Can There Be A General Equilibrium Liquidity Preference Demand For Money?

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Abstract for:

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In "Liquidity Preference as Behavior Towards Risk," Tobin suggests that risk aversion and expected utility maximization can provide a rigorous foundation for an equilibrium demand for money. In Tobin's model, money plays a risk reducing role in individual portfolios. This note considers whether a general equilibrium stochastic model can produce equilibrium yield distributions that allow money to play that role if money does not appear directly as an argument in the utility or production functions of the economy. The model examined, a stochastic production variant of Samuelson's model of overlapping generations, cannot produce such yield distributions.

Can There be a General Equilibrium Liquidity Preference Demand for Money?*

In his famous article, "Liquidity Preference as Behavior Towards Risk," Tobin (6) suggests that risk aversion in the presence of uncertainty can provide a rigorous foundation for an equilibrium demand for money. This possibility seems important, because economists are short of ways of modelling monetary economies. One way to model a monetary economy is to assume that money is the only store of value. Lucas (3), building on the model of overlapping generations introduced by Samuelson (4), studies the Phillips curve using such a model. Money may also be held in generalizations of the Samuelson model that allow real capital to be held as an asset. But, then, if there is an equilibrium with positive money holdings, the equilibrium is characterized by equality between the yield on money and that on capital and by individual indifference with regard to portfolio composition. [See Diamond (2), Cass-Yaari (1), and Thompson (5).] In contrast, according to liquidity preference theory, positive money holdings are consistent with the existence of many assets; with determinate, diversified individual portfolios; and with yield spreads among assets--all of which seem to be "facts." While there are hints that these facts may emerge from a model in which a medium of exchange role is attributed to money, unfortunately as yet there is no such model. We do seem to have agreement that any such model must, in some way, give rise to costly exchange and must, among other things, explain which potential markets operate and which do not.

Our investigation of liquidity preference money demand raises the question: Can a general equilibrium model produce equilibrium yield distributions that allow money to play a risk reducing role in individual portfolios?[1/] It raises rather than answers the question, because only an example of a stochastic model that cannot produce such yield distributions is provided. The example should be of interest, though, because it seems, on the surface at least, to contain the features that might have been thought sufficient to produce a liquidity preference money demand.

1. General Aspects of the Model

The model is a stochastic production variant of Samuelson's [4] in which N identical, two-period lived individuals are born each period. Each is endowed only with his labor--one unit in the first period, none in the second--and each maximizes the expected value of

$$U(c) = \Sigma u(c_i)$$

where $c = (c_1, c_2)$ and c_i is consumption at age i of a single non-storable good, and where $u' > 0$, $u'' < 0$ and $u'(0) = \infty$.

Production of the good is a function $f(n,k^*,x)$ of labor n, land k^*, and a random variable x. Each period there is an independent drawing of x from the same finite sample space distribution. For each value of x and $z > 0$, the set (n, k^*) for which $f(n,k^*,x) \geq z$ is strictly convex. Also, for each value of x, and $0 < n \leq 1$, $0 < k^* \leq K/N$ (K being the total amount of land in the economy), $f > 0$, $f_1 > 0$, $f_{11} < 0$, $f_{22} < 0$, and $f(0,k^*,x) = f(n,0,x) = 0$. These assumptions allow the marginal product of land to be negative for some values of x in some region of $0 < n \leq 1$, $0 < k^* \leq K/N$.

Something called money also exists. There is a fixed amount of it, denoted M. It neither depreciates nor appreciates physically and as implied by the above appears in neither the utility nor the production function. Thus, as regards most of its physical attributes, money is like land. Indeed, we should perhaps think of it as a second kind of land that is technologically distinguishable from the first kind only in that it is not usable in any production process.

In general, the young use part of their first period labor earnings to buy land and money from the older generation. In the second period of their lives, they produce output via f using some of the land previously acquired and labor that they hire. In this period, they receive output minus the wage bill and also the proceeds from selling their land and money holdings to the younger generation, their workers.

