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Cristina Arellano
Federal Reserve Bank of Minneapolis, University of Minnesota, and NBER

Yan Bai
University of Rochester and NBER

Gabriel Mihalache
Stony Brook University

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Cristina Arellano
Federal Reserve Bank of Minneapolis, University of Minnesota, and NBER

Yan Bai
University of Rochester and NBER

Gabriel Mihalache
Stony Brook University

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Abstract

The COVID-19 epidemic in emerging markets risks a combined health, economic, and debt crisis. We integrate a standard epidemiology model into a sovereign default model and study how default risk impacts the ability of these countries to respond to the epidemic. Lockdown policies are useful for alleviating the health crisis but they carry large economic costs and can generate costly and prolonged debt crises. The possibility of lockdown induced debt crises in turn results in less aggressive lockdowns and a more severe health crisis. We find that the social value of debt relief can be substantial because it can prevent the debt crisis and can save lives.

Keywords: default risk, pandemic mitigation, sovereign debt, partial default, debt relief
JEL classification: F34, F41, E52

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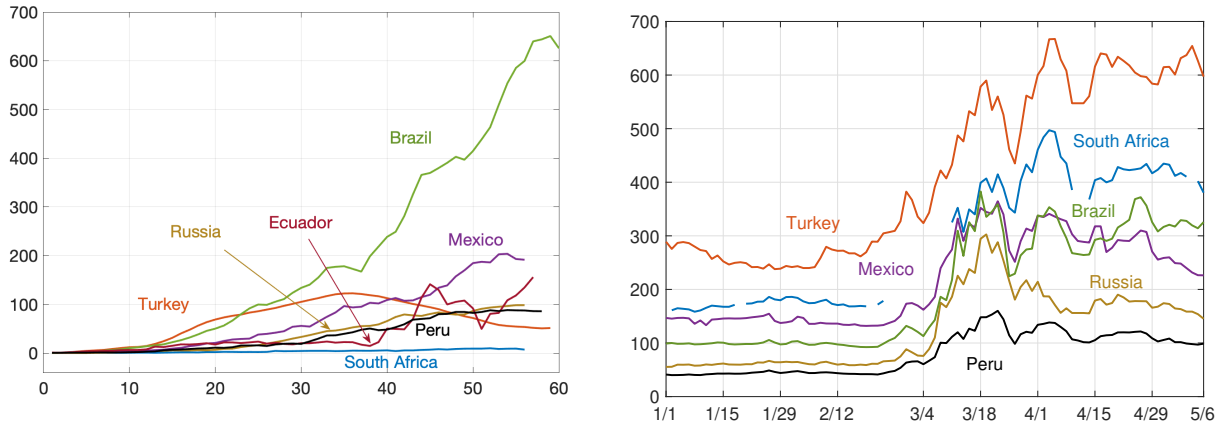
1 Introduction

The Coronavirus pandemic is blazing through the world and presents enormous challenges for society. The disease is highly contagious and causes significant loss of lives. Countries are imposing mitigation and lockdown policies that limit social interactions to control the disease. These lockdown policies have been effective in taming the spread of the disease but are associated with substantial economic costs, as productive activities shut down and unemployment soars. Many governments are engaging in large fiscal transfers to support consumption during the lockdowns.



(a) Fatalities (daily since 3 fatalities)

(b) CDS Spreads

Figure 1: Official COVID-19 Fatalities and Sovereign Spreads

The epidemic in emerging markets is growing and leading to a large human cost. As of May 13, the total number of official deaths across 35 emerging countries is over 39,000 and, as Figure 1 shows, the number of official COVID-19 daily fatalities across a few emerging markets continues to grow.[1] These countries are confronting the epidemic with additional challenges. As documented in Hevia and Neumeyer (2020), the pandemic is a tremendous external shock for emerging markets with collapsing export demand, tourism, remittances, and capital flows. These countries also have limited fiscal space, which has made it difficult for governments to extend substantial support to their citizens. A main impediment for these countries is their chronic problem with external debt and susceptibility to debt crises, as argued in Gourinchas and Hsieh (2020). Argentina and Ecuador have already defaulted on their sovereign debt, and all emerging markets are seeing their sovereign spreads rise in anticipations of more defaults, as shown in Figure 1.

In this paper, we study the COVID-19 epidemic in emerging markets that face a combined health,

[1] Current data is incomplete as to eventual human cost in emerging markets from COVID-19. The New York Times reported large underreporting of deaths in emerging markets; the ratio of excess deaths to official deaths is 2 in Istanbul and 15 in Ecuador. Also, the historical account from the Spanish Flu of 1918 in Barro, Ursúa, and Weng (2020) shows that developing countries had the highest death toll and warns of an even larger threat for the current epidemic.

economic, and debt crisis. We integrate a standard epidemiological susceptible-infected-recovered (SIR) model into a sovereign default model and study how the epidemic impacts both economic activity and the likelihood of a debt crisis. The epidemic creates a health crisis and generates time paths for the infected and deceased population. The government can impose lockdowns that save lives but depress output. The government borrows internationally and can default with varying intensity and a choice of duration for the episode. Borrowing smooths consumption during the lockdowns, but default risk limits fiscal capacity for supporting consumption. Default risk also increases the cost of lockdowns, because in addition to depressing output, they also lower the prospects for debt repayment, increasing the likelihood of a lengthy and costly debt crisis. The increased cost of lockdowns from default risk, in turn, leads to less aggressive lockdown choices and a more severe health crisis: default risk costs lives.

We study the dynamics of lockdowns, default, consumption, and fatalities following the outbreak of the epidemic. For the baseline, we consider the case when the economy starts with 30% debt to output ratio. We find that lockdowns start about two months after the outbreak and last for eight months. The first two months feature a high lockdown intensity, with about 50% of employment in lockdown, followed by gradually relaxed lockdowns. This optimal lockdown path reduces the death toll from the epidemic by half. Absent lockdowns, the death toll from the epidemic is 1% of the population; with optimal lockdowns, the death toll is 0.5% of the population. Lockdowns reduce output by 19% in present value and generate a debt crisis of 43 months with defaults. The first six months of the crisis feature high default intensity, the country defaults on more than 50% of the debt due; default intensity falls thereafter but remains elevated for 37 more months. Overall, the epidemic is costly for the country; welfare in terms of consumption equivalence falls 1.8% relative to pre-epidemic level.[2]

Debt alters the outcome of the epidemic because it changes the costs of lockdowns. A highly indebted economy, with a looming debt crises, chooses a less strict path of lockdowns because of its limited ability to use financial markets to support consumption. As a result, the highly indebted economy ends with a higher death toll from the epidemic. We find that economies that start the outbreak with more debt will suffer more severe health and debt crises: more fatalities and more prolonged defaults. Lockdowns are effectively an investment to save future lives. As such, our finding that high debt and default discourage lockdowns relates to the debt overhang literature, which has shown that high debt reduces investment as in Aguiar and Amador (2011).

