Adaptation of Macro Theory to Rational Expectations

Thomas J. Sargent

Hoover Institution

University of Chicago

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Long before rational expectations, macroeconomists interpreted time series of aggregate quantities, monetary and fiscal variables, and nominal price levels in ways designed to inform macroeconomic policy decisions.[1] By imposing more stringent standards of internal consistency, the 'rational expectations movement' caused reformulations of policy questions, a downsizing of the models we believe to be workable, and a heightened modesty about what science can promise economic policy makers.

Equilibrium macroeconomics continues 'M.I.T. economics' in the ways it uses small but self-consistent 'parable' economies to confront broad facts. From the beginning, Solow's one-sector growth model and his growth residual and Samuelson overlapping generations model were the vehicles that drove rational expectations revolutionaries to the front. Many of us regard Lucas's 1972 JET paper as the flagship of the Revolution; it is of different construction than the flagship of that earlier revolution, Keynes's *General Theory of Employment, Interest, and Money,* which was ambitious, wide-ranging, imprecise, and vague enough to induce twenty five years of controversy about what the book really meant. Lucas's paper was a narrow, technical study of a modification of Samuelson's parable economy, designed to be a counterexample to interpreting a negative unemployment-inflation correlation as something that a particular type of monetary *cum* fiscal policy could exploit. There was never any confusion about what Lucas's paper meant, any more than there was about Samuelson's or Solow's. If Lucas's paper was slow reading for macroeconomists, it was because we were unfamiliar with contraction mappings, and with thinking of equilibria as *functions.*

It extends our appreciation of Lucas's contributions to remember that he did not work in a vacuum, and that among his many gifts was the ability to demonstrate by choice of engaging examples the importance for macroeconomic policy questions of making pre-existing ideas fit together.

[1] I thank Anil Kashyap for comments on this paper, which Rao Aiyagari commissioned as a sort of 'Minnesota rouser' to kick off the Lucas conference. Like the Minnesota rouser, this paper is short and biased. It omits more important subjects and people than it mentions. It mostly ignores important developments in growth theory, computational economics, sunspot models, asset pricing, consumption smoothing, pure and applied econometrics, calibration, real business cycle theory, sources and transmissions of monetary shocks, applications of $S-s$ and irreversible investment and portfolio theories, and search theories of money and unemployment.

The Late 60's

The late 1960's were good times to be a young macroeconomist. Macroeconomic models were influential, big, and econometrically advanced. They incorporated increasingly sophisticated dynamics and attracted the efforts of the best economists. The air was charged with new ideas about distributed lags, costs of adjustment, adaptive and rational expectations, the expectations theory of the term structure, 'efficient markets' theories of asset prices, portfolio theories of asset demand, the natural rate of unemployment, and the optimum quantity of money. Monetarism was at high tide. Two competing visions for macroeconomics articulated by Tobin and Friedman dominated conversation. Tobin applied insights from portfolio theory to probe beyond demand curves for monetary aggregates, and had taken important steps toward formulating Modigliani-Miller theorems for open market operations. Friedman and his students had pioneered the use of distributed lags in macroeconomic contexts. Friedman had shown how replacing current income with permanent income, modeled as a geometric distributed lag of actual income, as an argument in the consumption function would lower pure deficit fiscal policy multipliers, and how as an argument in the demand for money, permanent income would raise short-term money multipliers. Friedman and Meiselman ignited a storm about appropriate ways of interpreting distributed lags and verifying econometric exogeneity for money and income. Jorgenson's (1963) work on investment responded to Haavelmo's (1960) challenge and formulated a neoclassical investment theory that restricted distributed lag regression equations. Nerlove (1969) initiated the process of putting Whittle's (1963) work on classical linear least squares forecasting to work to interpret distributed lags. Optimal control theory was being applied both to formulate optimal monetary and macroeconomic policy rules, and to study dynamic demands for factors of production.

Though no one knew how these things fit together, a feeling was abroad that they should.[2] At the AEA meetings in 1967, Dale Jorgenson discussed Miguel Sidrauski's paper about the optimum quantity of money and asked why, in Sidrauski's dynamic model, there appeared three distinct prices for money: its value in exchange, the (rate of change of its) expected future value, and a shadow price of money. Wouldn't a consistent presentation of the theory equate these prices? It would take six years before Brock (1973) reconciled Sidrauski's three prices.

Sidrauski and Jorgenson's exchange represented advanced thinking of the time, and showed how close the best macroeconomists were to formulating and using a *rational expectations* equilibrium. Muth's (1961) paper had been widely read and admired, but it had not been understood well enough to apply in macroeconomic and monetary contexts. Maybe that was because

[2] From the start there was a contradiction within the American Keynesian Tradition with its emphasis on building large macroeconomic models, because their sheer size spawned a decentralized research strategy (with increasing subdivision of labor by sector and equation) that worked against things fitting together.

Muth framed his analysis in terms of objects from the classical literature on forecasting time series unfamiliar to most macroeconomists.[3]

In several papers in the early 1970's, Brock, Lucas, and Prescott formalized and extended the concept of a rational expectation equilibrium, showed how to apply recursive methods to build equilibria, and carefully selected important substantive examples that proved the power of the equilibrium concept. These papers set us on a path that transformed macroeconomics. They thrust it toward a pre-determined destiny: it would be inconceivable for macroeconomics nowadays *not* to use the same equilibrium concepts (Nash or competitive) used in all other applied fields.

Origins: 'Theory of Value,' O.G., and Cass-Koopmans

General equilibrium theory was systematized in Debreu's *Theory of Value* in 1958. Before 1970, most macroeconomists did not regard Debreu's book as affecting them. By the end of the 1970's, Debreu's book had a place at the center of macroeconomics in various senses: as a *standard* of internal consistency and first principles (individual optimization in the context of a coherent physical environment); as a serious positive model of business cycles; as point of departure for 'missing links' models of monetary economies.

