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A MODEL OF NOMINAL CONTRACTS

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One of the most striking regularities of economic exchange, both over time and across different locations, is that a large number of trades occur employing contracts which pre-specify the nominal payment to be made for certain goods or services. In particular, such contracts specify certain dollar (or analogous accounting unit) payments, which are not contingent on intervening events. In light of the prevalence of such contracting practices, it is surprising that there has been so little effort to explain them. In fact, while the prevalence of nominal contracts has been viewed by many as a crucial feature of macroeconomic modelling, essential to the understanding of cyclical behavior,<sup>1/</sup> there has been surprisingly little effort devoted to understanding why exchanges should be accomplished in this manner.<sup>2/</sup> This is even more surprising in that it has been long understood that the appropriate role for monetary policy hinges on the manner in which nominal contracts are written and executed.<sup>3/</sup>

Perhaps the reason that so little effort has been devoted to understanding these contracting practices is that there has been wide acceptance of the following oral tradition, which is fairly explicit in Fama (1983) or White (1984). In particular, for purposes of economizing on calculations, it is convenient to have all prices expressed in terms of a unit of account which is also a medium of exchange, and to engage in contracts expressed in this same manner (i.e., to write unindexed contracts). Prior to embarking on a more ambitious study of nominal contracts, then, it should be shown that the above does not constitute an adequate explanation for the entire class of nominal contracts observed in practice. In fact, it does not. As an example of why it does not, it suffices to consider a class of nominal contracts that were prevalent in credit arrangements in the British North American colonies of the

18th Century. These contracts frequently took a form known as "barter contracts." In a barter contract, person A purchases a unit of commodity x on credit from person B. He agrees to deliver some quantity of commodity y to B at some future date. Note that money never appears in the transaction. Nevertheless, in the most usual form of barter contract in the colonial period, A would agree to deliver enough of good y so that its nominal value was at some pre-specified level. Moreover, the use of such contracts was widespread in both Pennsylvania, where the price level was quite stable, and in Massachusetts, where it was quite unstable. Notice, then, that nominal contracting cannot always be explained because money appears on at least one side of a transaction, or because it simplifies calculation. In barter contracts neither was the case. In a barter contract "indexation" of the contract is obviously a matter of the utmost simplicity, and yet this did not occur. Hence the explanations considered above will not account for all observed nominal contracts. A more complete explanation is required.

This paper offers an explanation of nominal contracts in which agents may choose to engage in nominal contracting, or to write completely indexed contracts (or equivalently, to operate in spot markets). The use of any form of contract between these extremes is also permitted. Moreover, the only risk in the model to which any agent is exposed is price-level risk. Nevertheless, some subset of agents in the model will, for a broad class of economies, choose to sell their labor for a pre-specified nominal wage. The reason for this is that the choice of contractual arrangement by which any agent is governed functions as a sorting device, and helps to overcome informational frictions. In particular, we consider an economy with production, firms, and two types of workers. One type is relatively more productive, and

there is no technological uncertainty. However, any worker's type is private information, ex ante. In equilibrium, firms must employ devices to induce self-selection of workers. These devices may take either of two forms in our analysis. In one form firms may restrict the hours of certain workers at an equilibrium real wage rate, i.e., underemployment may be used as a sorting device. Alternatively, some workers may opt for fixed nominal wages, and others for "indexed" wages. For some economies the latter is the dominant sorting mechanism. In fact, it will be seen that for some economies it permits a reproduction of the first-best allocation of resources. Moreover, this is true even though money need never appear in any transaction between firms and workers who are governed by nominal contracts.

More generally, both hours restrictions and nominal contracting would be employed as sorting devices by firms. In this case, our economies produce the following outcome. If all contracts were required to be fully indexed (or equivalently, if there were only spot markets), there would be unemployed labor but no Phillips curve. The introduction of nominal contracting produces a Phillips curve, but reduces unemployment in at least some states of nature. Also, in general nominal contracts work in our model only if the money supply varies randomly. Hence, our model generates an important role for rules which govern evolution of the money supply which are not constant growth rate rules. In fact, in the first set of economies considered, constant growth rate rules are suboptimal. However, attempts to vary the money supply need produce no contemporaneous effects on output or employment, and are typically suboptimal if they come as a surprise.<sup>4/</sup> Also, a general feature of our model is that variation in the money supply precedes in time some of the variation it produces in output. Thus, the model reproduces this

empirical regularity in the timing of changes in the money supply and in output.<sup>5/</sup>

Finally, the reason why nominal contracts function as a sorting mechanism for the economies considered is that they operate as a device for randomizing the consumption streams of parties to the contract. Since any number of randomizing devices might be imagined, it is reasonable to ask whether the use of nominal contracts enjoys any natural advantages over other types of randomizing devices. It will be argued that there are at least two such advantages. First, since nominal contracts employ price level variation as a randomizing device, and since price level variation is an aggregate phenomenon, such randomization is not manipulable ex post by parties to the contract. This is shown to be an important feature of such contracts. Second, in Section IV it is shown that classes of economies exist in which any equilibrium randomizing device must produce the same expected values of all endogenous (including nominal) variables as do nominal contracts. Thus, nominal contracts are an obvious candidate for the randomizing device that will be employed in equilibrium.

The scheme of the paper is as follows. Section I outlines a "real version" of the model in which the price level is constant over time. There is no role here for nominal contracts. Section II introduces nominal contracts for a special class of preferences, and shows that nominal contracting may eliminate unemployment and reproduce a first-best allocation of resources. Section III extends the set of preferences considered, and shows how the model can give rise to a Phillips curve which, in its qualitative features, is empirically reasonable. Section IV considers an example in which nominal contracts produce allocations which must be replicated (in expected value) by any other possible randomizing devices. Section V concludes.

## I. The Model

In order to study an economy in which nominal contractual arrangements function as a sorting mechanism, it is necessary to produce a monetary economy with heterogeneity and private information. The model presented here is the simplest one with all of these features.

The economy is an infinite horizon one, with time indexed by  $t = 0, 1, \dots$ . It is peopled with a sequence of two-period lived, overlapping generations. Each generation is identical in size and composition, and consists of the following four groups of agents. First, there is a group of entrepreneurs, or firms, which is endowed with access to a technology for converting labor into a single-consumption good. There are also two groups of workers, whom we shall term type 1 and type 2 workers, with type indexed by  $i = 1, 2$ . Finally, there is a set of agents who are neither firms nor workers. These are described below. Also, while it is not essential how we think of the population, to fix ideas we may think of there as being a fixed and countable number of firms, and a continuum of workers. Let  $\theta$  denote the fraction of workers who are of type 1.

As indicated, there is a single-consumption good. Agents receive some endowment of this when young, and it also may be produced using the labor of type 1 and 2 workers. Production is nonstochastic and constant returns to scale, so one unit of type  $i$  labor produces  $\pi_i$  units of the consumption good, with  $\pi_1 > \pi_2$ . As there is a continuum of workers and a countable number of firms, no individual worker's contribution to output is directly observable.

We will subsequently want to have workers and firms meet in their youth to contract, and then production takes place when old. For now, workers have no endowment of the consumption good. They are endowed with one unit of

labor when old, and no labor when young. Denote the consumption of a type  $i$  worker in his  $j$ th period of life by  $C_{ij}$ , and the labor of a type  $i$  worker (when old) by  $L_i$ . Then type  $i$  agents have preferences denoted by  $U_i(C_{i2}, 1 - L_i)$ , where  $1 - L_i$  is the leisure of an old type  $i$  agent.  $U_i$  is assumed to be strictly increasing in each argument, twice continuously differentiable and concave.

The set of agents who are neither workers nor firms will be called type 3 agents. Type 3 agents have an endowment of one unit of the good when young, and none when old. Their preferences are denoted by  $U_3(C_{31}, C_{32})$ , with  $U_3$  nondecreasing in each argument, concave, and twice continuously differentiable. Let  $\mu$  denote the measure of type 3 agents.

Finally, in addition to the consumption good and labor, there is a third good called fiat money. This is intrinsically valueless, and produced costlessly by the government. In this section it is supplied in the constant (for all time) aggregate amount  $M > 0$ , and money trades for the numeraire consumption good at rate  $S_t$ . We henceforth focus on steady states, and thus do not retain the subscript. Moreover, we focus only on equilibria with valued fiat money ( $S > 0$ ) in what follows. Also, we denote the rate at which type  $i$  labor trades for the consumption good as  $w_i$ .

It remains to describe the behavior and objective functions of firms, and to describe the initial old generation. Firms are risk neutral profit maximizers. They are also Nash imperfect competitors in labor markets. So long as  $S$  (the inverse price level) does not vary over time, there is no loss of generality in restricting them to announce wage-hours packages  $(w_i, L_i)$  to type  $i$  workers who choose to seek employment with the firm in question. Finally, all that need be said about the initial old generation is

that they are endowed at time 0 with the entire money stock, which for the remainder of this section is then held constant.

A. Equilibrium with a Constant (Inverse) Price Level

We henceforth assume that, while all workers know their own type (and hence productivity), this is private information ex ante. Thus, firms must announce wage-hours pairs subject to this informational asymmetry. Firms may adopt either of two strategies, then. In particular, a firm could announce two wage-hours pairs with  $(w_1, L_1) \neq (w_2, L_2)$  hoping to induce type 1 and 2 workers to accept different packages. This, in effect, enables the firm to price discriminate. Or, alternatively, a firm could announce a single wage-hours package and forego the opportunity to price discriminate. In either case, the announced wage-hours package of each firm must be incentive compatible, i.e., satisfy

$$(1) \quad U_1(w_1 L_1, 1-L_1) \geq U_1(w_2 L_2, 1-L_2)$$

$$(2) \quad U_2(w_2 L_2, 1-L_2) \geq U_2(w_1 L_1, 1-L_1).$$

Obviously if a firm announces a single package, (1) and (2) hold trivially. If it announces two distinct packages, (1) and (2) must hold in order for its announcement to be consistent with self-selection.

