Games and Economics: A General Solution and Particular Applications

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Abstract

Game theory addresses a problem which is central to economics. Yet, according to the folklore of economics, game theory has failed. This paper argues that this is an incorrect interpretation of the game theory literature. When faced with a well-posed problem, game theory provides a solution. Procedures for facing game theory with well-posed problems are suggested, and examples of economic applications provided. The applications are Samuelson's fiat money model, Phelps' capital overaccumulation problem, multiple rational expectations equilibria, and a bargaining problem.

Games and Economics: A General Solution and Particular Applications

The game theoretic conflict is at the heart of economics. Yet, according to the folklore of economics, game theory has failed economics; it has failed to provide solutions to the conflict situation. In the first section of this paper, it is argued that the situation is quite the reverse. It is economists who have failed to face game theory with well-posed problems. Game theory has reacted appropriately in the circumstance, it has not provided solutions. The first section concludes with a discussion of procedures for facing game theory with well-posed economic problems. The second section of the paper, "Minimax-Nash," presents one particular method of solution and discusses its application to some important and well-known examples in economics. The examples treated are Samuelson's [6] overlapping-generations model of fiat money (result--there is no unbacked fiat money), Phelps' [5] capital overaccumulation problem (result--there is no capital overaccumulation), and multiple rational expectations equilibrium generally (result--there is a unique equilibrium). The last section, "Fables of Specialization and Trade by Individuals: Some Solutions to a Bargaining Problem," presents solutions to a key bargaining problem where Minimax-Nash fails.

1. A General Method of Solution for Game Theory and Its Relevance for Economic Theorizing

The determination of the mechanism for ordering strategies in a game theoretic conflict is the keystone of economic science, at least insofar as economics is to remain an outgrowth of that (otherwise relatively minor) school of English philosophy, Utilitarianism. A method for the solution of the general game is presented in this section, and the implications for economic theorizing discussed.

1.1 The Structure of a Game

First, let us turn to the problem of game theory. Let A be the set of events, and let T be the set of individuals. For each $t \varepsilon T$, let X_t be the subset of A for which individual t has his unique complete preordering (preference ordering) $\geq_t$ on points in X_t. In game theory it is typically assumed that events are probability measures on outcomes and that $X_t = X \subset A$ for all t. Now we get to the conflict part of game theory. For each $t \varepsilon T$, there exists a collection of subsets of A, W_t, such that individual t can restrict events to belong to one of the sets in W_t, $S_t \varepsilon W_t$. The strategy of the individual t is choice of a member of W_t. In game theory it is typically assumed that for any collection of sets

$$\{S_t\}_{t \varepsilon T}$$

such that $S_t \varepsilon W_t$ for all t, $\bigcap_{t \varepsilon T} S_t$ is a point in X, and that for all t, $S_t \varepsilon W_t$ implies $S_t \subset X$. The problem in game theory is to determine a preference ordering on the sets $S_t \varepsilon W_t$ which, in an appropriate sense, is consistent with the preference ordering $\geq_t$ on points in X.

There is, of course, an obvious extension of the preference ordering on points to preference orderings on sets, dominance. For $S', S'' \subset X$, $S' \geq_t S''$ if $s' \varepsilon S'$, $s'' \varepsilon S''$ implies $s' \geq_t s''$. In practice this extension is not very useful, as dominance is a very strong condition. Moreover, dominance is the only noncontroversial extension of the preference ordering on points.

The resolution of this game theoretic conflict is the keystone of economics. Once one goes beyond the "Robinson Crusoe economy," one is in a game theoretic conflict situation. Whether one treats this problem explicitly or not, one's model must somehow resolve the conflict.

One obvious approach to the conflict situation is simply to start with preferences over sets. As we do not observe choices on points in X but on sets in X anyway, preferences on sets can be our primitive. One first has to determine desirable properties for such preferences. However, one immediately confronts a disadvantage to this approach. Independence of preference orderings is not a desirable property. In games that are solvable by the traditional game theoretic approach we know that strategies chosen by an individual depend upon other individuals' evaluations of their own strategies. Moreover, the desirable properties of the preference orderings over sets may be exactly those produced by assuming preference orderings over points in X, and an appropriate mechanism of resolving the conflict. In any case, we proceed with the traditional game theoretic approach and start with preferences over points in X.

