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Regulated, Insured Intermediaries

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by

John Bryant and Neil Wallace

In "The Inefficiency of a Nominal National Debt," we described models that account for positive interest on safe government debt. This was accomplished by assuming that such debt is costly to hold. In one version, we assumed (i) that all government debt has to be intermediated by way of a resource-using, constant-average-cost technology, (ii) that individuals hold as assets only currency and intermediary liabilities, and (iii) that at identical yields nothing distinguishes currency from intermediary liabilities.

The intermediaries of the earlier paper were like bond mutual funds. An open market purchase, a central bank purchase of outstanding government bonds, decreases the amount of bonds and the liabilities of these bond funds and increases the amount of currency held by the public. Under our assumptions, the only effect is to free some resources; less resources are used in intermediation. Hence, our conclusion: The open market purchase is deflationary and welfare-improving.

Here we show that this conclusion can survive in a model that is richer in three respects. First, individuals--at least at some relative prices--diversify between holdings of currency and holdings of intermediary liabilities. This is accomplished by making individual holdings of currency subject to a random proportional uninsurable loss. Second, there is investment in the form of storage of the consumption good. Third, in addition to uninsured intermediaries, there are intermediaries that have their liabilities insured by the government and that are subject to various regulations including a reserve requirement:

each insured intermediary must hold reserves in the form of currency equal to a fraction of its insured liabilities.

In Section I we describe our model, a version of Samuelson's pure consumption loans model with currency, government bonds that are titles to currency in the future, a storable consumption good with a stochastic return, and a resource-using intermediation technology. In Section II we describe what turn out to be several rich numerical examples. It is in the context of these examples that we study how the stationary equilibrium depends on the relative amounts of currency and government bonds outstanding, our way of studying open market operations. In the concluding section, we discuss our model against the background of the existing literature and the predominant view about the effects of open market operations.

## I. The Model

As noted above, our model is a complicated version of Samuelson's pure consumption loans model. We insist on taking up our questions in Samuelson's framework because it is the only one that generates valued fiat money. Our particular assumptions are chosen to be consistent with the existence of an equilibrium in which all prices remain the same from period to period. And, in order to conserve on notation, we will omit time subscripts except when to do so would be confusing.

### 1. Tastes, Technology, Resources, and Government

Time is discrete. At any date  $t$ , the model is peopled by  $M$  old people and  $M$  young people, the latter being the members of generation  $t$ . At  $t$ , each member of generation  $t$  acts to maximize expected utility, utility being common to all members of all generations and

given by  $U(e_1, e_2) = \sum u(e_j)$  where  $e_j$  is age  $j$  ( $=1, 2$ ) consumption of the single consumption good in the model. The function  $u$  satisfies  $u' > 0$ ,  $u'' < 0$ , and  $u'(0) = \infty$ .

Each member of generation  $t$  has an endowment at  $t$  of  $w$  units of the consumption good. This is perfectly divisible and may be consumed at  $t$  or traded.

Only intermediaries may store the consumption good, hold government bonds, and store currency safely. At each date, intermediaries acquire assets and sell claims to the payoffs that accrue at the next date. We express the intermediation technology in the form of a total cost function

$$(1) \quad G(y) = g_0 y + g_1 [\max(0, y - \bar{y})]^2; \quad g_0, g_1, \bar{y} > 0$$

where  $y$  is the total value of the assets acquired at  $t$ . Both  $y$  and  $G(y)$  are in units of the time  $t$  consumption good. The nonlinearity of  $G(y)$  is introduced to make a restriction on the number of insured intermediaries matter (see below).

The technology for storage of the consumption good is constant-returns-to-scale and stochastic. We let  $s \in [0, 1]$  be a state variable and  $x(s)$ --a nonnegative, nondecreasing bounded function--be the gross return function; if  $k$  units are stored at  $t$ ,  $kx(s)$  units are available at  $t+1$ . The value of  $s$  is drawn independently from period to period according to the density function  $f(s)$ . We assume that the drawing of  $s$  occurs so as to preclude intergeneration risk sharing. Thus, the realization of  $s$  that determines the return to storage from  $t$  to  $t+1$  occurs after generation  $t-1$  disappears and before generation  $t+1$  appears.

The government issues currency and one-period, zero-coupon bonds that are titles to specific amounts of currency. It also operates an insurance program for licensed intermediaries. The government behaves so that the stocks of currency,  $H$ , and of government bonds,  $B$ , are constant over time. At each date  $t$ , the government sells bonds with an aggregate face value of  $B$ ; that is, bonds that in the aggregate promise to pay  $B$  units of currency next period. If the bonds sell at less than face value, then the government levies taxes payable by the young equal to total interest on the debt.

While there is free entry into intermediation, there is not into insured intermediation. Instead, there are  $N$  licenses outstanding, each license being a permit to operate a limited liability corporation that issues insured liabilities subject to certain regulations and the cost function  $G$  described above.

We denote the price paid for each license by  $V$ . The government taxes the transfer of ownership of these licenses collecting  $N\gamma V$  each period where  $g_0/(1+g_0) < \gamma \leq 1$ . These taxes are returned to the current old via a lump-sum scheme. The insuring of the liabilities of these  $N$  intermediaries may for some realizations,  $s$ , imply payments by the government. These payments are financed via lump-sum (state-dependent) taxes levied on the old. All of this is spelled out in detail below.

## 2. The Choice Problem of the Young

Our first task is to describe the riskiness of individual currency holdings. We divide the  $M$  members of each generation into two groups:  $M_1 = M_2 = M/2$ . Each period a fair coin is tossed. If "heads," then each member of  $M_1$  loses a fraction  $\delta$  of his or her currency holdings. The loss shows up as a lump-sum transfer to one member of  $M_2$ . If "tails,"

then vice versa. The crucial restriction is that bets cannot be placed on the outcome of the coin tossing. Such bets or contingent contracts would prevent the coin tossing from producing uncertainty.

The symmetry between members of  $M_1$  and  $M_2$  is imposed for convenience. It implies that everyone is identical prior to the coin tossing. Therefore, each person maximizes

$$(2) \quad EU = u(e_1) + .5\int u[e_2(s,\delta)]f(s)ds + .5\int u[e_2(s,0)]f(s)ds$$

where

$$(3) \quad e_2(s,\delta) = q(s) + (1-\delta)Pc$$

$$(4) \quad e_2(s,0) = q(s) + Pc + \delta Pc'.$$

Here  $e_2(s,\delta)$  is second-period consumption in the state (s, "heads");  $e_2(s,0)$  is second-period consumption in the state (s, "tails"); one unit of  $q(s)$  is a claim on one unit of second-period consumption in state (s, "heads") and in state (s, "tails"),  $c$  (a choice variable) is currency holding,  $c'$  is per capita currency holding of the other group, and  $P$  is the price of currency in units of the consumption good.

EU is maximized by choice of  $e_1 \geq 0$ , a function  $q(s)$ , and  $c \geq 0$  subject to

$$(5) \quad e_1 + Pc + \int p(s)q(s)ds - (w-T) \leq 0$$

with  $P$ ,  $p(s)$ ,  $c'$ , and  $(w-T)$  treated as parameters. Here  $p(s)$  is the price in units of first-period consumption of one unit of  $q(s)$ , while  $T$  is the present value of direct taxes, an expression for which is given below.