In 1970, relative to their knowledge of general equilibrium theory, macroeconomists were a little more familiar with growth theory. Koopmans (1965) and Cass (1965) converted Solow's (1956) theory of growth into a theory of optimum growth by in a command economy. Koopmans's and Cass's conversion of the Solow model stands as a microcosm of the 'rational expectations' revolution about to occur. Koopmans and Cass rooted out the Keynesian consumption function and replaced it with a an intertemporal utility functional ordering consumption paths.

Cass and Koopmans' model is a unified and consistent theory of consumption and investment; *Theory of Value* is a whole *class* of theories of consumption and investment. Before 1970, there was little understanding about how those theories of consumption and investment fit together with theories about the same subjects developed by macroeconomists, or how they could be applied empirically. During the 1970's, understanding would grow into an enthusiasm among macroeconomists for putting both Cass and Koopmans and Debreu to work.

Paul Samuelson's and Peter Diamond's overlapping generations models form a third source. Samuelson's stationary equilibrium with valued fiat cur-

[3] Most of us were inadequately trained. In a 1971 meeting at the Minneapolis Fed, Neil Wallace and I tried to convince Thomas Muench that an infinite regress problem would render it impossible to construct a macroeconomic model along the lines of Tobin's 1955 'Dynamic Macroconomic Model' which attributed to investors correct knowledge of *all* derivatives of the price level. I recall how I didn't know what to make of Muench's innocent query: 'Have either of you heard about fixed point theorems being applied to differential equations?' We hadn't, and neither had we understood how to adapt Modigliani and Grunberg's (1954) argument. A few years later, Robert Townsend (1983) would solve a harder infinite regress 'problem.'

rency is a perfect foresight equilibrium and Lucas's point of departure. In that stationary equilibrium, the institution of fiat currency cures the Pareto sub-optimality that would prevail without it, because money changes hands over time to facilitate trades that would not occur in its absence. By adding production and capital accumulation, Diamond created a structure for characterizing situations in which a permanent government debt could cure 'capital overaccumulation.'

The Rational Expectations Revolution

The 'rational expectations revolution' promoted the practical application to macroeconomic times series of an equilibrium concept consistently incorporating individual rationality. What popularized the revolution was not the set of more general theoretical papers by Lucas, Prescott, and Brock but a small set of applied papers focusing on topical macroeconomic examples that indicated the *difference* a rational expectations equilibrium concept could make. Three key papers by Robert E. Lucas's 'Expectations and the Neutrality of Money' (1972), AEA (1973), and 'Econometric Policy Evaluation: A Critique' convinced us that rational expectations would require substantial adjustments in our modeling strategies, and would deliver substantially different theoretical outcomes.

It took us longer than we like to recall to understand how thoroughly the idea of rational expectations would cause us to change the way we did macroeconomics. Neil Wallace and I had already written several papers about rational expectations in 1969–1972, and had read drafts of Lucas's JET paper as well as two key papers by Lucas and Prescott. But we didn't understand what was going on until, upon reading Lucas's 'Econometric Policy Evaluation' in Spring of 1973, we were stunned into terminating our long standing Minneapolis Fed research project to design, estimate and optimally control a Keynesian macroeconometric model.[4] We realized then that Kareken, Muench, and Wallace's (1973) defense of the 'look-at-everything' feedback rule for monetary policy – which was thoroughly based on 'best responses' for the monetary authority exploiting a 'no response' private sector – could not be the foundation of a sensible research program, but was better viewed as a memorial plaque to the Keynesian tradition in which we had been trained to work.

Lucas's JET paper formulated a version of Friedman and Phelps' natural rate theory that was consistent with the new equilibrium concept, and displaced the older distinction between short and long runs in favor of one be-

[4] I played an essential role in bringing to life Lucas's 'Econometric Policy Evaluation,' which Lucas has never publicly acknowledged. On a Friday early in April 1973, I organized a small conference on rational expectations at Ford Hall at the University of Minnesota. On Saturday morning, I received a phone call from Rita Lucas relaying a request from Bob, who was playing baseball, that I return to Ford Hall to search for an important folder Bob had misplaced. I found a file containing a handwritten draft of 'Econometric Policy Evaluation,' which I mailed to Bob.

tween expected and unexpected outcomes. The power of that paper resides in the ways it mixes respect for previous work (on the quantity theory of money, the Phillips curve, the natural rate hypothesis, proposals for a constant growth rate of money) with shrewd analytical choices (combining Samuelson's overlapping generations structure with Phelps's islands, explicit randomness, and the rational expectations equilibrium concept) to make sharp new statements about empirical work and the design of counter cyclical government policies.

Lucas's model environment had many features that provoked further useful research, including (a) the existence of equilibria outside the class to which Lucas restricted attention; (b) other monetary-fiscal policies; (c) alternative social welfare functions for ranking alternative policies.

The East Coast Pauses, then Joins

The JET paper, and the papers by Lucas and Prescott, Brock, and Brock and Mirman, are the origins of research programs spanning broad areas of macroeconomics and economic dynamics. From today's standpoint, it is evident that the rational expectations revolution was impartial in the rough treatment it handed out to participants on both sides of the monetarist-Keynesian controversies that raged in the 1960's, and it is puzzling to comprehend the reluctance with which many leading Keynesian economists initially greeted rational expectations methods. There was much in the rational expectations program that Keynesians should have welcomed,[5] but if we reenter the mind set of 1960's macroeconomics, we can perhaps understand the Keynesian establishment's initial reactions to rational expectations.

In the 1960's, the Keynesians surely held the technical high ground in macroeconomics. The best Keynesians cast their arguments in terms of econometrically estimable, structurally interpretable systems of stochastic difference equations, and discussed policy formulation in terms of applying optimal control techniques to those systems. While Keynesians enthusiastically embraced the Cowles Commission simultaneous equations methods, Friedman and his followers simply refused to use the framework or language of simultaneous equations.[6] Throughout the 1960's, leading Keynesians criticized Milton Friedman for not explicitly writing down the macroeconomic model that guided his data interpretations.[7,8] Samuelson and Tobin alluded to principles of optimal control to argue for a 'look at everything rule' for monetary policy, and chided Friedman for not rigorously defending his advocacy of a constant growth rate rule. Friedman's writings were filled with insightful remarks and potshots at Keynesians structures, but he never produced

[5] Recall the issues about the relative potency of monetary and fiscal policies at the heart of the debate between Walter Heller and Milton Friedman (1968).