The International Monetary Fund, the Inter-Development Bank, and other international organizations are sponsoring debt relief programs to help countries get through the COVID-19 epidemic. We use our

[2]The lengthy and costly debt crises caused by the epidemic in our model are reminiscent of the Latin American debt crisis of the 1980s, which imposed sizable costs, as documented by Bergoeing, Kehoe, Kehoe, and Soto (2002) and Kydland and Zarazaga (2002).

model to perform debt relief counterfactuals and evaluate these programs on the health, economic, and debt crises. Debt relief helps countries because it enables them to avoid debt crises, giving the government the fiscal capacity to implement stricter lockdowns that save lives. In our baseline, a debt relief program that costs a financial assistance entity about 10% of output results in welfare gains to country of about 14% of output. Lenders also benefit through a capital gain on the market value of the debt, although in the baseline the gains are small and less than 1% of output. The social value of these programs is generally positive because the cost born by the financial assistance entity is more than offset by the gains to the country and its lenders. We also find debt relief programs that put more weight on the benefits to the country than the benefits to the lenders, should be targeted towards countries with intermediate levels of debt. Debt relief for these marginal economies can alter more profoundly the outcomes of the epidemic, as the program helps avoid the debt crisis and induces more lives saved.

The epidemiological model is the standard susceptible-infected-recovered framework. The epidemic starts when a fraction of the population is infected. The growth in the infected population depends on the infected population, on the susceptible population, and on how fast infections move from infected to susceptible. The infected population transit to either a recovered state or to the unfortunate deceased state. We follow Alvarez, Argente, and Lippi (2020) and assume that the death rate depends on the fraction of the infected population and that mitigation policies take the form of lockdowns that limit the growth of infections.

The sovereign debt and default framework we adopt follows the recent work in Arellano, Mateos-Planas, and Rios-Rull (2019). In this model, a sovereign in a small open economy chooses the intensity of default and the duration of the default episode. A fraction of the defaulted debt accumulates over time and new credit is endogenously restricted. Default in this framework amplifies shocks and leads to persistent adverse effects on the economy, resembling more closely historical emerging market data. Importantly, as the length of the debt crisis is endogenous, so are its associated costs, which enter into the calculation about the appropriate lockdown response to the epidemic.

The sovereign in our model values the lives of its citizens as well as consumption per capita. The government borrows internationally and can default on its debt. The epidemic lowers the value for the sovereign because it is associated with a loss of lives. The sovereign chooses policies for lockdowns, borrowing, and default to manage the epidemic dynamics and support consumption. Default risk interacts with optimal lockdown policies. High default risk restricts the country from supporting its consumption and makes lockdown policies more costly. The susceptibility of the debt crises, hence, makes the economic and health crises more severe. Our work suggests that the risks of a sovereign debt crisis are an additional first-order cost from the COVID-19 epidemic in emerging markets.

Our work also makes a methodological contribution. We develop a framework that integrates the dynamics of debt with the dynamics of the epidemic. We set up and solve a Markov problem where the government's choice on debt and lockdown affects the endogenous evolution of four state variables, namely the debt, and the three population groups: susceptible, infected, and recovered. The government lacks commitment and chooses these policies taking as given the future choices. We also provide an algorithm that can be adapted to other applications on epidemics with time-varying endogenous aggregate state variables.

Literature Our paper contributes to the fast-growing literature that studies the COVID-19 epidemic and its interactions with economics. Atkeson (2020) is the first to introduce to economists the classic epidemiology model of SIR to study the human costs of the COVID-19 epidemic for the United States. Alvarez, Argente, and Lippi (2020) study optimal lockdown policies in a simple production economy when the epidemic dynamics follow a SIR model. Their results highlight the trade-off of lockdowns: saving lives but costly for economic output. They also quantify their framework and find that an optimal lockdown policy has an inverted U shape over time and lasts about four months. Eichenbaum, Rebelo, and Trabandt (2020) modify the epidemic dynamics to depend on consumption and labor and show that these forces create negative externalities. In their framework, the optimal path of consumption and labor during the epidemic is more depressed than the laissez-fair outcome because the depressed production and consumption reduce infections and save lives. Farboodi, Jarosch, and Shimer (2020) also find that in the environment of Alvarez, Argente, and Lippi (2020), negative externalities apply and call for government imposed lockdowns.

A growing literature studies targeted mitigation strategies. Glover, Heathcote, Krueger, and Rios-Rull (2020) delved into the crucial distributional considerations, as the old are more at risk than the young from the epidemic, yet the young endures most of the economic costs from lockdowns. They find that the lockdowns mostly benefit the old and are used more extensively with better redistribution. Acemoglu, Chernozhukov, Werning, and Whinston (2020) studies unconstrained optimal lockdowns in an environment with multiple ages and sectors, a multi-risk SIR model, and also find that smart mitigation strategies that target the old are most helpful. Favero, Ichino, and Rustichini (2020) develop a framework with multiple sectors and ages, apply it to the case of Italy by comparing the performance of Lombardy with Veneto, and find similar results: smart mitigation strategies target the old and the at risk population. Baqaee, Farhi, Mina, and Stock (2020) and Azzimonti, Fogli, Perri, and Ponder (2020) study how networks across sectors and geography can be exploited in the design of optimal mitigation policies. Additional recent contributions on the optimal policy response to the COVID-19 pandemic are Hall, Jones, and Klenow (2020), Jones, Philippon, and Venkateswaran (2020), and the policy pieces in the volume by

Baldwin and Weber (2020). These papers focused on the epidemic costs for advanced economies and have abstracted from the additional challenges in emerging markets. Our paper highlights a main additional cost from the epidemic for emerging markets which are debt crises. With this focus in mind, we have abstracted from smart mitigation policies that depend on distributional considerations.

A few papers also share our focus on the impact of COVID-19 on emerging markets. Çakmaklı, Demiralp, Kalemli-Özcan, Yesiltas, and Yildirim (2020) construct a SIR-macro model considering the domestic and international input-output linkages and sectoral heterogeneity. They compare two types of exogenous lockdown: full and partial lockdown, under the calibration to the Turkish input-output network. They abstract from sovereign default risk. Espino, Kozlowski, Martin, and Sanchez (2020) study optimal fiscal and monetary policy of emerging markets in a sovereign default model with endogenous distortionary taxes and seigniorage. They model COVID-19 as an unexpected shocks to productivity, the disutility of labor, terms of trade and transfers, and find that default risk rises from the epidemic. They do not consider explicitly the epidemiological dynamics and hence are not concerned with the interaction between mitigation policies and debt.

The dynamic debt and default framework builds on the sovereign default literature in Aguiar and Gopinath (2006), Arellano (2008), and Chatterjee and Eyigungor (2012). We adopt the more recent approach of Arellano, Mateos-Planas, and Rios-Rull (2019) that model debt crises with partial default and an endogenous length for the debt crisis. They find defaults generate long lasting crises because the defaulted debt accumulates over time. This framework matches the empirical regularity of growing debt during debt crises and an exit from default when the economy recovers.

Our quantitative evaluation of debt relief contributes to the literature on debt buybacks. As in Bulow, Rogoff, and Dornbusch (1988) and Aguiar, Amador, Hopenhayn, and Werning (2019), we also find that international lenders would benefit from debt buybacks during the COVID-19 epidemic through capital gains. Nonetheless, we emphasize that the gains to the country net of financial assistance from debt buybacks are large and positive, including when the country is highly indebted. Reducing debt overhang can considerably shorten and lessen the debt crisis and save the output cost in default. Furthermore, debt reduction allows the country to adopt stricter lockdown policies, which are investments in future saves.