For  $P > 0$  and  $p(s) > 0$ , the unique maximizing values of  $e_1$ , and  $c$ , and the almost unique maximizing function  $q(s)$  satisfy (5) with equality and the first-order conditions

$$(6) \quad u'(e_1) - \lambda = 0$$

$$(7) \quad (1-\delta) \int u'[e_2(s,\delta)]f(s)ds + \int u'[e_2(s,0)]f(s)ds - 2\lambda \leq 0$$

$$(8) \quad u'[e_2(s,\delta)]f(s) + u'[e_2(s,0)]f(s) - 2\lambda p(s) = 0; \text{ almost all } s$$

where  $\lambda$  is the nonnegative multiplier associated with (5) and where (7) holds with equality if  $c > 0$ .

We will use (3) through (8) to find implicit demand functions for  $c$  and  $q(s)$ .

First, we examine what is implied by  $c = 0$ . By symmetry, if  $c = 0$ , then  $c' = 0$ . Then, (3) and (4) imply  $e_2(s,\delta) = e_2(s,0) = e_2(s)$ , so (7) becomes

$$(9) \quad (2-\delta) \int u'[e_2(s)]f(s)ds - 2\lambda \leq 0$$

while (8) becomes

$$(10) \quad u'[e_2(s)]f(s) - \lambda p(s) = 0.$$

If we integrate (10) over  $s$  and substitute the result into (9), we get

$$(11) \quad (2-\delta) \int p(s)ds - 2 \leq 0.$$

Letting  $1/r \equiv \int p(s)ds$ , we may write (11) as

$$(12) \quad r \geq (2-\delta)/2$$

where  $r$  has the interpretation of the safe gross rate of interest.

This proves that  $r < (2-\delta)/2$  implies  $c > 0$ . It is also easily shown that (12) implies  $c = 0$ .

Noting that  $c = c'$  at equilibrium, (5), (6), and (8) imply

$$(13) \quad u'[q(s)+(1-\delta)Pc]f(s) + u'[q(s)+(1+\delta)Pc]f(s) = \\ 2p(s)u'[w-T-Pc-\int p(s)q(s)ds] \text{ for all } s$$

while (5), (6), and (7) at equality imply

$$(14) \quad (1-\delta)\int u'[q(s)+(1-\delta)Pc]f(s)ds + \int u'[q(s)+(1+\delta)Pc]f(s)ds = \\ 2u'[w-T-Pc-\int p(s)q(s)ds].$$

If (12) holds, then  $c = 0$ , and we equate  $q(s)$  in (13) to the supply of  $q(s)$  which we describe below.

If (12) does not hold, then we equate  $q(s)$  and  $c$  in (13) and (14) to their respective supplies.

### 3. Uninsured Intermediaries

Free entry into uninsured intermediation provides bounds on  $S$ , the nominal market value of government debt per dollar of face value, and on certain functions of  $p(s)$ . Free entry implies that profits from holding any asset and operating on the linear portion of the cost function  $G$  must be nonpositive.

For storage of the consumption good, this implies

$$k\int p(s)x(s)ds - k - g_0k \leq 0, \quad k \leq \bar{y}$$

or

$$(15) \quad \int p(s)x(s)ds \equiv px \leq 1 + g_0$$

which holds with equality if uninsured intermediaries store some of the consumption good.

For holdings of currency,  $R'$ , the nonpositive profit condition gives

$$PR'/r - PR' - g_0 PR' \leq 0, PR' \leq \bar{y}$$

or

$$(16) \quad 1/r \leq 1 + g_0$$

which holds with equality if uninsured intermediaries store currency.

And, finally, for government bonds with face value  $b$ , we have

$$Pb/r - PSb - g_0 PSb \leq 0, PSb \leq \bar{y}$$

or

$$(17) \quad 1/r \leq (1+g_0)S$$

which holds with equality if uninsured intermediaries hold government bonds. It turns out that all government bonds are held by uninsured intermediaries, so (17) always holds with equality in equilibrium.

Notice that if uninsured intermediaries store currency so that  $1/r = 1 + g_0$ , then  $S = 1$ . More generally, we have  $S \leq 1$ .

#### 4. Insured Intermediaries

Assume that at  $t$  insured liabilities are committed to pay  $D \geq 0$  dollars at  $t+1$  and hold an arbitrary portfolio of the following assets: government bonds with face value  $b \geq 0$ ,  $R \geq 0$  units of currency, and storage of  $z \geq 0$  units of time  $t$  consumption good. Then the value of a permit at  $t$ ,  $V(t)$ , and that at  $t+1$ ,  $V(t+1)$ , satisfy

$$(18) \quad V(t) = \int p(s) \max[zx(s) + P(t+1)(R+b) + (1-\gamma)V(t+1) - P(t+1)D, 0] ds - \\ G[z + P(t)(R+S(t)b)] - [z + P(t)(R+S(t)b)] + P(t+1)D/r.$$

The RHS is simply revenue minus costs. One component of revenue is the last term  $P(t+1)D/r$ . (Since deposits are insured, they pay off in every state.) The second component of revenue is the integral; it is the value of state-specific net receipts. Note that  $(1-\gamma)V(t+1)$  is after-tax receipts from the sale of the license. Owners get all of this in state  $s$  only if the state  $s$  value of assets,  $zx(s) + P(t+1)(R+b)$ , exceeds the state  $s$  value of liabilities,  $P(t+1)D$ . Costs consist of operating costs,  $G[z + P(t)(R+S(t)b)]$ , and the cost of assets,  $z + P(t)(R+S(t)b)$ .

Before we can state the optimizing problem of the insured intermediary, we need a preliminary proposition.

For given nonnegative  $D$ ,  $b$ ,  $R$ , and  $z$ , denote the RHS of (18) by  $F[V(t+1)]$ . The proposition is about bounded  $V$  sequences that satisfy (18) for all  $t$  at constant prices and at an unchanging portfolio. (We are concerned only with bounded sequences, because, as will be seen below, any equilibrium  $V$  sequence must be bounded.)

Proposition 1. For a portfolio and prices that are constant over time:

- (a) If  $F(0) \geq 0$ , there exists one and only one nonnegative bounded  $V(t)$  sequence that satisfies (18). This is the constant sequence, the constant being the unique solution to  $V = F(V)$ .
- (b) If  $F(0) < 0$ , no nonnegative bounded  $V$  sequence satisfies (18).

This proposition, proved in the appendix, follows from the properties of  $F$ , inequalities (15) and (16), and our assumptions about the tax,  $\gamma$ , on the sale of  $V$ . In the absence of such a tax, the licenses,

which can be costlessly stored, would dominate currency. Indeed, as we shall see, even with the tax, their costless storage plays a role.

In light of Proposition 1 and our goal of describing stationary equilibria, we state the problem of an insured intermediary as follows. Choose nonnegative values of  $D$ ,  $b$ ,  $R$ , and  $z$  treating  $P$ ,  $S$ , and  $p(s)$  parametrically to maximize the solution to  $V = F(V)$  subject to two regulatory constraints,

- (i)  $R \geq \alpha D, 1 > \alpha > 0$
- (ii)  $z + P(R+Sb) \geq \beta PD, \beta \geq 1.$

The first regulation is a reserve requirement and the second is a capital requirement. The capital requirement at its weakest ( $\beta=1$ ) assures that a portfolio consisting entirely of reserves would allow the paying off of all claims. We will call any portfolio that satisfies (i) and (ii) a feasible portfolio.

We now state and prove several facts about optimal portfolios for an insured intermediary.

Fact 1. Any feasible portfolio with  $b > 0$  that implies a nonnegative solution to  $V = F(V)$  can be weakly dominated by one with  $b = 0$ .

Proof: We denote by "0" the given portfolio with  $b > 0$  and by "\*" the weakly dominating portfolio. We consider two cases.