1.2 Resolution of the Conflict: Consistency

Game theory provides one basic approach to solving the conflict situation, consistency. Any "mechanism" for resolving the conflict situation should not be, roughly speaking, self-contradictory. It should be consistent with the preference orderings. Knowledge that the conflict is to be resolved by the mechanism should not give the individual reason to diverge from the mechanism's implication for the individual's own decision. Rather, the individual's best choice, given the mechanism, is to take part in the mechanism. Any mechanism should obey a fixed-point property.

For noncooperative games, this consistency property takes a simple form, equilibrium: $s^e = \bigcap_{t \epsilon T} S_t^e$ for $S_t^e \epsilon W_t$ for all t is an equilibrium if and only if for all t

$$s^e \geq_t \left(\bigcap_{\substack{v \epsilon T \\ v \neq t}} S_v^e \right) \cap S_t$$

for all $S_t \epsilon A_t$.

Application of consistency to cooperative games is not so obvious. Indeed, there is no noncontroversial definition of consistency for the cooperative game. As a result, the cooperative game has proven intractable. There is always one possibility worth considering when a problem proves intractable. The problem is intractable because it is not a well-posed problem. We now argue that this is true for the cooperative game.

The cooperative game is, ultimately, nonsensical. Remember that the game as a complete model is taken to describe the entire relevant environment. In a game at a point in time all players decide which $S_t \epsilon W_t$ to restrict the world to. At this point in time all previous conversations, agreements, and so on, are irrelevant. Therefore cooperation is impossible. The fact of having to make independent choices at a point in time, the basic structure of the game, itself rules out cooperation in that choice. The cooperative game violates a basic assumption of Utilitarianism, individual choice.

Does this ultimate nonsensicalness of the cooperative game imply that game theory cannot confront the existence of coalitions? Not at all. One can have a coalition if individuals can bind themselves to strategies prior to the decision point of a game. But that decision to bind oneself must come at a particular previous point, a point of time in which the decision to bind can be analyzed as another noncooperative game. Coalitions appear in a sequence of noncooperative games in which decisions in early games determine the W_t's of

later games. The whole sequence of games should be analyzed at the initial point as a noncooperative super game. The fact that this procedure is conceptually justified does not, of course, imply that in practice it is tractable.

However, that there may be many possible sequences of noncooperative games corresponding to a single cooperative game does not invalidate this procedure. Quite the contrary. Rather, it implies that a search for a general solution to a cooperative game is misguided, the cooperative game is not a well-posed problem. The cooperative game provides only a partial description of the relevant environment. Loosely speaking, in removing the key assumption of independent choice by allowing binding contracts, the cooperative game renders the model incomplete. Additional structure must be provided to replace the deleted assumption.

There is another possible structure for the game that should be considered. Decisions are not made at a point in time, but on an open set, before a point in time. However, it is a basic fact of existence that events always occur, there is no empty set. Therefore, in such a game there must exist a null strategy of not making a decision. Then the game can be viewed as an uncountable "sequence" of games in which strategies, or the null strategy, must be announced at each point in time.

As all games reduce to the simple noncooperative game, this is what we consider from now on. We restrict our attention to equilibria.

1.3 Nonuniqueness of Equilibria

Any "mechanism" for resolution of the conflict situation should be consistent. Therefore, if there is a unique equilibrium in a game, it must be the solution generated by any mechanism. Our search for the resolution of the conflict is, then, over.

For a wide class of games there is at least one equilibrium, which is not surprising given the Brouwer fixed-point theorem of analysis. However, uniqueness of the equilibrium is much more special. Therefore, we concentrate on the problem of multiple equilibria.

The interpretation of multiple equilibria has not been resolved. Yet, the crucial problem in game theory, and therefore in economics, is the resolution of the problem of multiple equilibria. We now turn to a discussion of several approaches to the resolution of this problem.

One approach is to reject traditional theory entirely. It simply cannot address a host of interesting problems.

A second approach is to restrict ourselves to models with a unique equilibrium, as being the only possible models that describe reality. This approach may be more positive than it sounds. The need to produce a unique equilibrium may yield useful restrictions on technologies or information sets, for example, which in turn generate strategy sets of the required form.

A third, and common, approach is to assume that any of the equilibria is possible. This has the advantage that at least one will describe the outcomes under all possible "mechanisms."

