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A CONTRIBUTION TO THE THEORY OF PORK BARREL SPENDING

by

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PRELIMINARY AND INCOMPLETE

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Government policies frequently lead to inefficient outcomes. Many government programs deliver benefits to small, well-organized groups and impose costs on the rest of the population. They are sometimes inefficient in the usual sense that the costs exceed the benefits. In a representative democracy, like the United States, such programs require the assent of at least a majority of the legislators. Why do legislators consent to such programs? Furthermore, if a particular inefficient program benefits a small group, why can't legislators agree to an alternative policy that leaves this group at least as well off and makes the rest of the people better off?

The conventional wisdom in political economy (see for example, Downs 1957, Buchanan and Tullock 1962, Ferejohn 1974, and Ferejohn and Fiorina 1975) is that since small numbers of people are easier to organize into a group than large numbers, programs with concentrated benefits and diffuse costs are likely to be adopted. Why might it be more difficult for large numbers of people to defeat proposals which cost them individually a small amount, and collectively a large amount? One argument, due in large part to Olson (1965) is that there is a free rider problem which arises when each individual perceives the outcome to be largely unaffected by his decision and so chooses not to cooperate. In this paper, we present a formal model which captures the essence of this conventional wisdom. One advantage of our formal model is that it makes clear the assumptions under which the free rider problem emerges causing programs with concentrated benefits and diffuse costs are adopted.

We first consider a model where a central government provides local public goods financed by uniform taxation. The amounts of these goods are determined by majority voting in a legislature. We allow legislators to make payments to other legislators contingent on how these other legislators vote. We show that in such an environment, an inefficiently high level of local public goods is provided. This inefficiency is reduced if a supermajority vote is required for passage and the allocations are efficient if unanimous consent is required. We then allow legislators to make payments to bill proposers contingent on the nature of the bills that are proposed. We show that even allowing for such payments, an inefficiently high level of expenditures results if the legislature has a large number of members and if there is other legislators are uncertain about the proposer's preferences. Again, the inefficiency disappears with unanimous consent rules.

Since unanimous consent seems desirable for local public goods, we ask how such a rule works for global public goods allocations. We show that such a rule can work very badly. This result is familiar from the literature on mechanism design in public goods economies (see Rob 1989, Mailath and Postlewaite 1990, and Chari and Jones 1991). Thus, there is a conflict between desirable voting rules for allocating global public goods and those for allocating local public goods. A single voting rule cannot achieve desirable outcomes for both types of problems.

While our model has legislators making payments to each other, one interpretation is that interest groups whom a particular legislator represents make campaign contributions. We prefer this interpretation.

Three features of our environment play important roles in generating our results. First, interest groups must be able to influence the decisions of legislators through, for example, campaign contributions. Second, not all legislators need assent to a program for it to be adopted. For example, a bill can be passed if only a majority of legislators votes for it. Third, there must be some uncertainty about the benefits to interest groups from the program. The ability of interest groups to influence legislators' decisions combined with, say, majority voting allows interest groups to extract surplus from the minority without making the majority worse off. In order to accomplish this goal, it is important that voting decisions be observable, as they are in legislatures. The affected minority would like to be able to bribe interest groups to offer welfare enhancing proposals. But because the minority is not certain about the benefits to the interest groups, such groups have incentives to demand excessive bribes, leading each skeptical legislator to believe that his decision will not materially affect the outcome.

There is a large related literature on public choice, rent-seeking behavior and political economy. While we do not survey this literature, perhaps we should mention what is the closest paper to ours by Weingast, Shepsle, and Johnsen (1981). They offer an explanation for pork barrel projects based on differences in preferences between the legislators and voters. They plausibly argue for such differences in preferences but it is unclear why voters would elect such legislators in the first place.

The plan of the paper is as follows. In Section 1, we lay out a model of local public goods provision in which the beneficiary of the local public good proposes a bill and purchases votes from other representatives. In Section 2, we show that the results of Section 1 hold regardless of who owns the right to propose bills provided this right can be sold. In Section 3, we allow other representatives to offer payments to change the proposed bill. Section 4 contains an analysis of the provision of global public goods. Section 5 concludes.

1. Local Public Goods and Vote Buying

There are I symmetric districts, each of which has a single representative. Both districts and representatives are indexed by $i = 1, \ldots, I$. For notational convenience, we normalize the population of each district by unity. The government provides a local public good for each district. Let g_i denote the amount of the local public good provided in district i. The public goods are financed by a uniform tax τ on the entire population so that $\tau = (1/I)\Sigma_{i=1}^{I} g_i$. Each representative has identical preferences defined over the level of public expenditures and taxes in his district, as well as his own private consumption c_i, which are given by

$$(1.1) \quad U_i(g_1,\ldots,g_I,c_i) = f(g_i) - \tau + c_i.$$

We assume that f is strictly concave and twice differentiable $f(0) = 0$, $f'(0) > 1$, and $\lim_{g \to \infty} f'(g) = 0$.

We will say that an allocation of the public goods $(g_1,\ldots,g_I)$ is *efficient* if it satisfies

(1.2) $f'(g_i) = 1.$

There are a variety of ways of modeling the bargaining process among representatives which determines the provision of the local public goods. We model this bargaining process by considering an environment in which representatives can make side payments to influence each others' votes. These side payments can be interpreted as payments made by constituents in a district to other legislators. We take as a given institutional feature that the spending outcomes are determined by plurality voting separately on each local public good. Specifically, one of the representatives proposes a bill to spend g_i units in district i. If more than kI representatives vote for the bill it passes, where $k \in [0.5,1)$ is fixed exogenously as part of the prevailing political constitution. If not, spending in that district is set at zero.

Formally, we model the level of spending in each district as a two-stage game. In stage 1 each representative i simultaneously chooses a number $g_i \in \mathbf{R}_+$ and a vector $m_i \in \mathbf{R}_+^I$; with m_{ii} normalized to zero. The object m_{ij} is interpreted as the amount of money that representative i has pledged to pay representative j if j votes for representative i's proposal to spend g_i on local public goods in i's district. In stage 2 each representative chooses a vector $v_i \in \{0,1\}^I$. The object v_{ij} is interpreted as representative i's vote on j's spending proposal, where a one is interpreted as a "yes" and a zero as a "no" vote. If $(1/I)\Sigma_{i=1}^I v_{ij} > k$, then the proposal "passes," and the level of government spending in district i is g_i, and if $(1/I)\Sigma_{i=1}^I v_{ij} \leq k$ the proposal fails and the spending level in district i is zero.

The payoff to representative i from this game is given by

$$(1.3)\quad U_i(m,g,v) = f(\delta_i g_i) - \frac{1}{I}\sum_{j=1}^{I}\delta_j g_j + \sum_{j=1}^{I}[m_{ij}v_{ji} - m_{ji}v_{ij}],$$

where

$$(1.4)\quad \delta_i(v) = \begin{cases} 1 & \text{if } (1/I)\sum_{j=1}^{I} v_{ij} > k \\ 0 & \text{otherwise} \end{cases}.$$

An equilibrium of the game is a pair of vectors $m \in \mathbb{R}^{I^2}$ and $g \in \mathbb{R}^I$, and a set of functions $v_i(g,m) \to \{0,1\}^I$ for $i = 1, \ldots, I$, such that for all $i = 1, \ldots, I$:

(i) for all $g \in \mathbb{R}_+^I$ and $m \in \mathbb{R}^{I^2}$, v_i satisfies

$$(1.5)\quad f(\delta_i(v)g_i) - \frac{1}{I}\sum_{j=1}^{I}\delta_j(v)g_j - \sum_{j=1}^{I}[m_{ij}v_{ji} - m_{ji}v_{ij}]$$

$$\geq f(\delta_i(v_{-i},\hat{v}_i)) - \frac{1}{I}\sum_{j=1}^{I}\delta_j(v_{-i},\hat{v}_i)g_j - \sum_{j=1}^{I}[m_{ij}v_{ji} - m_{ji}\hat{v}_{ij}] \text{ for all } \hat{v}_i \in \{0,1\}^I, \text{ and}$$