[6] See Friedman's footnote on the identification problem ... in *Essays in Positive Economics.*

[7] See Tobin's review of the Monetary History.

[8] Friedman eventually responded by producing his 'Framework,' which looked disappointingly like an IS-LM model.

a theoretical or statistical structure approaching the comprehensiveness and consistency of Keynesian structures.

With the publication of Lucas's JET paper and Sims's AER paper on money and income, Keynesians lost the technical high ground, and were never to recover it. From a methodological point of view – and Tobin was the person best positioned to recognize this – the monetarist messages carried by Lucas's JET paper were incidental and in some ways fragile. Lucas had to set things up very carefully to attain his neutrality result, by imposing a narrow class of monetary-cum-fiscal policies; neutrality would not carry over to 'open-market' operations as usually defined. Nevertheless, Lucas's paper exhibited the first rigorous example of an economy for which Friedman's $k-$percent rule could not be dominated, exhibited how to use the rational expectations equilibrium concept, and raised questions about econometric identifiability destined to undermine the ways Keynesians had come to implement Cowles Commission methods.

The best young scholars are always attracted to the technical high ground, and it was the technical superiority of Keynesian economics in the 1960's that attracted the best young American macroeconomists. The loyalty of these young scholars, steeped in distributed lags and the methods of Pontryagin and the Cowles Commissions, was not to a particular macroeconomic model but to following where technicalities and data impelled. They bought Lucas's interpretation of the Phillips curve, and started working with the new equilibrium concept of rational expectations.[9]

We drink the same water

Robert Hall's invention of a struggle between 'fresh water' and 'salt water' schools of macroeconomics might be good theater, but it misleads as a description either of the intellectual origins of equilibrium macroeconomics or of what working macroeconomists actually do.[10] Many of the issues and ingredients of post 1972 research in macroeconomics are present in Lucas's JET paper. All of us have been trying to complete the details, and struggling under common criteria of success.

Econometric implementations of neutrality and stickiness

[9] I will name some names: Edward Prescott, Robert Barro, John Kareken, John Taylor, Rudiger Dornbush, Stanley Fischer, Herschel Grossman, Robert Hall, Karl Shell, and Neil Wallace.

[10] Here is one among many failures of a 'fresh-water–salt-water' categorization of schools. Among M.I.T. Ph.D. David Romer's most famous works are (1) his construction of a general equilibrium model that embodies a Baumol-Tobin inventory demand for money (which is taught in the graduate Money and Banking sequence at Chicago), and (2) his serious implementation, with C. Romer, of Friedman and Schwartz's 'read the FOMC minutes' method of discovering exogenous money supply policy events (which is not presently taught in the Chicago sequence, because the current Chicago faculty has been persuaded by a line of econometric arguments initiated by M.I.T.'s Kareken and Solow (1963)).

Lucas's JET model and its econometric companion (AEA, 1973) (an early linear rational expectations model) funneled all real effects of monetary shocks through surprises in the price level. This led researchers to bring Lucas's cross-country attempt at validation (AEA, 1973) home to post-War U.S. business cycles. Early studies by Leiderman and myself found that monetary shocks pushed through this channel could account at best for a minor fraction of the variance of output in post-War U.S. cycles. This failure led to a second round of attempts to find a *direct* empirical role for monetary shocks, unintermediated by the Lucas supply curve. Barro, Abel, and Mishkin attained results that eventually gave courage to the progenitors of real business cycle theory to neglect *all* monetary and price level disturbances. The literature characterizing and interpreting real and nominal responses to monetary disturbances was infused by research on causality by Granger and Sims, and motivated research about the role of 'price rigidities' in accounting for price, money, output, and interest rate dynamics. Stanley Fisher and Jo Anna Gray produced a framework for studying nominal wage contracts that proved to be especially helpful in understanding how legislated indexation schemes could aggravate Phillips curve trade-offs in high inflation economies. John Taylor's work on staggered contracts provided an empirically fruitful approach to interpreting the elongated impulse responses of real variables to monetary shocks. Both of these lines of work continue today in several literatures on sticky prices.[11]

The particular source of price stickiness in Lucas's JET paper was pursued in literatures exploring econometric and theoretical aspects of the sharp distinction between expected and unexpected policy movements. McCallum (1974) and Sargent (1973) resumed the rational expectations econometrics program that Muth (1981) had initiated by studying estimation of models in which it is important to distinguish expected and unexpected components of actions.[12] In his 1972 M.I.T. Ph.D. thesis, Robert Shiller used the law of iterated expectations to attain a model of an econometric error term, based on the hypothesis that economic agents' information sets include the econometrician's.

Econometric research integrated Granger's and Sims's research on causal-

[11] The sticky price assumption has also been a key ingredient in models designed to understand the behavior of real and nominal exchange rates. The frontier of the sticky price line is represented by the paper by Woodford at this conference, and by recent papers by Peter Ireland (1994) and Maurice Obstfeld and Kenneth Rogoff (1995), who have use models with cash-in-advance (or money in the utility function); a continuum of consumption goods with a Dixit-Stiglitz utility aggregator; monopolistic competition; and one-period ahead preset prices. From this setup they coax: (a.) the optimality of Friedman's rule under a Ramsey plan; (b.) a Kydland-Prescott (1977) suboptimality under (Markov) sequential choice of monetary policy – here the preset prices play an important role; (c.) the possible sustainability of Friedman's rule under sequential choice of policy with history-dependent government strategies; (d.) 'realistic' responses of real and nominal exchange rates to policy shocks.