In equilibrium, given the announcements of other firms, each firm must have no incentive to alter its announced wage-hours packages. Then, in light of the constant returns to scale assumption, it is willing to accept all workers who seek employment with it. Finally, it is convenient (although not necessary to the analysis) to impose a restriction on the announced wage-hours packages of firms. Thus, following Rothschild and Stiglitz (1976) and Wilson

(1977), we require that each announced package at least break even given the workers accepting it. Hence, if  $(w_1, L_1) \neq (w_2, L_2)$  then we require

$$(3a) \quad w_i \leq \pi_i.$$

If a firm announces a single (common) wage-hours package  $(w, L)$ , we require that

$$(3b) \quad w \leq \theta\pi_1 + (1-\theta)\pi_2,$$

i.e., that this at least breaks even if it attracts workers in their population proportions.

We are now ready to define a (stationary) Nash equilibrium for this economy.

Definition. A stationary Nash equilibrium is a set of announced wage-hours packages  $(w_i^*, L_i^*)$ ;  $i = 1, 2$ , satisfying (1)-(3), and a constant sequence  $\{S_t\}$  such that

- (i) given the announced wage-hours packages of other firms, no firm has an incentive to alter its announcements, and
- (ii) the money market clears, i.e.,

$$(4) \quad \mu\Psi(S_{t+1}/S_t) = S_t M \quad \forall t \geq 0,$$

where  $\Psi(S_{t+1}/S_t)$  is the savings function of type 3 agents.

Notice that attention is confined throughout to equilibria in pure strategies, and that firms do not employ lotteries over wage-hours pairs.

The qualitative features of this equilibrium for the constant value  $S = \mu\Psi(1)/M$  are identical to the equilibrium discussed by Rothschild and Stiglitz (1976). First, no equilibrium in pure strategies need exist. However, in the sequel we always choose parameter values such that existence is guaranteed. Second, equilibria need not be Pareto optimal. The third feature is important enough to state as a

Proposition. If

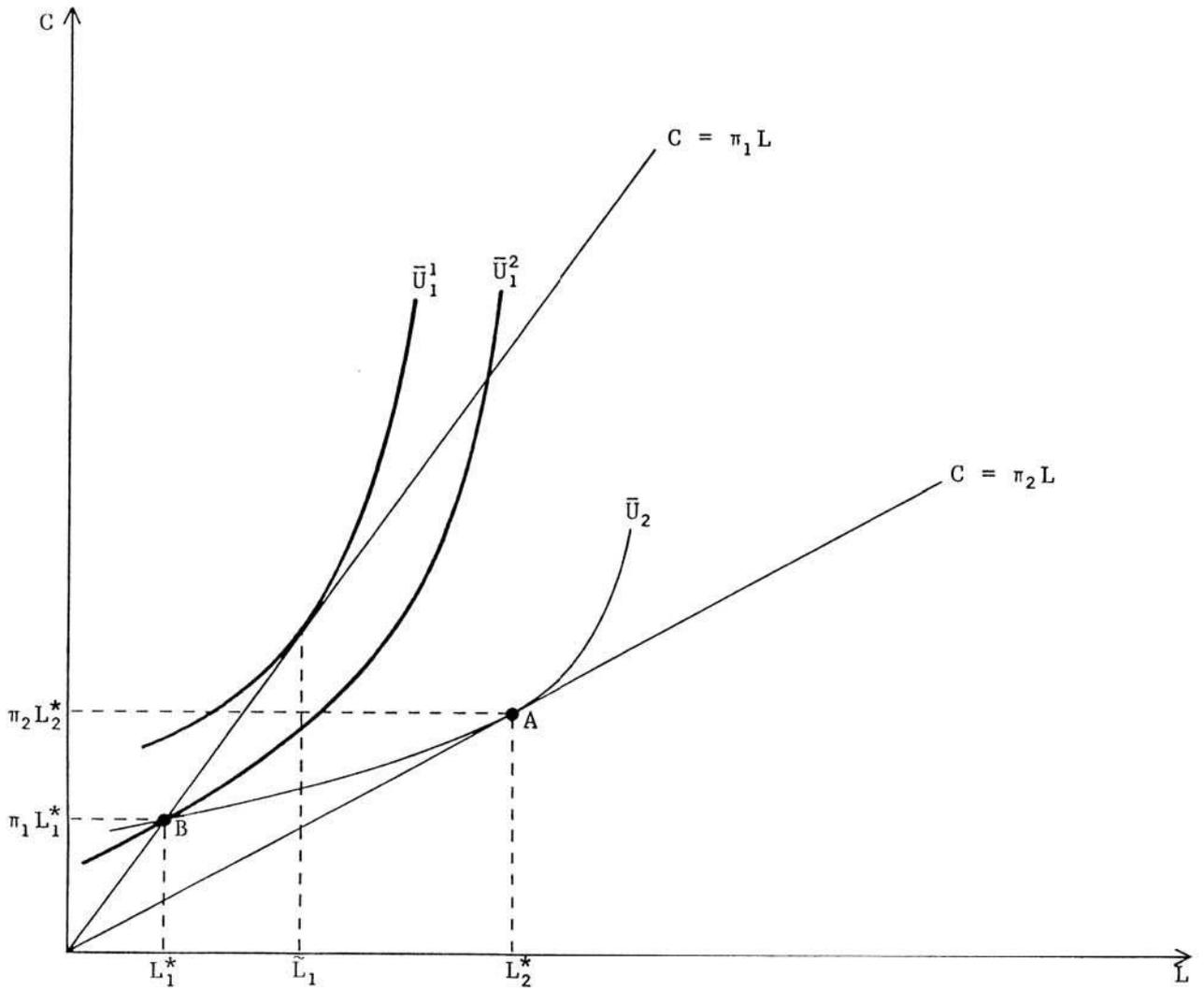
$$\frac{\frac{\partial U_1(C,1-L)}{\partial C}}{\frac{\partial U_1(C,1-L)}{\partial(1-L)}} > \frac{\frac{\partial U_2(C,1-L)}{\partial C}}{\frac{\partial U_2(C,1-L)}{\partial(1-L)}}$$

$\forall (C,1-L) \in [0,\pi_1] \times [0,1]$ , then any equilibrium displays self-selection of workers, i.e.,  $(w_1^*,L_1^*) \neq (w_2^*,L_2^*)$ .

The proof is exactly parallel to the argument given in Rothschild-Stiglitz (1976), and is omitted here. Finally, in light of (3), the proposition, and competition among firms for workers,  $w_i = \pi_i$ ;  $i = 1, 2$ .

It will be useful to display diagrammatically the determination of equilibrium values  $(w_i^*,L_i^*)$ . In Figure 1, then, consumption and hours worked appear on the vertical and horizontal axes, respectively. The loci labelled  $C = \pi_i L$  are the zero-profit loci for wage-hours packages which are taken by type  $i$  and only type  $i$  workers. In light of (3) and the competition among firms for workers, obviously each equilibrium wage-hours package will just break even. Finally, from the proposition we know that equilibrium announcements must induce self-selection. How, then, are the equilibrium hours levels of type  $i$  agents determined?

Figure 1  
An Unemployment Equilibrium



Consider first the hours of type 2 agents. We assert that  $L_2^*$ , the equilibrium hours level of type 2 agents, occurs at the tangency of the relevant zero-profit locus with a type 2 indifference curve, labelled  $\bar{U}_2$  in the figure. Suppose this were not the case. Define  $\hat{L}_2 = \operatorname{argmax} U_2(\pi_2 L_2, 1-L_2)$ . If  $L_2^* \neq \hat{L}_2$ , then there exists a pair of values  $(\hat{w}_2, \hat{L}_2)$  satisfying  $U_2(\hat{w}_2 \hat{L}_2, 1-\hat{L}_2) > U_2(\pi_2 L_2^*, 1-L_2^*)$  and  $\pi_2 > \hat{w}_2$ . Hence, this alternate wage-hours package would attract all type 2 agents, and since  $\pi_2 > \hat{w}_2$ , it would earn a profit. This contradicts the fact that no firm has an incentive to offer a wage-hours pair different from  $(w_2^*, L_2^*)$ . Thus,  $L_2^* = \hat{L}_2$ .

Now consider the determination of  $L_1$ , again using the proposition and the fact that  $w_1 = \pi_1$ . By the proposition, self-selection occurs in equilibrium, so that  $(C_{12}, L_1)$  must be incentive compatible, i.e., must lie on or below  $\bar{U}_2$ , the type 2 indifference curve through the point A in Figure 1. Since  $w_1 = \pi_1$ , this implies that  $(C_{12}, L_1)$  must lie along the ray  $C = \pi_1 L$ , and on or below the indifference curve  $\bar{U}_2$ . As before, competition among firms for workers implies that the equilibrium values  $(C_{12}^*, L_1^*) = (\pi_1 L_1^*, L_1^*)$  must be maximal for type 1 workers in this region. In Figure 1, the maximal  $(w_1^*, L_1^*)$  pair occurs at point B, where  $\bar{U}_2$  intersects  $C = \pi_1 L$ .

In general, it will be true that (1) holds with strict inequality in equilibrium. Otherwise, an equilibrium would fail to exist. (2) may hold with strict inequality as well. If (1) and (2) both hold with strict inequality an equilibrium exists, and coincides with the competitive equilibrium for the full-information analogue of our economy. If (2) holds with equality and  $L_1^* \neq \operatorname{argmax} U_1(\pi_1 L, 1-L)$ , then an equilibrium may or may not exist. While we will not be much concerned with existence issues here, it may be useful to say what existence issues hinge on. In Figure 1, we constructed the most

preferred allocation for type 1 and 2 workers consistent with (3) and self-selection. No other allocation consistent with self-selection and (3) is preferred by any agents. Hence no other set of wage-hours packages with  $(w_1, L_1) \neq (w_2, L_2)$  can attract any agents (and earn a profit). Thus, if the allocation depicted in Figure 1 is not an equilibrium allocation, there must be an allocation which does not induce self-selection and which results in the firm offering it earning a profit. Moreover, a wage-hours package not inducing self-selection has, by (3b),  $w < \bar{\pi} \equiv \theta\pi_1 + (1-\theta)\pi_2$ . Hence, in order for such a wage-hours package to attract type 1 workers (which is necessary since type 2 workers alone cannot be attracted in a profitable manner), it is necessary that there exist a value  $\hat{L}$  such that  $U_1(\bar{\pi}\hat{L}, 1-\hat{L}) > U_1(\pi_1 L_1^*, L_1^*)$ . Notice that there will exist such a value if and only if  $\bar{\pi}$  is sufficiently large. For given values of  $\pi_1$  and  $\pi_2$ , this hinges on  $\theta$ . Hence, by appropriate choice of  $\theta$  and other parameters existence can be guaranteed. This strategy is followed below to ensure existence of equilibria.