(ii) given $v(\cdot)$, g and m satisfy

$$(1.6)\quad f(\delta_i(v(g,m)) - \frac{1}{I}\sum_{j=1}^{I}\delta_j(v(g,m))g_j - \sum_{j=1}^{I}[m_{ij}v_{ji}(g,m) - m_{ji}v_{ij}(g,m)]$$

$$\geq f(\delta_i(v(g_{-i},\hat{g}_i,m_{-i},\hat{m}_i)) - \left[\frac{1}{I}\sum_{j\neq 1}\delta_j(v(g_{-i},\hat{g}_i,m_{-i},\hat{m}_i))g_j + \frac{1}{I}\delta_i(v(g_{-i},\hat{g}_i,m_{-i},\hat{m}_i))\hat{g}_i\right]$$

$$- \sum_{j=1}^{I}[\hat{m}_{ij}v_{ji}(g_{-i},\hat{g}_i,m_{-i},\hat{m}_i) - m_{ji}v_{ij}(g_{-i},\hat{g}_i,m_{-i},\hat{m}_i)]$$

for all $\hat{g}_i \in \mathbb{R}$ and $\hat{m}_i \in \mathbb{R}^I$.

As a first step, we construct an equilibrium of the model as follows. First, note that from (1.5), it is a weakly dominant strategy for representative i to set:

(a) $v_{ii} = 1$ if $f(g_i) - \frac{g_i}{I} \geq 0$ and,

(b) $v_{ij} = 0$ if $m_{ji} = 0$ and $v_{ij} = 1$ if $m_{ji} \geq \frac{g_j}{I}$.

So assume that $v(\cdot)$ satisfies (a) and (b). Now all that remains is to specify v_{ij} when $i \neq j$ and $0 < m_{ji} < g_j/I$. For a given j, let L_j equal the number of representatives, i, for whom (b) is satisfied. Then set $v(\cdot)$ as follows

(c) $v_{ij} = 1$ if $m_{ji} > 0$ and $L_j \geq K$, and

(d) if $L_j < K$, then for first $K - L_j - 1$ i's (not including $i = j$) for which $0 < m_{ji} < g_j/I$, $v_{ij} = 1$, and $v_{ij} = 0$ for the rest (again not including $i = j$).

Under our construction of $v(\cdot)$, no representative has any incentive to set m_{ij} so as to satisfy the condition in (b) on more than the minimum number of votes to secure passage. That number of votes, given that a representative will vote for his own proposal, is given by $K = \min\{n$ such that $n \in [1,2,\ldots]$, and $n+1 > kI\}$.

Now, consider the following maximization problem:

(P1) $$\max_{g_i, m} f(g_i) - \frac{g_i}{I} - Km$$

subject to

(1.7) $$m \geq \frac{g_i}{I}.$$

This concave programming problem has a unique solution given by

$$(1.8)\quad f'(\hat{g}_i) = \frac{K+1}{I},$$

and

$$(1.9)\quad \hat{m}_i = \frac{\hat{g}_i}{I}.$$

Let us summarize the previous discussion by giving a precise description of the equilibrium we are constructing. Let $g_i = \hat{g}_i$ for all $i = 1, \ldots, I$. Now, for each $i = 1, \ldots, I$, there must be exactly K j's $\in \{1,\ldots,i-1,i+1,\ldots,I\}$ (not including $j = i$) such that $m_{ij} = \hat{m}_i$, while $m_{ij} = 0$ for the rest. Since it doesn't matter how these j's are selected, assume for the sake of symmetry that they are selected randomly, with each $j \neq i$ having a $K/(I-1)$ chance of being selected. Finally we can completely determine $v(\cdot)$ by requiring it satisfy condition (a)–(d). The symmetric equilibrium we have just described has the property that should one of the positive voters turn negative on a spending proposal, it would fail. We shall refer to such an equilibrium as a *pivotal* equilibrium.

Proposition 1. The objects $\{g,m,v(\cdot)\}$ specified above constitute an equilibrium.

Proof. The proof consists of verifying that $\{g,m,v(\cdot)\}$ satisfies (1.5) and (1.6). Since $v(\cdot)$ satisfies (a)–(d), (1.5) is satisfied. Given the specification of $v(\cdot)$ a representative i would do strictly better by setting m_{ij} to be positive on no more than K j's for $j \neq i$. Given that this is the case, and that $\hat{g}_i$ and $\hat{m}_i$ are a solution to (P1), (1.6) is satisfied for all $i = 1, \ldots, I$. □

The equilibrium that we have just described is clearly not unique even among the class of pivotal equilibria, since there were many ways in which the choice of whose vote to "buy" could be specified. However, as the following proposition makes clear, within this class of equilibria there is a unique level of local government spending.

Proposition 2. There is a unique level of government spending in any pivotal equilibria given by $g = \hat{g}$.

Proof. Since by definition only K + 1 representatives vote "yes" on any proposal, the inequality in condition (b) must be satisfied for any $v_{ij} = 1$ and $i \neq j$ if condition (i) of the definition of an equilibrium is to hold. Note that the inequality in (b) cannot hold strictly in this case since i could lower m_{ij} by some amount while still strictly satisfying the inequality in (b), hence making himself better off. Now, without loss of generality set $m_{ij} = 0$ for all i, j such that $v_{ij} = 0$. Then note that for each g_i, and for each $j \neq i$ such that $v_{ij} = 1$, g_i, and m_{ij} must be a solution to (P) if (1.6) is to be satisfied. □

The level of government spending is inefficiently high in the pivotal equilibrium to the extent k is less than one. The simplest way to see this is to consider the level that would be chosen by an egalitarian social planner. The planner would set the level of spending in each district g, so that $f'(g) = 1$. Comparing this to the level in the pivotal equilibrium, $\hat{g}$ is such that $f'(\hat{g}) = (K+1)/I$. Note that k determines the magnitude of K and hence of $(K+1)/I$, and further that $(K+1)/I$ is smaller than one to the extent that k is less than one. (For large I, $(K+1)/I$ is approximately equal to k.) Thus, the extent to which a representative internalizes the effect of his proposed spending on other districts depends on the degree to which k is close to one.

It is also interesting to compare the pivotal equilibrium outcome to that which would occur if there were no side payments; that is, $m_{ij} = 0$ for all i, j, = 1, ..., I. In the voting stage $v(\cdot)$ would still have to satisfy (b), which would imply $\delta_i g_i = 0$ as the unique outcome for all i = 1, ..., I. Thus, the outcome under straight majority voting is for spending on local public goods to be inefficiently low.

There can also exist nonpivotal equilibria, since any representative who believes that his vote cannot affect the outcome of the proposal to spend g_j in district j will strictly prefer to set $v_{ij} = 1$ if $m_{ij} > 0$. Briefly, to construct a symmetric nonpivotal equilibrium first let J be the number of votes which must be purchased in equilibrium, where $J \geq K + 1$, and $\bar{m}$ is their price and $\bar{g}$ the level of spending. Then change condition (d) in the specification to be

(d′) if $L_j < K$:

(1) if the number of i's such that $m_{ji} \geq \bar{m}$ is greater than or equal to J, and $g_j \leq \bar{g}$, set $v_{ij} = 1$ if $m_{ji} \geq \bar{m}$ and equal to zero if $m_{ji} < \bar{m}$

(2) if the number of i's such that $m_{ji} \geq \bar{m}$ is less than J, or $g_j > \bar{g}$, then follow (d).

