demand function for money are in general very solid—perhaps the most solid findings in macroeconomics.

A particular expression for the function  $\phi(r_t)$  is the well-known squared root formula that Baumol and Tobin derived over half a century ago. In this paper, we use a theory that generalizes this Baumol-Tobin expression along a couple of dimensions. The first generalization allows for a technology to transform bonds into money that encompasses the linear technology assumed by Baumol and Tobin, but that also allows for non-linear relationships. The second generalization is the consideration of borrowing constraints, that affects the behavior of money demand at very low interest rates. In the empirical section we show that such a theory matches the behavior of the data remarkably well: out of the 38 countries, we declare just 4 blunt failures and 4 puzzling cases.

In his 2009 paper “Long Run Evidence on Money Growth and Inflation,” Benati used a band-pass filter in order to illustrate how the low-frequency components of money growth and inflation exhibit, in most cases, a nearly one-for-one relationship. The current paper goes beyond Benati (2009) in a few dimensions, but two are really central. First, in that paper, both M1 growth and inflation were treated simply as given time series. In this paper, we borrow from a vast post-World War II literature and take the interest rate as a given series — chosen by monetary policy — and assume that individual agents divide their work effort between producing goods and economizing in holding low-interest-bearing cash.<sup>1</sup> We address these elements of consumers’ decision problems in detail in Section 2, where we derive an equation like (1) that generalizes the familiar Baumol-Tobin specification. In Section 3 we plot the implied predictions of a particular case of the model against the data and let the graphics speak for themselves. We find this informal visual evidence quite remarkable. As mentioned above, in this paper we make central use of methods of cointegration. These methods replace the sharp distinctions between the high and low frequencies used in Benati (2009). They are

---

<sup>1</sup>That literature was led by Friedman, Schwartz, Brunner, Meltzer (1963), Baumol (1952) and Tobin (1956).

described in Sections 4 and 5, where we discuss the methodology that we use throughout the paper and discuss the results from cointegration analysis. Our results show that evidence of cointegration between real money demand and the short-term interest rate is widespread. In most cases in which cointegration is not detected, we show via Monte Carlo evidence that - conceptually in line with Engle and Granger (1987) - this result is what we should expect to obtain even if cointegration is truly in the data, because of the short sample length, the high persistence of the cointegration residual, or both. In Section 6, we discuss some extensions and section 7 concludes.

## 2 A Model of Money Demand

We study a labor-only, representative agent economy in which making transactions is costly. Preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t),$$

where  $\beta < 1$ ,  $x_t$  is consumption at date  $t$ , and the function  $U$  is differentiable, increasing, and concave. The agent is endowed in each period with a unit of time, with  $l_t$  allocated to goods production and  $1 - l_t$  used to carry out transactions. The goods production technology is given by  $y_t = x_t = z_t l_t$ , where  $z_t$  is an exogenous stochastic process.

We assume that households choose the number  $n$  of “trips to the bank” in the manner of the classic Baumol-Tobin model. At the beginning of a period, a household begins with some nominal wealth that can be allocated to money  $M_t$  or to risk-free government bonds  $B_t$ . During the first of the  $n$  subperiods, one member of the household uses money to buy consumption goods. During this same initial subperiod, another member of the household produces and sells goods in exchange for money. At the end of the subperiod, producers transfer to the bank the proceeds from their transactions. Thus, the situation at the beginning of the second subperiod exactly replicates the situation at the beginning of the first. This process is repeated  $n$  times during the period. The choice of this variable  $n$  will be the only economically relevant decision made by households. Purchases over a period are then subject to a cash-in-advance constraint,  $P_t x_t \leq M_t n_t$ .

Notice that  $n$  is the velocity of money, and its inverse in equilibrium is the money-to-output ratio, or the demand for real money, which is the concept

that we care about. Baumol-Tobin assumed that the cost of carrying out these transactions increases linearly in the number of trips. We consider a more general specification in which the total cost of making transactions, measured in units of time, is given by

$$\theta(n_t, \nu_t) = \gamma n_t^\sigma \nu_t, \quad (2)$$

where  $\gamma$  and  $\sigma$  are positive constants and  $\nu_t$  is an exogenous stochastic process. The natural interpretation of the stochastic shock  $\nu_t$ , is aggregate disturbances in intermediation technologies. The theory is of course silent with respect to it. However, this random component is essential to motivate the econometric analysis at the core of the paper. The expression in (2) becomes the Baumol-Tobin linear case when we set the curvature parameter  $\sigma$  equal to 1.

Equilibrium in the labor and goods markets implies

$$x_t = z_t l_t = z_t(1 - \gamma n_t^\sigma \nu_t),$$

so the equilibrium real wage is equal to  $z_t$ .

At the beginning of each period, the agent starts with wealth in real terms  $w_t$ , which can be allocated to money  $m_t$  or interest-bearing bonds  $b_t$ , both also measured in real terms. We can then write this constraint as

$$m_t + b_t \leq w_t. \quad (3)$$

In addition, we impose a productivity-adjusted borrowing constraint for the agent of the form

$$b_t \leq z_t b^* \quad (4)$$

for some arbitrary value of  $b^*$ .<sup>2</sup> At this point, we do not want to take a particular stand on where this constraint comes from. Rather, we want to show how the equilibrium money demand relationship is affected by a constraint of this type. We will discuss these implications in detail below.

We assume the nominal return on short term bonds,  $r_t$ , to be a process determined by monetary policy.<sup>3</sup> This implies that the behavior of the growth

---

<sup>2</sup>Our dataset contains long samples of over a century for a few countries. All of the countries had substantial increases in productivity over the length of the sample. We find it natural to assume that the borrowing constraint depends on the level of technology.

<sup>3</sup>When policy is described as a sequence of interest rates, the initial price level may be indeterminate. Real money balances will, however, be unique. In this paper we ignore issues regarding the determination of the price level.

rate of the money supply is restricted by other equilibrium conditions, as is well known and as we show in Online Appendix B. So far, we have been silent with respect to what our measure of money accounts for. For the theoretical analysis, one can allow for money to pay a nominal return,  $r_t^m$ .<sup>4</sup> In what follows, we will just consider the case in which  $r_t^m = 0$ . We discuss this choice below, together with the discussion of the empirical counterparts of the monetary aggregate.

The agent's wealth next period, contingent on the actions taken in the current period and the realization of the exogenous shock, is given by

$$w_t^{t+1} \leq \frac{m_t + b_t(1 + r_t) + [1 - \gamma n_t^\sigma \nu_t] z_t - x_t}{\pi_t^{t+1}} + \tau_t^{t+1}, \quad (5)$$

where  $\pi_t^{t+1}$  denotes the gross inflation rate between period  $t$  and period  $t + 1$  in that particular state, and  $\tau_t^{t+1}$  is the real value of the monetary transfer the government makes to the representative agent. Finally, the cash-in-advance constraint can be written in real terms as

$$x_t \leq m_t n_t. \quad (6)$$

We now consider the decision problem of a single, atomistic agent who takes as given the price level, the inflation rates  $\pi_t^{t+1}$ , the interest rate  $r_t$ , the real wage  $z_t$ , and the shock  $\nu_t$ . Given the initial wealth  $w_t$ , this agent chooses his consumption  $x_t$ , the number of bank trips  $n_t$ , and the assets  $m_t$ ,  $b_t$  that he chooses to hold. These choices then determine the wealth  $w_t^{t+1}$  that he carries into the next period, as a function of the state. These choices are restricted by equations (3), (4), (5), and (6).

In Online Appendix B, we show that as long as the borrowing constraint (4) does not bind, the equilibrium number of portfolio adjustments  $n$  depends on the interest rate  $r_t$  according to

$$r_t = \sigma \gamma n_t^{\sigma+1} \frac{\nu_t}{1 - \gamma n_t^\sigma \nu_t}. \quad (7)$$

The solution involves an extended quadratic formula for the equilibrium value of  $n$ . Using the cash-in-advance constraint (6), the following relationship must hold in equilibrium:

---

<sup>4</sup>That more general case and its details are presented in the working paper version of the Benati et al. (2017).

$$\frac{m_t}{x_t} = \frac{1}{n_t} = \frac{1}{n(r_t, \nu_t)}. \quad (8)$$

This solution has several empirical implications. First, notice that the solution for the money-to-output ratio does not depend on the technology level,  $z_t$ . In the model, income growth must be associated with a positive trend in technology. This implies an income elasticity of the real demand for money that is equal to one, which is the specification we will study.<sup>5</sup> Second, the stochastic properties of the money-to-output ratio,  $m_t/x_t$ , are inherited from the stochastic properties of  $r_t$  and  $\nu_t$ . This has testable implications as long as  $\nu_t$  is stationary, as we will assume throughout the paper. Specifically, if  $r_t$  is stationary,  $m_t/x_t$  should be too, whereas if  $r_t$  has a unit root,  $m_t/x_t$  should have a unit root as well. Our cointegration analysis below will address these issues.

## 2.1 Characterization of the solution

We first characterize the solution for the case in which the borrowing constraint (4) is not binding. Then, we provide a general characterization.

### 2.1.1 When the borrowing constraint does not bind

A simple inspection of equation (7) reveals that velocity is an increasing and continuous function of the interest rate differential  $r_t$ . As (8) makes clear, the ratio of money to income is then negatively related to  $r_t$ .

In order to obtain a simple parametric form that we can take to the data, we discuss one approximation. Note that  $\gamma\nu_t n_t^\sigma$  represents the welfare cost of inflation as a ratio of maximum potential output, which is arbitrarily close to zero when the interest rate  $r_t$  is small. For moderate interest rates, computations of the welfare cost are negligible. Even for interest rates as high as 20%, estimates of the welfare cost of inflation are barely above 4%, so the denominator in the expression above would range from 1 to 0.96.<sup>6</sup>

---

<sup>5</sup>In Online Appendix B, we allow for a more general specification that does not restrict the income elasticity to be one, and where we are able to test this unitary income elasticity implication.

<sup>6</sup>The approximation error in a model calibrated to match US data is very small, below 2%, even for interest rates as high as 50% per year.

We then use the approximation  $1 - \gamma\nu_t n_t^\sigma \simeq 1$  and write the solution as  $n_t^{\sigma+1} \sigma \gamma \nu_t \simeq r_t$ . Taking logs and using the fact that  $m/x = 1/n$ , we then obtain

$$\ln(m_t/x_t) = \frac{1}{(\sigma+1)} [\ln \sigma \gamma - \ln r_t + \ln \nu_t], \quad (9)$$

which is the log-log function typically used in the literature, with an interest rate elasticity of  $1/(1+\sigma)$ . The Baumol-Tobin case is the one obtained when  $\sigma = 1$ , so the elasticity is equal to  $1/2$ .

Notice that a property of this specification is that real money balances over output,  $m_t/x_t$ , go to infinity when the nominal interest rate differential,  $r_t$ , goes to zero. How can this be a solution for a representative agent with finite wealth? Inspecting the budget constraint (3) suggests that bond holdings must therefore be approaching negative infinite. But this may conflict with the borrowing constraint (4). Imagine that agents could run away with the cash they hold and keep some fraction of it. The borrowing constraints would naturally arise from an optimal contracting problem with enforceability constraints. We next turn to considering the case of the binding constraints.

### 2.1.2 When the borrowing constraint binds

When the borrowing constraint binds, the solution is trivial, since there is really no economic problem to be decided by the agent. Note that  $\lambda > 0$  implies that  $b = z_t b^*$ . But then, the budget constraint (3) implies

$$m_t = w_t - z_t b^*,$$

which fully determines the real quantity of money. The values for  $x_t$  and  $n_t$  are then determined by the equilibrium conditions  $x_t = z_t(1 - \theta(n_t))$  and  $x_t = n_t m_t$ .

### 2.1.3 A full characterization

In order to provide a full characterization of the relationship between money balances to output and the interest rate differential, given any value for the state variable  $\omega$ , it is useful to prove the following lemma.

**Lemma 1** *Given values for  $\omega$  and  $b^*$ , if constraint (4) binds for an interest rate differential  $r_0$ , then it also binds for any  $r < r_0$ . In addition, if constraint*

(4) is not binding for an interest rate differential  $r_1$ , then neither does it bind for any  $r > r_1$ .

The previous lemma implies that for any value of real initial wealth,  $w$ , there exists a value for the net interest rate  $\tilde{r}$  such that the money demand equation is given by the solution to (7), for all  $r \geq \tilde{r}$ , whereas it is equal to  $\frac{m}{x}(\tilde{r})$  for all  $r \leq \tilde{r}$ . Such a money demand is depicted as the dotted blue line, labeled A in Figure 1.