This approach is, of course, incomplete, as it does not show how any mechanism is generated, nor determine which equilibrium obtains. In particular, this approach leaves open the important question of the stability of the solution. What, if anything, might cause a shift from one equilibrium to another?

Nash has extended the notion of equilibrium to provide a broader solution concept (see [4], p. 106). If there are many equilibria, but with interchangeable strategies, then the game is solvable. Suppose $v \epsilon V$ indexes the equilibria $s^v = \bigcap_{t \epsilon T} S_t^v$, $S_t^v \epsilon W_t$, is an equilibrium for $v \epsilon V$. The game is solvable if

for any

$$\{v_t\}_{t \in T}$$

where $v_t \in V$ for all t, $\bigcap_{t \in T} S_t^{v_t}$ also is an equilibrium. However, unless $s^u \geq_t s^v$ and $s^u \leq_t s^v$ for all $u, v \in V$, and all t, this extension is not appealing. If this indifference does not hold, there still is a substantive issue of which equilibrium obtains. The Nash solution concept may, however, provide a criterion for deciding that there is no mechanism for resolving the conflict situation, and that theory provides only a partial description of behavior.

More generally, a fourth approach is to take seriously the result that there is no solution. The imposition of rationality is not, by itself, enough to determine economic agents' decisions. We must look elsewhere for a completion of the model. As behavior does occur, this suggests that completion is a necessary part of an economic model. Completion cannot be ducked on the grounds of being outside the purview of economics, as the conflict situation is the essence of economics. The implication is that a coherent economic model must include "noneconomic" elements.

A fifth approach is to impose an extra-model procedure for picking a particular equilibrium. This approach has appeal from the point of view of positive economics. However, it is ultimately unsatisfying, as it leaves unanswered the question of how market participants actually get to this result. One can advocate a particular equilibrium on the grounds of good properties which it exhibits. This suggests that another agent, or the same agents at a previous time, set in place restrictions on strategies that make the desired equilibrium the unique equilibrium. Such restrictions should, however, be included in the model. Moreover, the super game in which those restrictions are imposed should be explicitly analyzed. Perhaps this is expressed in saying that economics is innately political economy.[1/]

The most common tack taken in economics is the <u>assumption</u> of competitive equilibrium. As competitive equilibrium preceded game theory, it is not clear how this assumption is intended. Perhaps its advocates intend that it be used only in games in which it is inherently a unique equilibrium, but that seems to contradict its widespread, casual use. Rather, it seems to be an extra-model restriction. In some unexplained manner, strategies are restricted to make competitive equilibrium an equilibrium, and the only equilibrium.[2/]

The last approach to resolving the conflict situation which we consider is additional assumptions on the individual's psyche beyond just a preference ordering on points in X. There is a common theme running through the preceding discussion of multiple equilibria. Multiple equilibria exist because the model is incomplete, it is underspecified. One obvious possible incompleteness is that preference orderings on states of the world are just not a complete description of the individual as economic animal. Perhaps it is not at all surprising that a preference ordering is not a complete description of the individual in a conflict situation, and that as a result equilibria are not, in general, unique. We should not expect to predict behavior solely on the basis of orderings on states of the world. There simply is more to an individual in a social situation than a preference ordering. Utilitarianism must be supplemented.

This last approach suggests two possible procedures. One could impose additional attributes on the individual psyche. This is not something which economists have much experience with, there is little guidance in the economics literature as to what such attributes should be. Secondly, one could start with an observed equilibrium, and assuming conventional utility functions try to characterize the set of additional attributes that could produce that equilibrium.

2. Minimax-Nash

There is, however, some guidance in the game theory literature to useful restrictions on the individual's psyche. Indeed, the second major contribution of game theory, after consistency, is the recommendation of an additional attribute of the individual psyche. When all else fails, the individual maximizes security level. This is the concept of minimax.

It is worth stressing that minimax is a further additional description of the individual. It is, for example, totally consistent for the individual's preference ordering to exhibit risk preference, and for the individual to be a "maximiner." We now consider the use of the minimax concept to solve some conflict situations.