So long as the following conditions on J, $\bar{m}$, and $\bar{g}$ are satisfied then the above specification is part of a set of symmetric equilibrium strategies:

(1.10) $\bar{m} < \frac{\bar{g}}{I}$,

(1.11) $f(\bar{g}) - J\bar{m} \geq f(0)$,

(1.12) $f'(g) - \frac{1}{I} > 0$, and

(1.13) $f(g) - \frac{g}{I} - J\bar{m} \geq f(\hat{g}) - \frac{\hat{g}}{I} - K\hat{m}$

where $\hat{g}$ and m are a solution to (P1).

Condition (1.10) is a requirement that each representative whose vote is "purchased" actually weakly prefers that the proposal not pass, so that a representative cannot deviate and purchase only K votes while still ensuring passage. Condition (1.11) requires that no representative wishes to deviate, not purchase any votes, and have his proposal fail. Condition (1.12) ensures that the representative wishes to set $g_i = \bar{g}$. The last condition, (1.13), ensures that no representative wishes to deviate, choose a different level in his spending proposal, and so compensate K of the other representatives that they strictly prefer to vote for his proposal.

The reason that we have discussed these nonpivotal equilibria so casually is that we find them inherently implausible because of the extreme informational assumptions necessary for their existence. To see this implausibility, consider a slight variant of our game. In this variant, each representative, only observes payments he made to others, or others made to him. That is each representative i only observes m_{ij} and m_{ji} for $j = 1, \ldots, I$.

Since this is a game with incomplete information, we must introduce inferences for each representative about payments made to others from payments made to him. In order to introduce these inferences, we now have to distinguish between the history of the game in the second stage, $h = (g,m)$, and the private history which a given representative, say i, observes $h_i = (g,(m_{ij})_{j=1}^{I}, (m_{ji})_{j=1}^{I})$. We also have to specify each representative's beliefs about the actual history h, based upon his observed private history, h_i. Let $\mu_i(h|h_i)$ denote representative i's conditional probability distribution over h given h_i.

We now define a perfect Bayesian equilibrium of this model as a pair of vectors g and m, and a set of pairs of functions $\{v_i(h_i), \mu_i(h|h_i)\}_{i=1}^{I}$ such that (i′) for all $g \in \mathbf{R}^I$ and $m \in \mathbf{R}^{I^2}$, v_i satisfies

$$E_{\mu_i(h|h_i)}\left\{f(\delta_i(v)g_i) - \frac{1}{I}\sum_{j=1}^{I}\delta_j(v)g_j - \sum_{j=1}^{I}[m_{ij}v_{ji} - m_{ji}v_{ij}]\right\}$$

$$\geq E_{\mu_i(h|h_i)}\left\{f(\delta_i(v_{-i},\hat{v}_i)) - \frac{1}{I}\sum_{j=1}^{I}\delta_j(v_{-i},\hat{v}_i)g_i - \sum_{j=1}^{I}[m_{ij}v_{ji} - m_{ji}\hat{v}_{ij}]\right\}$$

for all $\hat{v}_i \in \{0,1\}^I$, along with condition (ii) from our complete information game, and (iii′) that μ_i be consistent with the specification of the strategies. Note that since in our definition the strategies are pure, μ_i must put probability one on the equilibrium history, so long as the private histories are consistent with it. Thus, the only issue with respect to μ_i will be specifying out of equilibrium beliefs.

It is easy to see how one would adjust the pivotal equilibrium to such a change in the game form. Now v(•) is set so that (a), (b), and the following condition hold:

(d″) $\quad v_{ij} = 0$ if $g_j > 0$ and $m_{ji} < \frac{g_j}{I}$,

while m and g are unaffected. The representatives' beliefs are that if $m_{ij} > 0$ then representative j believes that exactly K − 1 other representatives have been offered $\hat{m}$, and therefore votes "yes." Hence, his vote is pivotal and condition (d″) is consistent with condition (i′) in the definition. Given this specification of v(•), no representative purchases more than K votes. Hence, the price per vote that he pays, m_i, and the level of g_i he proposes are a solution to (P); given his beliefs about the magnitude of spending proposals which will pass excluding his own. Thus, there is a unique perfect Bayesian equilibrium which satisfies condition (d″) on v(•) since (P) has a unique fixed point.

However, no such generalization of a nonpivotal equilibrium is possible except in the case where $\bar{m} = 0$. This is because, if v(•) was such that (a) and the following condition held:

$$v_{ij} = 1 \text{ if } g_j \leq \bar{g} \text{ and } m_{ji} \geq \bar{m} < \frac{g_j}{I}, \text{ and } v_{ij} = 0 \text{ otherwise,}$$

then no representative would buy more than K votes, and hence v(•) would violate condition (i′) in the definition of a perfect Bayesian equilibrium. The nonpivotal equilibrium with $\bar{m} = 0$ seems implausible since the representatives are playing a weakly dominated strategy in the voting stage.

We now want to consider generalizing the model to allow for representatives having differential benefits over the level of local public expenditure which are not publicly observable. Let each representatives' benefits from the local public good be given by $\theta_i f(g_i)$ where θ_i is a random variable drawn from a c.d.f. $H(\theta)$. Representative i's preferences are now given by

$$(1.14)\quad U_i(g_1,\ldots,g_n,c_i,\theta_i) = \theta_i f(g_i) - \tau + c_i, \quad \text{for } i = 1, \ldots, I.$$

We construct the pivotal equilibrium of the model as follows. First, assume that $v(\cdot)$ is such that versions of (a)-(d) hold, where these conditions have been modified to take account of the fact that $t(\cdot)$ is linear. Thus, for example, condition (b) becomes:

$$\text{(b)}\qquad v_{ij} = 1 \text{ if } m_{ji} \geq \frac{1}{I} g_j, \text{ and } v_{ij} = 0 \text{ otherwise.}$$

Under our assumptions about $v(\cdot)$, once again no representative will acquire more than the minimum number of votes to assure passage, $\bar{K}$. Thus, the price per vote, m_i, and the level of g_i will still be solutions to a suitably modified version of the maximization problem (P1) given below:

$$\text{(P2)}\qquad \max_{g_i, m_i} \theta_i f(g_i) - \frac{g_i}{I} - km_i$$

subject to

$$m_i \geq \frac{g_i}{I},$$

$$g_i \geq 0.$$

Once again this concave programming problem has a unique solution given θ_i.

The strategies now consist of functions $g: \Theta^I \rightarrow \mathbb{R}^I$ and $m_i: \Theta^I \rightarrow \mathbb{R}^{I^2}$. Now, let $g_i(\theta_i) = \hat{g}(\theta_i)$, where $\hat{g}(\theta_i)$ is part of a solution to (P). Now, for each $i = 1, \ldots, I$ there are K j's $\in \{1,\ldots,i-1, i+1,\ldots,I\}$ chosen randomly, with each one having probability $K/(I-1)$ of being selected, such that $m_{ij}(\theta_j) = \hat{m}(\theta_j)$ (where $\hat{m}(\theta_j)$ is part of a solution to (P2)), while $m_{ij} = 0$ for the rest. We have already completely specified $v(\cdot)$.

Proposition 3. The objects $\{g(\cdot),m(\cdot),v(\cdot)\}$ specified above constitute an equilibrium.

Proof. The argument is similar to that in Proposition 1.

Proposition 4. For any realization $\theta \in \Theta^I$, there is a unique level of government spending in any pivotal equilibrium given by $\hat{g}(\theta)$.

Proof. The proof is similar to that in Proposition 2.