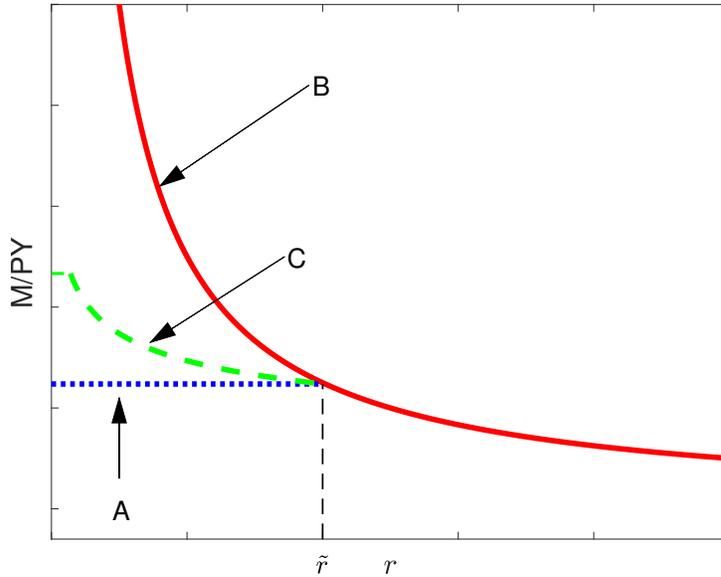


Figure 1 Borrowing constraint and money demand when interest rate is near 0

The location of the kink in the money demand depends on the value for  $w - zb^*$ . The larger that value, the lower will be the value for the interest rate  $\tilde{r}$ . Note also that if we let  $b^*$  go to infinity, real money balances also go to infinity as the interest rate goes to zero. Such a money demand is depicted as the solid red line, labeled B in Figure 1.

The log-log specification, widely used in applied work, assumes that agents, in equilibrium, do borrow arbitrarily large amounts from the government so as to hold arbitrarily large amounts of cash when the interest rate is very low. Such an outcome could not be an equilibrium if agents could run away with a fraction of the cash held, which can lead to a borrowing constraint that could resemble restriction (4). As it turns out, interest

rates had been very low for several years in some of the countries that we analyze. The role of this borrowing constraint will be very important in designing our empirical strategy, as discussed in detail in what follows. But this sharp characterization at near-zero interest rates depends on the representative agent assumption, and it is not robust to sensible generalizations. Thus, we may not want to take the flat portion of the money demand in schedule A of Figure 1 literally when pursuing our empirical analysis. We discuss this issue next.

#### 2.1.4 Connecting the theory to the data

Consider a model like the one above, with a continuum of agents that are alike in all respects except that they differ in their productivity. To be more specific, assume that idiosyncratic productivity for agent  $j$  is equal to  $\xi^j z_t$ , where  $\xi^j \in [\xi^l, \xi^h]$ , and where the mean of  $\xi^j$  is equal to one. It would be natural in this environment to impose agent-specific borrowing constraints, since agents' ability to pay would vary across types.

We explore such an economy in Online Appendix B. There we show that under certain conditions, there will be a threshold interest rate  $\hat{r}$  such that for interest rates higher than  $\hat{r}$ , no agent is constrained, so all individual money demand functions are well approximated by the log-log specification. It follows that the aggregate money demand function is also log-log. For interest rates lower than  $\hat{r}$ , the aggregate money demand is a combination of two types of agents. For the first type, the constraint binds, so their aggregate demand is insensitive with respect to the interest rate. For the second type, the constraint does not bind, and their elasticity is given by the log-log specification. As the interest rate keeps going down, the fraction of agents for which the elasticity is positive goes down, so the aggregate elasticity also goes down. The aggregate money demand is decreasing for this range, but with an interest rate elasticity that is lower than the log-log specification. Such a money demand is the dashed green line, labeled C in Figure 1.<sup>7</sup>

In light of this discussion, in our empirical strategy, we will follow two complementary approaches. First, we will ignore the borrowing constraints and use the log-log functional form implied by the theory. We expect this

---

<sup>7</sup>Similar results for the aggregate money demand arise in a model in which agents get the few first portfolio transactions for free. Such a model is developed and estimated in Alvarez and Lippi (2009).

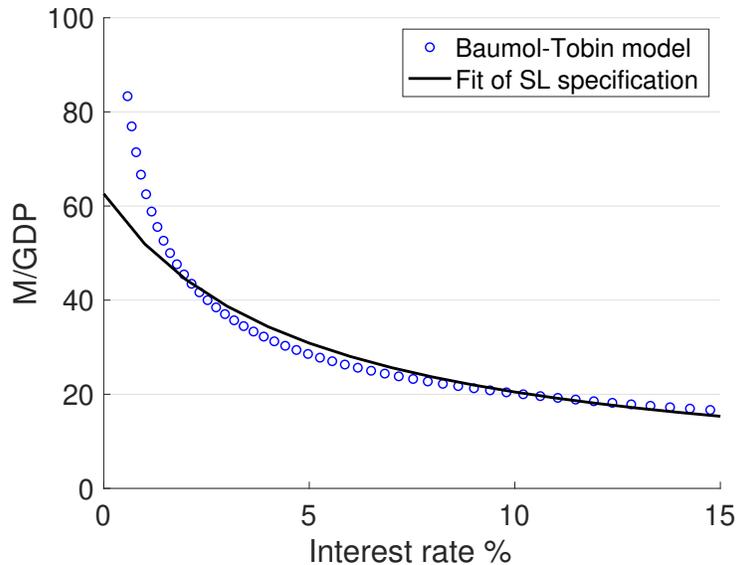


Figure 2 Fitting Baumol-Tobin model with Selden-Latané specification

strategy to work well in countries that did not experience low interest rates. Second, we will use a parametric form that is observationally very similar to the log-log specification for interest rates that are not too small, and which differs from that specification at very low levels of interest rates in a way that closely resembles the behavior of the real money demand with the borrowing constraints described above. This parametric form, used by Selden (1956), is given by

$$\frac{m_t}{x_t} = \frac{1}{a + br_t}. \quad (10)$$

In Figure 2, we plot two curves relating real money balances to the interest rate. The range of short-term interest rates is the relevant one for the United States in the last century: between 0% and 15%. The blue circles correspond to a log-log specification, with an elasticity equal to 1/2, as implied by the Baumol-Tobin linear technology discussed above. The constant in that equation has been chosen so that the ratio of money to output is 20% when the interest rate is 5%, which matches the US data reasonably well. The solid black line corresponds to the best fit of equation (10) to the blue circles in the figure.<sup>8</sup>

<sup>8</sup>The two parameters of the solid black line were calibrated using OLS.

As the figure makes clear, both functional forms behave in a remarkably similar way for interest rates between 15% and 2%. They are so similar, in fact, that it appears that the ability to identify one functional form from the other one would require a gigantic amount of data for interest rates within that range. On the other hand, the two functional forms do behave very differently at interest rates between 0% and 2%. We expect this formulation to work well in countries that did experience low interest rates in several periods. By comparing the empirical performance of the two specifications, we will be able to draw some conclusions regarding the quantitative relevance of the borrowing constraints.

### 2.1.5 The role of regulation and technology

The literature has long recognized that changes in regulation or technology can change the equilibrium relationship between interest rates and real money balances. For instance, Lucas and Nicolini (2015) argue that regulatory changes introduced in the United States in the early eighties can explain the apparent instability of real money demand in the United States. Alvarez and Lippi (2009) show that advances in banking technology are important in explaining their household level data on cash holdings. The theoretical implications of such changes can be analyzed with this model. In the working paper version of this paper, we show that when money is used to pay interest, the solution for the number of trips to the bank,  $n_t$ , is an equation equal to (7), except that on the right-hand side,  $r_t$  must be replaced by  $r_t - r_t^m$ . Thus, if banks are allowed to compete by paying interest on deposits, the optimal choices of  $n_t$  would change even if the interest rate  $r_t$  is unchanged. In addition, a change in the level parameter  $\gamma$  changes the optimal value for money balances, again keeping  $r_t$  constant.

As a first step, we will ignore both regulatory and technological changes in this paper. As it turns out, the data analysis shows that by and large, these theoretical considerations have little empirical relevance: the theoretical relationship in (8) is confirmed by the data for a large set of countries, even though we cover samples that are several decades long. As we document below, we found just a handful of cases for which M1 real money demand appears to have breaks that are suggestive of further analysis. Our general conclusion is that the apparent breakdown in the real money demand relationship observed in the United States, requiring a detailed and country-specific analysis, is an exception rather than the rule.

### 3 A First Look at the Data

In this section, we present the data and provide a visual comparison with the theory. To begin, we discuss how we map our theoretical construct  $M_t$  to the data. This choice is associated with the discussion of its nominal return. We have no data on the interest rate paid by deposits, so we choose to work with M1, which includes cash and only checking accounts. In deciding to set  $r_t^m = 0$  in the theory, we implicitly assumed that checking accounts pay no interest. This is a questionable assumption, but it is certainly more appropriate for M1 than for broader aggregates, which typically include interest-paying deposits.<sup>9</sup> Accordingly, we identify money in the model with M1.

Online Appendix A describes the data and the data sources in detail. All of the series are standard, with the single exception of the United States, where we also consider the adjustment proposed in Lucas and Nicolini (2015). Specifically, we add to the standard M1 aggregate the money market deposit accounts (MMDAs) that were created in 1982. We call this aggregate New M1.<sup>10</sup>

We first present the raw data in the form of cross plots between the short-term interest rate and the ratio of money to nominal income. The data were not manipulated in any way. Figures 3a to 3c are scatterplots of the short rate and the ratio between nominal M1 and nominal GDP (i.e., the inverse of M1 velocity). We also plot the theoretical curve that corresponds to equation (9), specialized to the case in which the elasticity is equal to 1/2. We allow the per-unit cost,  $\gamma^j$ , to be different across countries. Thus, for each country  $j$ , we plot the curve

$$\frac{M_t^j}{Y_t^j} = \frac{\gamma^j}{\sqrt{r_t^j}}, \quad (11)$$

where  $Y_t^j$  is nominal income at time  $t$  in country  $j$  and  $M_t^j$  is M1, except for the US, where we use New M1, as mentioned above. We calibrate the

---

<sup>9</sup>Deposits did pay interest in the United States after Regulation Q was modified in the early 1980s. Also, some deposits included in M1 did pay interest in very high-inflation countries such as Argentina and Brazil.

<sup>10</sup>The results are the same with an alternative aggregate in which currency has been adjusted along the lines of Judson (2017) to take into account the sizable expansion in the fraction of US currency held by foreigners since the early 1990s. See Benati (2019a) for details.

single free parameter for each country to be the one that minimizes the mean squared errors between the curve and the data, but imposing the elasticity to be  $1/2$ .

The criterion used to group the countries follow the results of the tests performed below. Details will be discussed then, but a rough approximation is that we start with the countries for which the evidence of comovement between velocity and the interest rate is very strong and show at the end the countries for which the evidence is weak or non-existent. In our view, it is remarkable how well this simple theory performs in this first inspection. Our own summary is the following. For the 12 countries in Figure 3a, the evidence is very strong. We offer two caveats: for both Brazil and Argentina, a few points clearly lay below the theoretical line. In both cases, those points correspond to the years following the successful stabilization of hyperinflations.<sup>11</sup> The model in the previous section is too simple to be used to address these cases, so we leave that topic for future research. The following 14 countries in Figure 3b also exhibit clear evidence of a negative relationship, though in some cases (such as Bahrain, Barbados, and Thailand), the slope seems to be different from the  $1/2$  implied by the linear technology. The 6 countries in the top row of Figure 3c also provide good evidence, though in some cases (such as Finland and West Germany), the picture is not as clear as before. Finally, the 6 countries in the bottom row of Figure 3c are the most problematic. Portugal and to a lesser extent Paraguay and Norway, also seem to display evidence of a negative relationship, but it is far from the theoretical curve. The last three cases are, from the viewpoint of the cross plots, blunt failures.

The plots in Figures 3a through 3c depict the data in a way that became traditional in empirical studies of money demand. In addition, the data can be visually compared with the theoretical curve indicated by the theory (11). But depicted in this way, the plots hide the behavior of each of the variables over time and it thus they fail to show the persistency that both series have in the data and how the persistent components of the two series move together over time. We find that information also very valuable as a visual motivation for the cointegration methods that we use in the rest of the paper. The figures also add the time dimension, which helps to explain

---

<sup>11</sup>Neither Bolivia nor Israel, which also experienced hyperinflations, exhibits the same puzzling behavior. But for those cases, we do not have data for many years before the hyperinflation, so they are not completely comparable.