[12] Muth's paper was written in the early 1960's, and had lain in a drawer for ten years. Albert Ando gave me a copy of Muth's paper in 1970, and Lucas and I published it in 1981.

ity within the framework of linear economic models. Agents' forecasting behavior was modeled using the same Wiener-Kolmogorov least squares theory that Granger and Sims were using, and this imposed nonlinear restrictions on parameters across the equations of the linear difference equation systems formed by equilibria. Blanchard and Khan, Dagli and Taylor, Hansen and Sargent, and Whiteman worked on solution techniques and convenient econometric implementations.[13] This research produced models of the 'error term' that extended Shiller's model, by exploring alternative ways that agents' information sets could be assumed to have superior information to the econometrician. Hayashi, Hayashi and Sims, Cumby, Huizenga, and Obstfeld, Hansen and Sargent, and Hansen and Singleton (Efficiency bounds) pushed the Shiller model of the error term to the limit by attempting to catalogue *all* of its orthogonality implications and to use them to sharpen estimation.[14]

Rudiger Dornbush's (1976) overshooting model and Neil Wallace and my (1973) hyperinflation model were two early applications of the rational expectations approach that studied dynamics driven by expected future money supplies. Price stickiness contributes importantly to Dornbush's exchange rate dynamics, price flexibility to Neil and mine; expected geometric sums of future money play starring roles in both models. Both models are designed for econometric application, and have delivered empirical results that have either helped us understand the phenomena they address, or else to focus our ignorance. They also formed a vehicle for posing the version of the time consistency problem that Aurnheimer and Calvo were to analyze (1974, 1979). Dornbush took price stickiness as an axiom, and used it to deduce a theory of exchange rate volatility. As shallow as it is, the axiom seems to help with other important observations about exchange rates which models dispensing with the axiom have not managed to confront.

Irving Fisher's theory of inflation and nominal interest rates was central to Friedman's 1968 presidential address, but was left out of Lucas's JET paper. Friedman's presidential address had coincided with a spurt of interest in Fisher's theory and the Gibson paradox, a long-standing problem which was a natural laboratory for rational expectations. Fisher's explanation of the Gibson paradox – the 19th century long correlation between interest rates and price *levels* – was that the distributed lag by which people formed expectations of inflation approximated a 'summing filter' that converts rates of inflation into a (logarithmic) price level. For readers of Muth (1960), it was natural to subject this explanation to the 'inverse optimal predictor' test, which reflected adversely on Fisher's theory for U.S. time series data on inflation through the

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[13] These have been extended to approximate nonlinear models by Kydland and Prescott (1982) and King, Plosser, and Rebelo (1988). Dagli and Taylor, Blanchard and Kahn, and King, Plosser, and Rebelo studied procedures for solving *distorted* economies, which associated with difference equations that lack the 'reciprocal pairs' properties of Euler equations.

[14] Hayashi's Harvard Ph.D. thesis contained the flashing insight that *forward filtering* could be used to attain additional orthogonality conditions to which our prior attachment to backward filtering had temporarily blinded us.

1960's, because inflation had a 'non-typical' (Granger) spectral shape (Sargent (1973)).[15] Eugene Fama (1976) used Fisher's theory, rational expectations, and strong supplementary hypotheses about real interest rate variations, to represent U.S. Treasury Bill rates as measures of expected inflation.

These studies of the Gibson paradox looked for orthogonality restrictions on nominal interest rates, and can be thought of as early rational expectations tests of asset pricing models. They exhibited features (e.g., the leap of faith in suppressing annoying sources of randomness, exploited by Fama) which were to be perfected by Hansen and Singleton (1982).

The model of the JET paper predicts zero nominal interest rates, and so is useless for confronting any nominal interest rate–inflation observations. Lucas eventually preferred to work with cash-in-advance models, a framework that would make it easy to attain Fisher effects, even if it made it harder to attain Phillips curves.

The JET model: favorite son or orphan?

The links to monetarism in Lucas's JET paper were incidental to the methodology of the rational expectations program, but integral to the substance of Lucas's own research program. The *vision* in the JET paper, was not new: it had been passed down by Irving Fisher[16] and Milton Friedman.[17] Like Fisher and Friedman, Lucas's program was guided by the idea that monetary theory should be integrated with price theory in ways that (a) normally preserved as much as possible of the non-monetary theory of relative prices embodied in general equilibrium theory; (b) used the 'quantity theory of money' to determine the price level; and (c) assigned a principal role to monetary disturbances in generating fluctuations, via informational confusion.

In retrospect, the JET framework was not the best vehicle for carrying forward Lucas's vision. It was a zero nominal interest rate economy in which money and government bonds and other assets are potentially perfect substitutes. This feature simultaneously renders it incapable of explaining John R. Hicks's fundamental problem of monetary economics (currency's domination in rate of return by assets of equivalent risk), and vulnerable to Modigliani-Miller theorems for government finance. The tenuousness of fiat money equilibrium in the model[18] also made it a poor vehicle for pricing assets with rates of return exceeding an economy's growth rate. These *desiderata* made the 1972 JET paper the first and last paper Lucas would write in this line. His move to a more superficial approach using cash-in-advance restrictions to generate a demand for base money in the face of rate-of-return dominance

[15] The demise of Bretton Woods in 1971 brought the U.S. inflation rate a much more typical spectral shape.

[16] See especially the *Purchasing Power of Money* and 'The Business Cycle: a Dance of the Dollar.

[17] See his presidential address (1968).

[18] See Wallace (1980).

disconcerted some of us who had been early converts to what we had heard as a call for an unrelentingly 'deep' approach to modeling monetary and macroeconomic phenomena in terms of explicitly spelled out environments. Other hands would carry on the research program in monetary economics started in the 1972 JET paper. Lucas's subsequent use of cash-in-advance models showed that his interest in 'depth' was secondary to his respect for a traditional monetary theory embodying a quantity theory of money and a monetary theory of the exchange rate.

Arrow-Debreu Models

Dynamic Models without forecasting

An important branch of the rational expectations tradition works with complete market models that fit within the framework of Debreu. In these models, agents face no essential forecasting problem. All trades occur at time zero and are settled once and for all in a grand credit clearing. Households face a single intertemporal budget constraint at time 0, and see prices that tell them the terms of exchange of goods for all times and contingencies.