- (a)  $b^0 \leq (1-\alpha)D^0$ . In this case, let the "\*" portfolio be identical except for  $b^* = 0, D^* = D^0 - b^0$ . Since  $S \leq 1$ , feasibility of the "0" portfolio implies feasibility of the "\*" portfolio.

As for dominance, since for any  $V(t+1)$  the integral in (18) is the same for both portfolios, we have by inequality (17)

$F^*(V) \geq F^0(V)$  for all  $V$ . The inequality is strict if the argument of  $G$  is greater than  $\bar{y}$ .

- (b) If  $b^0 > (1-\alpha)D^0$ , it follows that the maximum term in the integral in (18) is positive for all  $s$ . Then inequalities (15) and (16) imply that  $F^0(0) \leq 0$ .

Fact 1 allows us to limit consideration to portfolios with  $b = 0$ . Note that insured intermediaries would hold government bonds if they were subject to an additional constraint, a secondary reserve requirement that could be met by holdings of either currency or government bonds. With  $S < 1$ , insured intermediaries prefer government bonds to currency.

Fact 2. Any feasible portfolio with  $R > \alpha D$  that implies a nonnegative solution to  $V = F(V)$  can be weakly dominated by one with  $R = \alpha D$ .

Proof: If  $R^0 \leq D^0$ , define the "\*" portfolio by  $R^* = \alpha D^*$  and  $D^0 - D^* = R^0 - R^*$  and  $z^* = z^0$ . The rest of the argument is identical to that used to establish Fact 1.

Fact 2 allows us to limit consideration to portfolios with (i) at equality, that is, to portfolios without excess reserves.

Fact 3. Any feasible portfolio with (ii) at strict inequality that implies a nonnegative solution to  $V = F(V)$  can be weakly dominated by one with (ii) at equality.

Proof: Define the "\*" portfolio by  $D^* = D^0$  but with  $z^* < z^0$  so that (ii) holds at equality for the "\*" portfolio. At any given  $V(t+1)$ , the integral term for the "\*" portfolio is less than that for the "0" portfolio by no more than  $(z^0 - z^*)(px)$ . But, then, inequality (15) implies that for any  $V$ ,  $F^*(V) \geq F^0(V)$ .

Thus, insured intermediaries have the lowest capital/deposit ratio permitted.

Together these facts imply that an insured intermediary holds what is, in a sense, the riskiest feasible portfolio of assets.<sup>1/</sup>

Using Facts 1 through 3, we may write the stationary version of (18) as

$$(19) \quad V = \int p(s) \max[(\beta - \alpha)PDx(s) + \alpha PD + (1 - \gamma)V - PD, 0] ds - G(\beta PD) - PD(\beta - 1/r).$$

Therefore,

$$F(0) = PD \int p(s) \max[(\beta - \alpha)x(s) + \gamma - 1, 0] ds - G(\beta PD) - PD(\beta - 1/r).$$

By our specification of  $G$ ,  $F(0) \leq (PD)\Psi$  where

$$(20) \quad \Psi = \int p(s) \max[(\beta - \alpha)x(s) + \alpha - 1, 0] ds - g_0\beta - \beta + 1/r.$$

In the appendix, we prove

Proposition 2.

- (a) If  $\Psi > 0$ , there exist one or more values of  $D$  that maximize the solution to  $V = F(V)$ . Any maximizing  $D$  implies  $V > 0$  and is such that  $0 < \beta PD \leq \beta \bar{D}$ , where  $\bar{D}$  maximizes  $F(0)$ .
- (b) If  $\Psi = 0$ ,  $V \leq 0$  for all  $D$ . Any  $D$  such that  $0 \leq \beta PD \leq \bar{y}$  implies  $V = 0$ , and, hence, is a maximizing  $D$ .
- (c) If  $\Psi < 0$ ,  $D = 0$  is the unique  $D$  that maximizes  $V$ .  $V = 0$  and insured intermediaries do not operate.

Our last task in this section is to derive an expression for the value of net taxes implied by the operation of insured intermediaries.

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<sup>1/</sup>A similar result is found in [3].

Letting  $L(s)$  denote the insurer's payout in state  $s$ , we have

$$-L(s) = \min[zx(s)+P(R+b)+(1-\gamma)V-PD, 0].$$

Upon subtracting  $\int p(s)L(s)ds$  from both sides of the stationary version of (18), we have

$$V - \int p(s)L(s)ds = z(px-1) + PR(1/r-1) + Pb(1/r-S) + (1-\gamma)V/r - G[z+P(R+Sb)].$$

Using Facts 1 through 3, we may write this as

$$V - \int p(s)L(s)ds = (\beta-\alpha)PD(px-1) + \alpha PD(1/r-1) + (1-\gamma)V/r - G(\beta PD)$$

or as

$$\int p(s)L(s)ds - \gamma V/r = (\beta-\alpha)PD(1-px) + (\alpha PD+V)(1-1/r) + G(\beta PD).$$

This is the value in terms of period  $t$  consumption of the net taxes levied on the old at  $t+1$  attributable to the operation of each insured intermediary. Using the definition of  $G$ , we can rewrite this as

$$(21) \quad \int p(s)L(s)ds - \gamma V/r = (\beta-\alpha)PD(1+g_0-px) + \alpha PD(1+g_0-1/r) + V(1-1/r) + g_1[\max(0, \beta PD - \bar{y})]^2$$

where each term on the RHS has a straightforward interpretation.

$PD(\beta-\alpha)$  is the amount of the consumption good stored by the insured intermediary. It contributes to taxes by an amount proportional to the discrepancy between  $1 + g_0$  and  $px$ . Thus, by inequality (15), this contribution is nonnegative and is zero if uninsured intermediaries are also storing the consumption good.

The second term has an analogous interpretation for currency,  $\alpha PD$  being the real value of currency held by the insured intermediary. By inequality (16), this term is nonnegative. It is zero only if uninsured intermediaries hold currency. By inequality (17), it is necessarily positive if bonds bear interest.

The third term is negative if  $r < 1$ . Note that by (16),  $1 - 1/r \leq -g_0$  with equality if uninsured intermediaries hold currency. If the licences have value, they provide for the economy as a whole (but not for individuals) a costless means of carrying wealth from period to period in the form of a safe asset. In general, the alternative cost per unit of safe second-period consumption is given by inequality (17) at equality. This implies  $1/r - 1 = (1+g_0)S - 1$ . This is a maximum,  $g_0$ , at  $S = 1$ . The presence of the term in  $V$  gives rise to the possibility that the LHS of (21) is negative. When this happens the existence of insured intermediaries implies a net transfer and is socially beneficial. This possibility arises only because the claims to intermediaries are a superior form of money for the economy.

The fourth term is simply the total of resource costs in excess of those that would be borne if the same assets were held by uninsured intermediaries.

Although it is not true that the LHS of (21) is always positive, it is true that if  $L(s) = 0$  for all  $s$ , then  $V \leq 0$ . In other words, in order that licenses have value, it is necessary that the insurer have a positive gross liability. (This follows from (21) and the lower bound imposed on  $\gamma$ .)

### 5. Equilibrium

To begin we write out an expression for the value in units of time  $t$  consumption of the total of net lump-sum taxes levied on members of generation  $t$ :

$$(22) \quad MT = PB[1-S] + N[(\beta-\alpha)PD(1-px)+(\alpha PD+V)(1-1/r)+G(\beta PD)]$$

where the first component represents interest on government bonds and the second arises from the operations of  $N$  insured intermediaries.