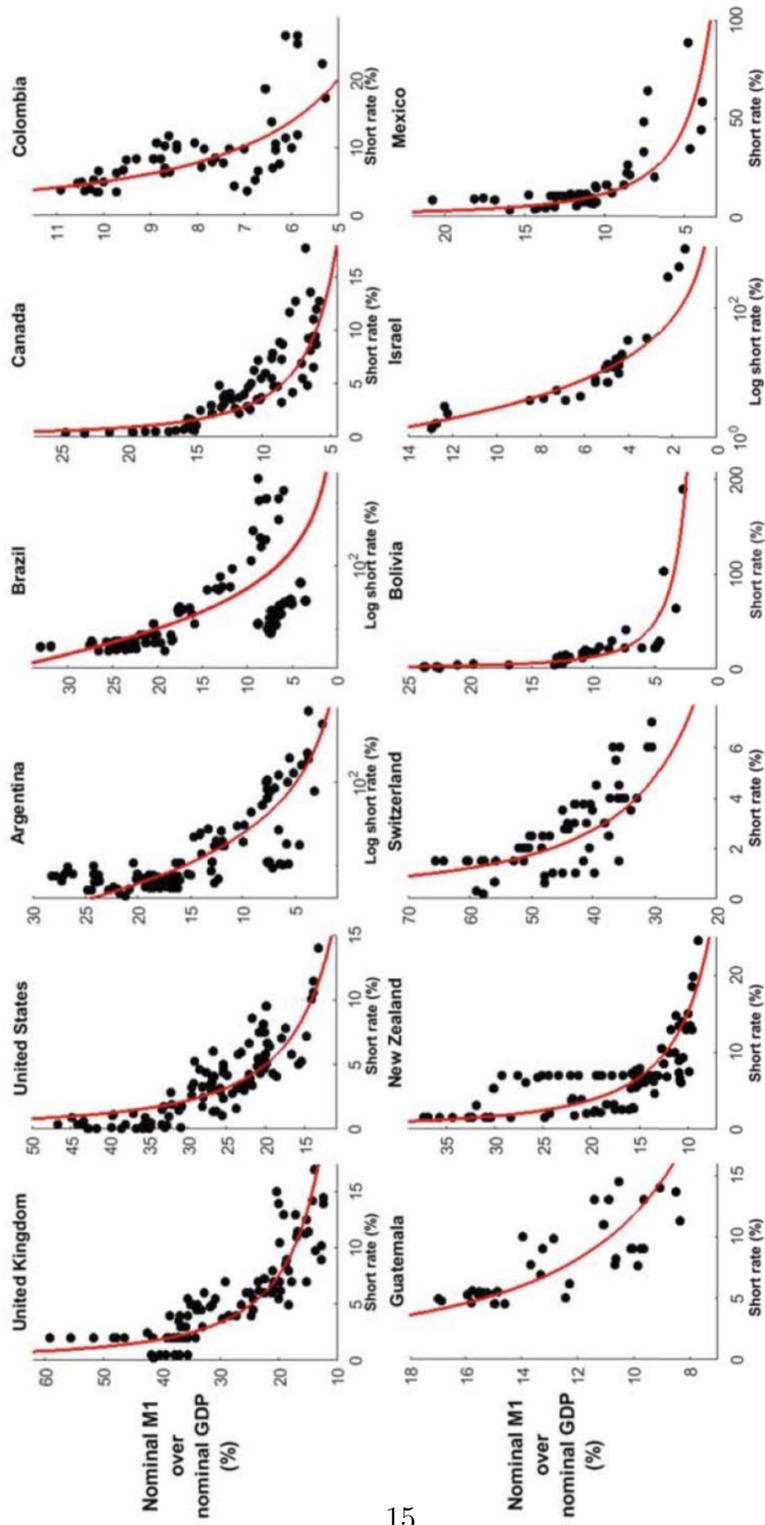


Figure 3a The raw data: scatterplots of the short rate and M1 as a fraction of nominal GDP

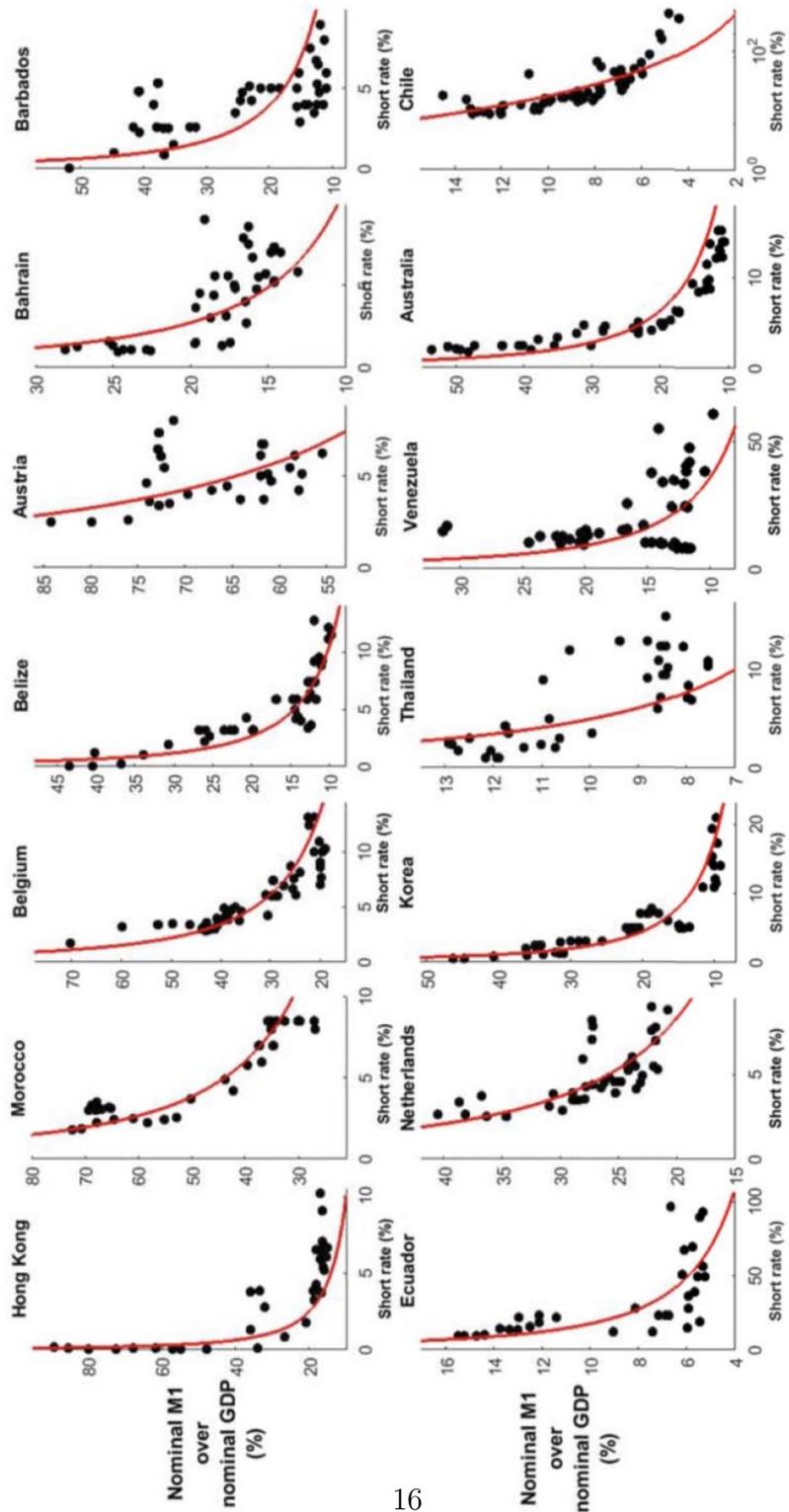


Figure 3b The raw data: scatterplots of the short rate and M1 as a fraction of nominal GDP

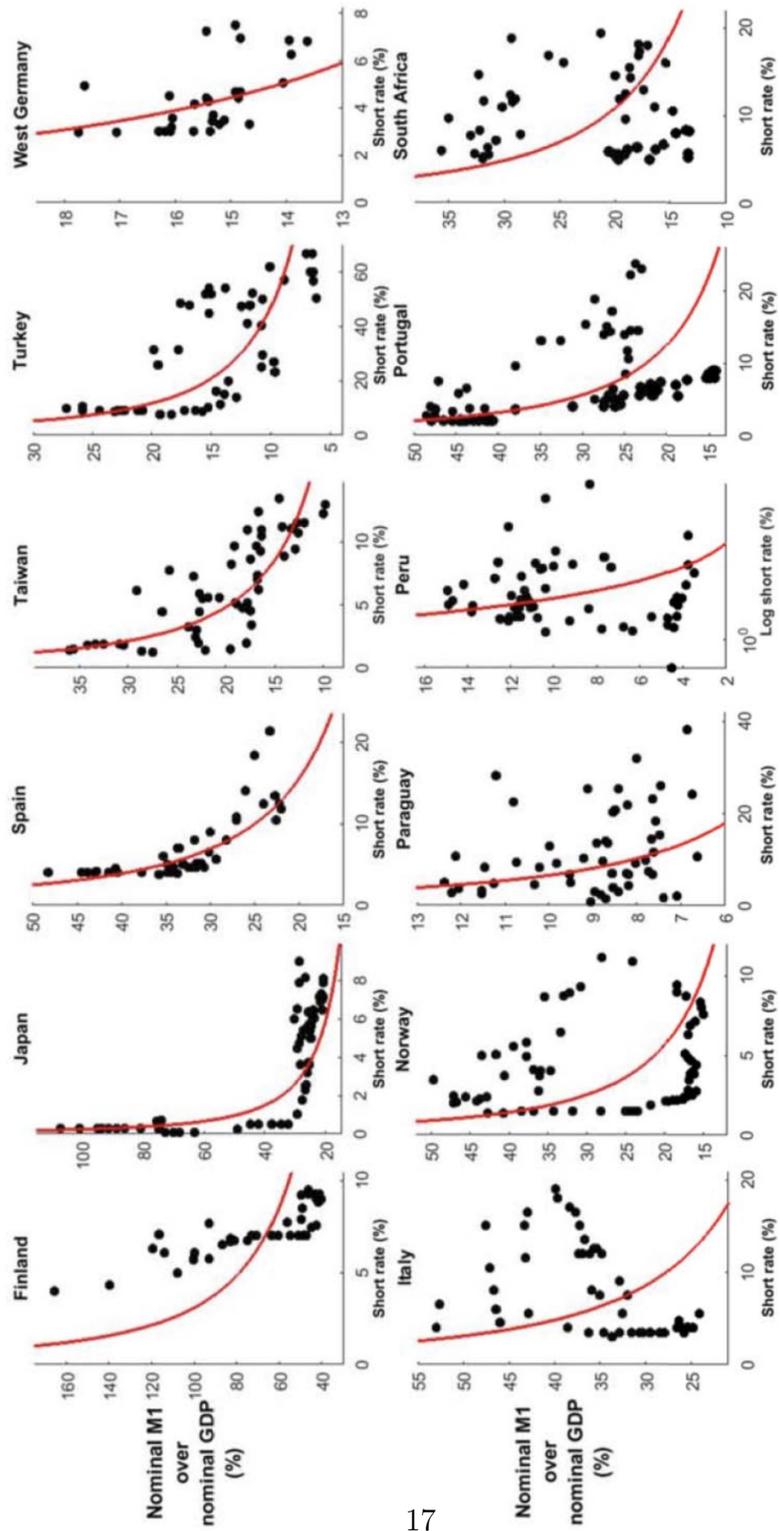


Figure 3c The raw data: scatterplots of the short rate and M1 as a fraction of nominal GDP

some of the apparent failures discussed above.