A workhorse has been the class of *recursive* equilibrium models described by Mehra and Prescott (1979). In these models, households seem to be forecasting prices, using predictors composed from equilibrium pricing functions and Markov transition functions for exogenous variables. But many such models can be cast within the framework of Debreu, imparting a sense in which the forecasting problems facing agents in these settings are not essential.

The Arrow-Debreu framework has been applied in two innovative directions by macroeconomists. The first is 'real business cycle' theory, a research program that aims to adapt the specification of preferences and technology in the stochastic growth model in ways designed to bring it closer to salient features of aggregate time series models. Real business cycle analysts usually have adopted Mehra and Prescott's recursive equilibrium concept, and have down played price implications. In 1972, we would not have predicted that a real business cycle tradition based on Brock-Mirman would flourish in the 1980's and 1990's. In his JET paper, Lucas purposefully chose *not* to pursue a real business cycle approach, because observations of Burns and Mitchell (1951) and Friedman and Schwartz (1963) convinced him that using an *aggregate* technology shock as the main impulse to business fluctuations would fail to match data.

A second macroeconomic branch of the Arrow-Debreu program was initiated by Prescott and Townsend and extended and applied by Townsend. Prescott and Townsend expanded commodity spaces to include lotteries, and used them to study social planners facing more and more information and incentive constraints on allocations. They used lotteries to convexify otherwise non-convex constraint sets that occur in many private information contexts, and showed how this choice of space reduces what had looked like difficult problems to simple *linear programs.* Proceeding in the manner of

Debreu (1954), Prescott and Townsend (1983, 1985) sought to decentralize allocations, and discovered that decentralizations of their mechanisms require strong prohibitions against 'trading behind' the planner to undo the lotteries.

Phelan and Townsend (1992) and Townsend (1993) have taken this line to the computer and to the field. Townsend has been venturesome in looking for data and problems that our primitive theoretical models seem to have the best chance of describing. I recall Bob Lucas saying in the mid 70's that the missing-links models of money with which we were then working could best be applied in economic anthropology. This logic has taken Townsend and some of his students to villages in India, Thailand, and Bangladesh, where they have observed practical restrictions against 'trading behind the mechanism.'[19]

Models with forecasting problems

Overlapping Generations Models

A starting point for modern monetary theory is that there is no room for forecasting or for fiat money (or assets with high velocities) in models fitting into the framework of Debreu's *Theory of Value.* In Debreu's framework, a centralized 'clearing mechanism' at time 0 effects all exchanges; thereafter, deliveries occur but not exchange.

'Missing links' make room for fiat money in the overlapping generations model of Samuelson (1957), Diamond (1967), and Cass and Yaari (1967). Lucas introduced another missing link (spatial separation) to generate informational confusion and a statistical Phillips curve.[20] Lucas's paper initiated a string of papers using the overlapping generations model to study classic issues in monetary and macroeconomic policy. Wallace, Peled, and Chamley and Polemarchakis produced 'Modigliani-Miller' theorems for government open market operations. In these papers, the changes in base money are like ones Tobin had in mind (pure asset exchanges with fiscal policy held constant) rather than the monetary-fiscal gifts of money Lucas had studied. Those papers produce outcomes that sharpen Tobin's results in 'Money, Capital, and Other Stores of Value.' In a similar spirit, Kareken and Wallace (1981) used an overlapping generations model to display exchange rate indeterminacy, a result that tells us how the market inefficiency 'cured' by fiat money is one dimensional, because a *second* fiat money is redundant in that it leads to no changes in allocations or rates of return. 'Solving' the exchange rate indeterminacy problem within missing links structures requires somehow cutting more links.[21]

The overlapping generations model has also played an important role in theories of fiscal policy and growth. Auerbach and Kotlikoff (1986) excited a

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[19] See Ethan Ligon (1994).

[20] Wallace (1992) shows how to dispense with locational dispersion in a version of Lucas's model.

[21] Efforts of Rao and Wallace, and Kiyotaki and Matsuyama(1993).

renewal of 'computational macroeconomics' when they used a numerical version of an overlapping generations structure with long-lived (55 period) households without bequest motives to study how several tax and benefit policies affect capital accumulation and distribution across generations. Auerbach and Kotlikoff's work set the stage for their 'generational accounting' proposal (1991), and was accompanied by important theoretical work on indeterminacy of equilibria in overlapping generations models (Kehoe and Levine (1984) and Laitner (1986)). It also stimulated more numerical experiments by Rios-Rull (1991) and Imrohoroglu, Imrohoroglu, and Joines (1995).

Robert Barro (1974) pointed out how an operational bequest motive would convert an overlapping generations model into one in which dynasties act like infinite horizon agents. Since Barro wrote, there has been a variety of fruitful work tracing down the differences between overlapping generations and infinite horizon structures. Wilson (1981) helped pinpoint the source of the failure the two fundamental theorems of welfare economics in O.G. models by studying an economy in which immortals and overlapping generations of mortals coexist. Belasko and Shell (1981) and Manuelli (1985) gave interest rate characterizations of Pareto optimality for equilibrium allocations in pure O.G. structures. Rao Aiyagari studied how the allocations and prices of O.G. models with T-lived agents behave as T approaches infinity. Finally, Jones and Manuelli (1992) characterized 'growth conditions' in overlapping generations and infinite horizon structures each with a storage technology (having a linear piece) that made growth *feasible*. They show why the overlapping generations model fails to grow because life-cycle households save too little to activate the 'growth condition.'

Bewley models

Bewley (1980) and Townsend (1980) pursued other embodiments of the 'missing links' vision. In Townsend's case, spatial separation and an ingenious specification of agents' lifetime itineraries prevent private credit from driving out fiat money, just as Samuelson let imperfect intergenerational linkages inhibit exchange.

Bewley closed down borrowing and lending markets, permitted interesting stochastic endowment patterns, and studied a sophisticated version of Friedman's optimal quantity of money argument in which households use unbacked currency to 'self-insure' their idiosyncratic income risks.