2 Model

We consider a small open economy model with a continuum of identical agents and government that borrows from the rest of the world and can default on its debt. Output in the economy depends on labor input and productivity. We evaluate the dynamics of this economy after it is hit with an epidemic (COVID-19) unexpectedly. The epidemic dynamics follow a standard epidemiological SIR (Susceptible-

Infectious-Recovered) model. During the epidemic a subset of the population endogenously transitions from being susceptible to infected. The infected eventually either recover or die. The government jointly sets lockdown policies and uses international debt and default to manage the epidemic.

We first describe preferences, technology, and the environment of sovereign debt and default. We then set up the recursive formulation for the economy before the epidemic. We proceed to describe the evolution of the disease and formulate the dynamic problem during the epidemic. The outbreak starts when a subset of the population exogenously becomes infected.

2.1 Preferences and Technology

The government values both the consumption and life of agents in the economy. As in Alvarez, Argente, and Lippi (2020) and Farboodi, Jarosch, and Shimer (2020), the value for the government increase with consumption per capita c_t and decrease with fatalities $\phi_{D,t}$. We assume that preferences over consumption are concave and that each fatality imposes a loss of χ. The lifetime value of the government is

$$v_0 = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \chi \phi_{D,t} \right] \tag{1}$$

where β is the discount factor. The utility function $u(c)$ is given by $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$, with σ controlling the intertemporal elasticity of substitution.

Output in the economy Y_t is produced using labor, with a linear technology. Labor input depends on lockdown policies. Absent lockdown policies, each surviving agent provides one unit of labor and hence total labor supply equals the mass N_t. During a lockdown of intensity L_t, agents cannot supply all their labor endowment, which reduces total labor to $(1 - L_t)N_t$.[3] The economy output equals

$$Y_t = z_t(1 - L_t)N_t \tag{2}$$

where z_t is the economy-wide productivity. It depends on an underlying level $\tilde{z}$ and falls with government default.

2.2 Government Debt and Default

The government issues long-term debt internationally and lacks commitment to repay it. We consider a flexible sovereign default model where the government can choose to partially default on the debt every period and decides whether to start or end the default episode. We study long-term debt in a

[3] In this baseline model we have assumed that all consumers, infected or not, can work with equal productivity. It is easy to consider an extension with infected consumers are unable to work or work with a reduced productivity.

tractable way by considering random maturity bonds, as in Hatchondo and Martinez (2009). The bond is a perpetuity that specifies a price q_t and a quantity ℓ_t such that the government receives $q_t \ell_t$ units in period t. In the following period, a fraction δ of the debt matures. Every period, conditional on not defaulting, each unit of debt calls for a payment of $\delta + r$.[4] The government can choose to default on a fraction d_t of the current debt obligation. The government transfers to consumers all of its proceeds from operating in international debt markets. The resource constraint of the economy is given by

$$N_t c_t + (\delta + r)(1 - d_t) B_t = Y_t + q_t \ell_t. \tag{3}$$

The equilibrium bond price q_t is determined by a schedule that depends on the face value of the debt as well as epidemic dynamics because, as we will see below, default intensity depends on next-period states including the severity of the infection.

In this model with accumulation of long-term defaulted debt, the face value of the debt due next period B_{t+1} depends not only on new issuances ℓ_t but also the legacy debt B_t and the share of debt defaulted on over time. Following Arellano, Mateos-Planas, and Rios-Rull (2019), we assume that partial default reduces the current debt service payment to $(1 - d_t)(\delta + r) B_t$ but increases future debt obligation by a κ fraction of the defaulted debt. We annuitize these future debt obligations such that the next period debt obligations increase by $\kappa d_t (\delta + r) B_t$. Default also depresses productivity to $z_t = \tilde{z}\gamma(d_t)$, where the function $\gamma(d_t)$ satisfies $0 \leq \gamma(d_t) \leq 1$ and $\gamma'(d_t) < 0$. The evolution of long-term debt is controlled by the new issuance ℓ_t, the legacy debt that has not matured, $(1 - \delta) B_t$, and the defaulted debt that is carried over,

$$B_{t+1} = \ell_t + \left[(1 - \delta) + \kappa(\delta + r) d_t\right] B_t. \tag{4}$$

International lenders are risk-neutral and competitive. They take as given the world risk-free rate r, their opportunity cost. The bond price q_t compensates lenders in expectation, for their losses due to future defaults,

$$q_t = \frac{1}{1 + r} \left\{ (\delta + r)(1 - d_{t+1}) + \left[1 - \delta + \kappa(\delta + r) d_{t+1}\right] q_{t+1} \right\}. \tag{5}$$

This expression reflects that partial default tomorrow d_{t+1} reduces the value that lenders get in period $t + 1$ but increases the subsequent value for lenders as the defaulted debt accumulates at rate κ and becomes due in the future.

[4] We set this coupon payment as to normalize the price of a default-free bond to 1. This normalization does not alter the maturity of the debt.

2.3 Epidemic Dynamics

We now describe the outbreak of the epidemic and subsequent dynamics. We build on the classic SIR model in Kermack and McKendrick (1927) and Atkeson (2020). Following the outbreak of the disease, a subset of the population will transition endogenously from being susceptible to infected and, eventually, to either recovered or deceased. During the epidemic, the population N_t is partitioned in three epidemiological groups, susceptible, infected, and recovered, with the mass of each group denoted by μ_t^S, μ_t^I, and μ_t^R, respectively. We assume that the initial population size is 1. The total mass of deceased is $\mu_t^D = 1 - N_t$. The epidemic starts when an initial mass from the population becomes infected exogenously, $\mu_0^I > 0$. The rest are susceptible, except possibly for measure of agents already recovered $\mu_0^R \geq 0$, so that $\mu_0^S = 1 - \mu_0^I - \mu_0^R$.

The epidemic dynamics can be modulated with lockdown policies that the government implements as in Atkeson (2020) and Alvarez, Argente, and Lippi (2020). We consider untargeted lockdowns, where the government can dictate the fraction of time agents must refrain from participating in labor markets, independent of their epidemiological state. A lockdown policy of size L_t reduces labor input by L_t and social interactions by θL_t. The parameter θ controls the effectiveness of lockdowns in reducing social interactions and therefore preventing infections.

A key component of the SIR dynamics concerns how likely susceptible agents become infected. We follow the standard approach that their probability of infection depends on the mass already infected μ_t^I and lockdown policies and their effectiveness in dampening social interactions θL_t. We denote by μ_t^x the mass of newly infected and assume that it is determined by

$$\mu_t^x = \pi_{SI}(1 - \theta L_t)\mu_t^I(1 - \theta L_t)\mu_t^S \tag{6}$$

The presence of $1 - \theta L_t$ twice in the above expression reflects the fact that the social interactions of both the infected and susceptible are reduced by lockdowns. The mass of susceptible agents in period $t + 1$ depends on that in period t net of the newly infected,

$$\mu_{t+1}^S = \mu_t^S - \mu_t^x. \tag{7}$$

Infected agents remain in this state with probability π_I. The mass of infected in period $t + 1$ equals a π_I fraction of the infected in period t plus any new infections. These dynamics are given by