Now for equilibrium, we equate  $c$  and  $q(s)$  in (13) and (14) to supplies given as follows:

$$(23) \quad C \equiv Mc = H - R' - \alpha ND$$

$$(24) \quad Mq(s) = x(s)[K+(\beta-\alpha)NPD] + P[R'+\alpha ND+B] + NV$$

where  $R'$  and  $K$  denote total currency and total goods holdings, respectively, of uninsured intermediaries. Note that two restrictions on  $D$  and  $V$  are implied by the solution to the optimizing problem of each insured intermediary. We get  $V$  as a function of  $D$ . And we get a correspondence between  $D$  and the prices,  $p(s)$  and  $P$ .

A stationary equilibrium in which currency has value consists of positive values of  $P$  and  $S$ , nonnegative functions  $p(s)$  and  $q(s)$ , and nonnegative values of  $R'$ ,  $K$ ,  $C$ ,  $D$ , and  $V$  that satisfy the two restrictions just mentioned, (13), (23), (24), and (15) through (17) with the following provisos: if  $c > 0$ , then (14) holds; if  $R' > 0$ , then  $1/r = 1 + g_0$ ; if  $B > 0$ , then  $1/r = (1+g_0)S$ ; and if  $K > 0$ , then  $px = 1 + g_0$ .

The parameters are the per capita endowment,  $w$ ; the size of a generation,  $M$ ; the possible loss on individual currency holdings,  $\delta$ ; the regulatory parameters  $N$ ,  $\alpha$ ,  $\beta$ , and  $\gamma$ ; those that determine the functions  $u$ ,  $G$ ,  $x(s)$ , and  $f(s)$ ; and  $H$  and  $B$ .

6. "Neutrality," Open Market Operations,  
and Welfare Comparisons

It is easily verified that only the ratio of H to B matters in our model. Put formally, if  $(X^*, P^*, D^*)$  is a stationary equilibrium for the vector of parameters  $(H^*, B^*, \mu^*)$ --where X, here, represents the vector of all endogenous variables other than P and D and  $\mu$  represents the vector of all parameters other than H and B--then for any  $\sigma > 0$   $(X^*, \sigma P^*, \sigma D^*)$  is a stationary equilibrium for the vector of parameters  $(\sigma H^*, \sigma B^*, \mu^*)$ . In the light of this property--which we will henceforth call "neutrality"--we will be describing how the stationary equilibrium for a given set of parameters  $\mu$  depends on the ratio  $h = H/(H+B)$ . Moreover, as we now argue, subject to strict qualifications we can interpret these alternative stationary equilibria as achievable in a single economy through open market operations.

For us, an open market operation proceeds as follows. If  $B(t-1)$  is the face value of bonds sold at  $t-1$  and  $B(t) = B(t-1) - \Delta$  is the amount sold at  $t$ , expenditures and receipts for the government at  $t$  are as follows:

<u>Expenditures</u>	<u>Receipts</u>
$B(t-1)$	$S(t)[B(t-1)-\Delta]$ $[1-S(t)][B(t-1)-\Delta]$

The expenditure item is what must be handed out to pay off maturing bonds. The first receipt item is the proceeds from the sale at  $t$  of bonds with face value  $B(t)$ . The second receipt item is the component of taxes levied on the young at  $t$  for interest on government bonds. As we assume, this is interest implied by the new stock of bonds and the new price. Note that expenditures minus receipts equal  $\Delta$ , thus giving us  $H(t) = H(t-1) + \Delta$  and, therefore,  $H(t) + B(t) = H(t-1) + B(t-1)$ .

So our open market operation holds the sum  $H + B$  constant.<sup>2/</sup>

In the examples described below, the different values of  $h$  are achieved in this way.

Given our assumptions about how the young are taxed, the past matters hardly at all in our model. Aside from consumption of the old at time  $t$ , all the endogenous variables at time  $t$  are determined solely by  $H$ ,  $B$ , and  $N$  at  $t$ , so long as it is believed that these values will prevail forever. Thus, we can interpret different stationary equilibria as achievable by way of the open market operation described above if we assume that the change from  $B(t-1)$  and  $B(t)$  is viewed as a once-for-all surprise. To be specific, at  $t-1$  it must have been believed that  $B(t-1)$  was going to be maintained forever, while at  $t$  it must be believed that  $B(t)$  is going to be maintained forever.

This suggests a way to evaluate the welfare effects of once-for-all (surprise?) changes. The new stationary equilibrium values imply a value of expected utility for the current and future young. They also imply asset values different from those that would have prevailed, values that affect the consumption of the current old. Since open market operations (and changes in regulatory parameters) do not affect the return on or amount of the consumption good stored from the last period, the implied change in the consumption of the current old is

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<sup>2/</sup>One ordinarily thinks of an open market operation as one that holds the sum  $H + SB$  constant. But, as is well known, such an open market operation is not a complete description of the actions of a consolidated Treasury-Federal Reserve. Such an operation necessarily implies a change in the net interest obligations of the Treasury--gross interest payments minus payments by the Federal Reserve to the Treasury. Therefore, a change in fiscal policy must accompany such an open market operation. We assume a change in taxes such that the interest payment continues to be financed by taxes.

equal to the change in  $P(H+B) + NV$ . And since we hold  $H + B$  constant, the value of money,  $P$ , and the value of licenses,  $V$ , are the determining factors.

## II. Some Examples

We now report on how the monetary stationary equilibrium depends on the relative quantities of  $H$  and  $B$  in three different economies.

Tastes, endowments, and the technology are the same for all three economies and given by:

$$U(e_1, e_2) = -(e_1^{-6} + e_2^{-6}), \quad w = 1, \quad M = 100;$$

$$\delta = .1, \quad g_0 = 1/12, \quad g_1 = 1, \quad \bar{y} = .15;$$

$$x(s) = \begin{cases} x_1 = .5 & \text{for } 0 \leq s < .5 \\ x_2 = 2 & \text{for } s \geq .5 \end{cases} \quad \text{and } f(s) \equiv 1.$$

Since  $x(s)$  takes on only two values, it follows that each of  $p(s)$  and  $q(s)$  takes on two values. Therefore, we let

$$p_1 = \int_0^{.5} p(s) ds, \quad p_2 = \int_{.5}^1 p(s) ds;$$

$q_1 = q(s)$  for  $0 \leq s < .5$  and  $q_2 = q(s)$  for  $s \geq .5$ . The regulatory parameters are also the same in all the economies. We set  $(\alpha, \beta, \gamma) = (.8, 1.15, .5)$ .

The three economies differ with regard to the number,  $N$ , of insured intermediaries. There is one economy without insured intermediaries; i.e.,  $N = 0$ . There is one with  $N = 30$ . And there is one with  $N$  so large that there is, in effect, free entry into insured intermediation.

These economies were designed to have equilibria with the following properties:

- (a) There is a monetary equilibrium ( $P > 0$ ) with positive storage of the consumption good.
- (b) Individuals hold some currency and some safe intermediary liabilities.
- (c) With  $N > 0$ , insured intermediaries operate at most values of  $H/(H+B) \equiv h$ .

Properties (b) and (c) guided our choices of  $(\alpha, \beta)$ . To keep individual holdings of currency positive, the gross return offered on insured deposits must remain below .95, the average of  $1-\delta$  and 1. That requires what might be called stringent regulation that among other things holds down the size and profitability of insured intermediaries. But if regulation is too stringent, then insured intermediaries do not operate. For our specification of  $x(s)$ ,  $\alpha + \beta < 2$  is necessary for  $\Psi > 0$ . (See (20) and inequalities (15) and (16).) But  $\alpha + \beta$  close to two is necessary in order that insured intermediaries not be able to offer a gross return on deposits  $\geq .95$ .

