Recession and Demand Management:
An Illustrative Example

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The use of demand management by the government has come under severe attack recently. In particular, varying the government's financing mix between taxing and money and bond creation in order to affect the business cycle has been discredited. This paper will illustrate through simple example that, in contrast, expenditure demand management may, indeed, be justified.

Before one can analyze expenditure demand management, one must first have a model of recession. One attribute of the economy that is widely accepted is that the economy is not inherently unstable. If this property is to be adhered to in one's model, then one of the first tasks is to determine the source of shocks that generate recessions.

There are three possible sources of shock that must be considered. First, there are shocks to technologies. Second, there are shocks to preferences. Third, there are stochastic nonneutral government policies. We will discuss the second two, reject them, and turn to the first source of shocks for our model.

A common way to introduce shocks to a model is through random preferences. The major problem with this source is that it just does not seem believable. Can one really explain recessions by sudden massive changes in taste? Certainly if individuals' costs are random but independently distributed, the law of large numbers is at work in spades in an economy the size of ours. One can easily think of examples of fads which are not independently distributed. But these fads typically involve a trivial proportion of GNP and involve switches among easily substitutable consumer items, not switches between major categories of output. Moreover, explaining recessions by shocks to preferences really is
giving up on economics. The very purpose of economics is to explain economic behavior in a given environment. The subject of the study, economic behavior, should not be the primitive of the model! This really is just a version of Keynes' unhelpful assumption that it is "animal spirits" that determine the business cycle.

A second possibility is stochastic nonneutral government policy. The problem with this explanation is that there should not be such policies. Whether one has the Lucas (8) model of nonneutral unanticipated money shocks or the Bryant-Wallace (6, 7) model of permanently nonneutral monetary policy, the conclusion is the same. Systematic policies dominate random ones. It seems unreasonable to write about the optimal expenditure policy response to suboptimal policy. If one can convince the government to follow optimal expenditure policy, then why cannot one convince the government to cease the suboptimal policy generating the recessions in the first place? If it is the expenditure policy itself that is generating the shocks, then our model of optimal expenditure policy is one without recessions.

The last possibility is that it is shocks to technology that disturb the economy. This explanation must address the fact that all our data suggest that it is demand shocks not supply shocks that precede a recession. This fact is, of course, one of the reasons that shocks to preferences have been a popular assumption. The tack taken in our model is the obvious answer to this problem. It is an anticipated future shock to technology that changes demand. Such anticipated future shocks do not, of course, appear in the current data set.

A model of recession need not only specify the source of shocks. One naturally has certain stylized facts concerning the "business cycle" that one wants the model to generate. There are three stylized facts which this model is designed to produce. First, the economy generally moves along a full employment
path. Secondly, employment occasionally falls rapidly and then converges back to the full employment path. Lastly, the economy has the possibility of having a deep recession from which it does not converge back to the full employment path.

In the model of recession, we consider government expenditure demand management. Demand management through financing policies is not considered because it has elsewhere been found to be unjustified. It is true that in a Bryant-Wallace (6, 7) framework changing transactions costs over the business cycle could justify an active Federal Reserve policy. However, such a policy would be more in the nature of an accommodation than demand management.

Expenditure demand management is not justified in our model by market failure. This contrasts to recent Keynesian attempts to resurrect demand management (see Azariadis (2)). Such models assume that an incompleteness in financial markets restricts risk sharing by individuals. Demand management can then redistribute risk in a way unavailable to the private market. One problem with this approach is that the implementation of such policies seems to require much sophisticated knowledge of the financial markets, and of individuals' preferences and behavior. On the face of it, it seems unlikely that the policies would "look" anything like simple demand management policies. Moreover, it is not clear that direct interference in an incomplete market is not a better solution, and it seems unlikely that demand management would "bridge" very many incompletenesses.