$$\mu_{t+1}^I = \pi_I \mu_t^I + \mu_t^x. \tag{8}$$

With probability $1 - \pi_I$ each infected agent either recovers or dies. As in Alvarez, Argente, and Lippi (2020), we assume that the probability of dying from the disease conditional on being infected $\pi_D(\mu_t^I)$ depends on the measure of current infections, resulting in $\phi_{D,t} = \pi_D(\mu_t^I)\mu_t^I$ fatalities every period. We assume $\pi_D'(\mu_t^I) > 0$ to capture the role of health care capacity for the fatality rate of the disease; more infections put a strain on the health care system and diminish its ability to successfully treat cases. The mass of agents that die or recover from the epidemic depends on the fraction of the infected that transition into these states, as well as the population in each of these absorbing states the previous period. These dynamics are

$$\mu_{t+1}^R = \mu_t^R + \left[1 - \pi_I - \pi_D(\mu_t^I)\right] \mu_t^I, \tag{9}$$

$$\mu_{t+1}^D = \mu_t^D + \pi_D(\mu_t^I)\mu_t^I. \tag{10}$$

The dynamics of the epidemic induce an evolution of the population size N_t, given by

$$N_{t+1} = \mu_{t+1}^S + \mu_{t+1}^I + \mu_{t+1}^R. \tag{11}$$

The loss of life from the epidemic is the cost that we focus on.

As is well known in epidemiology models, the epidemic continues until the mass of infected consumers asymptotes to zero. Absent lockdown policies, the SIR parameters $\pi_{SI}, \pi_I, \pi_D(\mu_t^I)$ determine the duration and severity of the epidemic. Lockdown policies L_t can alter these outcomes. In practice, we adopt an assumption that the epidemic ends H periods after it starts because a vaccine becomes available. With the discovery of a vaccine all susceptible agents vaccinate and functionally become recovered, so that no new infections can occur.

2.4 Government Problem

We first set up the recursive formulation for the government prior to the epidemic and then move to the corresponding problem during the epidemic.

Recursive Formulation Before the Epidemic The recursive problem for the government before the epidemic resembles the problem in Arellano, Mateos-Planas, and Rios-Rull (2019). We also assume that the government does not expect the epidemic arising in the future, the probability of dying is expected to be always zero and the measure of population remains at one. We study a Markov problem for the government. At period t, with debt holding B_t, the government chooses new issuance ℓ_t and partial

default intensity $d_t \in [0,1]$ to solve the following problem

$$V^{\text{pre}}(B_t) = \max_{\ell_t,\, d_t \in [0,1]} \; u(c_t) + \beta V^{\text{pre}}(B_{t+1}) \tag{12}$$

subject to the evolution of the debt in equation (4), the resource constraint of the economy (3) with $N_t = 1$, and the bond price function $q^{\text{pre}}(B_{t+1})$. The Markov structure generates a time invariant bond price function that depends on future default and borrowing decisions,

$$q^{\text{pre}}(B_{t+1}) = \frac{1}{1+r}\left\{(\delta+r)(1-d_{t+1}(B_{t+1})) + \left[1-\delta+\kappa(\delta+r)d_{t+1}(B_{t+1})\right] q^{\text{pre}}(B_{t+2}(B_{t+1}))\right\}. \tag{13}$$

This government's problem results in pre-epidemic decision rules for the evolution of government debt $B_{t+1} = \mathbf{B}^{\text{pre}}(B_t)$, default $d_t = \mathbf{d}^{\text{pre}}(B_t)$, and per capita consumption $c_t = \mathbf{c}^{\text{pre}}(B_t)$. It also gives the bond price function $q^{\text{pre}}(B_{t+1})$ and value function $V^{\text{pre}}(B_t)$ pre-epidemic. We will use these results to set initial conditions for the following problem during the epidemic. In the baseline experiment, we use the steady state values for debt B_t of this problem as initial condition.

Dynamic Problem During the Epidemic We now integrate the epidemic dynamics into the government's problem. The government and international lenders learn about the epidemic in period 0. During the epidemic, the government continues to borrow and default from international financial markets and also chooses lockdown policies L_t to reduce the loss of lives from the epidemic. We impose an upper limit $\overline{L}$, reflecting the presence of essential activities that cannot be suspended. The epidemic changes the prospects for the economy because it increases the probability of losing lives and reduces the output due to potential lockdowns. In studying this problem, we assume that a vaccine will be available in a future period H when all the susceptible population vaccinates and the epidemic quickly winds down. We study a Markov problem for the government, by solving the problem backward starting from the vaccine period H.

Consider first the problem of the government for any period before the vaccine arrives $t < H$. The state variable for the government consists of the measures of each group $\mu_t = (\mu_t^S, \mu_t^I, \mu_t^R)$ and its debt B_t. The deceased population is $\mu_t^D = (1 - \mu_t^S - \mu_t^I - \mu_t^R)$. The value function for the government depends on these states and on time $V_t(\mu_t, B_t)$. The bond price function depends on future states and time, $q_t(\mu_{t+1}, B_{t+1})$, because default decisions will depend on these variables.[5] The government takes as given future value functions $V_{t+1}(\mu_{t+1}, B_{t+1})$ and the bond price function and chooses optimal borrowing ℓ_t, partial default

[5] The time-dependency of the functions specified in the t subscripts reflects time-dependency of the vaccine. They would be time-invariant absent a terminal condition. In the baseline quantitative analysis the results do not depend on the terminal condition.

d_t, and lockdowns L_t to maximize its objective given by

$$V_t\left(\mu_t, B_t\right) = \max_{\ell_t,\, d_t \in [0,1],\, L_t \in [0,\overline{L}]} \left[u(c_t) - \chi\phi_{D,t}\right] + \beta V_{t+1}\left(\mu_{t+1}(\mu_t, L_t), B_{t+1}\right) \tag{14}$$

subject to the resource constraint,

$$N_t c_t + (1-d_t)(\delta + r)B_t = \tilde{z}\gamma(d_t)N_t(1-L_t) + q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1})\ell_t, \tag{15}$$

the evolution of debt (4), the SIR dynamics (6)-(9) that map current population groups and lockdown policies to future population groups $\mu_{t+1}(\mu_t, L_t)$, fatalities given by these dynamics $\phi_{D,t} = \pi_D(\mu_t^I)\mu_t^I$, and the total population constraint (11). The government internalizes that its choices for debt and lockdown impact the states tomorrow and the bond price.

The government trades off the potential benefits of savings lives against the consumption costs of lockdowns when choosing lockdown L_t. The consumption cost increases with the output disruptions from lockdown and it is amplified with limited availability of international credit and default risk. If international credit is ample and default risk is low, the output disruption only matters for consumption through the reduction of lifetime wealth. Consumption adjusts down to lower lifetime value, but the period-by-period consumption does not need to adapt to the contemporaneous declines in output from lockdowns.

The consumption costs from lockdown, however, are elevated when international credit is restricted and the default crisis deepens. Lockdowns spur default. The optimal interior partial default decision for the government satisfies

$$-\tilde{z}(1-L_t)N_t\gamma'(d_t) = \left[1 - \kappa q_t(\mu_{t+1}, B_{t+1})\right](\delta + r)B_t. \tag{16}$$

Partial default benefits the economy by lowering its debt payment but it is costly because it induces a loss of productivity. The right-hand side is the marginal benefit of default due to debt reduction: current debt payment minus the increase in the future payment of κ fraction of the defaulted debt, with the future market value depending on $q_t(\mu_{t+1}, B_{t+1})$. Our model shares the same feature with the standard sovereign default models that default incentive increases with current debt B_t. The left-hand side reflects the costs from partial default and shows that both lockdowns and a lower population from the disease lower the marginal cost of defaulting and hence generates more partial default. With lockdowns, high future default risk, and restrictive credit, consumption is substantially depressed, and the government will prefer to default more on the debt. Defaults bring extra productivity costs and lead to persistent debt

crises. The debt crises, in turn, magnify the costs from the epidemic and lockdowns.