Thus, in Figures 4a to 4c, we present the time series for both the short term interest rate and velocity.<sup>12</sup> In this case, we did some —minimal — manipulation by plotting the two variables in different axes. This amounts to making a linear transformation of one of the variables, which is consistent with the theoretical constructs in (9) and (10). We also go beyond the previous comparison with the theory, where we used the log-log specification for all countries. The theory suggests that the formulation (10) is more likely to be a better description of the data when the borrowing constraints are quantitatively relevant. Thus, we classify countries into two groups. For the first group of countries, we plot the log of the interest rate and the log of velocity, as specified in equation (9). For the second group of countries, we plot only the levels of the same variables, as specified in equation (10). This second group of countries comprises those that never had their interest rate too high and even had several years of interest rates close to zero, as the theory suggests.<sup>13</sup>

For the 26 countries in Figures 4a and 4b, the comovement between velocity and the interest rate is remarkable. The same caveats regarding Argentina and Brazil apply, and notice that in both Israel and Bolivia, our data mostly cover the years following the stabilization. Three more puzzles appear clearly in the time series: Venezuela, and to a lesser extent Ecuador, seem to exhibit different behavior during the first half of the sample relative to the second. In addition, the data for Chile show, as do the data for Argentina and Brazil mentioned above, that the behavior right after the stabilization of a very high inflation does not conform to the theory. The countries in the top row of Figure 4c also exhibit solid evidence of comovement between velocity and the interest rate, though it is less clear-cut than the previous cases. Finally, among the group of countries in Figure 4c, 4 of them suggest blunt failure, whereas both Norway and Portugal seem to conform to the theory in the last few decades, but not before.

To summarize, the first data inspection suggests the following : 26 countries exhibit remarkable evidence of comovement between velocity and the interest rate, 6 countries offer good evidence, and for the final group of 6

---

<sup>12</sup>We find convenient to show velocity - the inverse of real money demand according to the model - since it ought to be positively related to the interest rate.

<sup>13</sup>We use the following criterion: we present the data in levels for all countries with some observations below 5%, and no observation beyond 100%, while we present the data in logs for all other countries.

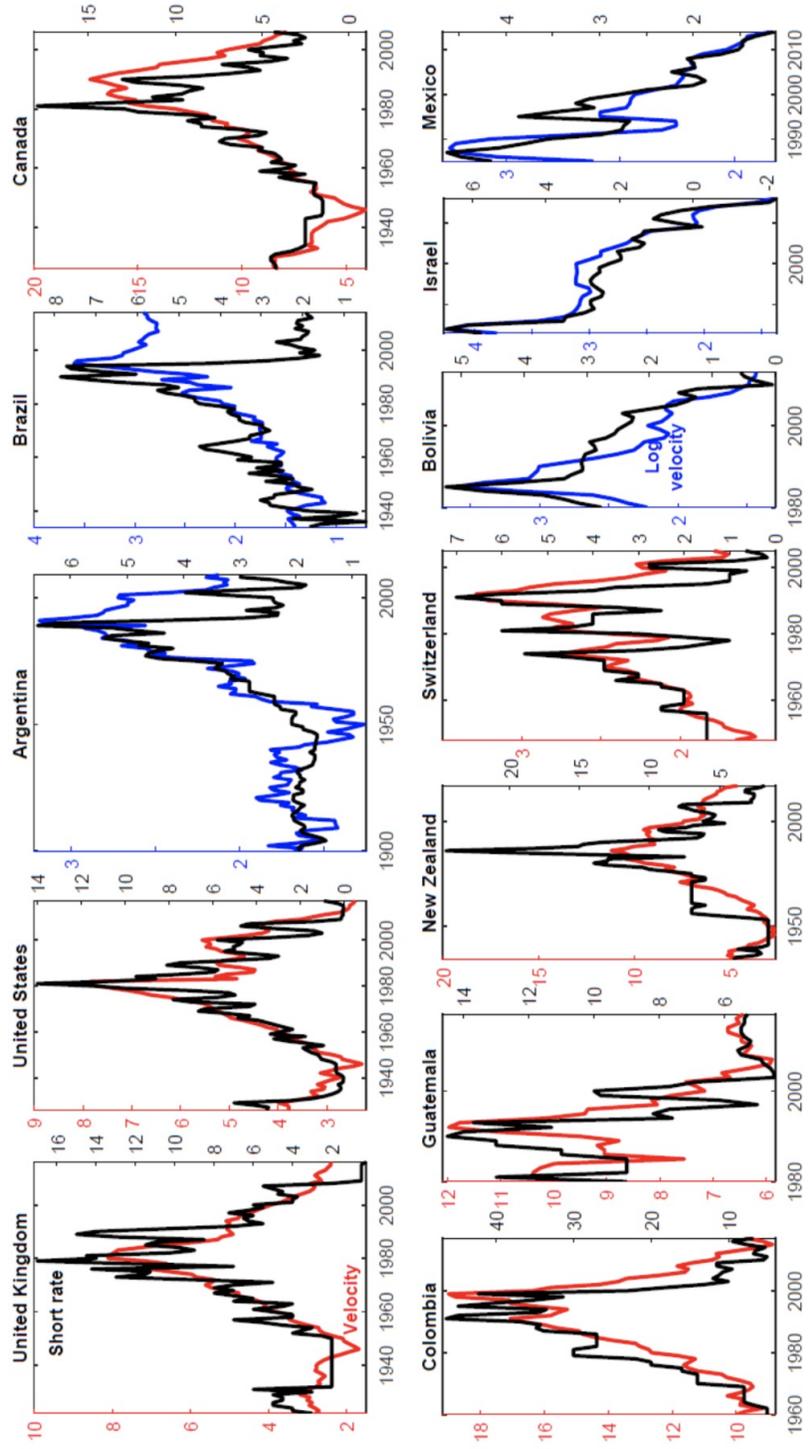


Figure 4a. The raw data: M1 velocity and the short rate

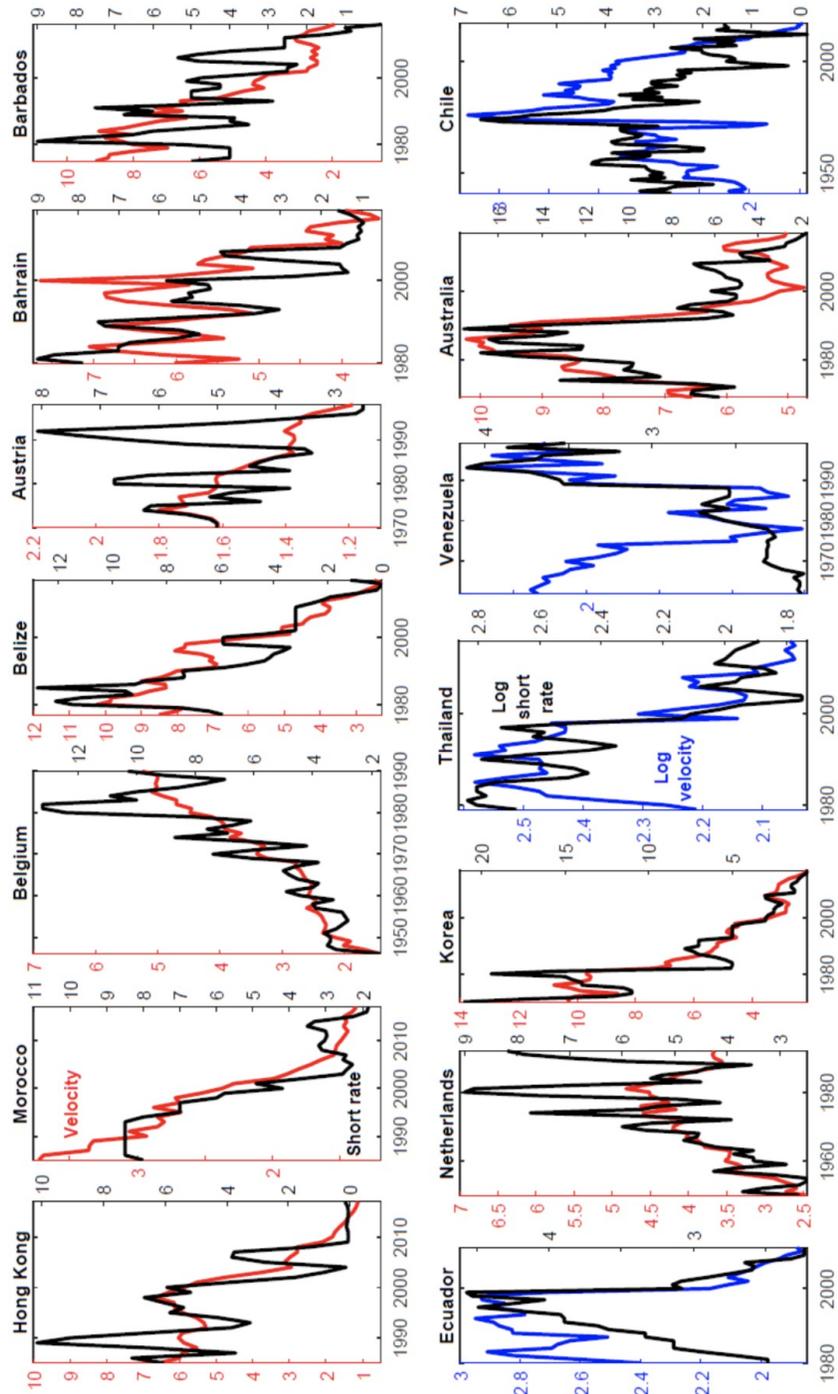


Figure 4b The raw data: M1 velocity and the short rate

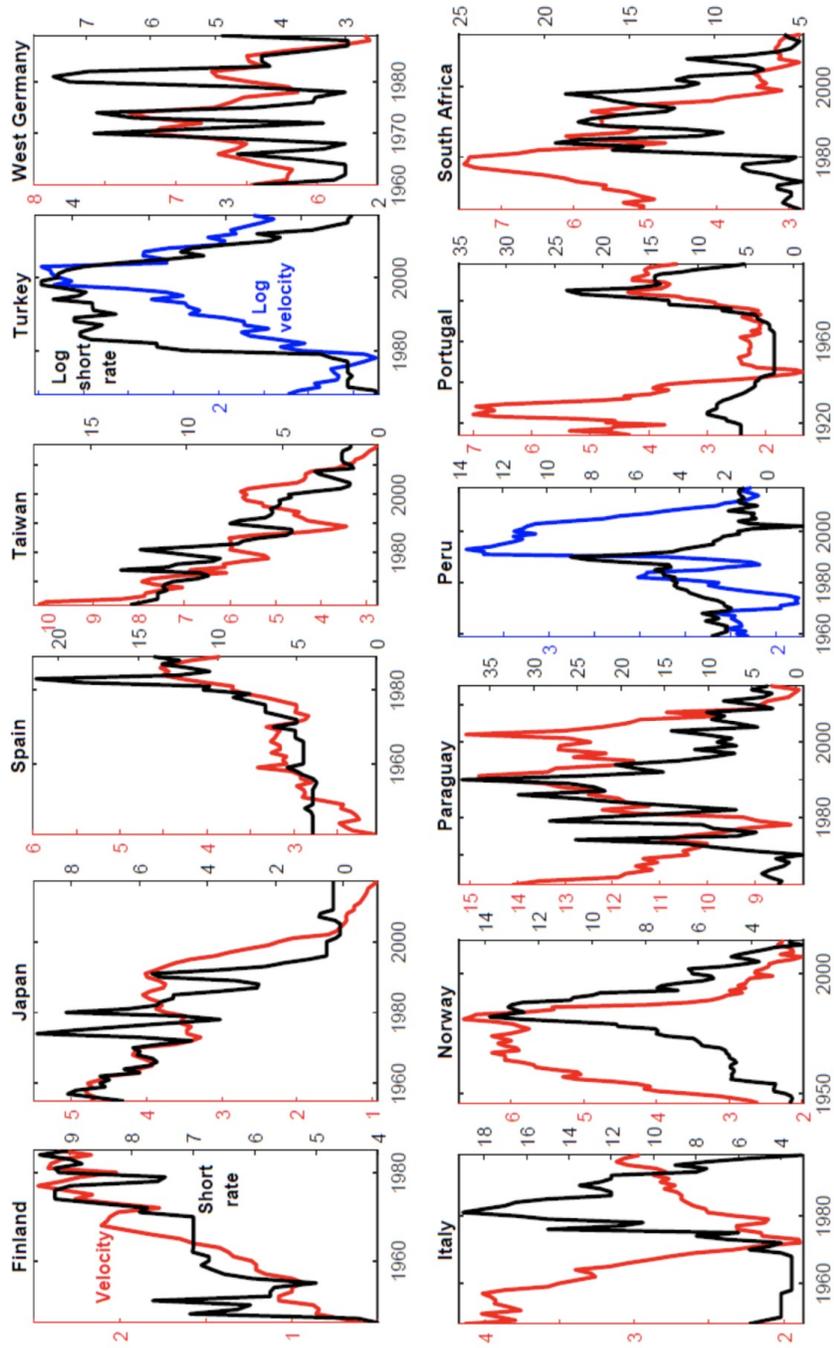


Figure 4c The raw data: M1 velocity and the short rate

countries, the evidence is either weak or non-existent. In addition, an inspection of the countries that experienced extremely high inflation — in the several 100s or even 1000% per year — suggests that it takes several years after nominal interest rates have returned to normal for real money demand to recover to its previous levels. Finally, we identify only three candidates (Venezuela, Norway and Portugal) that seem to exhibit a breakdown in the money demand relationship that could justify further analysis of whether regulation may have played a role— an analysis along the same lines discussed in Lucas and Nicolini (2015) for the United States.