Finally, three points should be made about the equilibrium depicted in Figure 1. Let  $\hat{L}_1 = \operatorname{argmax} U_1(\pi_1 L_1, 1-L_1)$ . Then, if the relevant indifference maps are as depicted in Figure 1,  $L_1^* < \hat{L}_1$ , i.e., there is unemployment (underemployment) of type 1 labor. Hence, the hours restrictions which are used to induce self-selection may give rise to permanent unemployment. Second, while it may seem unusual to have high productivity workers unemployed, there is substantial empirical support for models in the class under consideration (Smith [1984, 1985]). And finally, it will be noted that since the value of money is constant over time, all agents are indifferent as to whether contracts are specified in real or nominal terms. We now turn our attention to economies for which this is not the case.

## II. Nominal Contracts

In order to prevent agents from being indifferent regarding whether contracts are specified in real or nominal terms, it is necessary that the money supply be made stochastic. To this end, let there be a finite set  $E$  of possible states of the world, which correspond to different realizations of the money supply. Let  $e \in E$  denote a typical element of  $E$ . Also, for our purposes, it is sufficient to let  $E = \{1,2\}$ . Then there are two possible realizations of the money supply, denoted by  $M(1)$  and  $M(2)$ . Let  $M(1) \equiv M$ , and  $M(2) \equiv qM$ ;  $0 < q < 1$ . Finally, let  $S(e)$  denote the inverse price level in event  $e$ , let  $L_i(e)$  be the quantity of labor a young type  $i$  agent contracts to deliver next period if the current state of the world is  $e$ , let  $e'$  denote next period's state of the world, and let  $C_{i2}(e,e')$  denote type  $i$  old-age consumption if the sequence of states  $e, e'$  is realized. Notice that the quantity of labor supplied,  $L_i(e)$ , is contractually pre-specified when young. This permits considerable simplification over allowing hours to depend on the realization  $(e,e')$  without affecting the basic argument. The modifications required to the analysis if hours depend on  $e$  and  $e'$  are discussed below.

All agents are assumed to have von-Neuman-Morgenstern preferences, and the realization of  $M(e)$  in each period is independent of the realization in other periods, and is identically distributed across periods. The prevailing state of the world at each date is common knowledge, and as is implicit in our notation, we restrict attention to stationary states. In addition, something should be said about the way in which monetary injections (or withdrawals) occur here. It is assumed that all monetary injections are made via proportional transfers to old type 3 agents. Also, let  $p$  denote the probability of  $e' = 1$  occurring next period (so that  $1 - p$  is the probability of  $e' =$

2 occurring next period). Then, let  $\Psi(\cdot)$  denote the savings function of type 3 agents as before, which depends (in principle) on  $S(1)/S(e)$ ,  $S(2)/S(e)$ , and  $p$ , where  $p$  is the probability that  $e = 1$  in this period's event.

Finally, it remains to describe firm behavior. As before, firms are imperfect Nash competitors in labor markets. However, now firms may offer wage-hours packages in which wages are specified in either real or nominal terms (or in any combination of the two). Hence firms may use hours restrictions, the choice between contracts denominated in real or nominal terms, or both to induce self-selection of workers. Formally, then, let a wage-hours package consist of a 4-tuple  $(x_i, w_i, \lambda_i, L_i(e))$ . As previously,  $w_i$  denotes the number of goods to be received per hour of labor, and  $L_i(e)$  denotes hours of labor as a function of the young period state of nature. Let  $x_i$  denote the number of dollars paid per hour of labor, and let  $\lambda_i$  be a scalar constant satisfying  $\lambda_i \in [0,1]$ . The role of  $\lambda_i$  in the contract is that the ex post real wage received by a worker accepting a type  $i$  contract is  $z_i(e') = \lambda_i S(e') x_i + (1-\lambda_i) w_i$ . Thus  $\lambda_i$  represents the degree of indexation.

Finally, in order to retain comparability with the set-up of the previous section, we impose an analogue of assumption (3). In particular, we rule out contracts involving cross-subsidization of type 2 workers by type 1 workers. Hence, if a firm specifies wages in real terms, these must satisfy (3). Moreover, as above, let  $z_i(e)$  denote the ex post real wage offered to type  $i$  agents. Then if type 1 and 2 agents are distinguishable, the announcements  $z_i(e)$  are required to satisfy

$$(5a) \quad pz_i(1) + (1-p)z_i(2) \leq \pi_i.$$

If these agents are not distinguishable, then the common ex post wage rates  $z(e)$  must satisfy

$$(5b) \quad pz(1) + (1-p)z(2) \leq \theta\pi_1 + (1-\theta)\pi_2.$$

Hence, as before, each individual contract offered must at least break even in expected terms.

It is also the case that announced wage-hours packages must be incentive compatible. This requires that the announced contracts satisfy

$$(6) \quad pU_1[\lambda_1 S(1)x_1 L_1(e) + (1-\lambda_1)w_1 L_1(e), 1-L_1(e)] + (1-p)U_1[\lambda_1 S(2)x_1 L_1(e) + (1-\lambda_1)w_1 L_1(e), 1-L_1(e)] \geq pU_1[\lambda_2 S(1)x_2 L_2(e) + (1-\lambda_2)w_2 L_2(e), 1-L_2(e)] + (1-p)U_1[\lambda_2 S(2)x_2 L_2(e) + (1-\lambda_2)w_2 L_2(e), 1-L_2(e)]; e = 1, 2,$$

$$(7) \quad pU_2[\lambda_2 S(1)x_2 L_2(e) + (1-\lambda_2)w_2 L_2(e), 1-L_2(e)] + (1-p)U_2[\lambda_2 S(2)x_2 L_2(e) + (1-\lambda_2)w_2 L_2(e), 1-L_2(e)] \geq pU_2[\lambda_1 S(1)x_1 L_1(e) + (1-\lambda_1)w_1 L_1(e), 1-L_1(e)] + (1-p)U_2[\lambda_1 S(2)x_1 L_1(e) + (1-\lambda_1)w_1 L_1(e), 1-L_1(e)]; e = 1, 2.$$

If workers of both types are governed by real contracts (i.e., if  $\lambda_i = 0$ ;  $i = 1, 2$ ), incentive compatibility continues to require satisfaction of (1) and (2). In particular, notice that if contracts specify payments in real terms no workers face any risk from inflation, and  $L_i(1) = L_i(2)$ . Thus (6) and (7) reduce to (1) and (2).

We are now prepared to define an equilibrium for this economy.

Definition. A (stationary) Nash equilibrium is a set of announced wage-hours packages by firms which is incentive compatible and which satisfies (5), and a set of values  $S(e)$ ;  $e \in E$ , such that

(i) no firm has an incentive to change its contractual specifications given the announcements of other firms, with any announcements subject to (5), (6), and (7).

(ii) the money market clears, i.e.,

$$(8) \quad \mu \Psi \left[ \frac{S(1)}{S(e)}, \frac{S(2)}{S(e)}, p \right] = S(e)M(e); \quad e \in E.$$

As before, any equilibrium involves sorting of workers, and  $\theta$  can be chosen to ensure existence of equilibrium. Also, it should be noted that while we have used the term real contracts if  $\lambda_i = 0$ , these can be viewed as either fully-indexed contracts, or as a choice of making trades on a spot market. However, we retain the term real contracts below.

As already indicated, any equilibrium involves self-selection of workers. This is clear intuitively since if sorting did not occur, a contract which broke even would involve firms earning an expected profit on type 1 workers and making an expected loss on type 2 workers. Hence, an incentive would exist for some firm to attempt to attract only type 1 workers in such a situation, which could always be done if the condition in the statement of Proposition 1 were satisfied. Given the fact that sorting occurs, competition among firms for workers ensures that type 2 workers obtain the maximal contract for them consistent with (5a), and with the process generating the inverse price level. Similarly, type 1 workers receive the maximal contract for them consistent with (5a), consistent with self-selection occurring, and consistent with the process generating the price level. Finally, competition

guarantees that in equilibrium each package earns zero-expected profits in real terms. We do not require that profits of firms be zero in each event  $e$ . Since  $L_i(e)$  is specified contractually when young, then, zero-expected profits in a nominal contract requires that the nominal wage paid to type  $i$  workers,  $x_i$ , satisfy

$$x_i = \frac{\pi_i}{ES(e^i)}.$$

It will now prove useful to consider some specific economies in order to analyze the role played by nominal contracts in this world. We begin with a set of economies where nominal contracting permits attainment of a first-best allocation of resources.

A Class of Economies Where Nominal Contracting Produces a First-Best Allocation of Resources

We begin by specifying a class of parametric utility functions for each type of agent (other than firms). Let

$$U_1(C, 1-L) = \eta C + 1 - L$$

$$U_2(C, 1-L) = \lambda n C + \lambda n [(1-L)+k]$$

$$U_3(C_{31}, C_{32}) = C_{32}.$$

Productivity parameters are  $\pi_1$  and  $\pi_2$  as before, and let the measure of type 3 agents  $\mu = 1$ . Then an economy in this class is a set of parameters  $(k, \pi_1, \pi_2, \eta, \theta)$  with  $\pi_1 > \pi_2 > 0$ ,  $\eta\pi_1 > 1$ ,  $\theta \in (0, 1)$ , and  $k > -1$ .

We begin by analyzing as a benchmark the Nash equilibrium for this economy under full-information and a constant inverse price level ( $M(1) = M(2)$ ), which obviously coincides with the competitive equilibrium for the full-information version of the economy.