Recently much attention in economics has been focused on models which are not solvable in the sense of Nash. There is more than one equilibrium, and the strategies of these equilibria are not interchangeable. This is considered to be a severe problem, as it is taken to imply that the decentralized economy may exhibit pathological behavior. We argue that the problem lies with the solution concept, not the economy. The use of another solution concept is suggested, one which in some relevant economic models yields a unique equilibrium. The examples treated are a fiat money model, the capital overaccumulation problem, and multiple rational expectations equilibria, generally. In these examples we accept on faith that the competitive equilibria are, indeed, equilibria.

When a game does not have a unique equilibrium, there is a natural approach to consider. In this approach one treats a contraction of the game in which only strategies of equilibria are considered. Specifically, we assume that each individual is involved in a two-person game in which the other "person" is all other economic agents who choose the set of strategies of any one of the

equilibria. The individual herself can play any strategy of the original game. Further, we assume that the individual maximizes her security level in this contracted game, she is a maximiner. The individual may pick one strategy or be indifferent between a set of strategies. Call individual i's chosen strategy set S_i. If there is an element e of XS_i which is an equilibrium of the original game, we say e is an equilibrium. If all such equilibria have interchangeable strategies, then we say the game is solvable. Naturally, not all games are solvable. But the examples given below are.

The intuitive rationale for the above solution concept is as follows. We treat equilibria because we believe that the economy will achieve one, and we believe the individual believes this as well. But with multiple equilibria the individual has to decide which equilibrium the rest of the economy will get to. As she has no idea what process determines the chosen equilibrium, she maximizes security level. Then she decides that everyone else will do the same.

Now we turn to our examples. We consider three: fiat money, capital overaccumulation, and multiple rational expectations equilibria.

Our fiat money example is completely unbacked fiat money in an overlapping generations model as introduced by Samuelson [6]. In such models, there are always at least two equilibria, one in which fiat money has value and one in which it does not. One could assume that an individual of a given generation simply looks at what previous generations have done to decide which equilibrium obtains. This approach has at least two drawbacks. First, it does not tell the economist which equilibrium will have obtained. Second, it implies that earlier generations' decisions can bind later generations. Earlier generations cannot have made a mistake about which equilibrium obtains. But there is nothing in the structure of the problem which suggests such power. A more reasonable interpretation is that the individual takes previous decisions as initial conditions, and

views herself as playing a game with the rest of the current and future generations. Then her maximin strategy is obvious. The worst she can do is trade goods for fiat money and then have the next generation not trade goods for fiat money. So the individual does not trade goods for fiat money. With everyone playing this strategy, the solution is valueless fiat money.

Our conclusion in the previous paragraph is that completely unbacked fiat money has no value. This could be taken to mean that Samuelson's model exhibits an unsolvable problem for the (almost) decentralized economy. This, however, is not the case. Fiat money can be backed by a promise to tax future generations, for example. In many models, such a promise, if believed, never has to be exercised. Moreover, the value of the money can exceed the value of the promise. Of course, such a promise is a promised action of some coalition (the government), and that the issuance of this believable promise is the solution strategy of some cooperative game remains to be shown.

Our second example is the well-known capital overaccumulation problem as introduced by Phelps [5]. The basic difficulty in the capital overaccumulation problem is that in a competitive equilibrium the shadow price of capital may not obey the infinite horizon analog of the transversality condition. However, in such a world, our maximiner picks the best strategy for the lowest competitive shadow price on capital. With everyone doing this, the lowest shadow price obtains, and it is the one which obeys the transversality condition. We conclude that the capital overaccumulation problem does not occur.

Lastly, consider multiple rational expectations equilibria, generally. There are different self-fulfilling price processes. In this world, our maximiner acts assuming the least favorable process from her (myopic) point of view. As these models typically are symmetrical, everyone has the same least favorable process and it obtains. Whether this is the least preferred equilibrium is another matter, as we saw in the previous paragraph.

What does happen if "Minimax-Nash" does not produce a solution? One can assume that the individual has additional attributes of her psyche which do result in an equilibrium being chosen. The individual is a "maximiner" only when that behavior generates an equilibrium. For example, one can take maximizing security level as more nearly absolute. The individual's psyche is such that she maximizes security level in the contracted game if that yields equilibrium, but if that procedure does not yield equilibrium, the individual maximizes security level in the original game. This is, of course, a much stronger imposition of "maximin" behavior as it overrides the consistency criterion. It is worth noting that such a strong imposition of the minimax concept also can be used to solve games when there is no equilibrium. We now turn to a key bargaining problem in which such additional attributes are invoked. This problem also illustrates our approach to the cooperative game.