In our view, it is remarkable how well this simple theory performs in this first inspection for a large set of countries — though not all. Despite the attractiveness of looking at simple plots, however, the previous analysis has several limitations. We would like to formally test whether, as the simple model above implies, the ratio between real money balances and output inherits a unit root when the short-term interest rate exhibits a unit root. We also want to formally estimate the interest rate elasticities and see how good  $1/2$  is as an approximation, the value implied by the linear technology implied by original Baumol-Tobin specification. We would also like to let the data indicate the quantitative effect of the borrowing constraints when interest rates are very low, as it has been in countries such as Japan, the United States and the United Kingdom. If this were the case, the Selden-Latané specification ought to deliver better results.

The plots in Figures 4a through 4c show how persistent the series are and provide support for the use of the cointegration methods that we use below. In fact the statistical tests overwhelmingly identify unit roots both in the ratio of money to output and in the interest rates (see Appendix C). In spite of the results of the unit root tests, one may have theoretical reasons to believe that the interest rate, being a policy variable, ought to be stationary. One such reason is that in most monetary models used to justify inflation targeting policies, the policies that stabilize the economy around the inflation target deliver a stationary series for the equilibrium interest rate, even if the economy is subject to unit root shocks, as long as the real interest rate is stationary. And these models approximate very well the behavior of inflation rates in the data in countries that have successfully managed to have inflation very close to its target. Clearly, temporary but persistent deviations from that policy, as one may interpret the US experience from 1965 to 1985, may imply very persistent movements in the interest rate. And it is indeed quite difficult to distinguish that behavior from a unit root, statistically speaking,

given the sample sizes we have.

The good news is that those speculations are of little quantitative relevance: a crucial feature of one of the methods we use, owing to Wright (2000), is that they perform very well for highly persistent series, even if they are not exactly unit roots in Section 6.1. We illustrate this property in the specific context of the model described above. There we run Monte Carlo experiments with model-generated data. We simulate stationary but very persistent series and show that these methods identify the true parameters remarkably well. It is because we are handling very persistent series that we are fully comfortable embracing the cointegration techniques that follow.

We now turn to a brief discussion of the main features of our approach and several methodological issues.

## 4 Main Features of Our Approach

The cointegration techniques we use were justified above on statistical grounds: the series we work with are highly persistent, to the point where, in nearly all cases, it is not possible to reject the null of a unit root. But this approach also highlights a discussion that is at the heart of the debate over the stability of money demand: the distinction between the short run and the long run. This distinction is entirely absent in our model, but a large theoretical literature has been developed to try to understand the large and sustained deviations of observed real money balances from their theoretical counterparts: the “short-run” deviations of money demand.<sup>14</sup>

The notion of cointegration boils down to the existence of a long-run relationship between series driven by permanent shocks: those shocks are the source of identification of the relationship between the short rate and velocity. The existence of the cointegration relationship implies that, in the long run, any permanent increase in the interest rate maps into a corresponding permanent increase in velocity and therefore a decrease in real money balances: the exact amount will be captured by the cointegration vector. Further, any deviation of the two series from their long-run relationship—that is, the cointegration residual—is transitory and bound to disappear in the long run. The persistence of the residual is therefore a measure of how long-lived these short-run deviations are. As we will show, these estimated

---

<sup>14</sup>See Grossman and Weiss (1983) for an early contribution, or Alvarez and Lippi (2014) for a recent one.

residuals are indeed very persistent themselves. Our analysis therefore leaves unexplained a substantial fraction of the dynamic interactions between the short-term interest rate and the money-to-output ratio.

We present the analysis in two steps. In the first step, discussed in this section, we take the results of the unit root test literally—that is, we interpret the lack of rejection of the null hypothesis as evidence that the series contain *exact* unit roots—and present evidence from Johansen’s cointegration tests, which take no cointegration as the null hypothesis. Then, in a second step, we present the results from Wright’s (2000) tests, which take cointegration as the null hypothesis.<sup>15</sup> There are (at least) two reasons for also considering the Wright test, in addition to the Johansen test. First, although the overwhelming majority of the papers in the money demand literature have been based on Johansen’s procedure, there is no reason to regard no cointegration as the “natural null hypothesis”. Rather, it might be argued that, since we are searching here for a long-run money demand for *transaction* purposes, cointegration should be the natural null.<sup>16</sup> Second, as discussed by Wright (2000), Wright’s test works equally well both when the data contain exact unit roots, and when they are local-to-unity. On the other hand, as shown by Elliot (1998), when the data are local-to-unity, tests (such as Johansen’s) that are predicated on the assumption that the data contain exact unit roots can perform poorly.

Once cointegration is detected, we can use standard methods to estimate the parameter that governs the elasticity of the money demand relationship. For reasons of robustness, we consider Johansen’s just-mentioned procedure, as well as Stock and Watson’s (1993). We also compare the results of using the log-log specification and the Selden-Latané one, and use those results to discuss the behavior of real money demand at very low interest rates.

---

<sup>15</sup>The Wright (2000) test searches across the parameter space for all the values of  $\beta$  in the normalized cointegration vector  $[1 - \beta]'$  such that the null hypothesis that  $[1 - \beta]'.y_t$  is  $I(0)$  cannot be rejected, where  $y_t$  features the levels of velocity and the short rate for the Selden-Latané specification, and their logarithms for the log-log. A  $(1 - \alpha)\%$ -coverage confidence interval for  $\beta$  is computed as the set of all values of  $\beta$  for which the null hypothesis that  $[1 - \beta]'.y_t$  is  $I(0)$  cannot be rejected at the  $\alpha\%$  level.

<sup>16</sup>Basic economics logic suggests that, up to fluctuations in the opportunity cost of money, the nominal quantity of money demanded should be proportional to the nominal volume of transactions (proxied by nominal GDP).

## 4.1 Integration properties of the data

Online Appendix C reports evidence from our extensive investigation of the integration properties of the data based on Elliot et al.’s (1996) unit root tests. Our main results can be summarized as follows.

*First*, there is overwhelming evidence of unit roots in the vast majority of the series, with the bootstrapped  $p$ -values being near-uniformly greater than the 10% threshold, which, throughout the entire paper, we take as our benchmark significance level, and in most cases markedly so. In the very few instances in which this is not the case, we eschew the relevant specifications (e.g., if we can reject the null of a unit root for the logarithm of the short rate but not for the level, we eschew the log-log specification, and we uniquely focus on the Selden-Latané specification).

*Second*, for both the first difference and the log-difference of either velocity or the short rate, the null of a unit root can be rejected almost uniformly. In the few instances in which this is not the case—so that the relevant series should be regarded, according to Elliot et al.’s (1996) tests, as  $I(2)$ —we do not run cointegration tests.<sup>17</sup> As for nominal M1 and especially nominal GDP, on the other hand, the opposite is true, with the null of a unit root not being rejected most of the time. In all of these cases, we will therefore eschew unrestricted specifications for the logarithms of nominal M1, nominal GDP, and a short rate.

## 4.2 Methodological issues pertaining to cointegration tests

### 4.2.1 Issues pertaining to bootstrapping

Everything in this paper is bootstrapped. In this section, we briefly discuss details of the bootstrapping procedures we use, and how such procedures perform. In our discussion, we extensively refer to online Appendix D, which contains the Monte Carlo evidence motivating both our choices and the way in which we will interpret the evidence based on the actual data.

**Details of the bootstrapping procedures** As for Johansen’s tests, we bootstrap trace and maximum eigenvalue statistics via the procedure pro-

---

<sup>17</sup>Both Johansen’s and Wright’s tests are predicated on the assumption that the series contain (near) unit roots, but that their order of integration is at most one.

posed by Cavaliere et al. (2012; henceforth, CRT). In a nutshell, for tests of the null of no cointegration against the alternative of one or more cointegrating vectors, the model that is being bootstrapped is a simple, noncointegrated VAR in differences (for the maximum eigenvalue tests of  $h$  versus  $h+1$  cointegrating vectors, on the other hand, the model that ought to be bootstrapped is the VECM estimated under the null of  $h$  cointegrating vectors). All of the technical details can be found in CRT, to which the reader is referred. We select the VAR lag order as the maximum<sup>18</sup> between the lag orders chosen by the Schwartz and the Hannan-Quinn criteria<sup>19</sup> for the VAR in levels.

As for the Wright (2000) test, since its test has been designed to be equally valid for data-generation processes (DGPs) featuring either exact or near unit roots, we consider two alternative bootstrapping procedures, corresponding to either of the two possible cases. (In practice, as a comparison between the results reported in Table 2 in the text and Table E.1 in online Appendix E makes clear, the two procedures produce near-identical results.) The former procedure involves bootstrapping—as detailed in CRT, and briefly recounted in the previous paragraph—the cointegrated VECM estimated (based on Johansen’s procedure) under the null of one cointegration vector. This bootstrapping procedure is the correct one if the data feature exact unit roots. For the alternative possible case in which velocity and the short rate are near unit root processes, we proceed as follows. Based on the just-mentioned cointegrated VECM estimated under the null of one cointegration vector, we compute the implied VAR in levels, which, by construction, features one, and only one eigenvalue equal to 1. Bootstrapping this VAR would obviously be equivalent to bootstrapping the underlying cointegrated VECM, that is, it would be correct if the data featured exact unit roots. Since, on the other hand, here we want to bootstrap under the null of a near unit root DGP, we turn such an exact unit root VAR in levels into its near unit root correspondent, by shrinking down the single unitary eigenvalue to  $\lambda=1-0.5\cdot(1/T)$ , where  $T$  is the sample length.<sup>20</sup> The bootstrapping procedure we implement

---

<sup>18</sup>We consider the maximum between the lag orders chosen by the SIC and HQ criteria because the risk associated with selecting a lag order smaller than the true one (model misspecification) is more serious than the one resulting from choosing a lag order greater than the true one (overfitting).

<sup>19</sup>On the other hand, we do not consider the Akaike Information Criterion since, as discussed by Luetkepohl (1991), for example, for systems featuring I(1) series, the AIC is an inconsistent lag selection criterion, in the sense of not choosing the correct lag order asymptotically.

<sup>20</sup>In particular, we do that via a small perturbation of the parameters of the VAR

for the second possible case, in which the processes feature near unit roots, is based on bootstrapping such a near unit root VAR.

We now turn to discussing Monte Carlo evidence on the performance of the two bootstrapping procedures.

**Monte Carlo evidence** Tables D.1 and D.2 in online Appendix D report extensive Monte Carlo evidence on the performance of the bootstrapping procedures, which is discussed in detail in Sections D.1.1 and D.1.2 of online Appendix D. We perform the Monte Carlo experiments based on two types of DGPs, featuring no cointegration and cointegration, respectively. For either DGP, we consider several alternative sample lengths, from  $T = 50$  to  $T = 1,000$ . For the DGP featuring cointegration, we also consider several alternative values for the persistence of the cointegration residual, which we model as an AR(1). Finally, whereas for the experiments pertaining to Johansen’s tests we only consider DGPs with exact unit roots, for those pertaining to Wright tests we also consider the corresponding DGPs with roots local-to-unity, which we obtain by replacing, in the former DGPs, the exact unit root with  $\lambda=1-0.5\cdot(1/T)$ . In the case of cointegrated DGPs featuring exact unit roots, we bootstrap Wright’s test statistics based on the first procedure discussed in the previous subsection (that is, based on bootstrapping the VECM estimated conditional on one cointegration vector, as in CRT). In the case of cointegrated DGPs featuring near unit roots, on the other hand, we bootstrap the tests via the alternative procedure, based on bootstrapping the corresponding near unit root VAR in levels.

Our main results can be summarized as follows.

As for the Johansen test, if the true DGP features no cointegration, CRT’s procedure performs remarkably well irrespective of sample size, with empirical rejection frequencies (ERFs) very close to the nominal size. This is in line with the Monte Carlo evidence reported in CRT’s Table I, p. 1731, and with the analogous evidence reported in Benati (2015). If, however, the true

---

matrices  $B_j$ ’s in the cointegrated VECM representation  $\Delta Y_t = A + B_1\Delta Y_{t-1} + \dots + B_p\Delta Y_{t-p} + GY_{t-1} + u_t$ , where  $Y_t$  collects (the logarithms of) M1 velocity and the short rate, and the rest of the notation is obvious. By only perturbing the elements of the VAR matrices  $B_j$ ’s—leaving unchanged the elements of the matrix  $G$  (and therefore both the cointegration vector and the loading coefficients)—we make sure that both the long-run equilibrium relationship between velocity and the short rate, and the way in which disequilibria in such a relationship map into subsequent adjustments in the two series, remain unchanged.