Finally, we assume that government expenditure is for public goods, not for goods producible by the private market. Otherwise, without market failure, we would just reach the conclusion that there should not be such government expenditure. Our particular assumption on the public good is that it is a good the individual's consumption of which depends upon the total amount produced. As each individual is infinitesimally small, none is produced privately.
The fact that the model does not have market failure may convince some readers that it is not a reasonable model of recessions. The defense of the model is its consistency with the stylized facts mentioned above. Some readers may also feel that the model is really addressing the issue of the optimal provision of public goods, not demand management. This is, indeed, correct. However, the crucial observation here is that the resulting expenditure policies "look" like demand management. Only their motivation is different.
The Example

The example is a simple overlapping generations model. Time is discrete and is without beginning or end. Each period an equal number of individuals are born and they live two periods. There are three goods in the model, leisure time, a transferable but nonstorable consumption good, and the nonstorable public good. The individual is endowed with leisure only in his first period and can use his leisure time to work, up to a fixed constraint \( \bar{W} \). Working in the private good technology for \( W_p \) hours yields the individual \( W_p \) goods next period independent of the number of hours worked by the individual or all individuals. Working in the public good technology for \( W_g \) hours yields \( W_g \) public goods to the government. These technologies are known to the individual when he makes his decisions in his first period.

There also is a futures market. The individual when young can buy goods in his youth with promises to deliver goods next period. Similarly, when this generation meets its obligations next period, the following generation buys them with promises. We do not worry about how this market got here, it always existed. Nor do we worry about individuals meeting their contracts, they just do (but see Bryant (3)).

The individual maximizes a strictly concave, two-smooth utility function of a particular form. Let the individual work \( W \leq \bar{W} \) hours, and purchase \( g \) goods on the futures market with a promise of \( P_g \) goods tomorrow. Let government expenditures be \( G \). Let the individuals' second-period consumption be \( C_2 \). Then the individuals' utility function for \( W \leq \bar{W} \) is

\[
U_1(-W + l) + U_2(C_2) + U_3(G)
\]

where \( U_1(-\bar{W}) = U_3(0) = \infty \). The utility function is additive except that leisure and first-period consumption goods are perfect substitutes for \( W < \bar{W} \). One way to
view this is that leisure does not enter the utility function for $W < \bar{W}$, and the individual can produce current consumption goods one for one "at home" (out of the economy) in normal working time. In any case, it is this assumption which gets us the stable full employment path.

The government hires individuals to work in the public goods industry at the private industry wage rate, $w_p$. As we do not wish to discuss financing issues, we assume that the government runs a balanced budget each period. It uses equal, costless lump-sum taxes in the second period of an individual's life to pay workers in the second period of their life with the private consumption good. Moreover, only the public goods produced by a given generation enter that generation's utility function. One way to view this is that only the old consume the public good, and it takes a period to make the public good just as it takes a period to make the private good. We also assume that the government never chooses to hire the entire work force, $W < W_g < W$ where $W_g$ is hours worked for the government.

We conclude that $C = w_p W - T - P = w_p (W - W_g) - P$ and $G = w_g W_g$. The reader doubtless has noticed that "unemployment" in this model consists of all individuals working part-time, not a mix of fully employed and unemployed workers. In addition, unemployment is "voluntary" not "involuntary." It is the view of the author that these points are minor technicalities that have been adequately treated in the "new-new" labor economics (see, for example, Azariadis (1) and Bryant (5)), and it is useful to abstract away from such complications.

Now let us examine a few properties of the model and verify that it can generate our stylized facts. Time subscripts will be used only when necessary. The individual takes the "wage rates," $w_p$ and $w_g$, government expenditure and tax, and the value of futures contracts, $P$, as given. His problem is

$$\max_{l, W} U_1(-W+\bar{W}) + U_2[w_p W - T - P] + U_3(G).$$
The first-order necessary conditions are

\begin{align*}
(1) \quad & -U'_1(-W+\ell) + \omega_p U'_2(\omega_p W-T-\ell) \geq 0, \quad \text{if } W < \bar{W} \\
(2) \quad & U'_1(-W+\ell) - \omega_p U'_2(\omega_p W-T-\ell) \leq 0, \quad \text{if } \ell > 0.
\end{align*}