We chose a utility function that would imply positive holdings of safe intermediary liabilities for all values of  $h$ . When  $h = 1$  and  $N = 0$ , individuals face a safe gross rate of return of  $12/13$ . (See (16).) Substantial risk aversion is required to make individuals hold positive amounts of an asset with this rate of return given that holdings of currency offer gross returns of .9 and unity each with probability  $1/2$ .

#### 1. No Insured Intermediaries

The economy with  $N = 0$  is described in Table 1. In the tables, EU is expected utility of the individuals.  $i$  is  $100[1/S-1]$ , the interest

rate on bonds.  $\bar{P}/P$  is the price of currency in terms of the consumption good at  $N = 0$ ,  $B = 0$  divided by its price at the respective setting.  $\bar{P}/P$  can be interpreted as the "price level."  $T$  is total tax payments, 100 times individual tax payments.  $I$  is total storage of the consumption good, the sum of storage by insured and uninsured intermediaries.  $M_1/\bar{M}_1$  is the sum of individual currency holdings and insured deposits  $(100c+ND)$  divided by individual currency holdings at  $N = 0$ ,  $B = 0$ . All other variables are as defined in the text.

The example is consistent with the results of our earlier paper. If bonds bear interest, then they constitute a distortion. When bonds bear interest ( $S < 1$ ), their presence amounts to a subsidy, financed by taxes, on the provision of safe assets by intermediaries. Too many resources are used in that activity, and expected utility is lower than it is in the absence of bonds.

Note that bonds do not bear interest at  $h = .95$ . This is because bonds serve as well as currency in the "vaults" of uninsured intermediaries. Bonds begin to bear interest--that is, to sell at a discount--only when uninsured intermediaries must charge a service fee on their safe deposits lower than  $g_0$  in order to attract sufficient such deposits to enable them to hold all the bonds supplied.

We find it helpful to relate Table 1 (and, even more so, subsequent results) to Figure 1. There, in the  $(p_1, p_2)$  plane, we have depicted by the shaded area  $(p_1, p_2)$  pairs that satisfy inequalities (15) and (16). (Note that  $g \equiv 1 + g_0$ .) For now, the reader should ignore the line  $\Psi = 0$ . We have also plotted the line  $1/r = 2/(2-\delta)$ . Only  $(p_1, p_2)$  pairs that lie above this line are consistent with positive individual holdings of currency.

We noted above that we have imposed sufficient risk aversion so that individuals want to hold some safe intermediary liabilities at  $r = 1/g = 12/13$ . This and the favorable risky opportunity offered by the distribution of returns on storage of the consumption good give us an equilibrium at point A when  $h = 1$ . As  $h$  declines, the equilibrium point moves along the  $px = g$  line from A toward A'''. It does not reach A''', since with  $S < 1$  (and  $N = 0$ ), equilibrium requires that individuals hold all the currency. This requires a rate on safe deposits that is less than  $(2-\delta)/2$ .

The movement along the  $px = g$  line from A toward A''' constitutes a change in the relative prices of consumption in the two states: the lower is  $h$ , the lower is  $p_1/p_2$ , and the lower, both absolutely and relatively, is the amount of storage of the consumption good. If we interpret the storage as a simple form of investment, open market sales drive the interest rate up and drive investment down.

The only surprising aspect of this scenario is the behavior of the price level and, perhaps, of expected utility. The price level moves hardly at all and in what many would regard as a perverse direction. The expected utility results are not surprising once it is understood that the presence of interest-bearing bonds in this economy amounts to a subsidy on the return on safe assets financed by lump-sum taxation.

It is obvious from Table 1 and our earlier discussion that a once-for-all (surprise?) change from  $h < 1$  to  $h = 1$  in this economy is a welfare-improving move. Since the move increases the value of money, the consumption of the current old increases. The effects on the current and future young are given by the expected utility outcomes in Table 1.

## 2. Insured Intermediaries

We begin our discussion of insured intermediaries with the  $N = 30$  economy, the results for which are described in Table 2. It is helpful to refer again to Figure 1. For our choices of  $\alpha$ ,  $\beta$ , and  $x(s)$ , the equation of the line  $\Psi = 0$  is

$$p_2[(\beta-\alpha)x_2+\alpha-1] - \beta(1+g_0) + 1/r = p_2(2\beta-\alpha) - \beta g + p_1 = 0$$

it being the case that  $(\beta-\alpha)x_1 + \alpha - 1 < 0$ .

By Proposition 1, points above this line imply  $V > 0$ , while points on it imply  $V = 0$ . Points below it imply that insured intermediaries do not operate.

It turns out that for these parameters and  $N = 30$ , insured intermediaries in the aggregate are quite small. Thus, at  $h = 1$ , the equilibrium in terms of Figure 1 is at point A. As  $h$  declines, the equilibrium  $(p_1, p_2)$  pair moves along the  $px = g$  line toward A'''. While this behavior of  $(p_1, p_2)$  is similar to what happens with  $N = 0$ , the two economies differ in other respects.

At  $h = 1$ , the presence of 30 insured intermediaries implies negative taxes and is beneficial in terms of expected utility.<sup>3/</sup>

In terms of Equation (21), since uninsured intermediaries hold currency (note that  $C/H + R/H < 1$ ) and store some of the consumption good, the first and second terms on the RHS are zero. It turns out that the third term which is  $-g_0V$  outweighs the fourth term despite the fact that the latter is positive.

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<sup>3/</sup> Indeed, a once-for-all switch from  $h = 1$  and  $N = 0$  to  $h = 1$ ,  $N = 30$  at the parameter values of this example is a move to a Pareto superior allocation if the licenses are handed out to the current old;  $P(M+B) + NV$  is higher for the  $N = 30$  economy than for the  $N = 0$  economy even though  $P$  is lower for the former than for the latter.

At all other values of  $h$  listed in Table 2, the presence of insured intermediaries is not beneficial; for all  $.25 \leq h \leq .95$ , expected utility is lower with  $N = 30$  than with  $N = 0$ . The reason is that once  $S < 1$ , the holding of currency by insured intermediaries is wasteful in terms of resources. Bonds and currency held by intermediaries are equivalent in terms of payoffs (see the second term on the RHS of (24)), but resource costs are smaller for bonds if  $S < 1$ . In terms of taxes, the second term on the RHS of (21) is positive when  $S < 1$ .

Only this effect is at work between  $h = .30$  and  $h = .25$ . At both these values of  $h$ , the equilibrium  $(p_1, p_2)$  pair is at point A" in Figure 1. At  $h = .30$ , each insured intermediary is operating on the linear portion of its cost curve. Further declines in  $h$  in this neighborhood produce primarily a shift of assets from insured intermediaries to uninsured intermediaries, while leaving  $(p_1, p_2)$  and, hence,  $r$  unaffected. Since  $r$  is unaffected, individuals have no inducement to give up additional amounts of their currency holdings. Note the sharp increase in  $C/H$  between  $h = .30$  and  $h = .25$ . Indeed, since  $(p_1, p_2)$  is constant over this range and insured intermediaries are on the linear portion of their cost curves, the only nonzero term on the RHS of (21) is the second term. This declines as  $h$  declines, which produces both a decline in taxes and an increase in expected utility. Indeed, although the effect is not revealed to four decimal places, the price level falls between  $h = .30$  and  $h = .25$ . That the decline in  $h$  may be beneficial is not surprising. Insured intermediaries are, in general, distorting in this model and they are smaller the lower is  $h$ . For some parameter values the gain from reducing the size of insured intermediaries ought to offset the potential loss from a decrease in  $h$ .