2. Local Public Goods and Proposal Right Auctions

Thus far we have assumed that each representative owns the right to set the level of proposed spending in his district. We now extend the model to allow for a more general initial assignment of these rights, where these rights are assumed to be tradable commodities. For simplicity we focus on one district's spending level. We establish conditions under which the outcome in terms of the spending level is invariant to the initial assignment of the proposal rights. It then follows that if these conditions are satisfied, then this invariance property holds in a more general game in which the initial proposal right is random for all of districts simultaneously.

Consider an arbitrary district, say district j, and assume that representative 1 initially owns the proposal right in this district. The benefit to representative j from the public good is f(g). In the auction stage each representative i chooses a price p_i for $i = 1, \ldots, I$, where for $i \neq 1$ p_i can be interpreted as i's purchase offer to representative 1 for this right, while p_1 is the reservation price of representative 1. The representative whose price is the highest ends up owning the right and pays representative 1 the amount that he offered. If there is a tie then the outcome is decided by a coin toss, except in the case where the tie involves j or 1. In the case of a tie with j ownership is transferred to j, while in the case of a tie with 1 ownership is transferred to the other agent.

After the auction stage the voting game continues as before, except that the proposal right need not be owned by representative j. In the case where it is not, it should be clear that the equilibrium outcome of the voting stage is that the representative who owns the right, say i, will set $g_j = 0$ and his promised payments equal to zero, and everyone votes either yes or no. Thus, the net benefit to any representative $i \neq j$ from owning this right is the reduction in government spending less the expected level of his compensation in case when representative j owned the right: $(1 - (K/(I-1)))\hat{g}$ (where $\hat{g}$ is the solution to (P2) when $\theta = 1$). Symmetrically, this is also the opportunity cost to representative 1 of selling the right.

The value of the right to representative j is simply his payoff from the district j component of the voting game that we derived before, or $f(\hat{g}) - [(K+1)/I]\hat{g}$. Thus, an equilibrium of the model would be for

$$(2.1) \quad p_i = \left[1 - \frac{K}{I-1}\right]\hat{g} \quad \text{for } i = 1, \ldots, I,$$

and representative j to end up owning the right so long as:

$$(2.2) \quad f(\hat{g}) \geq \frac{\hat{g}}{I}\left[(K+1) + 1 - \frac{K}{I-1}\right].$$

Notice that as $I \rightarrow \infty$ and holding fixed $(K/I) = k$, the condition (2.2) implies

$$f(\hat{g}) \geq k\hat{g},$$

which from our assumptions holds strictly at $\hat{g}$.

Thus, so long as I is sufficiently large that condition (2.2) is satisfied, the value to representative j of owning the right to set the proposed spending level for his district is larger than for any other representative. The unique outcome of the auction stage in this case is for him to buy this right from representative 1 at the price specified by (2.1).

3. Local Public Goods and Proposal Buying

We turn now to an analysis of local public goods provision in environments where representatives can make side payments to influence the bills that can be proposed. One motivation for this analysis comes from our result that in plurality voting systems, vote-buying results in inefficiently high levels of local public goods expenditures. The representatives may then have an incentive to bribe the proposer to offer a bill with lower levels of local public goods. Since the public goods expenditures is inefficiently high in the vote-buying equilibrium, it is possible to construct such bribes or side payments which make everybody better off than in the vote-buying equilibrium. The problem, however, is as long as all other representatives make such side payments, each individual representative's contribution has a negligible effect on the outcome. Thus, each representative has an incentive not to bribe the proposer to reduce expenditure. That is, there is potentially a free rider problem. The purpose of this section is to show that this free rider problem results in an outcome where each bribe is close to zero and government expenditure is essentially the same as in the vote-buying equilibrium.

For simplicity, we consider the determination of spending in one district only, say district 1. Similar arguments apply to spending in other districts. We model the process for determining spending by a particular noncooperative game though, as will become clear, our results generalize to a large class of games.

Our game has three stages. The first stage is the proposal buying stage and the last two are the same as in our vote-buying game. In the first stage, representative 1 proposes to each of the other representatives a level of spending and a request for a side payment. Formally, representative 1 chooses nonnegative numbers (g_i,n_i), $i = 2, \ldots, I$ for each of the other representatives. Each of the other representatives then agrees or disagrees with the request. We denote this decision by y_i, $i = 2, \ldots, I$ where $y_i = 1$ signifies agreement and $y_i = 0$ signifies disagreement. The interpretation is that if representative i agrees to a proposal (g_i,n_i) and if representative 1 proposes a bill to spend g_i at the second stage, representative i must then pay representative 1 n_i units of the consumption good. Note that representative i's payments are not contingent on the subsequent vote. We have made these payments uncontingent to allow representative i to influence the outcome without having to commit to a voting decision.

The second and third stages of the game are exactly the same as in the vote-buying game. That is, representative 1 chooses a number G and a set of numbers, m_j, $j = 2, \ldots, I$. Then the representatives choose a vote vector v_j, $j = 1, \ldots, I$. A strategy profile for this game consists of pairs of numbers $(g,n) = \{g_i,n_i\}_{i=2}^{I}$ denoting offers by representative 1, a set of functions $y(g,n) = \{y_i(g,n)\}_{i=2}^{I}$ mapping offers into $\{0,1\}$ denoting agreement decisions, functions $G(g,n,y)$; $m(g,n,y) = \{m_i(g,n,y)\}_{i=2}^{I}$ denoting second stage decisions by representative 1 and voting functions $v(g,n,y,G,m)$ for the representatives. Let s denote the collection of strategies. The payoff to representative 1 is then given by

$$(3.1) \quad U_1(s) = f(\delta G) - \frac{1}{I}\,\delta G - \sum_{i=2}^{I} m_i v_i + \sum_{i=2}^{I} n_i y_i \alpha_i$$

where $\delta = 1$ if the vote is successful and $\delta = 0$ otherwise and $\alpha_i = 1$ if $G = g_i$ and zero otherwise. The payoffs to the other representatives are given by

$$(3.2) \quad U_i(s) = -\frac{\delta G}{I} + m_i v_i - n_i y_i \alpha_i.$$

A (subgame perfect) equilibrium of this game is defined in the usual fashion. In the discussion of the vote-buying game, we argued that there were strong reasons to restrict attention to the pivotal equilibrium. We will therefore restrict attention to equilibria such that for every history of the game at the second stage, the continuation strategies constitute a pivotal equilibrium for the vote-buying game. Recall that in that game, representative 1 randomly picked K representatives and purchased their votes for $m_i = G/I$ each. The payoff of representative 1 is then given by

$$(3.3) \quad U_1(s) = f(G) - \frac{(K+1)G}{I} + \sum_{i=2}^{I} n_i y_i \alpha_i.$$

The other representatives each perceive the probability of receiving a payment of G/I to be $K/(I-1)$ and with the complementary probability receive zero payment. The expected payoffs at the second stage for the other representatives are then

$$(3.4) \quad U_i(s) = -\frac{G}{I} + \frac{K}{I-1}\left[\frac{G}{I}\right] - n_i y_i \alpha_i.$$

An equilibrium (for the first stage) is a set of strategies for proposals, requests for payments, agreement decisions, and a spending decision on the local public good which satisfy the usual conditions. We begin our analysis by considering symmetric equilibria in which the offers (g_i, n_i) are the same for all i, and in which the agreement decisions $y_i(g,n)$ are also the same for all i. Consider the outcome when $n_i = 0$ for all i. In this case, G is determined by the solutions to the following problem

$$(3.5) \quad \max f(G) - \frac{(K+1)}{I} G.$$

We denote the solution to this problem by G_v.