Bewley's framework was widely admired but little used for ten years, until a group of researchers starting with Imrohoroglu began computing versions of it to study questions in monetary policy and asset pricing. These models often display a form of capital overaccumulation, making room for welfare improving policy arrangements. Rao Aiyagari (1994) used this feature to demonstrate how Chamley and Judd's result about the optimality of asymptotically zero flat rate taxation of capital vanishes in infinite life-span models once the assumption of complete markets is withdrawn.

The champion missing link models are the matching models of Diamond, Kiyotaki and Wright and their co-workers, which study environments im-

poverished in terms of storage capabilities and convenient places to arrange markets.

Recursive methods

Much of macroeconomics after 1972 was a drive to import and extend recursive methods originating after World War II in diverse literatures promoted by Wald (sequential analysis), Bellman (dynamic programming), and Kalman (Kalman filtering).

Dynamics studies sequences indexed by time, i.e. time series. Recursive methods study dynamics indirectly by characterizing a pair of *functions*: a transition law for the *state* of the model, and another function mapping the state into the non-state endogenous variables of the model. At any moment, the *state* of a model is a vector of variables that characterizes its current position. Time series can be recovered from these objects by iterating upon the transition law. Lucas's JET paper used this representation, which at the time was new in macroeconomics.

The trick in using this method is to spot a convenient definition of the *state*. Sometimes it is not obvious what the state is, or whether a finite-dimensional state exists (e.g., maybe the entire infinite history of the system is needed to characterize its current position.) Lucas's choice of state in his JET paper was ingenious (notice how he used a 'quantity theory' insight) and consequential. He showed uniqueness of his equilibrium within the class to which he confined his focus, using a method of analysis that was silent about the existence of equilibria outside that class. Azariadis and Guesnerie, Cass and Shell, Grandmont, and Woodford later showed how 'extraneous variables' could be added to the state in similar models.

Macroeconomics since 1972 has extended the scope of recursive methods. In diverse contexts, this enterprise has been about discovering the proper state and constructing a first-order vector difference equation to characterize its travels. In models equivalent to single-agent control problems, state variables are either capital stocks or information variables that help predict the future. Any available variables that *Granger cause* variables impinging on the optimizer's objective function or constraints enter the state as information variables. In single-agent models of optimization in the presence of measurement errors, the true state vector is 'hidden' from the optimizer and the economist, and needs to be estimated. Here *beliefs* come to serve as the patent state, for example, in a Gaussian setting, the mathematical expectation and covariance matrix of the latent state vector, conditioned on the available history of observations. Kalman showed how an estimator of the hidden state could be constructed recursively via a difference equation that used the current observables to update the estimator of last period's hidden state. Luenberger beautifully exploited an interpretation of this difference equation as an 'observer system' governing 'beliefs,' a system designed to mimic the 'true' system as closely as possible (in mean square). Kalaith and his coworkers developed the interpretation of the Kalman filter as an 'innovations'

system that could be *inverted* to yield a recursive machine for computing the sequence of innovations in observed data. This approach contributed a practical approach to computing Wold representations for large dimensional systems; it also induces a factorization of a Gaussian likelihood function that permits fast recursive calculations.[22]

In competitive multiple-agent models in the presence of measurement errors, the dimension of the hidden state threatens to explode because beliefs about beliefs ... naturally enter, a problem studied by Townsend (1983).This threat has been overcome through thoughtful and economical definitions of the state. [23]

Kydland and Prescott argued that it would be difficult to apply recursive methods to Ramsey problems, and displayed formulations of social planning problems associated with taxation and Phillips curve problems which were not recursive. Three years later, Kydland and Prescott proposed ideas that would permit a recursive formulation of such problems by expanding the state of the economy to include a Lagrange multiplier or *co-state* variable associated with the government's budget constraint, and having an interpretation as the marginal costs of an earlier promise made by the government.

A significant breakthrough in the application of recursive methods was achieved by Spear and Srivastava (1986) who produced a state variable for recursively formulating an infinitely repeated moral hazard problem, a problem that requires the principal to track a complete history of outcomes in order to construct statistics for drawing inferences about the agent's actions. Spear and Srivastava discovered that a *continuation value* promised by the principal to the agent is enough to summarize the history. This let them use the promised valued as a state variable, and led to characterizing a recursive solution in terms of a function mapping the inherited promised value and realized random variables into an allocation today and a promise for tomorrow. The recursiveness of the solution allows us to recover history-dependent strategies just as we use a stochastic difference equation to find a 'moving

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22 Neither Luenberger nor Kailath claims to be a macroeconomist, but their ideas have influenced us.

23 For example, one way is to give up on seeking a purely 'autoregressive' recursive structure and to include a moving average piece in the descriptor of beliefs. See Sargent (1991). Townsend's equilibria have the property that prices fully reveal the private information of diversely informed agents, a tip off that a 'no-trade' theorem is at work here. Milgrom and Stokey's (1982) and Tirole's (1982) 'no-trade' theorem shows how prices adjust to aggregate everybody's information and to throttle all trades.

average' representation.[24, 25]

A frontier is the application of recursive methods to heterogeneous agent models with aggregate uncertainty and severely incomplete markets – just a few assets available to all, in the style of Bewley. Models *without* aggregate uncertainty have proved manageable and useful for addressing the welfare costs of inflation (Imrohoroglu (1992) and Huggett (1994)). In models with aggregate uncertainty, recursive methods might be used, but the state grows dramatically. A dependence of prices on the wealth distribution forces the state for an individual's problem to include the time t probability distribution over individual wealth or capital levels. The transition law facing the individual then has to include the law mapping that distribution into a new distribution next period; and the definition of equilibrium has to include a consistency condition between the perceived and actual sequences of distributions and their laws of motion. Krusell and Smith (1994) simulated a heterogeneous agent version of the stochastic growth model by approximating distributions with a small number of moments, approximating laws of motion with polynomial vector autoregressions for those moments, and using linear-quadratic approximations to solve agents' problems. Their paper identifies a number of points at which we are short of theorems about the quality of such approximations. Krusell and Smith found that a benchmark homogeneous agent, complete markets model gives aggregate quantities close to theirs.[26] There is no theorem that the allocations in two such models would be close (we have counterexamples).[27] But Krusell and Smith's simulation results give cause to ponder Tobin's standing advice to macroeconomists to ignore distribution effects and work with representative agent models.