This setup is such that an equilibrium ought to have the property that the values of the endogenous variables at any time depend at most on the value of x at that time. This is so because neither the endowments nor the preferences of the old or the young alive at a given time depend on previous values of x. Clearly, this is so for the young. As for the old, no matter what they experienced when young, each old person maximizes the expected value of $u(c_2)$ subject to the constraints implied by the assets acquired during the first period of his life and the production function f. Since x is chosen independently from period to period, neither f nor the distribution over which this expected value is computed depend on previous values of x. And, since in equilibrium, asset holdings acquired when young must equal the constant per capita supplies, they do not depend on previous values of x.

Our approach, then, is to derive the implications of the existence of such an equilibrium with positive prices of land and money in terms of the consumption good. In each of several different versions of the model, two implications emerge: (A) land must be in surplus as an input in production, and (B) there is another equilibrium identical in all respects except that the price of money is zero and the price of land is proportionally higher. Put differently, land and money can both have positive prices in an equilibrium for this model only if their yield distributions are identical.

2. Version 1: Poststate Contracting

We begin with a version in which the young and the state (the value of x) appear simultaneously so that there is no opportunity for members of the two generations alive at the same time to make contracts prior to the occurrence of the state. We assume that individuals act as price-takers. Everyone faces a wage distribution $W(x)$, a price of land distribution $P(x)$, and a price of money distribution $S(x)$, all in units of the consumption good.

The young face the following problem. Letting x' be the value of x drawn in the second period of their lives, they maximize

$$E_x[U(c)] = u[c_1(x)] + \sum_{x'} p_{x'} u[c_2(x,x')]$$

subject to

(1) $\quad C1x = c_1(x) + P(x)k(x) + S(x)m(x) - W(x) \leq 0$, for all x;

(2) $\quad C2xx' = c_2(x,x') + W(x')n(x,x') - f[n(x,x'),k^*(x,x'),x']$

$\qquad\qquad -P(x')k(x) - S(x')m(x) \leq 0$, for all (x,x');

(3) $\quad C3xx' = k^*(x,x') - k(x) \leq 0$, for all (x,x');

by choice of $c_1(x)$, $c_2(x,x')$, $k(x)$ (land purchases), $m(x)$ (money purchases), $k^*(x,x')$ and $n(x,x')$. The probability that x takes on a particular value is denoted p_x.[2/]

We express necessary conditions for a maximum in terms of the LaGrangian

$$L = E_x[U(c)] - q_{1x}C1x - \sum_{x'} (q_{2xx'}C2xx' + q_{3xx'}C3xx')$$

where the C's stand for the constraints given in (1)-(3) and the q's are multipliers.

In a competitive equilibrium, a maximum to this choice problem must occur at positive values of the choice variables; c_1, c_2 and k* must be positive because of the assumptions made about u and f, while as conditions of equilibrium we must have $n(x,x') = 1$, $k(x) = K/N$ and $m(x) = M/N$. It follows that among the necessary conditions for a maximum are

(4) $$L_{c_1} = u'[c_1(x)] - q_{1x} = 0, \text{ all } x;$$

(5) $$L_{c_2} = p_{x'}u'[c_2(x,x')] - q_{2xx'} = 0, \text{ all } (x,x');$$

(6) $$L_k = -q_{1x}P(x) + \sum_{x'} (q_{2xx'}P(x') + q_{3xx'}) = 0, \text{ all } x;$$

(7) $$L_m = -q_{1x}S(x) + \sum_{x'} q_{2xx'}S(x') = 0, \text{ all } x;$$

(8) $$L_n = q_{2xx'}[f_1(n(x,x'),k^*(x,x'),x') - W(x')] = 0, \text{ all } (x,x');$$

(9) $$L_{k^*} = q_{2xx'}f_2(n(x,x'),k^*(x,x'),x') - q_{3xx'} = 0, \text{ all } (x,x').$$