It turns our that there are a large number of equilibria for our game. We characterize the equilibrium set by developing necessary conditions for equilibrium outcomes. Since representative 1 can always deviate to G_v and achieve the payoffs given by (3.5) it follows that if (G,n) is a symmetric equilibrium outcome, it must satisfy

$$(3.6) \quad f(G) - \frac{(K+1)G}{I} + (I-1)n \geq f(G_v) - \frac{(K+1)G_v}{I}.$$

Next, note that G_v solves (3.5). Thus, $f(G) - (K+1)G/I \leq f(G_v) - (K+1)G_v/I$. It follows that $n \geq 0$. Now consider the decision problem of one of the representatives, say i, who has been asked to contribute n units of the consumption good given that all the other representatives agree. Denote the payoff to this representative if he agrees by $U_i(1)$ and is given by

$$(3.7) \quad U_i(1) = -n - \frac{G}{I} + \left[\frac{K}{I-1}\right] \frac{G}{I}.$$

The first term on the right side of (3.7) is the payment that must be made, the second term is the taxes that must be paid to finance G and the last term is the expected payment at the voting stage. To see that the last term is the expected payment, recall that the probability of being chosen to have one's vote bought at the voting stage is $K/(I-1)$ and the payment conditional upon being chosen is G/I. The representative receives no payment if he is not chosen at the voting stage.

Now consider this representative's payoff if he disagrees. There are two cases to consider here. If $f(G) - (K+1)G/I + (I-2)n > f(G_v) - (K+1)G_v/I$, subgame perfection requires that representative 1 choose G even though representative i disagrees. In this case, $n > 0$ so representative i is better off by disagreeing since the government spending is unaltered and representative i does not have to pay representative 1. Therefore, a necessary condition for (G,n) to be an equilibrium is given by

$$(3.8) \quad f(G) - \frac{(K+1)G}{I} + (I-2)n \leq f(G_v) - \frac{(K+1)G_v}{I}.$$

The last necessary condition is given by the requirement that each representative must be better off agreeing then disagreeing. This condition is given by

$$(3.9) \quad -n - \frac{G}{I} + \left[1 - \frac{K}{I-1}\right] \frac{G}{I} \geq -\frac{G_v}{I} + \left[1 - \frac{K}{I-1}\right] \frac{G_v}{I}.$$

Conditions (3.6), (3.8), and (3.9) are also sufficient for (G,n) to be an equilibrium outcome. We have then

Proposition 5. Suppose (G,n) satisfies (3.6), (3.8), and (3.9) and $I \geq 4$. Then (G,n) is a (symmetric) subgame perfect equilibrium outcome of the proposal buying game.

Proof. Consider a proposal (G,n). We first argue that the strategy of disagreeing by representatives 2, ..., I is part of an equilibrium. To see this, note that if all other representatives disagree, regardless of the choice made by a given representative, the outcome at the vote-buying strategy is G_v. Thus, it is part of an equilibrium for that representative to disagree as well. Now construct the equilibrium strategies as follows. If representative 1 offers $(\hat{G},\hat{n}) \neq (G,n)$ all the other representatives disagree. If representative 1 offers (G,n) where (G,n) satisfies (3.6), (3.8), and (3.9), all the representatives agree. Given these strategies, representative 1 cannot do better than to offer (G,n). Thus, (G,n) is a subgame perfect equilibrium outcome. □

It is worth pointing out that $(G_v,0)$ satisfies (3.7) through (3.9) and is therefore an equilibrium outcome.

The plethora of equilibria for the proposal buying game is troublesome. In Figure 1 we plot the isoutility lines when (3.7), (3.9), and (3.9) hold with equality. The shaded area is the set of equilibrium outcomes. This set includes among others the efficient level of public goods provision. The theory appears to provide no clear outcome when representatives are allowed to buy proposals. It turns out, however, that with a small amount of private information, the equilibrium outcomes are close to $(G^v,0)$ when the number of representatives is large.

Proposal Buying with Private Information

We now examine the proposal buying outcomes when other representatives are uncertain about the benefits to representative 1 from the public good. We model this uncertainty by assuming that the benefit to representative 1 is given by $\theta f(G)$ where θ is a random variable drawn from a distribution $H(\theta)$ with strictly positive density $h(\theta)$ on an interval [a,b]. This uncertainty means that the other representatives are uncertain about G_v. We show that this uncertainty interacts with the free rider problem implicit in (3.8) and leads to spending levels of G_v.

A strategy profile for the proposal buying game now consists of (measurable) functions $(g_i(\theta),n_i(\theta))$ which map [a,b] into offers, agreement or disagreement decisions y(g,n) for the other representatives and a spending level G(g,n,y). To ensure that the other representatives have well defined decision problems, we also need a probability distribution $\mu_i(\theta|g,n)$ which describes representative i's beliefs about the representative 1's type. A perfect Bayesian equilibrium for this game is defined in the usual fashion.

We will show that as I gets large, the government spending level $G(\theta)$ converges to $G_v(\theta)$ for all θ. First, we show this result informally. Consider a sequence of economies indexed by I and a sequence of equilibrium outcomes functions $G(\theta;I)$, $n(\theta;I)$. Suppose that G and n are differentiable and suppose by way of contradiction that for all I sufficiently large $G(\theta;I) \neq G_v(\theta;I)$ for all θ in some interval $(\underline{\theta},\bar{\theta})$. Suppose also that the equilibrium is separating in this interval so that $G(\theta;I) \neq G(\hat{\theta};I)$ for $\theta \neq \hat{\theta}$. Now, maximization by representative 1 requires that $G(\theta)$, $n(\theta)$ (suppressing the index) satisfy the following

$$(3.10)\quad \theta \in \arg_{\hat{\theta}} \max \left\{ \theta f(G(\hat{\theta})) - \frac{(K+1)G(\hat{\theta})}{I} + (I-1)n(\hat{\theta}) \right\}.$$

Since G and n are differentiable, a necessary condition is

$$(3.11)\quad \left[\theta f'(G(\theta)) - \frac{(K+1)}{I}\right] G'(\theta) + (I-1)n'(\theta) = 0.$$

Next, we can use the same arguments as in the complete information environment to show that in a separating equilibrium, the analogues of conditions (3.6), (3.8), and (3.9) must continue to hold. That is, we have

$$(3.12)\quad \theta f(G(\theta)) - \frac{(K+1)G(\theta)}{I} + (I-1)n(\theta) \geq \theta f(G_v(\theta)) - \frac{(K+1)G_v(\theta)}{I}$$

$$(3.13)\quad \theta f(G(\theta)) - \frac{(K+1)G(\theta)}{I} + (I-2)n(\theta) \leq \theta f(G_v(\theta)) - \frac{(K+1)G_v(\theta)}{I}$$

$$(3.14)\quad n(\theta) \leq \left\{\frac{G_v(\theta)}{I} - \frac{G(\theta)}{I}\right\} \left\{1 - \frac{K}{I-1}\right\}$$

where, again, we have suppressed the index I for notational convenience. Recall that $G_v(\theta)$ maximizes the right side of (3.11) and is therefore bounded. We assume that as I increases $k = K/I$ remains constant. That is, the voting rule remains unchanged. From (3.14) it then follows that, since $G_v \geq G$, $n(\theta) \to 0$, and $(I-1)n(\theta)$ is uniformly bounded for all I. Next, note that the left sides of (3.12) and (3.13) differ only by $n(\theta)$ which goes to zero. Thus, the utility of representative 1 is arbitrarily close to utility under G_v for I sufficiently large. Define $G^*(\theta;I)$, $n^*(\theta;I)$ by the levels that set representative 1's utility equal to utility under G_v. That is, suppressing the I index, let $G^*(\theta)$, $n^*(\theta)$ satisfy

$$(3.15)\quad \theta f(G^*(\theta)) - \frac{(K+1)G^*(\theta)}{I} + (I-1)n^*(\theta) = \theta f(G_v(\theta)) - \frac{(K+1)G_v(\theta)}{I}.$$

Differentiating (3.15) with respect to θ and using the fact that G_v maximizes the right side of (3.15), we have

$$(3.16)\quad f(G^*(\theta)) + \left[\theta f'(G^*(\theta)) - \frac{(K+1)}{I}\right] G^{*\prime}(\theta) + (I-1)n^{*\prime}(\theta) = f(G_v(\theta)).$$

Now, we have argued that the utility of representative 1 approaches utility under G_v as I goes to infinity. Thus, $G(\theta)$ and $n(\theta)$ converge uniformly to $G^*(\theta)$ and $n^*(\theta)$. Comparing the first order condition of representative 1's maximization to (3.16) it follows that both can hold if and only if $G_v(\theta) = G(\theta)$ for all $\theta \in [\underline{\theta},\bar{\theta}]$.