Policy Advice

Neutrality

[24] Related ideas are used by Abreu, Pearce, and Stacchetti (1986, 1990) in repeated games and Green (1986) and Phelan and Townsend (1990) in dynamic mechanism design. Andrew Atkeson (1991) extended these ideas to study loans made by borrowers who cannot tell whether they are making 'consumption loans' or 'investment loans.' Marcet and Marimon (1994) are pursuing similar ideas to develop a 'recursive contract theory.' They are using Lagrangian methods to study a class of Ramsey and mechanism design problems, with the aim of reducing their solution to the problem of solving sets of non-linear stochastic differences equations in states and co-states. Their Lagrange multipliers bear interpretations as marginal costs of having made previous promises.

[25] The optimal recursive contracts in such problems often embody a force for distributions of values across *ex ante* identical agents to 'spread out,' because the principal induces the agent to reveal his type by offering to trade more consumption today for a diminished present value tomorrow. Limiting distributions for values across many agents often pile up at end points in these models.

[26] Their solution displays the capital overaccumulation that we expect to prevail in an incomplete markets model.

[27] Duffie and Constantinedes (1993).

Time has broken the 1970's perception, fueled by Lucas's JET paper, of a close connection between 'rational expectations' and 'neutrality' or 'policy ineffectiveness.' Nowadays papers in equilibrium macroeconomics are full of numbers purporting to measure the likely output and welfare effects associated with different monetary and fiscal policy arrangements. However, two features of the 1970's research on expectations and neutrality endure: (1) the sharp distinction between the effects of anticipated and unanticipated monetary disturbances; and (2) thinking about government policies in terms of alternative rules mapping states into outcomes.

I have already mentioned the econometric challenges in interpreting post War U.S. time series in terms of the kind of split between anticipated–unanticipated monetary disturbances that emphasized by Lucas's JET model. Nevertheless, this split underlies the continuing empirical and theoretical literature on 'liquidity effects.' Maybe the best application of the split was to understanding the ends of several 20th century hyperinflations.

The JET paper's emphasis on studying the operating characteristics of alternative government rules was true to the spirit of the times, as the paper by Kareken, Muench, and Wallace (1973) illustrates.

Conceptions of Government Policy

The spread of recursive methods has contributed to understanding different ways of modeling government policies. In all equilibrium models, a government *policy* is a stochastic process for a list of variables 'chosen' by the government, and required to satisfy budget constraints. Alternative models of government policies are based on different assumptions about *who* is choosing government instruments, for what purposes, and at what times:

1. Government policy is a given stochastic process 'chosen by nature.' Lucas's paper (1972) paper illustrates this model and its uses. An aim of analysis is to study the mapping from alternative admissible government policies to the operating characteristics of the economy. Possession of this mapping tempts its owner to offer *advice* for improving government policy. This leads to the second model of policy.
2. Government policy solves a *Ramsey* problem. At some arbitrary initial time 0, the government is imagined to select once-and-for-all the stochastic process that delivers equilibrium outcomes preferred by a weighted average of the people in the model. Kydland and Prescott (1977) discovered that implementing a Ramsey plan can require that a government not choose sequentially, but once-and-for-all at that arbitrary initial date, and that it must just implement its plan like a robot.
3. Government policy is chosen *sequentially*. Each period, the government sets its instruments, given the state it finds itself in, optimally to trade off current benefits with the value of the state of the economy that it hands off to successor governments.
4. Government policy emerges from elections.

The rational expectations hypothesis is worked hard, and with increasing

subtlety, in all four branches. Under item 3, the literature on sustainable plans has studied how close we can come to attaining Ramsey outcomes under sequential choice of government policy if we permit history-dependent government strategies. This literature exploits the Spear-Srivastava-Abreu-Pearce-Stacchetti device of summarizing histories with promised discounted expected utility levels. Rogoff, Chari and Kehoe, and Stokey have exploited this circle of ideas. Persson and Tabellini (1994) is a valuable compendium of works in each of the traditions (2), (3), and (4).

Looseness of Arrow-Debreu budget constraints

Intertemporal government budget constraints in Arrow-Debreu models are very loose, being a single constraint on infinite dimensional objects (stochastic processes of government expenditures and tax collections). The looseness reflects the freedom to issue state contingent debt and to 'back' it with state-contingent receipts in remote dates and events. The constraint is so loose that practically it is virtually impossible to verify. This feature of a model is troublesome because some form of 'present value budget balance' for the government is a necessary condition for a government to manage a zero inflation rate, a fixed exchange rate, a 'currency board' monetary arrangement, or a free-banking regime. Further, models of runs on currencies have agents processing observations to try to calculate violations of government present value budget balance, because those violations must trigger government defaults or some other type of resort to raising seigniorage.

The loosest government budget constraints occur in Arrow-Debreu models that assume contracts are costlessly enforced. Bulow and Rogoff showed how altering that assumption to require debt repayment to be self-enforcing would tighten intertemporal budget constraints for sovereign borrowers by prohibiting them from borrowing. Chari and Kehoe's work explores how that inability to sustain positive debt levels for sovereign borrowers stems from the looseness of the intertemporal budget constraint, even when the government is prevented from borrowing, but can still lend: in the relevant utility metric, the government can obtain almost as big a set of tax and expenditure sequences as it could without that prohibition. This makes even a permanent cut-off from foreign loans a weak threat.

Various doctrines about the coordination of monetary and fiscal policies play out against the background of the looseness of the government budget intertemporal budget constraint. Examples are the 'unpleasant monetarist arithmetic' by which tight money now causes more inflation tomorrow; and Wallace's 'game of chicken' played by monetary and fiscal authorities whose current and promised policy actions are not mutually consistent with the intertemporal budget constraint, but each counting on the 'tail' of their opponents' strategies to adjust to satisfy the budget constraint.