Full Information: Constant Price Level

Obviously, real versus nominal contracting is a matter of indifference to all. Hence, let all wages be set in real terms. Then  $\lambda_i = 0$  and  $w_i = \pi_i$ ;  $i = 1, 2$ . Given these real wages,  $L_1$  solves

$$\max_{0 \leq L_1 \leq 1} (\eta\pi_1 - 1)L_1.$$

We assume throughout that  $\eta\pi_1 > 1$ , so that  $L_1 = 1$ . Also,  $L_2$  solves

$$\max_{0 \leq L_2 \leq 1} \ln \pi_2 L_2 + \ln(1+k-L_2).$$

For an interior optimum this involves setting  $L_2 = \frac{1+k}{2}$ . (Clearly, an interior optimum involves  $-1 < k \leq 1$ .) Finally, type 3 agents obviously save their entire endowment when young. Hence,

$$\Psi(-) = 1.$$

Utility levels, then, are as follows.

$$U_1 = (\eta\pi_1 - 1)L_1 + 1 = \eta\pi_1$$

$$U_2 = \ln\left[\pi_2\left(\frac{1+k}{2}\right)^2\right]$$

$$U_3 = 1.$$

As this is a standard steady state competitive equilibrium with valued fiat money, the above is a first-best allocation of resources.

Private Information: Constant Price Level

As a second benchmark, it is useful to consider what happens here when nominal contracting cannot alter resource allocations (as when the price level is constant). Also, obviously, private information does not change

matters unless (2) fails for the full-information version of the economy. Now if type 2 agents were to mimic type 1 agents they would work  $L_1$  hours, and earn the wage rate  $\pi_1$ . Hence, the full-information allocation is not incentive compatible iff

$$(9) \quad \ln \pi_1 + \ln k > \ln \left[ \pi_2 \left( \frac{1+k}{2} \right)^2 \right],$$

or iff

$$(9') \quad k > \left( \frac{\pi_2}{\pi_1} \right) \left( \frac{1+k}{2} \right)^2.$$

(9') is henceforth assumed to hold.

This economy, under (9'), is now in the situation depicted in Figure 1. Hence, the equilibrium hours level of type 2 agents is the same as under full-information, so  $L_2^* = \frac{1+k}{2}$ .  $L_1^*$  is determined by the self-selection condition (2) at equality, along with  $w_1 = \pi_1$ , i.e.,

$$\ln \left[ \pi_2 \left( \frac{1+k}{2} \right)^2 \right] = \ln \pi_1 L_1^* + \ln (1+k-L_1^*).$$

(10) may be solved directly for  $L_1^*$  to obtain

$$L_1^* = \left( \frac{1+k}{2} \right) \left[ 1 - \left( \frac{\pi_1 - \pi_2}{\pi_1} \right)^{1/2} \right].$$

Finally, the price level is as before. Notice that, by (9')

$$U_1 = \eta \pi_1 L_1 + (1-L_1) < \eta \pi_1.$$

### Nominal Contracts

Now return to the setting in which  $M(1) \equiv M$ ,  $M(2) = qM$ ;  $0 < q < 1$ . We show that  $p$  and  $q$  can be chosen (in fact for any  $p \in (0,1)$   $q$  can be chosen) such that even with private information, a first-best allocation of resources is attained.

First consider determination of the inverse price level. The aggregate demand for real balances is always one unit (type 3 agents save their entire endowment of one unit, and they are the only savers). Hence,  $S(e)$  satisfies

$$(11) \quad S(e)M(e) = 1 \quad \forall e.$$

Thus,  $S(1) = \frac{1}{M}$  and  $S(2) = \frac{1}{qM}$ . Then consider the utility of type 3 agents. Since all monetary injections are through proportional transfers, we have the following. Young type 3 agents accumulate real balances  $S(e)M(e)$ . Then, when old their real balances are  $S(e')M(e)$  times one plus the proportional transfer, i.e., times  $M(e')/M(e)$ . Hence their old age real balances are, net of transfer,

$$S(e')M(e') = S(e)M(e) \left[ \frac{M(e')}{M(e)} \right] \left[ \frac{S(e')}{S(e)} \right] = S(e)M(e)$$

by (11). Since  $S(e)M(e) = 1 \quad \forall e$ ,  $U_3 = 1$  as before.

Second, we show that type 2 agents always have their compensation governed by real contracts ( $\lambda_2 = 0$ ). To do so, it suffices to suppose that type 2 agents are governed by nominal contracts in equilibrium ( $\lambda_2 = 1$ ). Then zero-expected profits would imply  $x_2 = \pi_2/ES(e')$ . It is easy to check that  $L_2(e) = \frac{1+k}{2} \quad \forall e$ . Hence, expected consumption for type 2 agents would be

$$E\left(\frac{1+k}{2}\right) \frac{\pi_2}{ES(e')} [S(e')] = \pi_2 \left(\frac{1+k}{2}\right).$$

But since type 2 agents are risk averse, obviously some firm could offer type 2 agents a strictly preferred wage-hours package  $(w_2, (\frac{1+k}{2}))$  with  $w_2 < \pi_2$ , attract all type 2 agents, and earn a profit. Hence, no type 2 agent is governed by a nominal contract in equilibrium, contradicting the

supposition. Thus, type 2 agents are in the same situation as previously, and  $L_2^* = \frac{1+k}{2}$ .

Third, we ask what is necessary for  $L_1(e) = 1 \forall e$  to be consistent with self-selection under nominal contracting. Thus let  $\lambda_1 = 1$ . Zero-expected profits implies that  $x_1 = \pi_1/ES(e')$ . Thus, if  $L_1(1) = L_1(2) = 1$ , type 1 agents receive state 1 consumption

$$C_{12}(1) = \frac{S(1)\pi_1}{ES(e')}$$

and state 2 consumption

$$C_{12}(2) = \frac{S(2)\pi_1}{ES(e')}.$$

Now  $S(1) = \frac{1}{M}$ ,  $S(2) = \frac{1}{qM}$ , and

$$ES(e') = pS(1) + (1-p)S(2).$$

Hence,

$$C_{12}(1) = \frac{\pi_1 q}{pq + (1-p)}$$

$$C_{12}(2) = \frac{\pi_1}{pq + (1-p)}.$$

Now any type 2 agent accepting a type 1 nominal contract would work one unit, and receive the same state contingent consumption levels as above. Hence,  $L_1(e) = 1 \forall e$  is incentive compatible (consistent with self-selection) iff

$$(12) \quad p \ln \left[ \frac{\pi_1 q}{pq + (1-p)} \right] + (1-p) \ln \left[ \frac{\pi_1}{pq + (1-p)} \right] + \ln k \leq \ln \left[ \pi_2 \left( \frac{1+k}{2} \right)^2 \right].$$

Exponentiating both sides of (12) and rearranging terms we obtain the equivalent expression

$$(12') \quad \left(\frac{\pi_2}{\pi_1}\right)\left(\frac{1+k}{2}\right)^2 > \frac{kq^p}{pq + (1-p)}.$$

Now by (9')

$$\lim_{\substack{q \rightarrow 1 \\ q < 1}} \frac{kq^p}{pq + (1-p)} > \left(\frac{\pi_2}{\pi_1}\right)\left(\frac{1+k}{2}\right)^2.$$

Also, clearly

$$\lim_{\substack{q \rightarrow 0 \\ q > 0}} \frac{kq^p}{pq + (1-p)} = 0 < \left(\frac{\pi_2}{\pi_1}\right)\left(\frac{1+k}{2}\right)^2.$$

Hence, by the intermediate value theorem for any  $p \in (0,1)$  there exists a value  $q'(p) \in (0,1)$  such that

$$\left(\frac{\pi_2}{\pi_1}\right)\left(\frac{1+k}{2}\right)^2 = \frac{k(q')^p}{pq' + (1-p)}.$$

Then  $\forall q \in (0, q'(p)]$ , it is incentive compatible to set  $L_1(e) = 1 \forall e$ .

Now consider the utility of type 1 workers if  $L_1(e) = 1 \forall e$ . Their expected utility is given by

$$\eta \left[ p \frac{S(1)}{ES(e')} + (1-p) \frac{S(2)}{ES(e')} \right] \pi_1 L_1(e) + [1-L_1(e)] = \eta \pi_1.$$

Moreover, this is incentive compatible for any  $q \in (0, q'(p)]$ . Hence, a choice of  $q$  in this range (for given  $p$ ) allows all workers to obtain the same utility as they would under full-information. Also, since type 1 workers attain their first-best levels of expected utility, for such a choice of  $q$  an equilibrium always exists.

For type 2 and 3 agents (and firms), then, the existence of nominal contracts and price level variation is not detrimental. For type 1 agents, the use of nominal contracts under private information is strictly welfare improving for appropriate choices of  $p$  and  $q$ . This type of welfare improvement occurs precisely because more productive workers in this economy are less risk averse than other workers. Hence nominal contracting, which produces risk due to the possibility of price level variation, can be used as a sorting mechanism. The empirical plausibility of this type of sorting mechanism is discussed below.

At this juncture several points should be made. First, nothing requires that workers governed by nominal contracts need be paid in money. Our nominal contracts could simply be of the form of the barter contracts discussed in the introduction.

Second, it is interesting to contrast the result on nominal contracting just exposted with other explanations of the role of nominal contracts based on private information. In particular, Azariadis and Cooper (1985) and Cooper (1982) produce models in which private information prevents the occurrence of complete contingent claims markets. Nominal contracts may then arise as a type of risk-sharing device. This seems contrary, however, to the received wisdom that nominal contracts actually expose agents to more risk than indexed contracts. In the model of this section nominal contracts are a risk-producing device which arises endogenously in the model as a sorting mechanism.

Third, one puzzle regarding nominal contracts has been why their use persists under high rates of trend inflation. For the class of economies discussed in this section, a trend could be introduced into the money supply pro-

cess. In particular, suppose  $M_t(1) = (1+\sigma)^t M$ , and  $M_t(2) = (1+\sigma)^t qM$ . Then if  $\sigma > 0$ , type 1 agents would be worse-off under purely nominal contracting than if  $\sigma = 0$  and  $q$  were chosen so that  $q < q'(p)$ . However, there is a critical rate of trend inflation  $\sigma' > 0$  such that if  $\sigma > \sigma'$ , type 1 agents would be strictly better-off under purely nominal than under purely real contracts. Hence this puzzle is partially resolved here. In this case, of course, some indexation of type 1 contracts would typically be desirable.