At  $h = .20$ , insured intermediaries do not operate at all. In terms of Figure 1, the equilibrium is to the left of  $A''$  along the line  $px = g$ . Note that at  $h = .25$ , insured intermediaries do not hold enough reserves to support a further decrease in  $h$  by  $.05$ . Thus, individuals must be lured away from currency into safe intermediary deposits. This requires an increase in  $r$ , which, in turn, makes  $\Psi < 0$ . The outcomes for  $h \leq .20$  are, therefore, identical to the corresponding Table 1 results.

Although we have discovered here a range for  $h$  in which a once-for-all move from a higher to a lower value of  $h$  is a welfare-improving move, it is still true as in Table 1 that  $h = 1$  is at least as good as any  $h < 1$ . And, as in Table 1, we still find the price level increasing as  $h$  declines over most values of  $h$ . Where open market sales increase the interest rate, they increase the subsidy on the return on safe assets, thereby increasing the price level and decreasing expected utility.

We next turn to Table 3 where we display the results for large  $N$  or, what amounts to the same thing, free entry into insured intermediation. In terms of Figure 1, free entry implies that the  $\Psi = 0$  line forms an upper boundary on the equilibrium  $(p_1, p_2)$  pair. Qualitatively, the equilibrium  $(p_1, p_2)$  for  $h = 1$  is on the  $\Psi = 0$  line a little to the left of  $A'$ . (We know the equilibrium is close to  $A'$  because  $S$  is close to one.) As  $h$  declines, the equilibrium  $(p_1, p_2)$  pair moves along the  $\Psi = 0$  line toward  $A''$ . At  $h = .35$  it is at  $A''$ , and then at lower values of  $h$ , outcomes are identical to those in Table 2.

At  $h = 1$ , we have what might be anticipated to be the main sort of distortion to be caused by our insurance scheme. Substantially

different amounts of risky assets are held than in the Table 1 and Table 2 economies. Here, the holding of risky assets for the economy is about 18 percent higher than in the economy without insured intermediaries. Accompanying this are higher taxes and lower expected utility.

As  $h$  declines, the aggregate size of insured intermediaries declines. We find it somewhat surprising that expected utility declines at about the same rate with respect to  $h$  as it does in Table 1. While declines in  $h$  must as in Table 1 force  $r$  upward and, hence, induce too little holding of currency by individuals, one might expect some off-setting gain from the contraction of insured intermediaries and the implied decrease in storage of the consumption good. This seems not to happen.

The reason is, perhaps, explainable in terms of Equation (21). With free entry, the third and fourth terms on the RHS are zero for all  $h$ . At  $h = 1$  the main contribution to taxes is from the first term, that attributable to too much storage of the consumption good. As  $h$  declines, that term declines, both because the amount of such storage declines and because  $1 + g_0 - px$  declines. But the second term need not decline as  $h$  declines. Although the real value of reserves held does decrease as  $h$  decreases, the degree of distortion per unit increases as the interest rate increases. Figure 1 exhibits these effects. As the equilibrium point moves along  $\Psi = 0$  toward  $A''$ , the distance from the  $1/r = g$  line increases, while that from the  $px = g$  line decreases.

As regards welfare-improving moves, two effects stand out. First, within the free-entry economy,  $h = 1$  is again a better position than any  $h < 1$ . (The price level is lower at  $h = 1$  than at any  $h < 1$ .) Second, at any  $h$ , a move away from free entry to  $N = 30$  is a welfare-improving move.

### III. Concluding Remarks

It is widely believed that the Federal Reserve by way of open market operations exerts an important effect on aggregate demand with open market purchases of government securities being expansionary and open market sales contractionary. What is the basis for this belief?

Certainly, there is no broad theoretical presumption in favor of it. In a sense, our model simply makes explicit what has been known for a long time. In a model with both fiat money and nominal government bonds, neutrality holds for proportional increases in both. Only in a very special world would a similar effect be produced by an increase in fiat money matched, instead of by a proportional increase in bonds, by a decrease in such a way as to hold the sum  $H + B$  constant.

Perhaps, then, there is strong empirical support for the belief. After all, (i) does not an open market operation increase the money supply, and (ii) is it not well established empirically that increases in the money supply result--although, perhaps, with a long and variable lag--in increases in the price level? We grant an affirmative answer to the first question for some definitions of the money supply. Indeed, the sum of currency held by individuals and deposits of insured intermediaries, a version of  $M_1$ , is an increasing function of  $h$  in our model. But as regards the second question, we think the evidence has been badly misinterpreted.

Some of the misinterpretation takes the following form: Under commodity standards--for example, a gold standard--exogenous increases in the supply of the standard commodity result in decreases in its value. Since money in our present system consists of  $M_1$ , does it not follow that increases in  $M_1$  (brought about no matter how?) result in decreases in its value? That the premise of this supposed syllogism is

presumed to have some implication for the effects of an open market operation is astounding. After all, the premise is an implication of no more than the simplest pure exchange competitive general equilibrium model. In such a model, it is not strange to find that the equilibrium price of a good is a decreasing function of the aggregate endowment of that good. Can such an experiment in a nonmonetary economy have implications for the effects of an open market operation in nominal government bonds in a model with valued fiat money?

Moreover, the data provide nothing like the sort of experiments carried out in the last section, so that other seemingly more direct evidence is also subject to misinterpretation. For example, in post-World War II U.S. data, there is not much variation in  $h$ . And, of course, it is not an easy matter to draw inferences from the variation that is there. One must be concerned with specifying the government rules that generated the observations on  $h$  and with individuals' views about those government rules.

Our view, then, is that there is neither theoretical nor empirical support for the notion that Federal Reserve open market purchases are expansionary and that sales are contractionary.

This is not to say that we have the right detailed model of open market operations. We can certainly be accused of presenting a model that is too simple. Indeed, when compared to the stories that are claimed to underlie other analyses, our model is simple in several respects--for example, in the way individual currency demand is modeled and in the way intermediation costs are modeled. But although simple, our model is logically consistent.

The alternative analyses posit demand and supply functions. Underlying those functions are, supposedly, solutions to complicated optimizing problems in which various kinds of transactions costs play a prominent role.<sup>4/</sup> But those costs do not appear in the analyses that use the demand and supply functions. Are the costs and changes in them of minor importance? Maybe so, but then why do estimated versions of those functions display so little elasticity with respect to rates of return and why do safe government bonds bear interest? We think any response to such questions must be a modeling effort that in form resembles ours.

As regards substance, we believe two features displayed by our model ought to be taken seriously. First, if nominal government bonds bear interest, it is because their absorption imposes real costs. As a consequence, their presence is beneficial only if they serve to limit the size of some distorting activity. In our model the insured intermediaries are distorting, and the reserve requirement allows open market sales to reduce their size. In this regard we are, perhaps, too harsh on insured intermediaries. For us they serve almost no useful purpose. But if they are made more useful, then bonds will have a more limited role. Only by imposing a sort of "second best" situation do we see any hope of finding a beneficial role for nominal government bonds. Second, our earlier paper suggested that changes in the composition of the sum  $H + B$  are of minor importance in terms of aggregate demand, minor compared to the effects of changes in the sum itself. This paper has shown that this conclusion can survive in a model with an investment good

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<sup>4/</sup>See [2], [4], and [5].

and with a binding reserve requirement on insured deposits for which only the H component qualifies as a reserve. Even though the level of investment and the amount of insured deposits respond in the expected way to the composition of the sum  $H + B$ , it is the sum rather than the composition of the sum which is important for aggregate demand.