These inequalities imply \( P < \omega_p \), = if \( W < \bar{W} \). While the individual chooses \( \ell \), in the aggregate \( \ell \) is determined by the previous generation's decision. \( P \) is determined in equilibrium by \( \ell_t = P_{t-1} \ell_{t-1} \). Suppose \( P_{t-1} \ell_{t-1} > 0 \) and \( \omega_p > 1 \). Suppose the individual's decision is \( W < \bar{W} \). Then \( P = \omega_p > 1 \). Therefore, \( \{\ell_t\} \) approaches infinity at the rate \( \omega_p \). We conclude that \( W = \bar{W} \) after a finite number of periods. At the point where \( W \) just equals \( \bar{W} \), \( P = \omega_p > 1 \), so \( \{\ell_t\} \) must continue to grow from this point until \( U'_1 - U'_2 = 0 \), or \( P = 1 \). From this position a one-period change in \( \omega_p \) would have to change \( W \) substantially before it would reduce \( W \) below \( \bar{W} \).

Let us be more precise about shifts to the private technology. Suppose \( \omega_p = (1+\gamma)\omega_p \), where \( \gamma \) is a random variable bounded below by \(-1\), and in each period \( \gamma \) is an independent drawing with the same probability distribution. The realization of its \( \gamma \) is known to a generation when it is born. Once again let us consider the individual's problem. Let \( "\" \) mean the solution value. Assume \( W < \bar{W} \). Then differentiating (1) we have:

\[
U'_1 dW + \omega_p [U'_2 + (1+\gamma)\omega_p (W-W_s-\ell)U'_2] d\gamma = U'_1 dW + \omega_p [U'_2 + C_2 U'_2] d\gamma = 0.
\]

We conclude that if second-period consumption is a gross substitute for the other goods, then \( \partial W/\partial \gamma > 0 \), and if second-period consumption is a gross compliment, \( \partial W/\partial \gamma < 0 \).

Assume second-period consumption is a gross substitute for the other goods. Further assume that \( \omega_p > 1 \), and we start at the solution \( \tilde{W} = \bar{W} \), \( p = 1 \).
Realizations of $\gamma$ greater than zero, and realizations of $\gamma$ not too far below zero will not move the economy away from full employment, $\bar{W} = \bar{W}$. Such realizations will affect real output. However, the effect on real output is muted by the fact that employment is not influenced. Only large negative deviations will have the output effect magnified by employment fluctuations.

We have, then, an asymmetry with respect to the behavior of both employment and output. However, it may be that the generated asymmetry in the behavior of output is not extreme enough. The full employment path in reality is characterized by pretty steady output, and, in particular, is without significant "up jumps." One possibility is that this asymmetry is in the distribution of shocks to the technology. This may be pushing the problem under the carpet just like imbedding shocks in the utility function is. Instead of refusing to analyze the economic dynamics by attributing it to unpredictable individual behavior, we attribute it to forces outside the scope of economics. However, it may also just be the way it is. Shocks may be in the form of disasters. Alternatively, suppose, for example, that shocks are in the form of shifts in the environment. These shifts do not affect productivity much given optimal use of existing technologies, but do affect output given the preshock mix of use of existing technologies. The affect of such a permanent shift in environment could appear as a one-period, large negative shock to $\gamma$ in our model. The first generation facing the shift uses the existing mix of technologies because it does not have time to adjust. Following generations use the optimal mix of technologies and face an unchanged "aggregate technology." Any "up shocks" in technology appear in the form of gradual increases in $\omega_p$ resulting from relatively slow and steady technological improvement. $\gamma$ is distributed with an atom at zero and the remaining weight between zero and minus one.
We have, then, full employment occasionally disrupted by a temporary recession. What about a large enduring recession? Suppose in one period \( \gamma = -1 \), and for all future realizations \( \gamma = 0 \). Then the futures market is wiped out. For all future generations \( \lambda = 0 \), and by \( U'_1(-\bar{W}) = \infty, \bar{W} < \bar{W} \) with \( U'_1(\tilde{W}) = \bar{W} U_2 [w \tilde{W} - T] \) for \( w < \tilde{W} \).
Demand Management