From the assumption that the equilibrium values of the second period choice variables--c_2, n and k*--do not depend on the state in the first period, it follows from (5) and (9) that the equilibrium values of the multipliers $q_{2xx'}$ and $q_{3xx'}$ do not depend on x. It follows, then, from (6) that $q_{1x}P(x)$ does not depend on x and from (7) that $q_{1x}S(x)$ does not depend on x. Thus $S(x)/P(x)$ is a constant, say s. Now summing (7) over x we have

$$\sum_x q_{1x}S(x) - J\sum_{x'} q_{2xx'}S(x') = 0$$

where J is the number of states. This may be written as

$$(10) \qquad (q_1 - Jq_2)'S = 0$$

where q_i is the column vector of the q_{ix}'s and S is the column vector of $S(x)$'s. Summing (6) over x, we have

$$(11) \qquad (q_1 - Jq_2)'P = J\sum_{x'} q_{3xx'}$$

where P is the vector of $P(x)$'s. Since $S(x)/P(x)$ is a constant, it follows from (10) and (11) that $\Sigma q_{3xx'} = 0$. But since $q_{3xx'} \geq 0$, this says that $q_{3xx'} = 0$ for all x', or by (9) that for each value of x' there must be a value of $k^*(x') \leq K/N$ such that $f_2[1,k^*(x'),x'] = 0$. This is the variant of proposition (A) that emerges from this version of the model.[3/] To prove proposition (B), it is enough to observe that if (1)-(9) hold for a value of $s > 0$, call it s^*, and a vector of positive land prices, P^*, and a set of values for all the other endogenous variables, they also hold for $s = 0$, a vector of land prices $[1 + s^*(M/K)]P^*$, and the same values for all the other variables.[4/] In particular, the distributions of consumption by state and age are the same in the two equilibria.

We should note as an aside that it is easy to prove--whether or not $f_2(1, K/N, x) \leq 0$ for all x--that an equilibrium with $S(x) = 0$

for all x exists, and, furthermore, that it is characterized, in general, by a yield distribution for land with both positive and negative values in the following sense: there are pairs (x,x') for which

$$[(K/N)P(x') + f(1,k^*(x'),x') - W(x')]/(K/N)P(x)$$

is less than one and other pairs for which it is greater than one.

3. Version 2: Prestate Determination of Inputs

We now assume that the young appear prior to the occurrence of the state which allows us to impose the technological constraint that inputs be determined prior to the occurrence of the state. But this also allows members of the age 1 generation to trade state contingent claims with members of the age 2 generation. We shall assume that such markets exist, an assumption we shall defend below.

Letting $Q(x)$ be the price (in terms of some abstract unit of account) of one unit of output if state x occurs and nothing in all other states, an age 1 individual maximizes

$$E[U(c)] = \sum_{x}\sum_{x'} p_x p_{x'} U(c) \tag{12}$$

subject to

$$C1 = \sum_{x} Q(x)[c_1(x) + P(x)k(x) + S(x)m(x) - W] \leq 0; \tag{13}$$

$$C2x = \sum_{x'} Q(x')[c_2(x,x') + Wn(x) - f(n(x),k^*(x),x') - P(x')k(x) - S(x')m(x)] \leq 0, \text{ all } x; \tag{14}$$

$$C3x = k^*(x) - k(x) \leq 0, \text{ all } x. \tag{15}$$

In (13)-(15), $P(x)$ and $S(x)$ are defined as above, as poststate or "spot" prices in units of the consumption good. Note that the young receive a wage in every state that is independent of the state. Its value, therefore, is $W\Sigma Q(x)$, which is first period income in abstract units of account. According to (13), total first period expenditure--made up of spending on consumption $\Sigma Q(x)c(x)$, spending on land $\Sigma Q(x)P(x)k(x)$, and spending on money $\Sigma Q(x)S(x)m(x)$--cannot exceed this income. Note that consistent with our definitions of $Q(x)$ and $P(x)$, $Q(x)P(x)$ is the price (in abstract units of account) of a claim on one unit of land in state x, the price of a contingent claim on land. Note also that although n and k* are second period choice variables, they are dependent at most on the state in the first period, because they must be chosen before the drawing of the state for the second period.