DGP features cointegration, the tests perform well only if the persistence of the cointegration residual is sufficiently low, the sample size is sufficiently large, or both: if the residual is persistent, the sample is short, or both, the tests fail to detect cointegration a nonnegligible fraction of the time. This is in line with some of Engle and Granger's (1987) evidence, and it has a simple explanation: as the residual becomes more and more persistent, it gets closer and closer to a random walk (in which case there would be no cointegration), so that the procedure needs larger and larger samples to detect the truth (i.e., that the residual is highly persistent but ultimately stationary).

As for the Wright test, evidence is qualitatively the same, and quantitatively very close, in the case of either exact or near unit root DGPs. Specifically, if the true DGP features cointegration, the procedure works remarkably well if either the sample size is sufficiently long, the persistence of the cointegration residual is sufficiently low, or both, with ERFs very close to the tests' nominal size. As the sample size decreases and/or the persistence of the cointegration residual increases, however, the ERFs increase systematically, to the point where, for example, for  $T = 50$  and the autoregressive parameter of the cointegration residual equal to 0.95, the test incorrectly rejects the null at about twice the nominal size. The explanation for this is straightforward, and it has to do, once again, with Engle and Granger's (1987) previously mentioned point: when the cointegration residual is highly persistent, only sufficiently long samples allow the test to detect the truth (i.e., that the deviation between the two series is ultimately transitory, so that they are in fact cointegrated). But, under these circumstances, the shorter the sample period, the more likely it will be to mistakenly infer that the deviation between the series is permanent, so that they are not, in fact, cointegrated. If, however, the true DGP features no cointegration, the test tends to reject the null at about twice the nominal size, essentially irrespective of sample length.

These results can be summarized as follows. If the Johansen test detects cointegration, we should have a reasonable presumption that cointegration is there. If however it does not detect it, a possible explanation is that the sample is too short, the cointegration residual is highly persistent, or both. As for the Wright test, lack of rejection of cointegration does not represent very strong evidence that cointegration truly is there, as this also happens with a comparatively high frequency for DGPs featuring no cointegration.

We now turn to the issue of how persistent cointegration residuals in fact are.

### 4.2.2 Evidence on the persistence of cointegration residuals

Tables H.1 and H.2 in online Appendix H report Hansen’s (1999) “grid bootstrap” median-unbiased (henceforth, MUB) estimates of the sum of the AR coefficients in AR(2) representations for the “candidate cointegration residuals” in our dataset.<sup>21</sup> By “candidate cointegration residual” (henceforth, CCR), we mean the linear combination of the variables in the system which will indeed be regarded as a cointegration residual *if* cointegration is detected.<sup>22</sup> For reasons of robustness, for either the Selden-Latané specification (Table H.1) or the log-log specification (Table H.2) we consider two alternative estimators of the cointegration residual: either Johansen’s or Stock and Watson’s (1993).

Evidence points toward both a nonnegligible extent of persistence of the CCRs, and a wide extent of heterogeneity across countries. Focusing on results based on the log-log specification, the MUB estimate based on Johansen’s estimator of the cointegration vector—let’s label it as  $\hat{\rho}_{MUB}^J$ —ranges from a minimum of 0.27 for Belize to a maximum of 1.17 for the Barbados islands. By classifying the  $\hat{\rho}_{MUB}^J$ ’s, in an admittedly arbitrary fashion, as “highly persistent” ( $\hat{\rho}_{MUB}^J \geq 0.8$ ); “moderately persistent” ( $0.4 < \hat{\rho}_{MUB}^J < 0.8$ ); and “not very persistent” ( $\hat{\rho}_{MUB}^J \leq 0.4$ ), we end up with 22  $\hat{\rho}_{MUB}^J$ ’s in the first group, 14 in the second, and 4 in the third. Results based on Stock and Watson’s estimator are qualitatively the same, with the three groups comprising, respectively, 25, 13, and 2 countries.

Under these circumstances, statistical tests will often have a hard time detecting cointegration even if it truly is present, especially when  $\hat{\rho}_{MUB}$  is high and the sample period is comparatively short. This implies that results from cointegration tests should not be taken strictly at face value, but rather should be interpreted in the light of the previously mentioned Monte Carlo evidence in Tables D.1 and D.2 in online Appendix D.

---

<sup>21</sup>Results are based on 2,000 bootstrap replications for each possible value of the sum of the AR coefficients in the grid. Bootstrapping has been performed as in ?. For reasons of robustness, we report results based on two alternative estimators of the cointegration vector, Johansen’s and Stock and Watson’s (1993).

<sup>22</sup>We label it as a candidate cointegration residual because, as the Monte Carlo evidence in the previous section has shown, if a residual is highly persistent, cointegration might well not be detected even if it is present, which would prevent the candidate from being identified as a true cointegration residual.

## 5 Results

In presenting the results, we first discuss the cointegration tests and then show the parameter estimates. We finish with a comparison between the log-log and Selden-Latané specifications.

### 5.1 Cointegration tests

In this section, we discuss the results from bivariate systems for velocity and the short rate, as implied by equation (7).<sup>23</sup> Table 1 reports results from Johansen’s maximum eigenvalue test of 0 versus 1 cointegration vectors, together with the Monte Carlo-based ERFs computed conditional on the null of one cointegration vector. We highlight in yellow all  $p$ -values for maximum eigenvalue tests smaller than 10% and all ERFs smaller than 50%, corresponding to a less-than-even chance of detecting cointegration if this is truly present in the data.

The table reports the cointegration test results for only one of the functional forms when the unit root tests for either the level or the log of one of the variables were rejected. For two of the countries, Morocco and Hong Kong, the series were identified as I(2) for both the level and the log of the variables, so no cointegration tests are reported. Thus, Table 1 reports results for only 36 countries.

We ordered the countries according to the test results. Within each category, we ordered the countries alphabetically. For each country, we first mention the time period for which we have consistent data. In some cases (Australia, Canada, and Chile), we have two different datasets, for long enough periods, but they do not completely overlap. The series are not exactly the same, so they cannot be used to construct a single series that can suitably be analyzed using cointegration. We report the results using both series. The third and fourth columns report the  $p$ -values of the tests for both the Selden-Latané and the log-log specifications. Finally, we show the ERF of the Monte Carlo exercises for both the Selden-Latané and the log-log specifications.

We first report the results for the United Kingdom and the United States, for which we have close to a century of data. For the case of the US, we use both M1 and New M1 ( the monetary aggregate proposed in Lucas and

---

<sup>23</sup>In Online Appendix F, we discuss test results for unrestricted specifications between the log of the interest rate, the log of nominal output, and the log of M1.

**Table 1 Bootstrapped  $p$ -values<sup>a</sup> for Johansen's maximum eigenvalue<sup>b</sup> test and empirical rejection frequencies of the tests under the null**

<i>Country</i>	<i>Period</i>	I: Bootstrapped $p$ -values		II: Empirical rejection frequencies	
		<i>Selden-Latané</i>	<i>Log-log</i>	<i>Selden-Latané</i>	<i>Log-log</i>
United Kingdom	1922-2016	<b>0.003</b>	0.793	0.975	0.661
US – M1 + MMDAs	1915-2017	<b>0.063</b>	0.212	0.859	0.625
US – M1	1915-2017	0.869	0.218	<b>0.099</b>	<b>0.275</b>
Argentina	1914-2009	–	<b>0.031</b>	–	0.789
Brazil	1934-2014	–	<b>0.093</b>	–	<b>0.341</b>
Canada	1967-2017	<b>0.015</b>	<b>0.028</b>	0.965	0.939
	1926-2006	<b>0.007</b>	0.361	0.968	0.628
Colombia	1960-2017	<b>0.032</b>	<b>0.027</b>	0.648	0.593
Guatemala	1980-2017	<b>0.007</b>	<b>0.038</b>	0.536	<b>0.448</b>
New Zealand	1934-2017	<b>0.099</b>	<b>0.030</b>	0.690	0.819
Switzerland	1948-2005	<b>0.000</b>	<b>0.017</b>	0.923	0.769
Bolivia	1980-2013	<b>0.053</b>	0.681	0.686	<b>0.125</b>
Israel	1983-2016	<b>0.000</b>	0.252	0.767	<b>0.197</b>
Mexico	1985-2014	<b>0.007</b>	0.505	0.537	<b>0.200</b>
Belgium	1946-1990	0.361	<b>0.062</b>	0.699	0.721
Belize	1977-2017	0.704	<b>0.007</b>	<b>0.107</b>	<b>0.394</b>
Austria	1970-1998	0.203	0.180	<b>0.220</b>	<b>0.265</b>
Bahrain	1980-2017	0.401	0.335	<b>0.082</b>	<b>0.085</b>
Barbados	1975-2016	0.542	0.677	<b>0.348</b>	<b>0.335</b>
Ecuador	1980-2011	0.838	0.686	<b>0.043</b>	<b>0.053</b>
Netherlands	1950-1992	0.349	0.568	<b>0.463</b>	<b>0.325</b>
South Korea	1970-2017	0.364	0.955	<b>0.086</b>	<b>0.169</b>
Thailand	1979-2016	0.101	0.212	<b>0.101</b>	<b>0.091</b>
Venezuela	1962-1999	0.776	0.922	<b>0.087</b>	<b>0.059</b>
Australia	1969-2017	0.134	0.960	0.720	<b>0.425</b>
	1941-1989	0.642	0.722	<b>0.168</b>	<b>0.198</b>
Chile	1941-2017	0.442	0.307	0.824	0.628
	1940-1995	0.133	0.175	<b>0.111</b>	0.859
Finland	1946-1985	0.246	–	<b>0.286</b>	–
Japan	1955-2017	0.567	0.142	<b>0.363</b>	0.590
Spain	1941-1989	0.120	0.196	0.636	<b>0.205</b>
Taiwan	1962-2017	–	0.909	–	<b>0.139</b>
Turkey	1968-2017	0.460	–	<b>0.229</b>	–
West Germany	1960-1989	–	0.352	–	<b>0.219</b>
Italy	1949-1996	0.171	–	0.629	–
Norway	1946-2014	<b>0.035</b>	<b>0.043</b>	0.749	0.789
Paraguay	1962-2015	<b>0.074</b>	0.168	<b>0.443</b>	<b>0.366</b>
Peru	1959-2017	<b>0.003</b>	0.171	0.992	<b>0.389</b>
Portugal	1914-1998	0.857	<b>0.047</b>	<b>0.048</b>	<b>0.059</b>
South Africa	1965-2015	0.116	0.157	0.563	<b>0.329</b>

<sup>a</sup> Based on 10,000 bootstrap replications. <sup>b</sup> Null of 0 versus 1 cointegration vectors.

Nicolini 2015). The second group of countries contains the ones for which both  $p$ -values (or the only one that we could run) are below 10%. The next two groups include countries for which one and only one of the  $p$ -values is below 10%. The fifth and sixth groups contain countries for which both  $p$ -values are above 10%, but either the two ERFs are below 50% (fifth group) or only one is below 50% (sixth group).<sup>24</sup> Finally, the last group includes the six countries for which we believe the evidence is weak or nonexistent based on the visual evidence, in spite of the test results.

We first discuss how to interpret the United States and United Kingdom results in detail. The other numbers in the table are to be interpreted in a similar way. Recall that for both countries, the evidence of the simple graphs in Figures 3 and 4 is quite remarkable. The results of the tests confirm that notion. In using M1 for the United Kingdom and New M1 for the United States, the  $p$ -values for the Selden-Latané specification are below 10%, but the ones for the log-log specification are both above 10%. For both countries, the ERFs are substantially larger than 50%. This strong preference of the data for the Selden-Latané specification is consistent with the fact that both countries had several periods with interest rates very close to zero. Taken together, these results provide evidence of a satiation point at zero in the aggregate real money demand. Finally, when using the standard M1 aggregate for the United States, both  $p$ -value are higher than 10%, although the ERF are below 50% for both specifications, indicating that the predictive power of the test is low.