Having seen that our model does generate our stylized facts, let us turn to government policy. We will consider two polar cases. First, \( w_p = (1+\gamma)w_p \) and \( w_g = w_g \). Second, \( w_p = (1+\gamma)w_p \) and \( w_g = (1+\gamma)w_g \). What we will be examining is \( \frac{dW_g}{d\gamma} \) and \( \frac{dW_g}{d\gamma} \). Because of the symmetry between \( W \) and \( L \), we will not consider \( \frac{dW_p}{d\gamma} \) and will only treat the first period of a shock. Clearly, the same analysis will hold for subsequent periods where the effects come through \( L \).

The objective function of the government is not obvious. We will assume that the government wants to maximize individual utility, but has no desire to redistribute income between generations from that generated by the market. The government decision this period influences future generations only by its effect on \( P_L \). We will consider the government decision in the current period rather than the government decision functions. The government is assumed to maximize the sum of this generation’s utility and a valuation function on \( P_L \), \( V(P_L) \). At the moment the only constraint we put on \( V \) is that it be increasing and continuous. We will, however, be taking the derivative of \( V \) below. The reader can interpret this as the derivative at the points where \( V \) is differentiable, almost everywhere, and not worry about the (hopefully) zero probability event of being at a nondifferentiable point. Alternatively, the reader can interpret \( V'(P_L) \) as a number appropriately bounded by the right- and left-hand derivatives of \( V \) which exist everywhere.

The government's problem can be written:

\[
\max \ W_1(-W-\xi) + U_2[w_1(\bar{W}-W_0)-P_L] + U_3[w_2 W] + V(P_L).
\]

The first-order necessary condition is that

\[
-w_1 U_1 + w_2 U_2 + [-U_1+w_1 U_1] d\bar{W}/dW + \xi[-U_1+w_1 U_1] dP/dW = 0.
\]
Note that if $\bar{W} = \bar{W}$, then $d\bar{W}/dW_1 = 0$ and if $\bar{W} < \bar{W}$, $-U'_1 + W_1 U'_2 = 0$ by (1) so the third term on the LHS is zero. We now impose that the government does not desire to redistribute income between generations. This implies that $-U'_2 + V' = 0$, that the marginal return to redistribute this generation's second-period consumption to next generation's first-period consumption is zero. As a result a necessary condition for the government maximization problem is: $-W_1 U'_1 + W_1 U'_2 = 0$. Marginal utilities are equated to relative prices, certainly the intuitive result. For $\bar{W} < \bar{W}$ this can be written as:

(3) 
$$-U'_1 (\bar{W}+\lambda) + W_1 U'_1 (w_{-1} \bar{W}) = 0$$

and for $\bar{W} = \bar{W}$ as

(4) 
$$-\bar{W}_1 U'_1 (\bar{W}+\lambda) + W_1 U'_1 (w_{-1} \bar{W}) = 0.$$ 

I. $w_p = (1+\gamma)w_p$, $w_1 = w_1$.

Now we are ready to examine our first polar case. The private technology is subject to random shocks, while the public technology is not. Note that because the shocks are independently distributed, $V$ is independent of the value of $\gamma$. We will only treat the case $\bar{W} < \bar{W}$. The same qualitative results for $\bar{W} = \bar{W}$ can be derived manipulating (4) instead of (3) (except that $d\bar{W} = 0$).