Viewed as a counter-example, our model shows there is no general result to the effect that open market purchases are expansionary and open market sales contractionary.

Table 1  
No Insured Intermediaries

h	EU	i	$\bar{P}/P$	T	$p_1$	$p_2$	I	C/H	R/H	$M_1/\bar{M}_1$	V
1.00	20.4	.00	1.0000	.000	.722	.361	3.18	.915	--	1.00	--
.95	20.4	.00	1.0000	.000	.722	.361	3.18	.963	--	1.00	--
.90	20.3	.04	1.0010	.002	.722	.361	3.18	1.000	--	.98	--
.85	19.7	.17	1.0042	.012	.720	.362	3.16	1.000	--	.93	--
.80	19.1	.30	1.0073	.028	.718	.362	3.15	1.000	--	.87	--
.75	18.5	.44	1.0103	.051	.716	.363	3.14	1.000	--	.82	--
.70	17.8	.59	1.0131	.081	.714	.363	3.12	1.000	--	.77	--
.65	17.1	.73	1.0159	.117	.712	.364	3.10	1.000	--	.71	--
.60	16.3	.88	1.0186	.161	.710	.364	3.08	1.000	--	.66	--
.55	15.4	1.04	1.0211	.212	.707	.365	3.07	1.000	--	.60	--
.50	14.5	1.20	1.0235	.270	.705	.365	3.05	1.000	--	.55	--
.45	13.5	1.36	1.0258	.336	.703	.366	3.03	1.000	--	.49	--
.40	12.5	1.52	1.0280	.410	.701	.367	3.00	1.000	--	.44	--
.35	11.4	1.69	1.0301	.491	.698	.367	2.98	1.000	--	.38	--
.30	10.2	1.86	1.0320	.580	.696	.368	2.96	1.000	--	.33	--
.25	9.0	2.03	1.0338	.676	.693	.368	2.93	1.000	--	.27	--
.20	7.6	2.21	1.0355	.780	.691	.369	2.91	1.000	--	.22	--
.15	6.3	2.38	1.0370	.892	.689	.370	2.88	1.000	--	.16	--
.10	4.8	2.56	1.0385	1.012	.686	.370	2.85	1.000	--	.11	--
.05	3.2	2.74	1.0398	1.139	.684	.371	2.83	1.000	--	.05	--

Table 2  
Thirty Insured Intermediaries

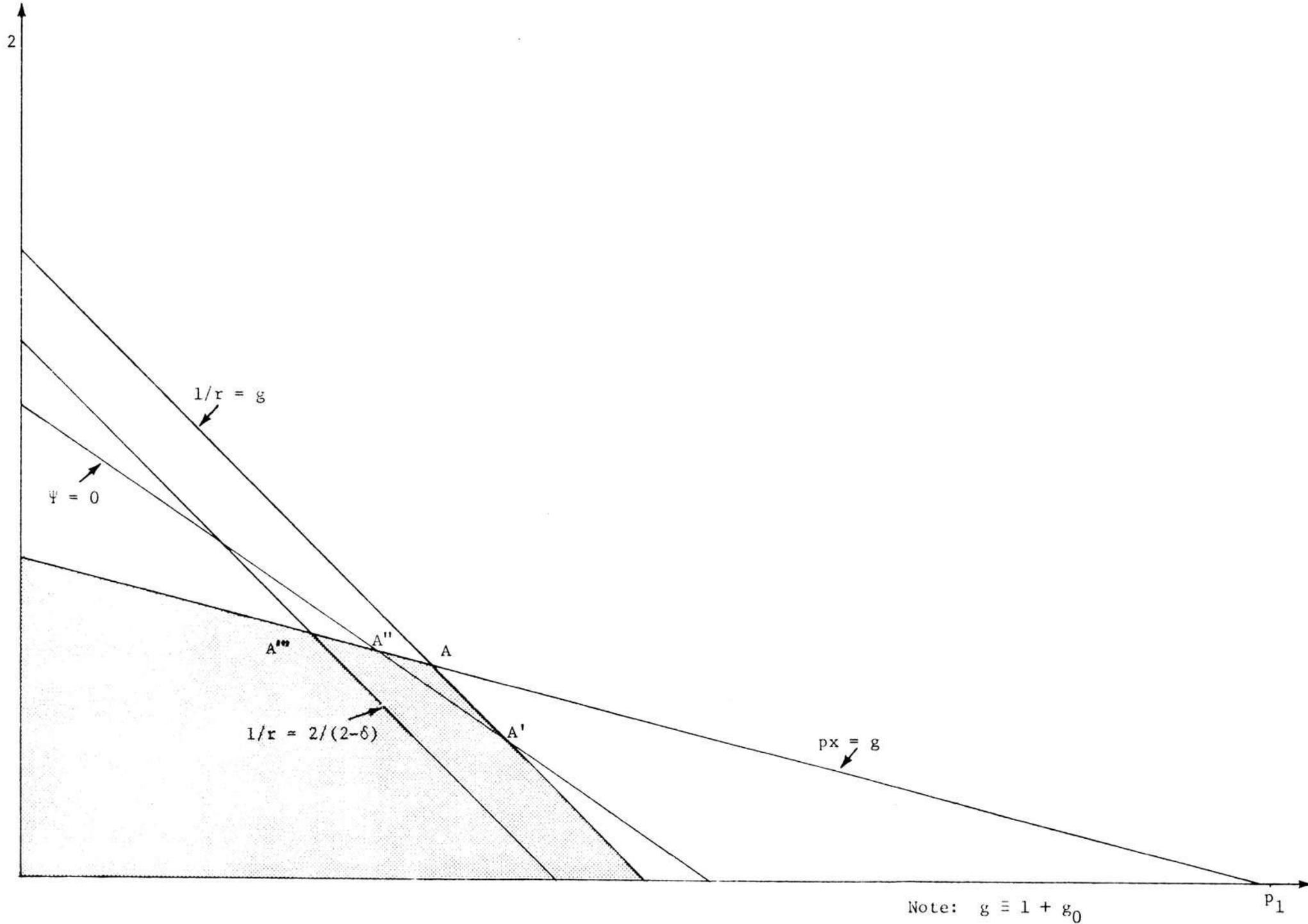
h	EU	i	$\bar{P}/P$	T	$P_1$	$P_2$	I	C/H	R/H	$M_1/\bar{M}_1$	V
1.00	20.5	.00	1.0018	-.006	.722	.361	3.18	.917	.071	1.10	.0029
.95	20.1	.10	1.0042	.000	.721	.361	3.17	.926	.074	1.06	.0028
.90	19.5	.23	1.0072	.014	.719	.362	3.16	.922	.078	1.00	.0026
.85	18.9	.36	1.0101	.033	.717	.362	3.14	.917	.083	.95	.0024
.80	18.2	.50	1.0129	.060	.715	.363	3.13	.912	.088	.89	.0022
.75	17.5	.65	1.0156	.093	.713	.363	3.11	.906	.094	.84	.0020
.70	16.7	.80	1.0182	.133	.711	.364	3.10	.900	.100	.78	.0018
.65	15.9	.95	1.0207	.181	.709	.365	3.08	.892	.108	.73	.0015
.60	15.0	1.10	1.0230	.235	.706	.365	3.06	.883	.117	.67	.0013
.55	14.1	1.26	1.0252	.297	.704	.366	3.04	.873	.127	.62	.0011
.50	13.1	1.43	1.0273	.367	.702	.366	3.02	.860	.140	.57	.0009
.45	12.0	1.59	1.0293	.444	.700	.367	2.99	.845	.155	.51	.0006
.40	10.9	1.76	1.0311	.528	.697	.367	2.97	.826	.174	.46	.0004
.35	9.7	1.93	1.0329	.621	.695	.368	2.95	.802	.198	.40	.0002
.30	8.9	2.04	1.0339	.684	.693	.368	2.93	.827	.173	.34	.0000
.25	8.9	2.04	1.0339	.680	.693	.368	2.93	.992	.008	.27	.0000
.20	7.6	2.21	1.0355	.780	.691	.369	2.91	1.000	--	.22	--
.15	6.3	2.38	1.0370	.892	.689	.370	2.88	1.000	--	.16	--
.10	4.8	2.56	1.0385	1.012	.686	.370	2.85	1.000	--	.11	--
.05	3.2	2.74	1.0398	1.139	.684	.371	2.83	1.000	--	.05	--