#### A Remark

Nominal contracting works in the class of economies above because nominal wage contracts cause price level variation to induce variance in the consumption levels of type 1 agents. This variance serves as a sorting device since type 1 workers are relatively less risk averse than type 2 workers. However, this mechanism relies on the government to produce appropriate randomness in the (inverse) price level. It is natural to ask whether firms could not produce this randomness on their own without the aid of price level variation?

Our answer to this question consists of two parts. The first is to note that one can construct economies for which any randomizing devices that can possibly be equilibrium randomizing devices must produce (in expected value) the same outcome as does the use of nominal contracts. This statement applies even to nominal wage rates, for instance. An example of such an economy is presented in Section IV below. However, this is not true for the class of economies considered in this section. Hence, we might ask again whether firms could not induce the requisite randomness in type 1 consumption without the aid of nominal contracts (and price level variation) for this class of economies?

The answer is that if private agents were to attempt to create the randomness in type 1 consumption observed above, this would give rise to a new class of incentive problems. In particular, once the old age state is realized, the ex post real wage is given by  $S(e')\pi_1/ES(e')$ . In the economies above, since  $q < 1$ ,

$$\frac{S(1)\pi_1}{ES(e')} < \pi_1 < \frac{S(2)\pi_1}{ES(e')}.$$

Thus, if firms were to attempt to reproduce the allocation of resources of the previous section, they would have to use a device which produced real wage  $S(1)\pi_1/ES(e')$  with probability  $p$ , etc. However, since  $S(1)\pi_1/ES(e') < \pi_1$ , firms have an incentive to "cheat," and attempt to enhance the probability of this state occurring. However, this would reduce the expected real wage below  $\pi_1$ , and thus reduce the expected utility of type 1 agents below that obtained through nominal contracting. Similarly, if type 1 workers were allowed to operate a randomizing device which reproduced the ex post real wages above, they would have an incentive to cheat by enhancing the probability of receiving the real wage  $S(2)\pi_1/ES(e')$ . This, in turn, would cause expected losses for firms. Hence, utility maximizing agents cannot be trusted to use randomizing devices with unobservable outcomes so long as the real wage is not constant across states of nature.

It is natural, then, to use as a randomizing device a contracting technology which ties the ex post real wage to observable, economy wide events<sup>6/</sup> such as realizations of the price level. Other random, generally observable objects could also be used--sunspots for instance. However, the price level seems a reasonable choice with other features in its favor, such as the oft-cited "convenience" of nominal contracts.

As a final remark, there would be no incentive for firms to cheat in the use of a randomizing device if  $L_1(e)$  were to vary across states while the realized real wage  $w_1(e) = \pi_1 \forall e$ . However, now for the class of economies above clearly  $L_1(e) < 1$  for some  $e$  occurring with positive probability. Hence, again nominal contracts possess a natural advantage.

For the class of economies considered in this section, then, the use of nominal contracts for type 1 agents can produce a first-best allocation of resources. This does not require the use of any hours restrictions. Neither of these outcomes holds across all classes of economies, however. Also, for other classes of economies hours vary in a manner which produces a Phillips curve. We now turn to a set of economies with the features that (a) a Phillips curve with the "proper" slope exists, (b) hours vary over a "business cycle," and (c) the first-best allocation of resources cannot be reproduced under private information.

### III. Nominal Contracting with a Phillips Curve

We now wish to relax the assumption that type 1 agents are risk neutral, and to do so in a way which allows hours to vary. Thus, we consider a class of economies in which type 2 and 3 agents (and firms) have the same preferences as in section II, and in which

$$U_1(C, 1-L) = \eta C - \left(\frac{\alpha}{2}\right)C^2 + (1-L); \alpha > 0.$$

Parameters will always be restricted in such a way that  $\frac{\partial U_1}{\partial C} > 0$ . Also, we now change our assumption on endowments so that type 1 agents are endowed with one unit of the good when young. This is obviously saved through the acquisition of real balances by these agents. We assume that agents' portfolios are not observable by firms.

Also, we continue to consider the case in which hours are contractually pre-specified as a function of the young period state alone ( $L_i(e)$ ). Finally, for simplicity we now exclude the possibility of partial indexation of contracts, i.e.,  $\lambda_i \in \{0,1\}$ ;  $i = 1, 2$ . Thus firms offer either fully indexed or completely unindexed contracts.

Despite the fact that the model of this section differs in a relatively minor fashion from that of Section II, the range of possible equilibrium outcomes is now sufficiently large that a general analysis of this class of economies would be quite difficult. Moreover, this is true even though many of the results of the previous section continue to apply. In particular, it will continue to be the case that generically any equilibrium displays sorting of workers by contract accepted and that in equilibrium type 2 agents will be covered by indexed contracts. Nevertheless, a large array of equilibrium outcomes is possible here. Thus we adopt the following strategy. The remainder of the section considers a numerical example which illustrates the range of possible equilibrium outcomes, and which gives rise to a Phillips curve. This will, hopefully, suggest the ability of models in this class to generate nominal contracts, and as a consequence of these contracts, non-trivial cyclical behavior.

Example. Let  $k = 1$ ,  $\pi_1 = 1$ ,  $\pi_2 = 3/4$ ,  $\eta = 5/2$ ,  $\alpha = 1/8$ ,  $p = 1/4$ , and  $q = 1/10$  (where  $M(2) \equiv qM$  with probability  $1 - p$ ). As a first benchmark we consider what we would observe for this economy under full information and a constant price level ( $q = 1$ ).

#### Full Information, Constant (Inverse) Price Level

As before,  $L_2 = \frac{1+k}{2} = 1$ , and  $U_2 = \ln[\pi_2(\frac{1+k}{2})^2] = \ln(3/4)$ . Also, recall that type 1 agents have one unit of the good when young, which they use

to acquire real balances that (with  $M(1) = M(2)$ ) bear a gross rate of return of unity. Hence

$$U_1 = \eta C_{12} - \left(\frac{\alpha}{2}\right) C_{12}^2 + (1-L) = \eta(\pi_1 L_1 + 1) - \left(\frac{\alpha}{2}\right) (\pi_1 L_1 + 1)^2 + (1-L_1),$$

which is to be maximized by choice of  $L_1$ . This results in setting  $L_1 = 1$  for our parameter values. Hence  $U_1 = \eta(\pi_1 + 1) - \left(\frac{\alpha}{2}\right) (\pi_1 + 1)^2 = 19/4$ .

#### Private Information, Constant (Inverse) Price Level

As a second benchmark, we consider what the equilibrium outcome would be for this economy under private information with a constant money supply, and hence with a constant inverse price level. Clearly, since  $L_1 = 1 = L_2$  under full information, the self-selection conditions (1) and (2) bind on the determination of the equilibrium value  $L_1^*$ . Thus, just as in Section II, we are in the world depicted in Figure 1. Hence, as before,  $L_2^* = \left(\frac{1+k}{2}\right) = 1$  and  $U_2 = \ln\left[\pi_2 \left(\frac{1+k}{2}\right)^2\right] = \ln(3/4)$ .  $L_1^*$  is now determined by (2) at equality along with  $\pi_1 = 1$ . As before, this implies that

$$L_1 = \left(\frac{1+k}{2}\right) \left[ 1 \pm \left(\frac{\pi_1 - \pi_2}{\pi_1}\right)^{1/2} \right]$$

and, of course,  $L_1 \in [0, 1]$ . Hence, for the parameter values of the example,  $L_1 = 1/2$ , and  $U_1 = \eta(\pi_1 L_1 + 1) - \left(\frac{\alpha}{2}\right) (\pi_1 L_1 + 1)^2 + (1-L_1) = 4.109$ .

We now wish to consider what happens when the price level varies according to the parameters given above. Again it will be useful to begin by considering what happens under full information, as this will allow us to check that incentive constraints do in fact bind on the determination of equilibrium contracts.

Full Information, Variable (Inverse) Price Level<sup>7/</sup>

Clearly under full information all agents will receive indexed contracts. Hence, as before,  $L_2(e) = \frac{1+k}{2} = 1 \forall e$ , where it will be recalled that hours are set when young. Then a type 1 agent who is young in state  $e$  faces the real wage rate  $w_1 = \pi_1$ , and competition among firms for workers implies that  $L_1(e)$  is chosen to solve the problem

$$\begin{aligned} \max_{0 \leq L_1(e) \leq 1} \quad & \eta \left[ \pi_1 L_1(e) + \frac{ES(e')}{S(e)} \right] + [1 - L_1(e)] - p \left( \frac{\alpha}{2} \right) \left[ \pi_1 L_1(e) + \frac{S(1)}{S(e)} \right]^2 \\ & - (1-p) \left( \frac{\alpha}{2} \right) \left[ \pi_1 L_1(e) + \frac{S(2)}{S(e)} \right]^2; \quad e = 1, 2. \end{aligned}$$

For our parameter values, this results in the choices  $L_1(1) = L_1(2) = 1$ . Hence the incentive constraint (2) continues to bind in the determination of type 1 contracts. We now consider how these are determined under private information.

Private Information, Variable (Inverse) Price Level

Since  $\lambda_1 \in \{0,1\}$ , type 1 workers will be offered either a fully indexed, or a purely nominal contract in equilibrium. As indicated previously,  $\lambda_2 = 0$ , or type 2 workers are always covered by indexed wages. Then, as before,  $L_2 = \frac{1+k}{2} = 1$ , and  $U_2 = \ln\left(\frac{3}{4}\right)$ .