With the exception of Hong Kong and Morocco - for which we could not run the tests — the order of the countries in the table is the same as the order of the countries in the figures in Section 3. The first four groups contain 14 countries in total. For all of them, the tests detect cointegration in at least one of the specifications, even though in several cases the ERFs are low. The next two groups contain a total of 16 countries for which cointegration is not detected, but the ERFs are low in the two tests (8 countries in group 5) or in one test (8 countries in group 6). For these 30 countries, the visual evidence is very good - with the caveat of the behavior right after the stabilization of very high inflations, as in Argentina and Brazil.

The final group contains 6 countries for which the visual evidence was problematic or nonexistent. Two problematic cases are Norway and Portu-

---

<sup>24</sup>In classifying countries for which we have more than one set of series, we chose the one that contains the most recent data.

gal. In both cases, the tests do detect cointegration in at least one of the specifications. However, the visual evidence suggests a different behavior over time, somewhat similar to what occurred with M1 in the United States. Exploring whether regulation could explain those 2 cases seems to be worth pursuing, but not in this paper. For the remaining 4 countries, the visual evidence does not suggest such a pattern (or any other pattern!). Even though in 2 of those 4 cases the tests do detect cointegration, we can only classify those 4 countries as failing to behave as the theory implies.

To summarize: we find the evidence quite remarkable for 32 out of the 38 countries analyzed (the 30 countries in groups 1 to 6, plus Hong Kong and Morocco). Of the remaining 6 countries, regulatory changes may explain the behavior of real money demand in 2, whereas for the other four are blunt failures.

In Table 2, we present the results for the Wright test. We report 90% confidence intervals for the second element of the normalized cointegration vector  $(1 - \beta)$ . As mentioned above, they represent the set of all values of  $\beta$  for which the null hypothesis that  $(1 - \beta) \cdot y_t$  is  $I(0)$  cannot be rejected at the  $\alpha\%$  level, where  $y_t$  is a vector that contains either the levels or the logs of the short rate and velocity. The order of the countries is the same as in Table 2. In those cases in which cointegration is not detected, the entry in the table is NCD. We highlight in yellow the cases in which the confidence interval lies entirely in the range of negative numbers, so that cointegration is detected in the data, and furthermore the relationship is negative as it is in the theory. The results in Table 2 are in general even stronger than the ones in Table 1, but the results are consistent with them. For the 6 countries we had identified as having weak or nonexistent evidence, the results are also bad in this case. On the other hand, for the 16 countries in groups 5 and 6 of Table 1, where the Joansen test failed to detect cointegration (the  $p$ -values were above 10%) but where at least one of the ERFs were low, the Wright test identifies cointegration in 15 of them (West Germany being the only exception). For the 12 countries for which we did identify cointegration using the Johansen test, there is conflicting evidence for only one country: results for Switzerland are very strong in Table 1 but not in Table 2.

## 5.2 The estimated coefficients on the short rate

The coefficients can be estimated using both Johansen's and Stock and Watson's procedures. In addition, the Wright test also delivers confidence in-

<b>Table 2 Results from the Wright (2000) test: 90% coverage confidence intervals for the second element of the normalized cointegration vector</b>			
<i>Country</i>	<i>Period</i>	Bootstrapped process: cointegrated VECM	
		<i>Selden-Latané</i>	<i>Log-log</i>
United Kingdom	1922-2016	<b>[-0.529; -0.417]</b>	NCD
US - M1 + MMDAs	1915-2017	<b>[-0.613; -0.393]</b>	<b>[-0.352; -0.108]</b>
US - M1	1915-2017	NCD	<b>[-0.506; -0.029]</b>
Argentina	1914-2009	<b>[-0.107; -0.087]</b>	<b>[-0.513; -0.245]</b>
Brazil	1934-2014	<b>[-0.065; -0.009]</b>	[-1.366; 0.276]
Canada	1926-2006	<b>[-1.490; -1.053]</b>	<b>[-0.719; -0.607]</b>
	1967-2017	<b>[-0.578; -0.494]</b>	<b>[-0.389; -0.345]</b>
Colombia	1960-2017	NCD	NCD
Guatemala	1980-2017	<b>[-0.752; -0.448]</b>	<b>[-0.678; -0.414]</b>
New Zealand	1934-2017	NCD	<b>[-0.589; -0.312]</b>
Switzerland	1948-2005	NCD	NCD
Bolivia	1980-2013	<b>[-0.369; -0.193]</b>	<b>[-0.520; -0.388]</b>
Israel	1983-2016	NCD	<b>[-0.388; -0.320]</b>
Mexico	1985-2014	<b>[-0.260; -0.184]</b>	<b>[-0.422; -0.314]</b>
Belgium	1946-1990	<b>[-0.465; -0.289]</b>	<b>[-1.146; -0.710]</b>
Belize	1977-2017	<b>[-0.840; -0.692]</b>	<b>[-2.567; 1.433]</b>
Austria	1970-1998	[-0.601; 0.080]	[-1.040; 0.618]
Bahrain	1980-2017	NCD	<b>[-0.254; -0.194]</b>
Barbados	1975-2016	<b>[-2.006; -0.748]</b>	<b>[-2.899; 0.101]</b>
Ecuador	1980-2011	NCD	NCD
Netherlands	1950-1992	<b>[-0.394; -0.290]</b>	<b>[-0.483; -0.331]</b>
South Korea	1970-2017	<b>[-0.617; -0.521]</b>	<b>[-0.639; -0.338]</b>
Thailand	1979-2016	NCD	<b>[-0.498; -0.386]</b>
Venezuela	1962-1999	NCD	[-0.249; 0.287]
Australia	1941-1989	<b>[-0.691; -0.526]</b>	<b>[-0.808; -0.704]</b>
	1969-2017	<b>[-0.484; -0.404]</b>	<b>[-0.506; -0.314]</b>
	1941-2017	[-0.106; 0.047]	[-0.215; 0.085]
Chile	1940-1995	<b>[-0.140; -0.028]</b>	<b>[-0.382; -0.278]</b>
Finland	1946-1985	<b>[-0.530; -0.414]</b>	<b>[-2.693; -1.780]</b>
Japan	1955-2017	<b>[-0.520; -0.312]</b>	<b>[-0.513; -0.125]</b>
Spain	1941-1989	<b>[-0.163; -0.159]</b>	NCD
Taiwan	1962-2017	<b>[-0.449; -0.341]</b>	<b>[-0.453; -0.253]</b>
Turkey	1968-2017	NCD	NCD
West Germany	1960-1989	[-0.963; 0.931]	[-0.489; 0.692]
Italy	1949-1996	[0.032; 0.204]	[0.159; 0.511]
Norway	1946-2014	[-0.961; 0.985]	[-0.227; 1.043]
Paraguay	1962-2015	[-0.328; 0.125]	<b>[-0.200; -0.024]</b>
Peru	1959-2017	[-0.042; 0.026]	[-0.493; 0.692]
Portugal	1914-1998	[-0.340; 0.433]	[-0.018; 0.210]
South Africa	1965-2015	[-0.170; 0.427]	[-0.052; 1.065]

NCD = No cointegration detected.

tervals for the parameters. The full set of results is presented in Online Appendix I, where we show the estimation using the three procedures for the two specifications. We will focus the discussion in this section on the estimates of the elasticity in the log-log specification using Stock and Watson’s procedure. The reason for focusing on the log-log specification is twofold. First, the coefficient on the log of the interest rate has the natural interpretation of an elasticity. Second, and most importantly, we can directly relate it to the transactions technology that is the key component of the theory. In that respect, our reference value has an elasticity equal to 0.5, which corresponds to the case of a linear technology, as in Baumol and Tobin. Higher values for the elasticity imply that the exponent  $\sigma$  in equation (2) is lower than 1, which implies that the marginal cost of making transactions is decreasing with the number of transactions. The opposite is true when the elasticity is lower than 0.5.

The reason for focusing on the Stock and Watson’s estimates is that they are based on a single equation, whereas Johansen’s is based on estimating an entire cointegrated VAR, so there are many more coefficients. In small samples this approach may produce less precise results. In almost all cases, Johansen’s estimates are broadly in line with Stock and Watson’s but typically deliver larger standard errors. In 4 cases the results are different, and we conjecture that this result might be a small-sample issue. For details, see Online Appendix I. We are aware that as long as the data-generating process corresponds to the model with the borrowing constraint, the estimate of the elasticity will be biased downward in countries with several observations of interest rates near zero. The reason is that the procedure will try to match those observations with low interest rates that all lie below the log-log curve that has a good fit with the observations for moderate and high interest rates. However, given that the number of observations at very low rates is not such a large fraction of the total sample, we expect this bias to be small.

In Figure 5, we present the results of the estimation for 33 countries using the Stock and Watson procedure. As explained above, for 2 countries (Hong Kong and Morocco), we could not run the tests, and for 3 of them (Finland, Italy, and Turkey), the test did not detect cointegration for the log-log specification. The horizontal axis represents the value for the estimator of the elasticity for the corresponding country, ranging from -1 to 1. For each country, we report the point estimator with a black rhombus and the 90% confidence interval with a dotted red line. We order the countries according to the point estimate, starting with the lowest one. Finally, in the figure

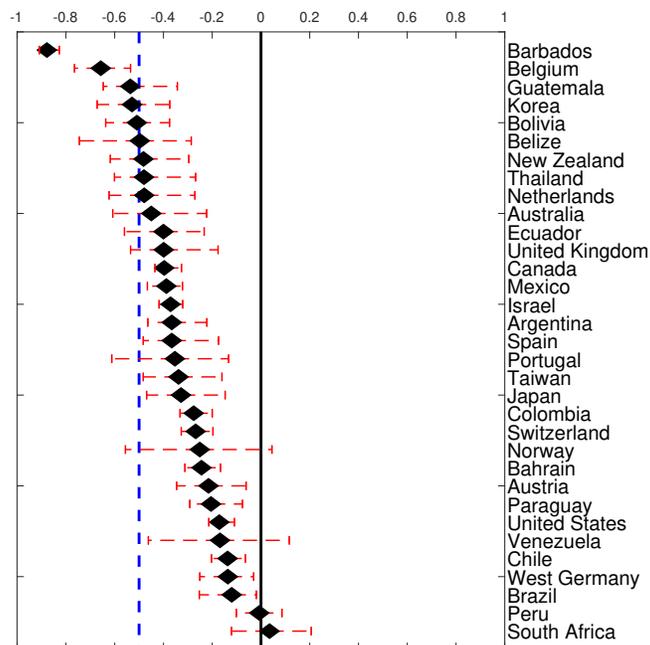


Figure 5 Estimation results using the procedure from Stock and Watson (1993)

we plot two vertical lines: one at zero, which corresponds to the null of no relationship between the log of the interest rate and the log of real money demand, and one at the value 0.5, which corresponds to the linear technology assumed by Baumol and Tobin.<sup>25</sup>

As a summary of the figure, the confidence interval includes zero for only 4 out of the 33 countries. Two of them (Norway and South Africa) belong to the group with either weak or nonexistent evidence. In no case is the estimate statistically larger than zero. Finally, for around 20 countries, the confidence interval includes 0.5 or is remarkably close to it. Table 2, together with the country plots in Figures 3a to 3c give very strong support to a rather simple theory that, in its essence, was developed over half a century ago.

### 5.3 Evidence on the functional form

Figure 6 provides simple, informal evidence on which specification – the Selden-Latané or the log-log – provides the most plausible description of the

<sup>25</sup>For the three countries for which we had two different sets of data, here we report the set that includes the latest observations.