Table 3  
Free Entry Into Insured Intermediation

h	EU	i	$\bar{P}/P$	T	$P_1$	$P_2$	I	C/H	R/H	$M_1/\bar{M}_1$	V
1.00	19.0	.29	1.0204	.202	.749	.331	3.77	.812	.188	1.14	.0
.95	18.5	.42	1.0223	.207	.745	.334	3.71	.805	.195	1.09	.0
.90	17.8	.55	1.0240	.218	.740	.337	3.64	.797	.203	1.03	.0
.85	17.2	.69	1.0255	.235	.736	.340	3.58	.789	.211	.98	.0
.80	16.5	.83	1.0270	.258	.732	.343	3.52	.779	.221	.92	.0
.75	15.7	.97	1.0283	.288	.727	.346	3.45	.769	.231	.87	.0
.70	14.9	1.11	1.0295	.324	.723	.349	3.38	.757	.243	.81	.0
.65	14.0	1.26	1.0305	.367	.718	.352	3.31	.744	.256	.76	.0
.60	13.1	1.41	1.0315	.417	.713	.355	3.24	.728	.272	.70	.0
.55	12.2	1.56	1.0323	.473	.708	.358	3.16	.710	.290	.64	.0
.50	11.1	1.71	1.0330	.537	.704	.362	3.09	.688	.312	.59	.0
.45	10.0	1.87	1.0335	.607	.699	.365	3.01	.662	.338	.53	.0
.40	8.9	2.03	1.0340	.685	.694	.368	2.94	.629	.371	.48	.0
.35	8.8	2.04	1.0340	.688	.693	.368	2.93	.708	.292	.41	.0
.30	8.9	2.04	1.0339	.684	.693	.368	2.93	.827	.173	.34	.0
.25	8.9	2.04	1.0339	.680	.693	.368	2.93	.992	.008	.27	.0
.20	7.6	2.21	1.0355	.780	.691	.369	2.91	1.000	--	.22	--
.15	6.3	2.38	1.0370	.892	.689	.370	2.88	1.000	--	.16	--
.10	4.8	2.56	1.0385	1.012	.686	.370	2.85	1.000	--	.11	--
.05	3.2	2.74	1.0398	1.139	.684	.371	2.83	1.000	--	.05	--

Figure 1

Attainable and Equilibrium Contingent Claims Prices



## Appendix

Proof of Proposition 1. Under the hypotheses of the proposition, we may write the RHS of (18) as

$$F(V) = \int \phi(s, V) ds + \phi_0$$

where  $\phi_0$  is a constant and  $\phi$  is the function of  $s$  which is integrated in (18). For a fixed  $s$ ,  $\phi(s, V)$  consists of two connected line segments:

$$\phi(s, V) = \begin{cases} 0 & \text{for } V \leq \bar{V} \\ p(s)(1-\alpha)V & \text{for } V > \bar{V} \end{cases}$$

where  $\bar{V}$ , which may be positive or negative, satisfies  $zx(s) + P(R+b-D) + (1-\gamma)\bar{V} = 0$ .

It follows that  $F(V)$  is continuous and monotone increasing for all  $V$ . Moreover, for fixed  $s$  and any  $V_2 > V_1$ ,

$$\phi(s, V_2) - \phi(s, V_1) \leq p(s)(1-\gamma)(V_2 - V_1)$$

so that

$$F(V_2) - F(V_1) = \int [\phi(s, V_2) - \phi(s, V_1)] ds \leq (1-\gamma)(1+g_0)(V_2 - V_1)$$

where the last inequality follows from inequality (16). Since  $(1-\gamma)(1+g_0) < 1$  by assumption, Proposition 1 is an immediate consequence.

Proof of Proposition 2. Parts (b) and (c) are immediate from Proposition 1 and the inequality  $F(0) \leq (PD)\Psi$ , which holds with strict inequality if  $\beta PD > \bar{y}$  and with strict equality if  $\beta PD \leq \bar{y}$ .

To begin the proof of part (a), it is helpful to define  $n(s) = (\beta-\alpha)x(s) - (1-\alpha)$  and to make the dependence of  $F(V)$  on  $D$  explicit by defining  $\bar{\phi}(V, D)$  to be the RHS of (19).

It follows that

$$\bar{\phi}(0,D) = \begin{cases} (PD)\Psi & \text{for } 0 \leq \beta PD \leq \bar{y} \\ (PD)\Psi - g_1(\beta PD - \bar{y})^2 & \text{for } \beta PD \geq \bar{y}. \end{cases}$$

It is immediate, then, that for  $\Psi > 0$ ,  $\bar{D}$  is unique and satisfies  $\beta \bar{P}\bar{D} > \bar{y}$  and  $\bar{\phi}(0,\bar{D}) > 0$ .

Our first task is to prove that for any  $D' > \bar{D}$ ,  $\bar{\phi}(V,D') - \bar{\phi}(V,\bar{D}) < 0$  for each  $V \geq 0$ . (This implies that any  $D$  that maximizes the solution to  $V = F(V)$  is no greater than  $\bar{D}$ .)

For any  $V > 0$ , let  $S'$  be the set of states for which  $PD'n(s) + (1-\gamma)V \geq 0$  and let  $\bar{S}$  be the set for which  $\bar{P}\bar{D} + (1-\gamma)V \geq 0$ . From  $D' > \bar{D}$ , we have  $S' \subset \bar{S}$ . Therefore,

$$\begin{aligned} \bar{\phi}(V,D') - \bar{\phi}(V,\bar{D}) &\leq (PD' - \bar{P}\bar{D}) \int_{S', p(s)} n(s) ds - \\ &[G(\beta PD') - G(\beta \bar{P}\bar{D})] - (PD' - \bar{P}\bar{D})(\beta - 1/r) \leq \\ (PD' - \bar{P}\bar{D}) \int_{p(s)} \max[n(s), 0] ds - [G(\beta PD') - G(\beta \bar{P}\bar{D})] - \\ (PD' - \bar{P}\bar{D})(\beta - 1/r) &= \bar{\phi}(0,D') - \bar{\phi}(0,\bar{D}) < 0. \end{aligned}$$

This shows that any  $D \geq 0$  that implies a maximal solution  $V$  to  $V = \bar{\phi}(V,D)$  is in the interval  $(0, \bar{D}]$ . Therefore, the sought-after solution is the solution to the following problem: choose  $D \in [0, \bar{D}]$  to maximize the solution to  $V = \bar{\phi}(V,D)$ . This problem has a solution, because since  $\bar{\phi}(V,D)$  is a continuous function of  $D$ , the solution  $V$  to  $V = \bar{\phi}(V,D)$  is a continuous function of  $D$ .

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