There are now two possibilities as regards type 1 contracts. The first is that wages are set in real terms. Then, as we have seen, incentive constraints bind, and  $w_1 = \pi_1$ . Thus  $L_1(e)$  is determined by condition (2) at equality along with  $w_1 = \pi_1$ . As before, this results in

$$L_1(e) = \left( \frac{1+k}{2} \right) \left[ 1 \pm \left( \frac{\pi_1 - \pi_2}{\pi_1} \right)^{1/2} \right] = \frac{1}{2}; \quad e = 1, 2.$$

The second possibility is that type 1 workers are covered by nominal contracts. In this case, they receive the nominal wage rate  $x_1 = \frac{\pi_1}{ES(e')}$ . Then the maximum possible expected utility level attained by a young type 1 worker born in state  $e$  would be generated by choosing  $L_1(e)$  to solve the problem

$$(P) \quad \max_{0 \leq L_1(e) \leq 1} \left\{ p\eta \left[ \frac{\pi_1 S(1)}{ES(e')} \right] + (1-p)\eta \left[ \frac{\pi_1 S(2)}{ES(e')} \right] \right\} L_1(e) + [1-L_1(e)] + \eta \frac{ES(e')}{S(e)}$$

$$- p \left( \frac{\alpha}{2} \right) \left[ \frac{\pi_1 S(1)}{ES(e')} L_1(e) + \frac{S(1)}{S(e)} \right]^2 - (1-p) \left( \frac{\alpha}{2} \right) \left[ \frac{\pi_2 S(2)}{ES(e')} L_1(e) + \frac{S(2)}{S(e)} \right]^2 ; e = 1, 2.$$

It will be useful to begin by considering price level determination. As before, the demand for real balances is constant consisting of young type 1 and 3 agents supplying their young period endowment in return for money. Denote this constant demand by  $y$ . Then  $S(e)M(e) = y \forall e$ , so that  $S(1) = y/M$  and  $S(2) = y/qM$ . Then, as before,

$$\frac{S(1)}{ES(e')} = \frac{q}{pq + (1-p)} = 4/31$$

$$\frac{S(2)}{ES(e')} = \frac{1}{pq + (1-p)} = \frac{40}{31}.$$

Finally, as is implicit in our description of type 1 agents' behavior, no old age monetary transfers are made to type 1 agents. All transfers are made to old type 3 agents. Parenthetically, it might be noted that, depending on the relative sizes of the type 1 and type 3 agent populations, feasibility of this transfer scheme may require that type 3 agents have some endowment of the good when old. We may assume their old age endowment is whatever we require, as this does not affect any of the analysis.

There are now two cases to consider.

Case 1.  $e = 1$ .

Suppose that a type 1 agent young when  $e = 1$  takes a nominal contract. Then it is easy to verify that the solution to the problem (P) sets  $L_1(1) = 1$ , and that the associated level of expected utility is 15.88. If, instead, a type 1 agent takes an indexed contract then  $L_1(1) = 1/2$ . Then the expected utility associated with an indexed contract is

$$\eta \left[ \pi_1 L_1(1) + \frac{ES(e')}{S(e)} \right] + [1 - L_1(1)] - p \left( \frac{\alpha}{2} \right) \left[ \pi_1 L_1(1) + 1 \right]^2 \\ - (1-p) \left( \frac{\alpha}{2} \right) \left[ \pi_1 L_1(1) + \frac{S(2)}{S(1)} \right]^2 = 15.923.$$

for the parameters of our example. Hence an indexed contract yields higher expected utility for type 1 workers than does any nominal contract. Moreover, the indexed contract satisfies (2) by construction. Thus if  $e = 1$  type 1 agents take indexed contracts and have  $L_1(1) = 1/2$ . At the equilibrium real wage  $\pi_1 = 1$ , these agents would choose  $L_1(1) = 1$  if unconstrained. Hence if  $e = 1$  type 1 agents experience unemployment.

Case 2.  $e = 2$ .

Suppose that type 1 agents are covered by nominal contracts. Then  $x_1 = \frac{\pi_1}{ES(e')}$ . We begin by considering the value  $L_1(2)$  that solves the problem (P) for this value of  $x_1$ . It is straightforward to verify that the maximizing value of  $L_1(2)$  is  $L_1(2) = 1$ . Moreover, this is an incentive compatible contract if (6) and (7) are satisfied. For this example, since  $L_1(2) = 1$ , and since type 2 agents are governed by real contracts, (7) is equivalent to (12'). It is straightforward to check that for the parameter values of this

example, (12') is satisfied. To check that (6) is satisfied, note that the maximized value of the objective in (P), conditional on  $e = 2$ , is  $E[U_1|e=2] = 4.19$ . A type 1 agent taking a type 2 (indexed) contract would receive the certain real wage  $w_2 = \pi_2 = 3/4$ , and work  $L_2 = 1$  units. This would result in expected utility

$$\eta\left[\pi_2 L_2 + \frac{ES(e')}{S(2)}\right] + (1-L_2) - p\left(\frac{\alpha}{2}\right)\left[\pi_2 L_2 + \frac{S(1)}{S(2)}\right]^2 - (1-p)\left(\frac{\alpha}{2}\right)\left[\pi_2 L_2 + 1\right]^2 = 3.658.$$

Hence both self-selection conditions are satisfied by this nominal contract.

It remains to consider the possibility that type 1 agents are covered by indexed contracts. This involves, as before, setting  $w_1 = \pi_1 = 1$ , and  $L_1(2) = 1/2$ . This contract yields type 1 agents expected utility given by

$$\eta\left[\frac{1}{2} + \frac{ES(e')}{S(2)}\right] + 1/2 - p\left(\frac{\alpha}{2}\right)\left[\frac{1}{2} + \frac{S(1)}{S(2)}\right]^2 - (1-p)\left(\frac{\alpha}{2}\right)\left[\frac{1}{2} + 1\right]^2 = 3.578.$$

Thus type 1 agents prefer nominal contracts, so that competition among firms for workers implies that this is the equilibrium outcome if  $e = 2$ .

### The Phillips Curve

Now we consider the Phillips curve generated by the economy of the example. In particular, if at time  $t$   $e = 1$ , then since  $S(1) < S(2)$ ,  $S(1) < ES(e')$ . Hence, the inverse price level rises on average, or the price level is expected to fall. At  $t + 1$   $L_1(e) = L_1(1)$  (with  $L_2$  constant), so hours (and hence output) are low. Hence, on average, deflation will accompany low output and conversely inflation will accompany high output. Hence, a Phillips curve arises.

Two points, which are completely general, should now be made about our economy. First, since hours at time  $t$  are determined at time  $t - 1$ , movements in the money stock tend to occur before movements in the real sector of the economy. This is a widely noted empirical regularity (Friedman and Schwartz [1963], King and Plosser [1984], Prescott [1983]). Second, since hours at time  $t$  depend only on the time  $t - 1$  money supply, and since the money supply is distributed independently across periods, the time  $t$  money supply and the time  $t$  level of output are uncorrelated. This is consistent with the evidence of King and Plosser (1984) that the money supply and the level of output are not strongly correlated contemporaneously. It also avoids a countercyclical price level, which would be contrary to observation [Prescott (1983)]. Hence, our model produces an empirically reasonable aggregate Phillips curve.

As an additional remark, we might note that the example just presented yields something that an outside observer might regard as "endogenous indexation" of contracts. In particular, in some periods all wage contracts will be indexed, while in others some contracts will be unindexed. Hence an outside observer might be led to conjecture that in states where  $e = 1$ , and all newly negotiated contracts are indexed, this was because the money supply had recently expanded, with the resulting period of inflation alerting workers to the potential "costs" of not indexing wage rates. While this conclusion would, of course, not be warranted, it would accord well with observations generated by the example economy.

The example just considered is indicative of the range of possible equilibrium outcomes for the economies at hand. In particular, in some states all wages may be indexed, while in other states some may be unindexed. Other

economies might generate equilibria where indexation always or never occurs, or, as another possibility, in some states incentive constraints need not bind at all. Finally, as the example illustrated, in  $e = 2$  with entirely real contracts, it is possible that both incentive constraints (i.e., (1) and (2)) bind simultaneously. Hence, in the absence of nominal contracts, no equilibrium would exist. Thus the presence of nominal contracts may also result in the existence of an equilibrium for economies where an equilibrium would otherwise not exist.

Hours Selected After Realization of Second Period State

In this section, we briefly consider how the analysis would be altered if hours could be specified as a function of  $e'$  as well as  $e$ . Such a possibility would greatly complicate the analysis, for the following reason. Consider a firm negotiating a contract with young type 1 agents if the current period state is  $e$ . (Of course, type 1 workers are the only agents who might receive nominal contracts.) Suppose further that for simplicity we impose  $\lambda_1 \in (0,1)$ , i.e., all contracts must be either real or nominal. If nominal contracts are to be used, the firm must set values  $x_1$  and  $L_1(e,e')$ . Then its zero (expected) profit condition is

$$(13) \quad p[\pi_1 - S(1)x_1]L_1(e,1) + (1-p)[\pi_1 - S(2)x_1]L_1(e,2) = 0; e = 1, 2.$$

In addition, given that type 2 workers will continue to be covered by real contracts, the relevant incentive compatibility constraint on the choices of  $x_1$  and  $L_1(e,e')$  is

$$(14) \quad pU_2[S(1)x_1L_1(e,1), 1-L_1(e,1)] + (1-p)U_2[S(2)x_1L_1(e,2), 1-L_1(e,2)] \\ \leq U_2(\pi_2L_2, 1-L_2); e = 1, 2.$$

Then competition among firms for workers implies that any set of candidate equilibrium values  $x_1$ , and  $L_1(e, e')$  must maximize

$$pU_1[S(1)x_1L_1(e,1),1-L_1(e,1)] + (1-p)U_1[S(2)x_1L_1(e,2),1-L_1(e,2)];$$

$$e = 1, 2,$$

subject to (13), (14), and  $0 \leq L_1(e, e') \leq 1$ , where for simplicity we have assumed that type 1 agents have no goods endowment when young.

The maximization problem faced by firms, then, is a fairly difficult one, since (14) is not a convex constraint, and both (13) and (14) are non-linear. Even for very special type 2 preferences, such as the logarithmic preferences assumed above, a closed form solution for  $x_1$ , and the values  $L_1(e, e')$  can generally not be obtained unless  $L_1(e, 1) = L_1(e, 2)$ ;  $e = 1, 2$ . In this case, of course, the analysis of the previous section applies. Hence non-trivial analysis of these economies can be done only by simulation, which we do not undertake here.