We now make the same argument more formally without assuming differentiability. We restrict attention to separating equilibria and we conjecture that similar results apply to pooling equilibria. Let $G_v(\theta)$ maximize $\theta f(G) - kG$.

<u>Proposition 6</u>. Suppose $G(\theta;I)$, $n(\theta;I)$ is a sequence of separating equilibrium outcomes. Then $G(\theta;I)$ converges almost surely to $G_v(\theta)$ and $(I-1)n(\theta;I)$ converges almost surely to zero.

<u>Proof</u>. The proof is by contradiction. Suppose not. Then, taking subsequences if necessary and dropping the subscript on subsequences we have that there exists some interval $(\underline{\theta},\bar{\theta})$ and a number $\delta > 0$ such that for almost all θ in this interval, $G(\theta,I)$ converges to, say, $\hat{G}(\theta)$ and $(I-1)n(\theta,I)$ converges to, say, $\hat{N}(\theta)$ where $\hat{G}(\theta) < G_v(\theta)-\delta$.

We claim that $\hat{G}(\theta)$ and $\hat{N}(\theta)$ must satisfy the following

$$(3.17)\quad \theta f(\hat{G}(\theta)) - k\hat{G}(\theta) + \hat{N}(\theta) = \theta f(G_v(\theta)) - kG_v(\theta).$$

To prove this claim, note that from (3.14), $n(\theta)$ converges to zero. Now taking limits (of subsequences, if necessary) in (3.12) and using (3.13) we obtain (3.17). Furthermore, it is also clear that this convergence is uniform in $\theta \in (\underline{\theta},\bar{\theta})$. We construct the rest of the argument assuming we are at the limit. The details of the (ϵ,δ) arguments are available upon request.

Choose θ_1, $\theta_2 \in (\underline{\theta},\bar{\theta})$ with $\theta_2 > \theta_1$. Since $\hat{G}$, $\hat{N}$ is an equilibrium, we have

(3.18) $\theta_1 f(\hat{G}(\theta_1)) - k\hat{G}(\theta_1) + \hat{N}(\theta_1) \geq \theta_1 f(\hat{G}(\theta_2)) - k\hat{G}(\theta_2) + \hat{N}(\theta_2).$

Using (3.17) we have that

(3.19) $\theta_1 f(G_v(\theta_1)) - kG_v(\theta_1) \geq \theta_1 f(\hat{G}(\theta_2)) - \theta_2 f(\hat{G}(\theta_2)) + \theta_2 f(G_v(\theta_2)) - kG_v(\theta_2).$

Now, recall that G_v maximizes $\theta f(G) - kG$. Let $U_v(\theta) = \theta f(G_v) - kG_v$. We then have $U_v(\theta_2) - U_v(\theta_1) = \int_{\theta_1}^{\theta_2} f(G_v(x))dx$. Using this result and rearranging (3.19) we have

(3.20) $(\theta_2-\theta_1)f(\hat{G}(\theta_2)) \geq \int_{\theta_1}^{\theta_2} f(G_v(x))dx.$

Now from the contradiction hypothesis $G_v(\theta) - \hat{G}(\theta) \geq \delta$ for all $\theta \in (\underline{\theta},\bar{\theta})$. Thus, for θ_2 sufficiently close to θ_1 we have $f(G_v(\theta_1)) - f(\hat{G}(\theta_2)) > 0$. The right side of (3.20) is at least as large as $(\theta_2-\theta_1)f(G_v(\theta_1))$ since G_v increasing in θ. Thus, we have a contradiction. □

We have established that, even with a small amount of uncertainty, government spending must be close to G_v if there are many representatives. The free rider problem is crucial to this result. It is this problem that forces (3.17) to hold. Put differently, if we required (3.12) and (3.14) to hold but not (3.13), it is possible to construct (G,n) that are different from $(G_v,0)$ even with private information and with a large number of representatives.

4. Global Public Goods

We are interested now in examining the implications of our basic political model for the determination of government spending upon a global public good. Let the scalar g denote the level of per capita spending on the public good. As before the spending on the global good is financed by a uniform tax on the entire population. Let representative i's preferences be given by

$$(4.1) \quad \theta_i f(g) - g \quad \text{for } i = 1, \ldots, I.$$

We allow for the representative's preferences over the optimum level of spending on this global public good by allowing the θ's to vary across representatives. Without loss of generality we assume that

$$\theta_I \geq \theta_{I-1} \geq \ldots \geq \theta_1 \geq 0,$$

with at least two of the inequalities holding strictly to make the model interesting.

We first analyze the voting game given an arbitrary assignment of the right to set the level of g in the proposal to be voted on. We then analyze the outcome of the auction stage in terms of the final allocation of the proposal right given an arbitrary initial allocation and that the continuation game is the voting game.

The voting game proceeds as follows. Given that representative j has the right to set the level of g in the proposal, he chooses $g \in \mathbb{R}_+$ and $m \in \mathbb{R}^I_+$, where m_j is normalized to zero. Each representative i picks $v_i \in \{0,1\}$ for $i = 1, \ldots, I$.

The payoff to representative $i \neq j$ is given by

$$(4.2) \quad \theta_i f(\delta g) - g + m_i v_i,$$

while that of representative j is given by

$$(4.3) \quad \theta_j(\delta g) - g - \sum_{i=1}^{I} m_i v_i,$$

where δ is given by the various analog to (1.4).

Given the assignment of the proposal right to representative j, an equilibrium of the voting game is a number $g \in \mathbf{R}_+$, a vector $m \in \mathbf{R}_+^I$, and a set of functions $v_i(g,m) \rightarrow \{0,1\}$ $i = 1, \ldots, I$ such that for all $i = 1, \ldots, I$:

(i) for all $g \in \mathbf{R}_+$ and $m \in \mathbf{R}_i^I$, v_i satisfies

$$\theta_i f(\delta(v)g) - \delta(v)g + m_i v_1 \geq \theta_i f(\delta(v_{-i},\hat{v}_i)) - \delta(v_{-i},\hat{v}_i)g + m_i \hat{v}_i$$

for all $\hat{v}_i \in \{0,1\}$, and (ii) given $v(\cdot)$, g and m satisfy

$$\theta_j f(\delta(v(g,m))g) - \delta(v(g,m))g - \sum_{i=1}^{I} m_i v_i(g,m)$$

$$\geq \theta_j f(\delta(v(\hat{g},\hat{m})g)) - \delta(v(\hat{g},\hat{m}))\hat{g} - \sum_{i=1}^{I} \hat{m}_i v_i(\hat{g},\hat{m})$$

for all $\hat{g} \in \mathbf{R}_+$ and $\hat{m} \in \mathbf{R}_+^I$.