The government's problem results in decision rules for economic variables in periods $t = 0, 1, \ldots, H-1$ for government debt $B_{t+1} = \mathbf{B_{t+1}}(\mu_t, B_t)$, default $d_t = \mathbf{d_t}(\mu_t, B_t)$, lockdowns $L_t = \mathbf{L_t}(\mu_t, B_t)$, and per capita consumption $c_t = \mathbf{c_t}(\mu_t, B_t)$. The problem also induces functions for the evolution of epidemiological variables that depend on the level of debt as well as the distribution of the population over types. Debt affects population dynamics through its impact on lockdowns. Let the equilibrium functions for the evolution of susceptible, infected, and recovered be $\mu_{t+1} = \boldsymbol{\mu_{t+1}}(\mu_t, B_t)$.

The bond price function during the epidemic, $q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1})$, depends on the debt levels as well as on lockdown policies because they impact epidemic dynamics. The bond price function satisfies

$$q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1}) = \frac{1}{1+r}\left\{(\delta + r)(1 - \mathbf{d_{t+1}}) + \left[1 - \delta + \kappa(\delta + r)\mathbf{d_{t+1}}\right] q_{t+1}(\boldsymbol{\mu_{t+2}}, \mathbf{B_{t+2}})\right\}.$$

where future defaults, borrowing, and lockdowns are given by the policy rules of the government problem. The problem from period H on, is similar to the problem described above, except that the vaccine at period H moves all the susceptible agents to the recovered state and resolves all infections. We solve this problem working backwards starting from period H. The appendix provides a definition of the dynamic equilibrium of the economy during the epidemic.

We define spreads in our model using a synthetic credit default swap (CDS) instrument. CDS spreads reflect the average default probabilities on underlying bonds without taking into account recovery. We can calculate a synthetic bond price for this security of duration controlled by δ with our default and borrowing policy function as follows

$$q_t^{\text{CDS}}(\mu_{t+1}, B_{t+1}) = \frac{1}{1+r}\left\{(\delta + r)(1 - \mathbb{1}_{\mathbf{d}_{t+1}>0}) + (1-\delta) q_{t+1}^{\text{CDS}}(\boldsymbol{\mu_{t+2}}, \mathbf{B_{t+2}})\right\}.$$

We measure the underlying spreads using the standard yield-to-maturity expression

$$\text{spread}_t = (\delta + r)[1/q_t^{\text{CDS}} - 1]. \tag{17}$$

3 Quantitative Analysis

This section contains the quantitative analysis of the model. We first discuss the choice of parameters, including those controlling SIR dynamics and the cost of default. We then describe model policy functions for lockdowns and defaults and the economy dynamics during the epidemic for varying initial levels of indebtedness. Finally, we conduct counterfactual debt relief experiment and show that these programs can have large social gains.

3.1 Parameterization

We consider a weekly model. The intertemporal elasticity of substitution is set at a standard value of 0.5, and the annual risk-free rate is 1%. The discount factor β is chosen to match an average 2% real domestic interest rate for emerging market inflation targeters, reported in Arellano, Bai, and Mihalache (2020). The real domestic interest rate is constructed using the domestic short-term rates and ex-post CPI inflation.

Following Alvarez, Argente, and Lippi (2020), we assume the case fatality rate $\pi_D(\mu_t^I)$ depends on the number of infected people in a linear way, to capture the congestion effect in the health care system,

$$\pi_D(\mu_t^I) = \pi_D^0 + \pi_D^1 \mu_t^I.$$

The SIR parameters are calibrated using epidemiological research findings. π_I determines the rate at which infected people either recover or die of the diseases. According to Wang, Wang, Dong, Chang, Xu, Yu, Zhang, Tsamlag, Shang, Huang, et al. (2020), the duration of illness is on average 18 days.[6] For our weekly model, this implies $\pi_I = (1 - 1/18)^7 = 0.67$. The parameter π_{SI} relates to the widely used measure $\mathcal{R}_0$, the expected number of additional infection caused by one infected person. Zhang, Diao, Yu, Pei, Lin, and Chen (2020) uses the data from the Diamond Princess cruise ship and estimates a $\mathcal{R}_0$ of 2.28 with a 95% confidence interval of $[2.06, 2.52]$. In the model $\mathcal{R}_0 = \pi_{SI}/(1 - \pi_I)$, which implies a value of $\pi_{SI} = 2.28 \times 0.33 = 0.75$.

The parameters π_D^0 and π_D^1 control the mortality rate of the infected. In the data, measured mortality from COVID-19 varies for many reasons, for example incomplete information on the number of infections and various mitigation policies across countries. We assume that, absent health care capacity constraints, the fatality rate is 0.5%, which within the range of parameters used in the recent papers studying COVID-19. This implies that $\pi_D^0 = 0.005(1 - \pi_I)$. We rely on Alvarez, Argente, and Lippi (2020) for setting π_D^1 and assume that $\pi_D^1 = 0.05(1 - \pi_I)$. The combined parameters for the SIR block imply that the fatality rate of the epidemic conditional on being infected ranges from 0.5% to 1.5% at the peak of the epidemic, absent lockdowns, when 20% of the population is infected. We adopt the values for lockdown effectiveness $\theta = 0.5$ and the upper bound on lockdown intensity $\overline{L} = 0.7$ from Alvarez, Argente, and Lippi (2020).

The cost of losing a life χ relates to the value of statistical life (VSL), which measures the marginal willingness to take on mortality risk. Viscusi and Masterman (2017) report estimates of the VSL across countries. In terms of annual consumption per capita, their estimation implies that the VSL is 184 for Argentina, 229 for Brazil, 224 in Mexico, 297 in Russia, 226 in South Africa, and 211 in Turkey.[7]

[6] This is also the duration used in Atkeson (2020) and Eichenbaum, Rebelo, and Trabandt (2020).

[7] For the U.S. Viscusi and Masterman (2017) find an VSL estimate of 9.6 million, which represents 207 times per capita consumption.

Using an annual interest rate of 2% and a residual life of 40 years, we can express the VSL in terms of weekly consumption. For example, a typical individual in Brazil is willing to give up 0.81% of weekly consumption forever to avoid a 0.1% increase in mortality rate. The median level of willingness to pay for 0.1% of mortality rate is 0.85% of weekly consumption. We use this calculation in setting the parameter χ, as the solution to the following equation,

$$\frac{1-\beta^{20\times 52}}{1-\beta}u(1) - 0.001\chi = \frac{1-\beta^{20\times 52}}{1-\beta}u(1-0.0085),$$

where we assume the representative COVID-19 victim has 20 years of residual life. The implied value for χ is 7295, given our parameters values for β, the utility specification and intertemporal elasticity. As Farboodi, Jarosch, and Shimer (2020) discusses, the relevant cost of the epidemic in terms of utility depends on the VSL and the fatality rate of the epidemic. Our parametrization is well within the range of values considered in the literature. Finally, we assume that a vaccines arrives 3 years after the outbreak of the epidemic, with $H = 156$. The arrival of the vaccines turns out to be irrelevant in our baseline model because herd immunity is reached before 3 years.