#### IV. Nominal Contracts and Other Randomizing Mechanisms

In the preceding sections the use of nominal contracts served as a mechanism for randomizing the consumption of certain sets of agents. Such a randomizing device, as has been seen, can serve to relax certain incentive constraints, and thus enhance the economic efficiency of equilibrium resource allocations. However, in the examples above, nominal contracts are just one of any number of conceivable randomizing devices that could be employed, even if attention is restricted to devices which cannot be manipulated by parties to specific contracts. Moreover, different randomizing devices may have very different implications for certain aspects of economic behavior in which we

are interested. In order to see this, consider the class of economies examined in Section II. Suppose that, in addition to randomness in the money supply, there is some other nonmanipulable randomness in the system. Let the state of the world be described by the vector  $(z,e) \in Z \times E$ , with  $Z$  and  $E$  finite. If the state of the world is  $(z,e)$ , then the money stock is  $M(e)$ , and let  $z$  be (for instance) the level of sunspot activity. Moreover let sunspot activity and the money stock be independent, and let the probability of state  $(z,e)$  be  $p(z)p(e)$ , with  $\sum_z p(z) = 1 = \sum_e p(e)$ . Moreover, suppose that firms now offer contracts in which the ex post real wage received by type 1 agents varies with  $z$ . Denote this wage  $w_1(z)$ . Then (with constant hours) the zero expected profit condition for firms implies that  $\sum_z w_1(z)p(z) = \pi_1$ . Also, if  $w_1(z)$  is variable in an appropriate way, it will be incentive compatible to set  $L_1(z,e) = 1 \forall z, e$ , so that a first best allocation of resources is obtained.

Now consider the expected value of the nominal wage for this economy. The ex post nominal wage is  $w_1(z)/S(e)$ , where it should be clear that the inverse price level can depend only on  $e$ . Then the expected nominal wage rate is

$$\sum_{z,e} \frac{w_1(z)p(z)p(e)}{S(e)} = \sum_z w_1(z)p(z) \sum_e \frac{p(e)}{S(e)} = \pi_1 \sum_e \frac{p(e)}{S(e)} \neq \frac{\pi_1}{ES(e)}.$$

Thus the expected nominal wage is not proportional to the expected inverse price level. In short, different randomizing mechanisms have very different implications for the behavior of nominal wages, for instance.

In this section, then, we wish to show that for some economies the types of randomizing mechanisms that can be employed in equilibrium are much more restricted than they are in the economies considered above. To do so, we

now present an example in which (i) the use of nominal contracts permits the attainment of a first best allocation of resources, and (ii) any other equilibrium randomizing device produces the same expected resource allocations, real wages, and nominal wages as does the use of nominal contracts.

In order to present such an example, it will prove convenient to slightly alter the economic environment being considered. Thus the environment is now as follows. All agents are three period lived. Workers have no endowment of the good at any date, and all workers have one unit of time in their middle period of life to be allocated between labor and leisure. Type 3 agents have some endowment of the good at each date, about which we will say more below. As before, the first period of life for firms and workers is a negotiating period, and all agents are retired when old. Preferences of agents are described by the utility functions.

$$U_1(C_{11}, C_{12}, C_{13}, 1-L) = \eta C_{13} + (1-L)$$

$$U_2(C_{21}, C_{22}, C_{23}, 1-L) = \beta C_{22} + \phi C_{23} + (1-L)$$

$$U_3(C_{31}, C_{32}, C_{33}) = C_{33};$$

where  $1 - L$  refers to middle age leisure, and where we impose the restrictions  $\beta > \phi$ ,  $\eta > \phi$ ,  $\eta\pi_1 > 1$ ,  $\beta\pi_2 > 1$ , and  $\phi\pi_1 = \beta\pi_2$ . As before,  $\pi_1$  and  $\pi_2$  are productivity parameters.

The other alteration to the environment is that we now assume that all agents' portfolio behavior is publicly observable. Thus if type 2 agents wish to take type 1 contracts, their savings behavior must also mimic that of type 1 agents. Given this, firms behave as previously.

Finally, we must describe the behavior of the money supply. As before  $E = \{1,2\}$ ,  $M(1) = M$ ,  $M(2) = qM$ , and  $e = 1$  occurs with probability  $p$ . All monetary injections/withdrawals are accomplished via transfers to old type 3 agents. This, of course, requires that type 3 agents have a sufficiently large old age endowment for the requisite transfers to be feasible. We can henceforth assume this as this parameter plays no other role in the analysis.

Having described the environment, we may now consider the behavior of this economy under various circumstances. We begin with the case of a constant price level.

#### Constant Price Level, Full Information

We first let  $M(1) = M(2) = M$ . Then the inverse price level is constant, type 1 agents accumulate their entire labor income  $\pi_1 L_1$  as real balances which they carry into old age, and since  $\beta > \phi$ , type 2 agents do not acquire real balances. Then, since  $\eta\pi_1 > 1$  and  $\beta\pi_2 > 1$ ,  $L_1 = L_2 = 1$ , and  $U_1 = \eta\pi_1$ ,  $U_2 = \beta\pi_2$ . Finally, we may consider price level determination. Denote the (constant) demand for real balances of type 3 agents by  $y$ , let  $\mu$  be the measure of type 3 agents, and let  $\gamma$  be the measure of type 1 agents. Then

$$SM = \mu y + \gamma \pi_1.$$

#### Constant Price Level, Private Information

Now suppose that type is privately observed. Then the full information allocation is incentive compatible. To see this note that a type 2 worker accepting a type 1 contract works one unit, and receives income  $\pi_1$  which must be saved at a gross rate of return of unity. Then type 2 agents taking type 1 contracts receive expected utility  $\phi\pi_1 = \beta\pi_2$  by our assumptions on parameter values. Since type 2 utility from a type 2 contract is  $\beta\pi_2$ , the full information allocation is incentive compatible.

Variable Price Level, Full Information

Suppose now that, for whatever reason, the money supply varies randomly according to the distribution discussed above. Suppose also that  $\eta\pi_1 ES(e)/S(e) > 1 \forall e$ , where we state what this requires below. Then  $L_1(e) = 1 \forall e$ . In addition, suppose that  $\phi ES(e)/S(e) < \beta \forall e$ . Then  $L_2(e) = 1 \forall e$ , and type 2 agents do not accumulate real balances. Hence  $U_2 = \beta\pi_2$ . Finally, since  $S(e)M(e) = \mu y + \gamma\pi_1 \forall e$ ,  $S(1) = (\frac{1}{M})(\mu y + \gamma\pi_1)$  and  $S(2) = (\frac{1}{qM})(\mu y + \gamma\pi_1)$ . Thus

$$\frac{ES(e)}{S(1)} = \frac{pq + (1-p)}{q}$$

$$\frac{ES(e)}{S(2)} = pq + (1-p).$$

Therefore,  $\eta\pi_1 ES(e)/S(e) > 1 \forall e$  requires  $\eta\pi_1(pq+1-p) > 1$ , and  $\phi ES(e)/S(e) < \beta \forall e$  requires that  $(pq+1-p)(\phi/q) < \beta$ . Given our assumptions, these conditions will be satisfied for any  $q$  sufficiently close to one.

Private Information, Real Contracts

Now that the price level is variable, the full information allocation is no longer incentive compatible. To see this, note that a type 2 agent taking the type 1 contract above works one unit, receives real wage  $\pi_1$  which must be saved, and viewed from the standpoint of period 1, earns the expected gross rate of return  $E[\frac{ES(e')}{S(e)}]$  on savings, where  $e'$  denotes the event in old age. Thus a type 2 agent taking this type 1 contract obtains expected utility

$$\phi\pi_1 E[\frac{ES(e')}{S(e)}] = \phi\pi_1 ES(e)E[\frac{1}{S(e)}] > \phi\pi_1 = \beta\pi_2$$

by Jensen's inequality. Thus, incentive constraints bind on the determination of  $L_1$ .

Now suppose that hours are set in youth (during the negotiating period), which is an assumption of convenience, and that firms are restricted to offer only indexed contracts. It is easy to check that the values  $ES(e)/S(e)$  are exactly as previously. Also, define

$$R(p,q) = E\left[\frac{ES(e')}{S(e)}\right] > 1.$$

Then  $L_1$  is determined from the relevant self-selection condition at equality:

$$\phi\pi_1 R(p,q)L_1 + (1-L_1) = \beta\pi_2.$$

Solving for  $L_1$  yields

$$L_1 = \frac{\beta\pi_2 - 1}{\beta\pi_2 R - 1},$$

as  $\phi\pi_1 = \beta\pi_2$ . Then a type 1 contract yields type 1 agents the expected utility level (with expectations taken in period one)

$$EU_1 = \eta\pi_1 R(p,q)L_1 + (1-L_1) = \frac{\eta\pi_1 R(\beta\pi_2 - 1) + \beta\pi_2(R-1)}{(\beta\pi_2 R - 1)}.$$

It is tedious but straightforward to check that

$$\frac{\partial EU_1}{\partial R} = \frac{(\phi - \eta)\pi_1(\beta\pi_2 - 1)}{(\beta\pi_2 R - 1)^2} < 0$$

since  $\beta\pi_2 > 1$  and  $\phi < \eta$ . Hence the utility of type 1 agents is diminished relative to what they would obtain if the money supply were constant.

#### Private Information, Nominal Contracts

Now suppose that firms offer type 1 agents a prespecified nominal wage  $x_1$  and a prespecified hours level  $L_1$  in the contracting period. Then

zero expected profits for firms requires that  $x_1 = \pi_1/ES(e)$ , and suppose  $L_1 = 1$ . The type 1 expected utility level from such a contract would be

$$E\left\{\eta\left[\frac{ES(e')}{S(e)}\right]S(e)x_1\right\} = \eta\pi_1,$$

since  $S(e)x_1$  is the ex post real wage received by type 1 agents under this contract.