Totally differentiating (3) we get

$$U'_1 dW + W_1 U'_1 (w_{-1} \bar{W}) dY = 0$$

while totally differentiating (1) yields

$$U'_1 dW + (1+\gamma)^2 w_p U'_1 w_{-1} dW = -w_p [U'_1 + C_2 U'_1] dY.$$ 

Solving these two equations simultaneously by Cramers rule we conclude that if second-period consumption is a gross substitute for the other goods, $d\bar{W}/dY > 0,$
\frac{dW}{dy} < 0, and, therefore, \frac{dW}{dy}/\frac{dW}{dy} < 0. If second-period consumption is a gross compliment, \frac{dW}{dy} < 0, \frac{dW}{dy} > 0, and it still holds that \frac{dW}{dy}/\frac{dW}{dy} < 0.

The government's hiring policy is countercyclical relative to aggregate employment. The government should hire some of the unemployed when the private economy suffers unemployment. Given gross substitutes we have the intuitively obvious result. If the private sector technology becomes less productive, but the public sector technology is unchanged, private sector employment should fall and public sector employment should rise. Now we turn to the case where both private and public sector technologies are hit by a real shock.

\[ w = (1+\gamma)w_p, \quad w_g = (1+\gamma)w_g. \]

Once again we treat only the case \( \tilde{W} < \bar{W} \), as similar manipulation of (4) yields similar results for \( \tilde{W} = \bar{W} \) as before.

Totally differentiating (3) we now get

\[ \frac{dW}{dy} + (1+\gamma)^2 \omega^2 \frac{dW}{dy}/\frac{dW}{dy} = -\omega_g \left[ U_1 + G U_1 \right] dy, \]

and the total derivatives of (1) is unaffected at

\[ \frac{dW}{dy} - (1+\gamma)^2 \omega_p \bar{U}_2 \frac{dW}{dy} = -\omega_p \left[ U_1 + C_2 U_1 \right] dy. \]

Solving by Cramers rule again we conclude that if second-period consumption is a gross substitute and the public good a gross compliment, then \( \frac{dW}{dy} > 0, \frac{dW}{dy} < 0, \) and \( \frac{dW}{dy}/\frac{dW}{dy} < 0 \). Similarly, for second-period consumption a gross compliment and the public good a gross substitute \( \frac{dW}{dy} < 0, \frac{dW}{dy} > 0, \) and \( \frac{dW}{dy}/\frac{dW}{dy} < 0 \) still. Otherwise the signs are ambiguous. The case for countercyclical policy is weaker here, as would be anticipated. For example, if both second-period consumption and the public good are gross substitutes, if both technologies become less productive, then there are offsetting effects and no general results on employment public or private. This is not surprising.
We have examined the optimal government expenditure response to the onset of a recession. But what if the recession turns into a depression, if the futures market is wiped out? If this occurs, $\lambda = 0$ in subsequent periods, which influences optimal government expenditure. Note, however, that the altered government expenditure does not move the economy out of the depression. That is achieved by reinstitution of the futures market.
Concluding Comments

We have produced a simple model of the "business cycle" where the technologies of producing goods are the sources of shocks to the economy. However, because the shocks are to (correctly) anticipated future output, the shocks first appear as shocks to demand not supply. The model is characterized by a stable full employment path which is randomly punctuated by recession and recovery. Moreover, the economy will not recover to the full employment path if the recession destroys the futures market.

In this simple model, if the shocks to technology generating recession do not affect the public goods technology, the government should follow a countercyclical policy. Activist expenditure "demand management" is justified. In the preceding analysis we assumed that the government observes the anticipated shocks to technology. In our simple world where there is only one technology observed by individuals, it seems reasonable that the government would have the same information as individuals. If the model is to be applied to our vastly more complicated real world, this assumption may not be justified. Individuals anticipate the shocks to their own individual technologies, which differ, and may well have motive not to reveal what those shocks are. The government has data on stocks and preceding flows but does not have data on the anticipated future shifts to technology. In our simple model, if the government does not observe \( \gamma \), it can still base countercyclical policy on the employment decisions of individuals if these are observed (of course, those employment decisions must anticipate this government action in order to estimate tax). The government will only lose the ability to adjust to shocks that do not move the economy away from the full employment path.
References