As in Arellano, Mateos-Planas, and Rios-Rull (2019), we assume that the default cost is a convex function of the default intensity,

$$\gamma(d) = [1 - \gamma_0 d^{\gamma_1}](1 - \gamma_2 \mathbb{1}_{d>0})$$

where the indicator $\mathbb{1}_{d>0}$ is 1 if d is positive so that a share γ_2 of productivity is lost if the country defaults at all, with any intensity. We adopt estimates for the default parameters γ_0 and γ_1 from Arellano, Mateos-Planas, and Rios-Rull (2019). The debt recovery κ is set to be 0.58 consistent with the evidence in Cruces and Trebesch (2013), once preemptive restructurings are excluded. Lastly, we choose the fixed cost parameter γ_2 to generate a 30% of debt-to-output ratio. Table 1 summarizes the parameter values and targeted moments. We set $\tilde{z} = 1$, a normalization.

Appendix B describes our computational algorithm. Briefly, we first compute the model without the epidemic. We then compute the epidemic-default model backwards, starting from the terminal period H when the vaccine arrives. As shown in the appendix, the period H problem is very similar to the pre-epidemic problem as no new infections occur. We use the equilibrium of the pre-epidemic model to set up the problem in the terminal period. The solution of the problem results in the time dependent functions for policies, bond price functions, and value functions.

Table 1: Parametrization and Moments

Parameters	Value	*Moments*
Preference		
Intertemporal elasticity $1/\sigma$	0.5	Standard value
Risk free rate r (annualized)	1%	International real rate of 1%
Discount factor β	0.9996	Domestic real rate 2%, emerging markets
Value of life χ	7295	VSL, Viscusi and Masterman (2017)
SIR and lockdown parameters		
SIR newly infected π_{SI}	0.75	Contagion rate $\mathcal{R}_0 = 2.28$
SIR resolution π_I	0.67	Mean recovery 18 days
SIR fatality π_D^0	0.165%	Baseline fatality rate 0.5%
SIR congestion π_D^1	1.65%	Alvarez, Argente, and Lippi (2020)
Lockdown effectiveness θ	0.5	Alvarez, Argente, and Lippi (2020)
Maximum lockdown $\overline{L}$	0.7	Alvarez, Argente, and Lippi (2020)
Debt and default parameters		
Long-term debt decay δ	0.003	Mean debt maturity 6 years
Debt recovery factor κ	0.58	Cruces and Trebesch (2013)
Default costs γ_0, γ_1	0.04, 1.62	Arellano, Mateos-Planas, and Rios-Rull (2019)
Default cost γ_2	0.0014	Mean debt-to-GDP 30%

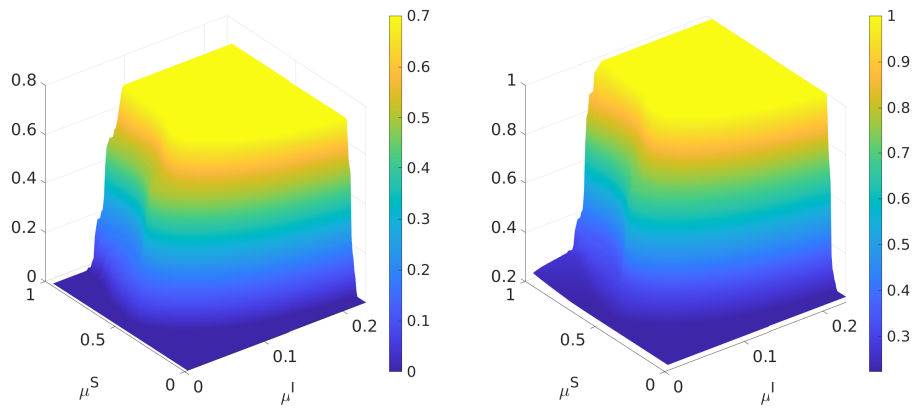
3.2 Policy Functions

We start by describing the policy functions at the start of the outbreak, $t = 0$. We consider the lockdown $\mathbf{L_t}(\mu_t, B_t)$ and partial default $\mathbf{d_t}(\mu_t, B_t)$ policies as a function of the epidemiological states for susceptible and infected (μ_0^S, μ_0^I) when the debt state is B_0 and no deceased, $\mu_0^D = 0$. The fraction of recovered is $\mu_0^R = 1 - \mu_0^S - \mu_0^I - \mu_0^D$.

Figure 2 plots lockdown and default as a function of the infected μ^I and susceptible μ^S for the pre-epidemic steady state level of debt. The disease is more contagious for higher levels of μ_0^S and/or μ_0^I. SIR dynamics (6) show that both μ^I and μ^S increase the number of newly infected μ^x, which in turn generates a sequence of future deaths. States of the world in which these two groups are large are therefore associated with higher benefits of lockdowns, because they can be most helpful in saving lives, which is reflected in the shape of the lockdown policy $\mathbf{L}$ that increases with both the fraction of infected and the fraction of susceptible.

The figure also shows that partial default responds to epidemiological states. High levels for (μ_0^S, μ_0^I) are associated with higher partial defaults. When the epidemic is very contagious, the government implements lockdowns that depress output dramatically. To support consumption, the government defaults on the debt.[8]

[8] Appendix C includes plots of the value V_0 and bond price q_0 as functions of the μ^S and μ^I states.



(a) Lockdown Intensity (b) Partial Default

Figure 2: Joint Behavior of Lockdowns and Default

Lockdown policies are also responsive to the debt holding of the economy. Governments with higher debt levels implement more relaxed lockdown policies. High debt comes with either lower consumption due to sizable debt repayments or with a default, which also depresses consumption, due to the cost of default. Lower consumption increases marginal utility which effectively makes lockdown less attractive. Therefore, lockdowns are more relaxed with higher levels of debt. Figure 3 shows the lockdown as a function of the number of currently infected (for a fixed number of susceptible μ^S) and two levels of debt, a low level, equal to 0, and a high one, equal to 60% of debt to output. As we show in Figure 2, lockdown intensity increases with the number of infected. At a higher debt level, lockdown starts when the infection rate is about 5.5% infected and climbs gradually to $\overline{L} = 70\%$ when the rate is around 12%. For a lower debt level, the economy can fight the infection with a tighter lockdown; it starts the lockdown when the infection rate is 4% and reacts more aggressively, reaching the peak of 70% lockdown when the infected is about 7%. We find that lockdowns tend to decrease with debt but their sensitivity is higher with respect to the epidemiological groups, μ^S and μ^I.

3.3 Baseline Dynamics under Optimal Lockdown

We now describe the dynamics of the economy as the epidemic evolves and the government jointly chooses borrowing, partial default, and lockdowns. Lockdowns help reduce fatalities, lowering the peak of infections, but generate a long debt crisis.