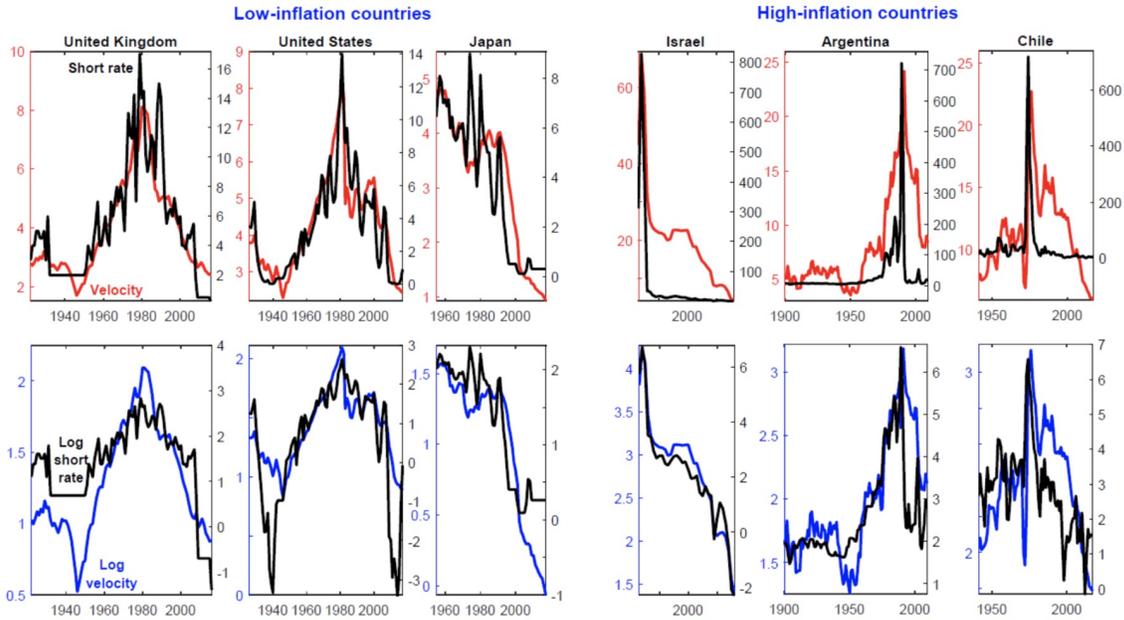


Figure 6 Comparing the Selden-Latané and log-log specifications: selected evidence for low-inflation and high-inflation countries

data at both low and high interest rates. For both low-inflation and high-inflation countries, the top row shows the levels of M1 velocity and the short rate, and the bottom row shows the logarithms of the two series. By plotting the series with different axes, we search for a linear relationship between either the levels or the logs. The evidence in the top row therefore corresponds to a Selden-Latané specification and the bottom row to the log-log specification.

Two broad patterns emerge from Figure 6. First, for the low-inflation countries, both formulations do a very good job at capturing the rise and fall of both velocity and interest rates in the United Kingdom and the United States, and the persistent fall of both in Japan. The Figure clearly shows, however, that the log-log specification is substantially worse when interest rates are close to zero for the three countries. This result is in line with our discussion of the borrowing constraints in the theoretical section. Second, for the high-inflation countries the opposite is true: the specification in logs appears to deliver a linear relationship, whereas the specification in levels does

not.<sup>26</sup> This overall pattern is consistent with the theory, where borrowing constraints play an important role in low interest rates.

We did not select countries in Figure 6 randomly; rather, they are the ones that either had interest rates close to zero for many periods or had very high inflation rates. The full comparison is presented in Online Appendix I, and while the general message is similar, the conclusion there is not as striking as the examples shown here.

## 6 Two Additional Issues

We now discuss two additional issues. First, we show that the estimators of the elasticity of money demand we used in Section 5.1—which are predicated on the assumption that the series feature exact unit roots—work equally well for series that are local-to-unity. Then, we discuss tests for the stability of the money demand cointegration relationship.

### 6.1 Robustness of the estimates of the elasticity of money demand to near unit roots

Table 3 reports results from the following Monte Carlo experiment. We simulate the following DGP for the logarithms of the short rate,  $R_t$ , and of M1 velocity,  $V_t$ :

$$\ln R_t = \lambda \ln R_{t-1} + \epsilon_t, \text{ with } \lambda = 1 - 0.5 \cdot (1/T), \epsilon_t \sim i.i.d. N(0, 1) \quad (12)$$

$$u_t = \rho u_{t-1} + v_t, \text{ with } 0 \leq \rho < 1, v_t \sim i.i.d. N(0, 1). \quad (13)$$

$$\ln V_t = \alpha_0 - \alpha_1 \ln R_t + u_t \quad (14)$$

We set  $\alpha_0=1$ , and  $\alpha_1$  equal to Baumol and Tobin’s benchmark value of 0.5. As for  $\rho$  we consider six possible values ranging from 0 to 0.95, corresponding to alternative extents of persistence of the cointegration residual. Finally, we consider four possible values for the sample length,  $T$ , ranging from 50 to 1,000. For each possible combination of values for  $T$  and  $\rho$  we simulate (12)-(14) 10,000 times, and based on each artificial sample, we estimate

---

<sup>26</sup>These results are in line with the evidence produced by Benati (2019b) based on either monthly or weekly data for 20 cases of hyperinflation, from the French Revolution to Venezuela’s episode: in nearly all cases, econometric evidence shows a clear and often overwhelming preference for the log-log specification.

$\rho$	0	0.25	0.5	0.75	0.9	0.95
T = 50	0.5002	0.4983	0.5018	0.4978	0.4889	0.4992
T = 100	0.5007	0.4996	0.4995	0.5002	0.5025	0.5010
T = 200	0.5000	0.4999	0.4997	0.4989	0.4990	0.4982
T = 1000	0.5000	0.5000	0.5000	0.4998	0.5002	0.5004

Table 3: Mean of Monte Carlo distribution for alternative values of T and  $\rho$

the elasticity of money demand as we did in Section 5.1, based on either Johansen’s or Stock and Watson’s (1993) procedures. Table 3 reports the mean of the Monte Carlo distribution for the estimates of  $\alpha_1$  based on Stock and Watson’s procedure (results based on Johansen’s procedure are qualitatively the same, and they are available upon request). The evidence in the table speaks for itself and shows that the estimates of the elasticity of money demand we discussed in Section 5.1 are in fact robust to the series being local-to-unity, rather than featuring exact unit roots.

## 6.2 Testing for stability in cointegration relationships

We test for stability in cointegration relationships based on the three tests discussed by Hansen and Johansen (1999): two Nyblom-type tests for stability in the cointegration vector and the vector of loading coefficients, respectively; and a fluctuation test, which is essentially a joint test for time variation in the cointegration vector and the loadings. In either case, we bootstrap the test statistics via CRT’s procedure, based on the VECM estimated conditional on one cointegration vector, and not featuring any break or time variation of any kind.

Table H.1 in the Online Appendix reports Monte Carlo evidence on the performance of the tests conditional on bivariate cointegrated DGPs, for alternative sample lengths and alternative degrees of persistence of the cointegration residual, which is modeled as an AR(1). The main results can be summarized as follows. The two Nyblom-type tests exhibit an overall reasonable performance, incorrectly rejecting the null of no time variation, most of the time, at roughly the nominal size. Crucially, this is the case irrespective of the sample length and of the persistence of the cointegration residual. The fluctuation test, on the other hand, exhibits good performance only if the persistence of the cointegration residual is low. The higher the residual’s

persistence, however, the worse the performance, so that for example, when the AR root of the residual is equal to 0.95, for a sample length  $T = 50$ , the test rejects at twice the nominal size. This result is clearly problematic since, as previously discussed, residuals are typically moderately to highly persistent. In what follows, we therefore focus on the results from the two Nyblom-type tests, but we eschew instead results from the fluctuation test (these results are reported in Tables H.2 and H.5 in the Online Appendix).

We now turn to the results from cointegration tests and tests for time variation in cointegration relationships.

Tables H.2 and H.3 in the Online Appendix report results from Hansen and Johansen's (1999) Nyblom-type tests for stability in either the cointegration vector or the vector of loading coefficients. The key finding in the two tables is that evidence of breaks in either the cointegration vector or the loading coefficients is weak to nonexistent. Specifically, for the United States, based on the Selden-Latané specification, the null of no breaks in either feature is never rejected for either New M1 or any of the other two "expanded" M1 aggregates. Stability in the cointegration vector is also never rejected based on semi-log and log-log specifications, whereas breaks in the loadings are detected based on the log-log specification, and in one case out of six based on the log-log. Evidence for other countries is qualitatively the same. For instance, based on the Selden-Latané specification, stability in the cointegration vector is rejected in three cases, whereas stability in the loadings is rejected in six cases. Results for the log-log specification are along the same lines.

## 7 Conclusions

Our review of real money demand behavior leads us to reach the following conclusions. First, there is overwhelming evidence of a long-run relationship between the ratio of money to nominal income and the short-term interest rate. Second, the log-log specification that implies a constant elasticity is a very good representation of the data, except when interest rates are very close to zero. Third, there is strong evidence of a satiation point at zero, implying that the elasticity of real balances with respect to the interest rate is lower in that range, approaching zero as interest rates go to zero.

## References

- [1] Alvarez, F., and F. Lippi (2009): “Financial Innovation and the Transactions Demand for Cash,” *Econometrica*, 77(2), 363-402.
- [2] Alvarez, F., and F. Lippi (2014): “Persistent Liquidity Effects and Long-Run Money Demand,” *American Economic Journal: Macroeconomics*, 6(2), 71-107.
- [3] Baumol, W.J. (1952), “The Transactions Demand for Cash: An Inventory Theoretic Approach, ” *Quarterly Journal of Economics*, 66(4), 545-556.
- [4] Benati, L. (2009): “Long-Run Evidence on Money Growth and Inflation,” European Central Bank Working Paper 1027, March.
- [5] Benati, L. (2015): “The Long-Run Phillips Curve: A Structural VAR Investigation, ” *Journal of Monetary Economics*, 76, 15-28.
- [6] Benati, L. (2019a): “Money Velocity and the Natural Rate of Interest,” mimeo, University of Bern.
- [7] Benati, L. (2019b): “Cagan’s Paradox Revisited,” mimeo, University of Bern.
- [8] Benati, L., R. E. Lucas, Jr., J.-P. Nicolini, and W. Weber (2017): “International Evidence on Long-Run Money Demand,” working paper 737, Federal Reserve Bank of Minneapolis.
- [9] Cavaliere, G., A. Rahbek, and A.M.R. Taylor (2012): “Bootstrap Determination of the Co-integration Rank in Vector Autoregressive Models,” *Econometrica*, 80(4), 1721-1740.
- [10] Diebold, F.X., and C. Chen (1996): “Testing Structural Stability with Endogenous Breakpoint: A Size Comparison of Analytic and Bootstrap Procedures,” *Journal of Econometrics*, 70(1), 221-241.
- [11] Elliot, G. (1998): “On the Robustness of Cointegration Methods When Regressors Almost Have Unit Roots,” *Econometrica*, 66(1), 149-158.
- [12] Elliot, G., T.J. Rothenberg, and J.H. Stock (1996): “Efficient Tests for an Autoregressive Unit Root,” *Econometrica*, 64(4), 813-836.

- [13] Engle, R.F., and C.W. J. Granger (1987): “Co-integration and Error Correction: Representation, Estimation, and Testing,” *Econometrica*, 55(2), 251-276.
- [14] Grossman, S., and L. Weiss (1983): “A Transactions-Based Model of the Monetary Transmission Mechanism,” *American Economic Review*, 73(5), 871-880.
- [15] Hansen, B.E. (1999): “The Grid Bootstrap and the Autoregressive Model,” *Review of Economics and Statistics*, 81(4), 594-607.
- [16] Hansen, H., and S. Johansen (1999): “Some Tests for Parameter Constancy in Cointegrated VAR-Models,” *Econometrics Journal*, 2(2), 306-333.
- [17] Judson, R. (2017): “The Death of Cash? Not So Fast: Demand for U.S. Currency at Home and Abroad, 1990-2016,” paper presented at the International Cash Conference 2017 - War on Cash: Is there a Future for Cash? April 25-27, 2017, Island of Mainau, Germany, Deutsche Bundesbank, Frankfurt a. M.
- [18] Latané, H.A. (1960): “Income Velocity and Interest Rates: A Pragmatic Approach,” *Review of Economics and Statistics*, 42(4), 445-449.
- [19] Lucas, R.E. Jr. (1988): “Money Demand in the United States: A Quantitative Review,” *Carnegie-Rochester Conference Series on Public Policy*, 29, 137-168.
- [20] Lucas, R.E. Jr., and J.-P. Nicolini (2015): “On the Stability of Money Demand,” *Journal of Monetary Economics*, 73, 48-65.
- [21] Luetkepohl, H. (1991): *Introduction to Multiple Time Series Analysis*, 2nd ed. Springer-Verlag.
- [22] Meltzer, A.H. (1963): “The Demand for Money: The Evidence from the Time Series,” *Journal of Political Economy*, 71(3), 219-246.
- [23] Selden, R.T. (1956): “Monetary Velocity in the United States,” in M. Friedman, ed., *Studies in the Quantity Theory of Money*, University of Chicago Press, pp. 405-454.

- [24] Stock, J.H., and M.W. Watson (1993): “A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems,” *Econometrica*, 61(4), 783-820.
- [25] Tobin, J. (1956): “The Interest Elasticity of Transactions Demand for Cash,” *Review of Economics and Statistics*, 38(3), 241-247.
- [26] Wright, J.H. (2000): “Confidence Sets for Cointegrating Coefficients Based on Stationarity Tests,” *Journal of Business and Economic Statistics*, 18(2), 211-222.