For reasons that are analogous to those in the previous section, we will restrict attention to pivotal equilibria. A pivotal equilibrium has the property that in equilibrium representatives only vote for proposals which they weakly prefer would pass and that the proposal setter never compensates more than K of the other representatives.

We can construct a pivotal equilibrium as follows. First note that from that part (i) of the definition of an equilibrium it is a weakly dominant strategy to set $v_i = 1$ if the payoff of representative i, conditional on passage (that is $\delta = 1$), is positive. Now, similar to the local public goods environment, let L denote the cardinality of the set of i's whose payoff conditional on passage is positive (where we are deliberately including j). If $L > K + 1$, then representative i sets $v_i = 1$ if $m_i > 0$ and set $v_i = 0$ if $m_i = 0$. If on the other hand $L < K + 1$, the first $K - L$ representatives whose conditional payoff is negative, but for whom $m_i > 0$ set $v_i = 1$, while the remaining representatives whose conditional payoff is negative, but for whom $m_i > 0$, set $v_i = 0$. This completely determines $v(\cdot)$.

Now let the set J(j) be such that

$$(4.4) \quad J(j) = \begin{cases} \{I-K-1,\ldots,j-1,j+1,\ldots,I\} & \text{if } j > I-K-1 \\ \{I-K,\ldots,I\} & \text{otherwise} \end{cases}.$$

Then consider the following maximization problem:

$$(P3) \quad \max_g \theta_j f(g) - g + \sum_{i \in J(j)} \min[0, \theta_i f(g) - g].$$

It is easy to see that if $j \leq I - K - 1$ or if $\theta_i \geq \theta_j$ for all $i \in J(j)$, the solution to (P3), $\hat{g}$, will be characterized by

$$(4.5) \quad \theta_j f'(g) = 1,$$

and that the left side of (4.5) will be strictly greater than one if these conditions aren't met.

The two key properties of this programming problem that we wish to establish for later use are that the solution is unique, and that the value of the solution to (P3) is increasing in the index of the proposal right holder (because θ_j is). To see the first point, note that this is a strictly concave objective. To see the second, let v_j denote the value of the solution to (P3) given that representative j holds the proposal right.

Proposition. $v_{j'} \geq v_j$ for all $j' > j$ and this inequality is strict if $\theta_{j'} > \theta_j$.

Proof. Let $\hat{g}_j$ denote the solution to (P3). If $j' \notin J(j)$, then the proposition follows trivially since the value of the objective function in (P3) is increasing in θ_j. So assume that $j' \in J$, and then note that since $\theta_j f'(\hat{g}_j) \geq 1$, $\theta_{j'} f(\hat{g}_j) - \hat{g}_j > 0$ since $\theta_{j'} \geq \theta_j$ and f is concave. Thus,

$$\theta_j f(\hat{g}_j) - \hat{g}_j + \min[0, \theta_{j'} f(\hat{g}_j) - \hat{g}_j] = \theta_j f(\hat{g}_j) - \hat{g}_j \leq \theta_{j'} f(\hat{g}_j) - \hat{g}_j$$
$$= \theta_{j'} f(\hat{g}_j) - \hat{g}_j + \min[0, \theta_j f(\hat{g}_j) - \hat{g}_j],$$

where the inequality holds strictly if $\theta_{j'} > \theta_j$. □

One can see from (P3) why we have referred to the equilibrium we are constructing as semi-pivotal. If for the smallest $i \in J$, $\theta_i f(\hat{g}_j) - \hat{g}_j < 0$, then the equilibrium will be pivotal. But if $\theta_j \leq \theta_i$, then the inequality will be reversed and more than K + 1 representatives may strictly prefer that the proposal to spend $\hat{g}_j$ pass.

We can now give a precise description of the equilibrium we are constructing. Given that representative j has the proposal right, let $g_j = \hat{g}_j$ (where $\hat{g}_j$ is a solution to (P3)), and let $m_i = \min[0, \theta_i f(\hat{g}_j) - \hat{g}_j]$ for all $i \in J(j)$. Finally, we can completely determine $v(\cdot)$ by requiring that it satisfy (a)-(c). It is easy to see that this equilibrium satisfies conditions (i) and (ii) of the definition, and that the equilibrium level of government spending in any semi-pivotal is unique since the solution to (P3) is.

Briefly, now consider adding the auction of the proposal right back in as the preliminary stage in our voting game. In the local public goods model we saw that the representative who had the most to gain from acquiring the proposal right end up with it as the unique outcome of this preliminary stage. Similar logic implies that one of the representatives with the highest θ's will acquire the right in the auction stage, and the only reason for the indeterminacy is if $\theta_I = \theta_{I-1}$, etc. However, the outcome in terms of the level of spending will be invariant since whether I or I − 1 acquires the right the solution to (P3) is the same if $\theta_I = \theta_{I-1}$.

In the case where representative I acquires the proposal right, representative I will only properly internalize the welfare consequences to those members of J(I) for whom the direct impact of the proposed spending is negative. This gives an upward bias to the spending level that will emerge from our game compared to the efficient level at which

(4.6) $$\sum_{i=1}^{I} (\theta_i f'(g) - 1) = 0.$$

Even if the voting outcome rule required unanimity for passage, so $J = \{1,\ldots,I-1\}$, the efficient level could only emerge as an outcome of our game if $\theta_{I-1}f(g) - g < 0$, or $\theta_1 = \theta_I$. Other than in these two special cases, the level of government spending on the global public good will be too high; that is $\hat{g}_I > g$, where g is such that (4.6) holds.

Global Public Goods, Private Information, and Unanimity

Our analysis thus far suggests that voting systems which require unanimity yield good outcomes. In local public goods environments with and without private information, proposers have incentives to internalize only the interests of those who must be persuaded to vote for the proposal. As the required number of consenting representatives rises, the proposer internalizes the interests of more people. Thus, unanimous consent appears to be a good mechanism. With global public goods and complete information, we have shown that similar results hold. The results of Rob (1989), Mailath and Postlewaite (1990), and Chari and Jones (1991) suggest, however, that in global public goods environments with private information, unanimous consent rules lead to poor outcomes. Since these results are available, we briefly set up the problem with global public goods and refer to the literature for the results.

We assume that each representative's benefits from the public good are given by $(\theta_i/I)f(h)$ where θ_i is an identically, independently drawn random variable from a distribution $H(\theta)$ with support $[0,\bar{\theta}]$ and a strictly positive density $h(\theta)$. Note that we have scaled the benefits by 1/I. We will want to increase the number of representatives. When we do so, we keep the efficient level of government consumption under full information unchanged with this device. We assume that $\theta - (1 - H(\theta))/h(\theta)$ is increasing in θ. We assume that $f'(0)$ is finite.

We consider a game in which one of the representatives, say representative 1, is chosen randomly. This representative then chooses some mechanism to allocate the public good. Without loss of generality, we restrict attention to direct mechanisms in which each representative reveals his type θ_i. Let $\theta = (\theta_1,\ldots,\theta_I)$. A direct mechanism is then a set of functions $m_i(\theta)$ denoting payments to 1 and a government spending function $G(\theta)$.