3. Fables of Specialization and Trade Between Individuals: Some Solutions to a Bargaining Problem

There is a unifying theme in the following four fables of specialization and trade. Specialization and trade depend upon the details of the available technologies of exchange and of the structure of the economy. One common structure in the fables is that the individual knows some salient features of the other individual(s') behavior, but lacks a complete description of the other individual(s). However, each fable has its own moral. The moral of the first fable is that specialization and trade do not occur in a vacuum. The possibility of trade is not by itself sufficient to engender trade. The model of the second fable is that even if binding contracts are feasible, specialization and trade need not occur. The possibility of trade and of binding contracts is not sufficient to engender trade. The moral of the third fable is that the ability to make a final offer can generate specialization and trade. However, the resulting allocation is not, in general, Pareto optimal. The moral of the fourth fable is that a competitive economic structure and appropriate exchange technology can yield specialization and trade and an optimal allocation.

3.1 Fable One

Now we turn to our first fable. The world is as follows. There are two individuals, one and two. There are two goods, apples and blueberries. There are two fields, (a) and (b). Individual one is in field (a), and individual two is in field (b). Field (a) can be used to produce 25 apples and 25 blueberries or 100 apples. Field (b) can be used to produce 25 apples and 25 blueberries or 100 blueberries. Fields (a) and (b) are separated by a high fence, except at two cliffs. One cliff has field (a) on top and one has field (b) on top. Fruit can be dropped from a cliff without harm, but cannot be thrown up a cliff. These technologies are known to both individuals.

Individual behavior is described as follows. Each individual acts to maximize her own independent utility function, which has as domain her own consumption of apples and blueberries. The individual's utility function is known only to herself. However, both utility functions are strictly increasing in apples and blueberries, and both put a higher value on 25 of each fruit than on 100 of one fruit and none of the other, and these facts are known to both individuals. Moreover, when faced with a game theoretic conflict situation, both individuals are conservative, and this fact, too, is known to both individuals.

Let us define "conservatism" precisely. If there is a unique equilibrium, that is the solution. Suppose there are multiple equilibria. Then each player plays Minimax-Nash. If the game is not solvable by the Minimax-Nash solution concept, as it is not in the fables, individuals maximize security level. Moreover, suppose there are several strategies with the same security level. If there is a single one of those strategies such that it generates an outcome above the security level in every circumstance in which any of those strategies does so, it is the chosen strategy.

Now we turn to the solution of our bargaining problem. Suppose one or both individuals specialized in the production of a single fruit. The strategies of trade involve unilaterally dropping fruit off a cliff. The only equilibrium in this set of strategies is dropping no fruit. Therefore, trade does not occur. Knowing this, neither individual specializes.

Moral: Specialization and trade do not occur in a vacuum.

3.2 Fable Two

Now we turn to our second fable. The world in the second fable has the same structure as the world in the first fable. However, in addition to the cliffs, there exists a device for lifting fruit across the fence in both directions in fixed amounts (or, alternatively, fixed proportions). The device is

activated only when both individuals set it for the same exchange (or proportion).

Now we turn to the solution of our bargaining problem. Suppose one or both individuals were to specialize. There are multiple equilibria settings of the device, namely, every feasible trade. However, the "conservatism" of individuals does not resolve which equilibrium setting is chosen. Therefore, both individuals fear that the null strategy of no trade will occur, and therefore do not specialize. Notice that we could also assume that each individual has his own two-way exchange device and reach the same conclusion.

Moral: Even if binding contracts are feasible, specialization and trade need not occur.

3.3 Fable Three

Now we turn to our third fable. The world in the third fable has the same structure as the world in the second fable. However, individual one has figured out a way to jam her setting of the exchange device so that she cannot change it afterwards. Moreover, individual one also has figured out a way to bind herself to specialize in apple production before individual two has to decide whether to produce only blueberries or both goods. For example, individual one can plant her trees early, but if she does so, she cannot return to the fence to reset the device or she will get no product. Individual one can play Stackleberg vis-a-vis individual two.[3/]

Now we turn to the solution of our bargaining problem. Suppose individual one sets the exchange device to trade 25 apples for 74 blueberries (or 26 apples for 75 blueberries, depending on her taste), and then commits herself to specialize in apple production. Then, the best that individual two can do for herself is to specialize and trade to get 26 of one fruit and 25 of the other. Moreover, individual two can be assured of getting this outcome, as individual

one is bound to specialize, and given that she has specialized, will trade. Therefore, individual two specializes and trades. Individual one realizes that this will be individual two's decision. However, to be sure of individual two, individual one does have to choose an exchange that gives individual two an allocation that dominates 25 of each fruit. So our supposed solution is indeed the solution.