The conditions for a maximum can be stated in terms of the LaGrangian

$$L = \sum_x p_x [\sum_{x'} p_{x'} U(c) - q_{2x} C2x - q_{3x} C3x] - q_1 C1$$

Again, assuming a competitive equilibrium with positive prices and, hence, the existence of a maximum at positive values of the choice variables, among the necessary conditions for such a maximum are

(16) $$L_{c_1} = p_x u'[c_1(x)] - q_1 Q(x) = 0, \text{ all } x;$$

(17) $$L_{c_2} = p_x [p_{x'} u'[c_2(x,x')] - q_{2x} Q(x')] = 0, \text{ all } (x,x');$$

(18) $$L_m = p_x q_{2x} \sum_{x'} Q(x')S(x') - q_1 Q(x)S(x) = 0, \text{ all } x;$$

(19) $$L_k = p_x [q_{3x} + q_{2x} \sum_{x'} Q(x')P(x')] - q_1 Q(x)P(x) = 0, \text{ all } x;$$

(20) $$L_n = -p_x q_{2x} \sum_{x'} Q(x')[W - f_1(n(x),k^*(x),x')] = 0, \text{ all } x;$$

(21) $$L_{k^*} = p_x [q_{2x} \sum_{x'} Q(x') f_2(n(x),k^*(x),x') - q_{3x}] = 0, \text{ all } x.$$

As above, from the assumption that the equilibrium values of the second period choice variables--c_2, n and k*--do not depend on x, the state in the first period, it follows from (17) and (21) that the equilibrium values of the multipliers q_{2x} and q_{3x} do not depend on x. It follows then from (18) and (19) that S(x)/P(x) does not depend on x. It also follows upon summing (18) over x that $q_{2x} = q_1$. Then, upon summing (19) over x, it follows that $q_{3x} = 0$. Also, from $q_{2x} = q_1$, it follows from (16) and (17) that $c_1(x) = c_2(x) = .5f(1,k^*,x)$, so that $q_{3x} = 0$ and (21) imply

$$(22) \qquad \sum_x p_x u'[.5f(1,k^*,x)]f_2(1,k^*,x) = 0.$$

Thus, an implication of an equilibrium with $S(x) > 0$ is that land must be in surplus as an input in the sense that there must be a value of $k^* \leq K/N$ such that (22) holds. This is the variant of (A) that emerges from this version of the model. Proposition (B) emerges in the same form as it did in Version 1 of the model.

4. Version 2a: Prestate Determination of Inputs, No Contingent Markets

We now want to show that we have not stacked the cards against liquidity preference money in Version 2 by including superfluous markets. To do this, it is instructive to examine aspects of the equilibrium that obtains if there are no contingent markets.

The problem facing the young in such a regime is similar to that posed in Version 2 except that the constraints C1 and C2x [equations (13) and (14)] break up into sets of constraints:

$$(23) \quad C'1x = c_1(x) + P(x)k(x) + S(x)m(x) - W \leq 0, \text{ all } x;$$

$$(24) \qquad C'2xx' = c_2(x,x') + Wn(x) - f(n(x),k^*(x),x') - P(x')k(x) - S(x')m(x) \leq 0, \text{ all } (x,x').$$

In contrast to the setup with contingent markets, here spending in any state is limited by income in that state.