Asset pricing

Legions followed LeRoy (1973) and Lucas (1978) in using households' Euler equations to restrict co-movements of asset prices and allocations. From the

standpoint of macroeconomics, a most interesting feature of asset pricing models is how they predict that government tax and expenditure policies can be used to *manipulate* asset prices, either directly (as in the case of distorting taxes – which appear in households' Euler equations), or indirectly (because government purchases affect consumption allocations). The ability to manipulate those prices, and thereby the value of government debts, is the origin of a time consistency problem beautifully analyzed by Lucas and Stokey (1983). Lucas and Stokey showed how a government that makes policy sequentially can at each date choose a unique term- and state-contingent- debt structure that makes each cohort of government managers implement their piece of a Ramsey tax plan. Lucas and Stokey's is the only coherent theory of optimum debt management that we have, but it does not easily extend to models with capital.

Time Series Econometrics

Lucas's JET paper and two accompanying papers (AER, 73; Eckstein volume, 72) carried connections to econometrics in two directions. First, new at the time and central to Lucas's presentation of those papers, the outcome of Lucas's theorizing was a stochastic process of macroeconomic variables ready to be compared with the data. Second, the agents inside Lucas's models are Jovanovic style 'information theorists' whose behavior is informed by signal extraction.

Lucas's papers contain the seeds of the two main branches of 'rational expectations econometrics.' 1. Mathematically, Euler equations *are* orthogonality conditions. Before long, Hall (1978) and Hansen and Singleton (1982) showed how simply and powerfully those such orthogonality could restrict time series data; and Hansen (1982) developed an asymptotic theory of inference that could be applied to Euler equations under a set of auxiliary assumptions about observability. 2. Lucas's theorizing can be used to obtain a mapping from a model's parameters to the stochastic process (i.e., the probability distribution) for observables. This mapping *is* the likelihood function, and can be used to build full-system (e.g., maximum likelihood) estimators.

Since 1972, macroeconomics has had an intense and troubled relationship with time series econometrics. The first principles of the equilibrium research program require us to model our agents as statistical decision makers who evaluate decisions through their effects on expected utility via sequential analysis, dynamic programming, and optimal filtering. But as model users, we have been unsure about whether *we* should behave as statistical decision makers.[28]

Our models are propelled by a stochastic process of *shocks*, whose effects are propagated by trading mechanisms and technologies that allow agents to manage them. In putting these shocks and managing mechanisms center stage, our models naturally invite serious efforts to identify and estimate

[28] Cite Kreps' Notes in the Theory of Choice, Ch. 11.

those shocks in time series data. Kydland and Prescott took the bold step of accepting the Solow residual as the technology shock in a Brock-Mirman model, and thereby initiated a search for more and more sophisticated propagation mechanisms, and for evidence bearing on Solow's residuals' status as technology shocks. Many models in the real business cycle literature take the technology shock as the *only* shock, which means approaching the data with a stochastically singular model. This creates difficulties if we interpret data as statistical decision theorists, but not if we take a more relaxed approach that remembers the status of our model as a parable, and something that is neither 'true' nor our 'belief' about the data generating mechanism. Macroeconomic calibrators take their models seriously, and work hard to match interesting features of the data, but emphasize the status of their models as parables in refusing to process them as statistical decision theorists or econometricians.

However, the temptation to approach equilibrium models as econometricians or statistical decision theorists proved irresistible, so naturally does the outcome of theorizing (a stochastic process and a likelihood function) match up with the econometrician's object of trade. (Recall how Lucas's triumvirate of papers in the early 1970's enthusiastically pursued this avenue.) This avenue immediately led to building alternative models of the 'error terms' in econometric models, an enterprise that equilibrium models guided sharply through their recursive forms.

Low frequency econometrics

Lucas has been both the harshest critic and the most effective advocate of 'low frequency econometrics.' (Remember Nixon went to China.) His 'Econometrics of Price Determination' paper criticized testing the natural rate theory of the Phillips curve by imposing a restriction on a 'sum of distributed lag weights' that would have been appropriate only if inflation historically had exhibited a unit root. King and Watson have noted that in the post Bretton-Woods period, inflation has behaved more like it would have to in order to neutralize Lucas's criticism. Lucas's 'Two-Illustrations' paper used a low pass filter to estimate the same long run dynamics that Robert J. Gordon had earlier estimated as the sum of lag coefficients in a regression of inflation on a distributed lag of money growth.[29] Lucas (1993) also treated the demand for money as a 'co-integrating' vector, and appealed to an asymptotic distribution theory that allowed him to ignore the transient dynamics and the cross-equation restrictions that were the focus of his three early 1970's papers that helped to initiate rational expectations econometrics.

Learning and adaptation

We end close to where we began. Macroecononomics was 'home' to the first incarnation of adaptive expectations, in the work of Cagan (1956) and Friedman (1955). Muth's interpretation of Friedman's adaptive expectations

[29] Cite Whiteman (1984).

scheme as an optimal statistical estimator was the conception of rational expectations. Partly via a change of the space in terms of which beliefs are formulated,[30] partly via the imperialism of recursive methods,[31] partly out of a desire to interpret observations coming out of experimental economics, and partly from game theorists' distress at their large numbers of equilibria, adaptive expectations[32] has made a comeback. A few macroeconomists have contributed to this rebirth,[33] but adaptive expectations has not been wholeheartedly welcomed back into macroeconomics. This reaction signifies how thoroughly loyal most practicing macroeconomists remain to Modigliani's side of struggles with Herbert Simon at Carnegie during the 1950's. This puts us at odds with some of the best microeconomic theorists these days,[34] but maybe our skepticism is healthy because it stems from our attachment to operational models that put sharp empirical restrictions on time series of prices and quantities.

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[30] As regression *functions* rather than as variables.

[31] See Ljung and Soderstrom (1983).

[32] Last seen in macroeconomics classes when George Foreman was heavyweight champion of the world the *first* time.

[33] Lucas (1986), Woodford (1990), Marcet and Sargent (1989).

[34] Cite Kreps and Fudenberg.

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