We focus first on baseline dynamics, when the economy in period $t = 0$ is hit by the epidemic and the government has outstanding debt at the steady-state level prevalent prior to the epidemic. Considering asymptomatic carriers, we start the outbreak with a small fraction of the population infected $\mu_0^I = 0.5\%$

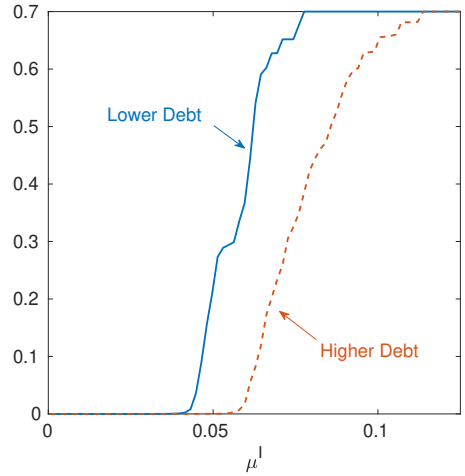


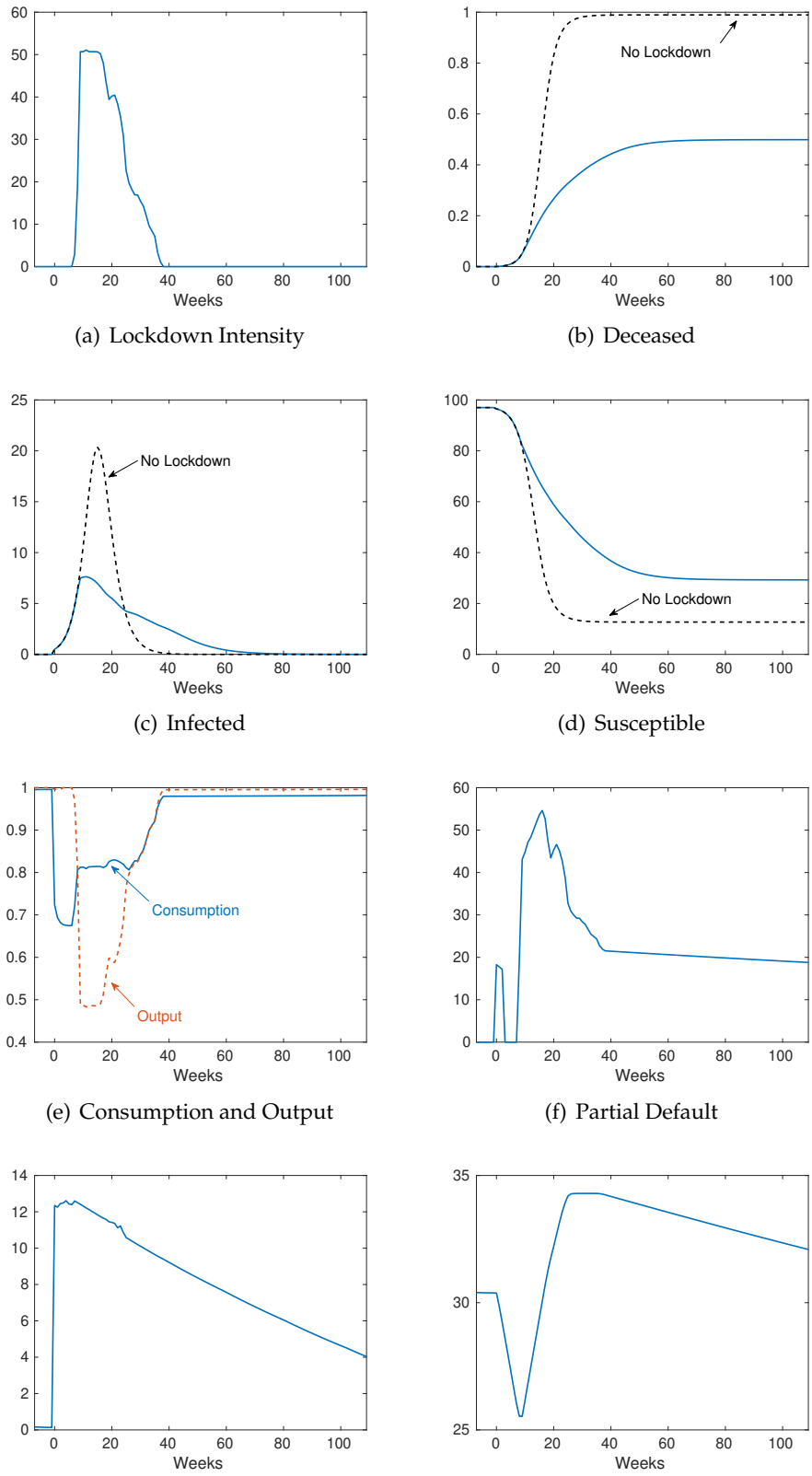
Figure 3: Lockdown Policy Function

and 3% recovered or immune to the disease $\mu_0^R = 3\%$. Figure 4 plots the time-path of lockdowns, the SIR dynamics, the economic variables of consumption and output, partial default, spreads, and debt.

Figure 4 (a) plots the time paths induced by the optimal lockdown policy. The lockdown starts two months after the outbreak. It remains at a 50% level for about two months, it then gradually winds down and lasts about eight months in total. Lockdown successfully saves lives. The solid line in Figure 4 (b) plots the evolution of the deceased μ_t^D under the optimal lockdown, while the dashed line shows the reference case without any lockdowns. Under this reference scenario, the number of deceased increases sharply and reaches 1% eleven months after the outbreak. Optimal lockdowns flatten the death curve; it takes about 16 months to reach the stable level of 0.5%. An optimal lockdown slashes the death toll by more than half.[9]

The next two panels of Figure 4, (c) and (d), plot the evolution of the infected and susceptible during the epidemic. In each panel, the solid lines are paths under optimal lockdown while the dashed lines are those without lockdowns. With the government's mitigation efforts in place, the number of infected reaches its peak, 7.6% of the initial population, three months after the outbreak. Without lockdowns, the number of infected would peak at 20%. As the epidemic progresses, the fraction of susceptible falls smoothly. After three years, the vaccine arrives, and all the susceptible become recovered, but the vaccine comes too late, and it is irrelevant for the outcomes both in our baseline and in the reference with no mitigation. For the no-lockdown reference paths, the fraction of susceptible level off at 13% about 9 months after the outbreak. The epidemiological parameters of $\mathcal{R}_0 = 2.28$ and 18 days to resolution of the infection imply a fairly rapid evolution of the disease. With optimal lockdowns under our baseline, the

[9]The decrease in fatalities with the optimal lockdown goes a long way towards the minimum feasible fatalities from the epidemic which equals 0.3% as derived in Hethcote (2000) and the pedagogical exposition in Moll (2020).



(a) Lockdown Intensity

(b) Deceased

(c) Infected

(d) Susceptible

(e) Consumption and Output

(f) Partial Default

(g) Spread

(h) Debt

Figure 4: Dynamics under Optimal Lockdown

fraction of susceptible level off at about 29% of the population at the end of the epidemic, or about 2.3 times higher, as a sizable share is spared from having to experience infection.