Notice that if individual two figures out how to build a new exchange device and jam it after individual one has jammed the original, she will not do so (unless she can destroy the original device). For if she did, no trade is a possibility, which is worse than her (25, 26) allocation. Notice also that the economy can consist of many of such pairs of individuals and yield the same solution, even if exchange between pairs is possible.

Moral: The ability to make a final offer can generate specialization and trade. However, the resulting allocation is not, in general, Pareto optimal.

3.4 Fable Four

Now we turn to our fourth and final fable. The world in the fourth fable has a different structure from the world in the third fable, a competitive structure. There are a countable infinity of fields like field (a), inhabited by one individual each, all having the same characteristics as our individuals one and two above. Each individual of type one (in a field (a)) has an exchange device which she can jam to offer any of a specified continuum of trades. There is the same single individual two whose field (b) meets all the other fields in the center of a "pie." Individual two can also commit herself to specialize early, and can commit herself to trade with only one of the type one individuals by allowing one exchange device to be operable, and destroying the others.

Now we turn to the solution of our bargaining problem. Suppose each individual of type one sets her exchange device to yield any final allocation to

herself that she just prefers to 25 of each fruit. Individual two picks the single trade device and particular trade she wants, destroys the other devices, and specializes. Then the chosen individual one specializes and the others do not. This is the solution. Individual two cannot pick to trade with more than one individual, because she cannot guarantee that it will be in her own interest to trade with more than one individual. No individual type one will specialize unless individual two is certain to trade with her. The individuals of type one do not offer a less favorable trading continuum to individual two, because they fear being undercut by the other type one's. The only equilibrium is each type one offering the best deal she can.

If individual two would not choose to trade with more than one individual of type one if she could, Pareto optimality is achieved. Moreover, if individual two can alter the exchange devices so that any chosen one works only if all chosen ones are used, then multiple trades are feasible and Pareto optimality obtains. Notice that having only two fields like field (a) or having a finite number of individual type two's with a pecking order for choosing individual type one's offers yields similar results.

Moral: A competitive economic structure and an appropriate exchange technology can yield specialization and trade and an optimal allocation.

3.5 A Concluding Comment on the Fables

It is worth noting that the linear "economy" which economists often seem to have in mind is not an economy at all. It is just a replication of Robinson Crusoes. The allocation of rents from specialization, as illustrated in these fables, is of the essence of economics. And the lesson of the fables is that there is no general solution to the allocation of these rents. Rather, it depends upon the details of the available technologies of exchange and of the structure of the economy.

4. General Summary and Concluding Comments

Every conflict situation should be modeled as a noncooperative game. When the noncooperative game does not yield a unique equilibrium, this should be treated as an incomplete specification of the game. To complete an incomplete model one may add more to the assumptions on the individual psyche than just a preference ordering, or alter the model in some other way.

The keystone of economics, the ordering of strategies in a game theoretic conflict situation, is conceptually feasible using the above method, although in practice it may be difficult. This method of solution may in many applications imply that "economics" is better termed "political economy." Purely, "economic" concepts do not make a complete economic model. This further suggests that use of this method may put us well on the way to the goal of coherent "micro-micro" modeling of political economy: models in which criteria like Pareto optimality are irrelevant. That is to say, any useful interpretation of Pareto optimality holds vacuously. What is is what can be.

Footnotes

[1/] See, for example, Bryant [2].

[2/] In fairness to economists, it should be mentioned that for some models, as the number of agents increases the core, a controversial definition of consistency for the cooperative game, converges to competitive equilibrium. That this is an inadequate defense for competitive equilibrium is testified to by the fact that game theorists are wont to defend the core on the grounds that it converges to competitive equilibrium.

[3/] Use of a Stackleberg strategy space to generate equilibrium appears in Bryant [1].

References

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