Truth-telling is an equilibrium of the direct mechanism if for all θ_i, $\hat{\theta}_i$,

$$(4.7)\quad E_{-i}\left\{\frac{\theta_i}{I} f(G(\theta)) - \frac{G(\theta)}{I} - m_i(\theta)\right\} \geq E_{-i}\left\{\frac{\theta_i}{I} f(G(\hat{\theta}_i,\theta_{-i})) - \frac{G(\hat{\theta}_i,\theta_{-i})}{I} - m_i(\hat{\theta}_i,\theta_{-i})\right\}.$$

We restrict attention to truth-telling equilibria. We say that a truth-telling equilibrium of the mechanism satisfies unanimous consent if

$$(4.8)\quad E_{-i}\left\{\frac{\theta_i}{I} f(G(\theta)) - \frac{G(\theta)}{I} - m_i(\theta)\right\} \geq 0, \quad \text{for all i.}$$

Representative 1's expected payoffs are then given by

$$(4.9)\quad U_1(\theta_1) = E_{-i}\left\{\frac{\theta_1}{I} f(G(\theta)) - \frac{G(\theta)}{I} + \sum_{i=2}^{I} m_i(\theta)\right\}.$$

This representative's problem is to choose a mechanism to maximize (4.9) subject to (4.7) and (4.8). We use standard results from the mechanism design literature to solve this problem. Let $U_i(\theta_i\hat{\theta}_i)$ denote the utility of representative i when his type is θ_i and he reports $\hat{\theta}_i$. This is given by the right side of (4.7). Let $V_i = U_i(\theta_i,\theta_i)$. Let $q_i(\hat{\theta}) = E_{-i} \int f(G(\hat{\theta}_i,\theta_{-i}))$. Then, it is straightforward, using the techniques in Myerson (1980), to establish that the incentive compatibility conditions are equivalent to the conditions that $q_i(\cdot)$ is increasing, and that $V_i(\cdot)$ satisfy

$$(4.10)\quad V_i(\theta_i) = V_i(0) + \int_0^{\theta_i} q_i(x)dx.$$

An informal argument to establish (4.10) is as follows. Incentive compatibility requires that θ maximize $U_i(\theta,\hat{\theta})$ over $\hat{\theta}$. Using the envelope theorem, we have $V'(\theta)$ equals the derivative of U_i with respect to its first argument. Integrating this condition yields (4.10).

Next, using the definition of $V(\cdot)$ and (4.10) we have that the expected payments from i are given by

$$(4.11)\quad E\bar{m}_i(\theta) = E\left\{\frac{\theta_i}{I}\, f(G(\theta)) - \frac{G(\theta)}{I} - \int_0^{\theta_i} q_i(x)dx - V_i(0)\right\}.$$

Integrating (4.11) by parts and substituting (4.11) into (4.9) yields that the expected utility of 1 is given by

$$(4.12)\quad EU_1(\theta_1) = E\left\{\frac{\theta_1}{I}\, f(G(\theta)) - \frac{G(\theta)}{I} + \sum_{i=2}^{I}\left\{\left[\theta_i - \frac{1-H(\theta_i)}{h(\theta_i)}\right] f(G(\theta)) - \frac{G(\theta)}{I} - V_i(0)\right\}\right\}.$$

Representative 1's problem is then to maximize (4.12) given that (4.8) implies that $V_i(0) \geq 0$. It is easy to show that the solution to this problem has $G(\theta)$ increasing in θ and has $V_i(\cdot)$ increasing in θ so the solution to this problem is also the solution to the original problem of maximizing (4.9) subject to (4.7) and (4.8). Let $G(\theta{:}I)$ denote the solution to this problem. We will show that as I goes to infinity, $G(\theta;I)$ converges to zero in probability. The argument is by contradiction, so suppose not. Then, choosing subsequences if necessary, it follows that there exists

δ and $\epsilon > 0$ such that Prob$\{G(\theta;I) \geq \delta\} \geq \epsilon$ for all sufficiently large I. Now, maximizing (4.12) it follows that $G(\theta) = 0$ if

$$(4.13) \quad \left[\frac{\theta_1}{I} + \frac{1}{I}\sum_{i=2}^{I}\left[\theta_i - \frac{1 - H(\theta_i)}{h(\theta_i)}\right]\right] f'(0) < 1$$

and $G(\theta) > 0$ if the inequality is reversed.

But the expected value of $(\theta - (1 - H(\theta)/h(\theta))) = 0$ which is the lower end of the support. The second term in the brackets in (4.11) converges to zero. Using Chebyshev's inequality, it follows that

$$(4.14) \quad \Pr\left[\frac{\theta_1}{I} + \frac{1}{I}\sum_{i=2}^{I}\left[\theta_i - \frac{1 - H(\theta_i)}{h(\theta_i)}\right]\right] > \frac{1}{f'(0)}$$

goes to zero as I goes to infinity. Thus, for large I, $G(\theta)$ converges to zero.

This result suggests that unanimous voting rules can perform poorly in global public good environments. In contrast, majority voting rules can perform much better. To see this, consider the following mechanism. Each representative submits a report of his type and the highest K types are required to make payments given by (4.11). These payments are incentive compatible by construction. The spending levels $G(\theta)$ is then chosen to maximize the analogue of (4.9). It is possible to show that such a mechanism yields spending levels which do not converge to zero.

5. Concluding Remarks

We have shown that representative democracies are prone to pork barrel spending. Such spending arises from institutional constraints like majority voting rules and from free rider problem which prevents Pareto-superior proposals from being adopted. We have also shown that unanimous voting rules lead to efficient allocations for local public goods, but that such rules are likely to do poorly for global public goods. One area for future research is to find institutional rules that yield desirable allocations for both global and local public goods.

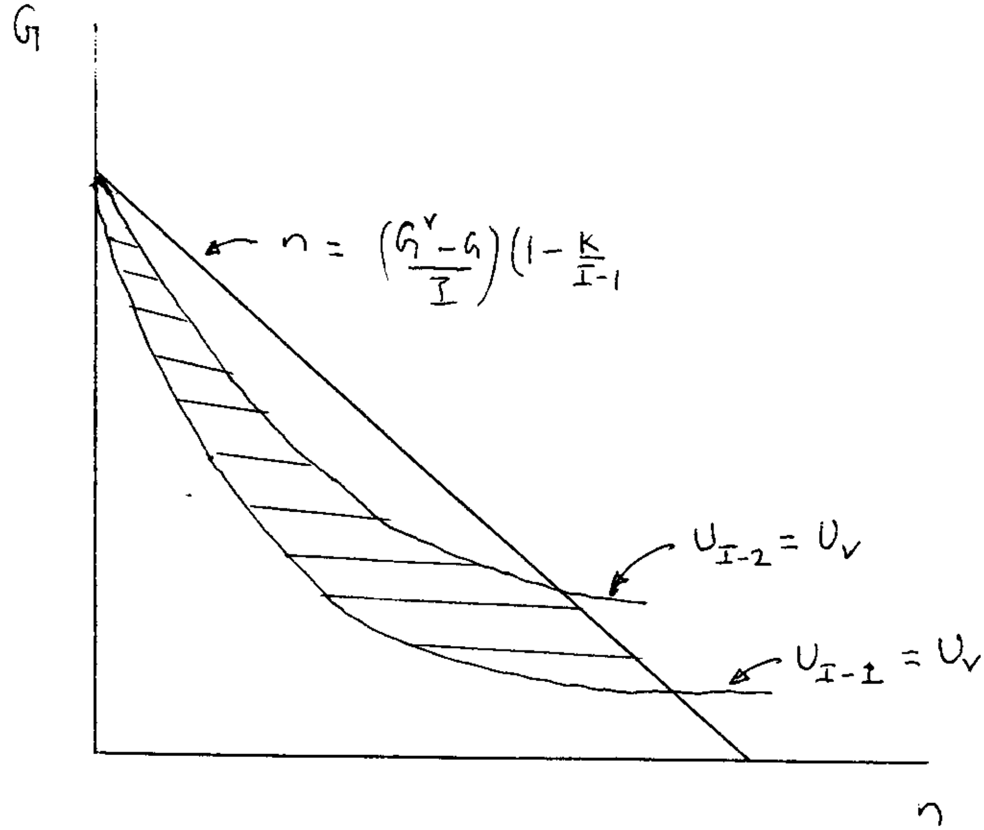

FIGURE 1