The conditions for a maximum can be stated in terms of the LaGrangian

$$L = \sum_x p_x[\sum_{x'} p_{x'}U(c) - \sum_{x'} q_{2xx'}C'2xx' - q_{3x}C3x] - \sum_x q_{1x}C'1x$$

where C3x is given in (15). Among the conditions implied by positive optimum values for the choice variables are

$$(25) \qquad L_{c_1} = p_x u'(c_1(x)) - q_{1x} = 0, \text{ all } x;$$

$$(26) \qquad L_{c_2} = p_x[p_{x'}u'(c_2(x,x')) - q_{2xx'}] = 0, \text{ all } (x,x');$$

$$(27) \qquad L_m = p_x \sum_{x'} q_{2xx'}S(x') - q_{1x}S(x) = 0, \text{ all } x;$$

$$(28) \qquad L_k = p_x[\sum_{x'} q_{2xx'}P(x') + q_{3x}] - q_{1x}P(x) = 0, \text{ all } x;$$

$$(29) \qquad L_n = p_x \sum_{x'} q_{2xx'}[W - f_1(n(x),k^*(x),x')] = 0, \text{ all } x;$$

$$(30) \qquad L_{k^*} = p_x[\sum_{x'} q_{2xx'}f_2(n(x),k^*(x),x') - q_{3x}] = 0, \text{ all } x.$$

Again assuming that the equilibrium values of second period choice variables do not depend on the first period value of x, it follows from (26) and (30) that the equilibrium values of the multipliers $q_{2xx'}$ and q_{3x} do not depend on x. Then, by (27) and (28), it follows that S(x)/P(x) does not depend on x. And, summing (27) over x we get

$$(31) \qquad (q_2 - q_1)'S = 0$$

where q_2 stands for the vector of $q_{2xx'}$'s, etc. Then, it follows upon summing (28) over x that $q_{3x} = 0$. In addition, it follows from (31) either that

(a) $$q_2 = q_1$$

or that $q_2 - q_1$ has some positive and some negative elements implying by (25) and (26) that

(b) $$u'[c_1(x_i)]/u'[c_1(x_j)] \neq u'[c_2(x_i)]/u'[c_2(x_j)]$$

for some pair (x_i, x_j).

If condition (b) obtains, there are gains from trade in contingent claims. Assuming that all nonsuperfluous markets not explicitly ruled out by the model exist, condition (b) forces us back to Version 2. As for condition (a), it implies condition (22). Thus, Version 2 does, indeed, fairly represent the situation with prestate input determination.

But, even if we simply rule out contingent markets, we still obtain implications (A) and (B). Land is in surplus in the sense implied by $q_{3x} = 0$ and equations (26) and (30). As for proposition (B), it follows from $q_{3x} = 0$ and the constancy of S(x) P(x) in the same way as in the other versions of the model.

5. Conclusion

It is well known that in order for individuals to diversify between two assets, the yield distributions on the assets must be such that neither one dominates the other; i.e., there must be pairs of states (x_i, x_j') such that the contingent yield on the first asset exceeds that on the second and other pairs for which the reverse is true. This our model does not produce. It either produces a yield distribution for land that dominates that for money or (in certain special circumstances) produces identical distributions. Indeed, in our model, the implications for equilibrium money holdings seem not to depend on whether production is or is not stochastic. The generality of this result remains to be determined, since we have produced only an example in which there is no liquidity preference demand for money.

FOOTNOTES

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1/ To ask this question is to go beyond the usual partial equilibrium treatment that takes as exogenous the yield distributions of assets. Typically, the macroeconomist has been satisfied with such a partial equilibrium micro foundation for the behavior relationships that constitute his macroeconomic model even though such an approach is bound to produce inconsistencies; for example, an inconsistency between the asset yield distribution generated by the macro model and that posited in the underlying partial equilibrium liquidity preference analysis.

2/ In writing the constraints as we have, we are assuming that equilibrium prices depend only on the current value of the random variable x. But, in conformity with a dynamic programming approach to the individual optimization problem, we are allowing second period choice variables--c_2, n, and k*--to depend on the state experienced in the first period, even though their equilibrium values will depend only on the current state.

3/ Thus, there could not be such an equilibrium if there is a value of x' for which $f_2(1, K/N, x') > 0$.

[4]More generally, if (1)-(9) hold for s* and P*, they also hold for all vectors of land prices and values of $s \geq 0$ that satisfy $[1 + s(M/K)]P = [1 + s^*(M/K)]P^*$; i.e., that give the same total value of asset holdings.

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