Now consider the expected utility of a type 2 agent taking a type 1 contract. Such an agent works one unit, receives the ex post real wage  $S(e)x_1$  in middle period state  $e$ , and in such a state faces the expected gross rate of return  $ES(e)/S(e)$ . Hence the expected utility for a type 2 agent taking a type 1 contract is

$$E\left\{\phi\left[\frac{ES(e')}{S(e)}\right]S(e)x_1\right\} = \phi ES(e') \frac{\pi_1}{ES(e)} = \phi\pi_1 = \beta\pi_2 \leq EU_2.$$

Recalling that a type 2 contract yields utility  $\beta\pi_2$ ,<sup>8/</sup> it is clear that this contract is incentive compatible, and results in type 1 agents obtaining the same expected utility as they would under a regime with a constant money supply.

### Other Randomizing Devices

As before, suppose that some other "noneconomic" randomness exists in the system. Let  $(z,e)$  denote the current period state, and let  $p(z)p(e)$  be the probability of event  $(z,e)$ . Further, let  $w_1(z,e)$  be the ex post real wage offered to type 1 agents, which may be state dependent, and let  $S(z,e)$  be the inverse price level in event  $(z,e)$ . Finally, for simplicity, we continue to have hours contractually prespecified when young. Then, in equilibrium, contracts offered to type 1 agents must provide at least the expected utility level of the nominal contracts examined above. In addition, they must be

incentive compatible, and satisfy the zero expected profit condition

$\sum_{z,e} w_1(z,e)p(z)p(e) = \pi_1$ . Moreover, such equilibrium contracts will, because they must maximize the expected utility of type 1 workers, set  $L_1 = 1$ . Thus the values  $w_1(z,e)$  must be chosen (in equilibrium) to solve the problem

$$\max E\left\{\left[\frac{ES(z',e')}{S(z,e)}\right]w_1(z,e)\right\}$$

subject to

$$(15) \quad \sum_{z,e} w_1(z,e)p(z)p(e) = \pi_1$$

$$(16) \quad \phi E\left\{\left[\frac{ES(z',e')}{S(z,e)}\right]w_1(z,e)\right\} \leq \beta\pi_2,$$

where the latter is the relevant incentive compatibility constraint if  $\phi ES(z',e')/S(z,e) < \beta \Psi(z,e)$ . Since

$$S(z,e)M(e) = \mu y + \gamma w_1(z,e),$$

$$\begin{aligned} \frac{ES(z',e')}{S(z,e)} &= \left\{ \mu y \left( \frac{pq+(1-p)}{q} \right) + \gamma \sum_z w_1(z,1)p(z)p(1) \right. \\ &\quad \left. + \left( \frac{\gamma}{q} \right) \sum_z w_1(z,2)p(z)p(2) \right\} \left[ \frac{M(e)}{M} \right] [\mu y + \gamma w_1(z,e)]^{-1}. \end{aligned}$$

Thus  $\phi ES(z',e')/S(z,e) < \beta \Psi(z,e)$  requires that

$$(17) \quad \max_{z,e} \frac{ES(z',e')}{S(z,e)} < \beta/\phi.$$

Since  $w_1(z,e) \geq 0 \forall z, e$ , and since  $w_1(z,e)$  is bounded above  $\forall z, e$ , this will be satisfied for  $\gamma$  sufficiently small relative to  $\mu$  and  $q$  sufficiently near unity.

Assuming that (17) is satisfied, then, the incentive constraint (16) may be written as

$$E\left\{\left[\frac{ES(z',e')}{S(z,e)}\right]w_1(z,e)\right\} < \frac{\beta\pi_2}{\phi} = \pi_1.$$

Therefore

$$E\left\{\left[\frac{ES(z',e')}{S(z,e)}\right]w_1(z,e)\right\} = ES(z',e')E\left[\frac{w_1(z,e)}{S(z,e)}\right] < \pi_1,$$

or

$$(18) \quad E\left[\frac{w_1(z,e)}{S(z,e)}\right] < \frac{\pi_1}{ES(z',e')}$$

Noting that  $w_1(z,e)/S(z,e)$  is the nominal wage in state  $(z,e)$ , (18) states that the expected nominal wage rate must not exceed  $\pi_1/ES(z',e')$ . Since we know that it is possible to choose the values  $w_1(z,e)$  so that  $ES(z',e') E\left[\frac{w_1(z,e)}{S(z,e)}\right] = \pi_1$ , (18) holds with equality in equilibrium. Hence any equilibrium randomizing scheme for this economy satisfies (15) and (18), i.e., produces expected real wages equal to  $\pi_1$  and expected nominal wages proportional to the expected inverse price level. Thus, in particular, any equilibrium randomizing scheme reproduces, in expected value, the real and nominal wage rates associated with the outcome under nominal contracting.<sup>9/</sup>

As a final remark, we might note here that the use of nominal contracts can reproduce a first best allocation of resources for this class of economies even though all agents are risk neutral. Thus the efficacy of nominal contracts does not generally require that type 1 workers be relatively less risk averse than type 2 workers, as was the case in Sections II and III above.

## V. Conclusion

While the analysis here is quite different from other analyses of nominal contracting, it retains the flavor of a number of earlier models. In

particular, the assumption of free entry into production implies that nominal wages are set so as to make expected real wages equal to some target level (marginal product) as in Fischer (1977a). Also, the nominal wages and the hours of type 1 workers cannot be chosen arbitrarily given the (real) wage of type 2 workers. This is similar in spirit to the model of Taylor (1980), where comparisons among agents covered by different contracts are used in specifying equilibrium wage rates. However, all nominal wages are set one period in advance here, as in Gertler (1982). Finally, we have largely followed the suggestion of Fischer (1977b) that informational asymmetries provide a role for contractual rigidities.

It will also be noticed that in many respects our analysis, while allowing the choice between real (indexed) and nominal contracts to be endogenous, supports conventional wisdom about nominal contracting. For instance, nominal wages are completely "sticky" here, remaining constant independent of variation in the money supply. Clearly, this feature of the model is also consistent with the empirical observation that nominal wages vary less (about trend) than the money supply does (Gertler [1982]). Finally, nominal contracting is often argued to give rise to a Phillips curve which would not exist in a pure spot-market economy. In the example of Section III, if there are only spot markets (real contracts only), then  $L_1$  is constant over time, so no Phillips curve results. Once the possibility of nominal contracts is admitted, it is suboptimal for the government to set  $q = 1$ , so now  $L_1(e)$  will typically vary and a Phillips curve results. Hence, the possibility of nominal contracting does create a role for a nonconstant money supply (or something other than a  $k$  percent growth rule), which then gives rise to a Phillips curve. Notice, though, that relative to the case of a constant price

level, nominal contracting (with the given values of  $p$  and  $q$ ) raises hours and output for some realizations of the money supply, and does not reduce hours or output for any such realization.

More generally, in contrast to conventional wisdom, for certain classes of economies (such as those of Sections II and IV), nominal contracting in conjunction with an appropriately variable money supply can reproduce a first-best allocation of resources even in the presence of private information. Hence, the model gives rise to a role for such contracts. This is true even though nominal contracts do enhance the risk faced by all parties to the contract (contrast with Azariadis-Cooper [1985] or Cooper [1982]). It is also the case that the model can easily explain unindexed barter contracts, which would seem anomalous according to most explanations of why nominal contracts are used in practice.

Finally, it remains to say something about the empirical plausibility of firms using the choice of nominal versus indexed wages as a sorting mechanism. Here any remarks must remain conjectural, since the empirical implications of the model in this respect are somewhat limited. In particular, it is tempting to suggest, based on the analysis of Sections II and III, that nominal contracts can function as a sorting mechanism because they exploit the fact that type 1 workers are less risk averse than type 2 workers. However, as Section IV indicates, nominal contracts may arise even when all workers are risk neutral. Hence the sole empirical implication of the analysis at the micro level is that type 1 workers, who are high average real wage earners, are covered by nominal contracts while type 2 workers receive indexed contracts. This implication turns out to accord well with experience. In fact, two relevant observations do suggest that workers who (on

average) earn higher real wages also tend to receive less protection against inflation than do workers who earn lower real wages. First, the one industry where cost-of-living adjustments are virtually nonexistent is the construction industry. This is also a relatively high wage industry. Second, Wilton (1980), in an examination of pre-1976 Canadian contracts, shows that among contracts with cost-of-living escalator clauses, high wage workers receive less inflation protection than low wage workers. Hence, this aspect of the model is at least not obviously contradicted by experience.

Footnotes

1/See, e.g., Fischer (1977a,b, 1984), Gertler (1981,1982), or Taylor (1980).

2/An exception is the work of Azariadis and Cooper (1985) and Cooper (1982). We will have more to say about this below.

3/See, e.g., Simons (1948) or Mints (1950).

4/That is, if agents do not anticipate even the possibility that certain levels of the money supply can occur.

5/See Friedman and Schwartz (1963) or King and Plosser (1984).

6/In particular, events which parties to the contract cannot influence.

7/The discussion that follows assumes that there are no markets for insuring against price level fluctuations. The presence of such markets would create some difficult modelling problems. Some examples of these problems are as follows.

(i) Clearly type 1 agents would like to insure when young against old age price level variation. Given the parameters of the example, it would not be feasible for type 2 agents to purchase the optimal insurance policy purchased by type 1 agents. Thus, if insurance purchases were publicly observable, the purchase of price level insurance would perfectly signal type. Of course the model could be altered to make such insurance purchases feasible for type 2 agents, say by giving them some endowment of the good in old age. However, the presence of insurance would then complicate the self-selection conditions to an extent that would be quite difficult to deal with. Hence considerations of tractability preclude this approach.

(ii) Alternatively, if insurance purchases were not observable by firms type 2 agents could use insurance markets to partially "undo" the effects of nominal contracts. Thus employers would want to make such insurance purchases observable, perhaps by offering their own insurance to employees. This would, of course, recreate the modelling problems discussed above.

In light of the fact that the presence of insurance markets creates these additional complications, then, for the present we simply dispense with them in the analysis.

8/This, of course, requires that  $\phi ES(e)/S(e) < \beta \forall e$ . We state sufficient conditions for this below.

9/By this we mean that the expected nominal wage rate is equal to the expected real wage divided by the expected inverse price level.

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