Figure 4 also plots the paths for output and consumption per capita, partial default, spreads, and debt. The epidemic alone, absent lockdowns, does not affect much output or consumption per capita. Combined with lockdown, however, the epidemic generates a debt crisis: partial default jumps to about 54% and spreads increase 300 basis points during the lockdowns. The lockdown generates a protracted debt crisis, with the economy continuing to default for 3.5 years. While the economy is in lockdown, government debt increases mainly due to the accumulated defaulted debt, arrears. The economy also receives new funding from its lenders, albeit at high-interest rates. These defaults and increasing indebtedness support consumption during the lockdown. In anticipation of the lockdown, the economy first goes through a short spell of austerity, by reducing consumption and debt before the lockdown begins. Once the lockdown starts, the economy smooths its consumption through partial default and borrowing, so that consumption declines considerably less than output. Consumption, however, remains depressed, about 1% below the initial steady-state, for much longer as the debt levels remained elevated for several years after the end of the lockdown.

These paths suggest that the epidemic, combined with mitigation policies that reduce output, can have long lasting detrimental consequences for debt crises. The lockdowns end in week 40, the economy continues to default until week 175, the debt level falls slowly and continues to be high relative to historical benchmarks past week 200. Figure 8 in Appendix D plots the same key outcome variables on a longer time frame, in excess of 200 weeks since the start of the pandemic, to capture more fully the longer-term consequences of the episode.

3.4 Dynamics under Exogenous Lockdown

Our baseline results features an optimal lockdown path, which starts two months after the outbreak, remains at about 50%, and gradually opens up. To highlight the importance of the timing of the start of the policy and the gradual opening, we consider an experiment with an exogenous lockdown, which starts one month after the outbreak and ends abruptly four months later. In Figure 5 we compare the time paths for this economy against our baseline. The exogenous lockdown shown in Figure 5 (a) does help reduce total deaths and the peak of the infection, though the reduction is smaller than under the optimal lockdowns of the baseline. Consumption decreases earlier but also recovers earlier than in the baseline.

In terms of the debt crisis, this intense 4 month lockdown generates a deep debt crisis that lasts over 5 years. The intense lockdown generates more intense defaults than in the baseline, which in turn cause a greater run-up in debt, as more of the defaulted debt accumulates. With higher debt, the government

defaults more and takes longer to recover from the debt crisis.

Comparing this exogenous lockdown policy to the optimal one can provide some lessons for emerging countries. Most emerging economies began school closings and restrictions in movement in early March, similar to U.S. states. These restrictions are still in effect as of early May. Our experiments suggest that it is best for these countries to open up only very gradually. Such policy would not only help with the health crisis but also it can potentially alleviate the severity of the debt crisis.

3.5 Dynamics under a Lower Debt Burden

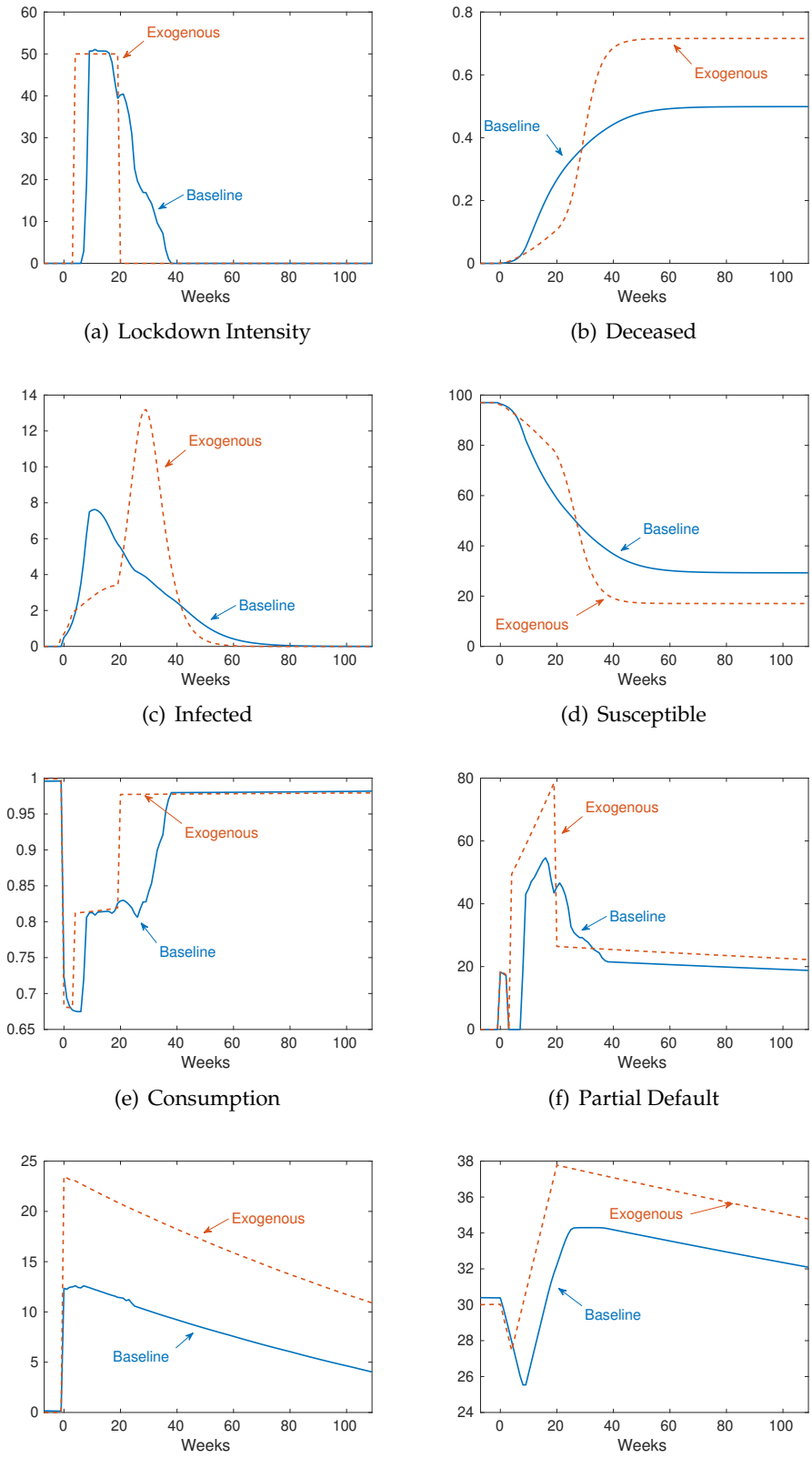
The fiscal capacity of the economy is a major determinant of the optimal lockdown policies in our baseline model. With ample fiscal space, the government can react with tighter and longer lockdowns that can save more lives. We explore here the time paths of the epidemiological and economic variables for an economy that starts the epidemic without outstanding debt. Figure 6 compares the dynamics of the baseline (solid lines) with those of an economy hit by the outbreak at a zero debt level (dashed lines). Without the initial debt burden, the economy responds with more aggressive lockdowns, reaching a peak of 57%, which is 6% higher than the peak in the baseline, and ends the lockdown later compared to the baseline. By week 37, the no-debt case still has a lockdown in effect at 23% intensity, while the baseline has already removed the lockdown. The combined effect of this more aggressive lockdown path is a reduction in deaths of about 0.05% of the initial population (10% of total deaths in the baseline).

Without any initial debt, the economy can maintain an almost constant level of consumption per capita. Lockdowns do not trigger any default for the first three months. Defaults only happens for a much briefer two month spell, with at most 22% intensity, as shown in Figure 6 (f). Spreads only rise slightly. Our model suggests that the indebtedness of emerging economies during the COVID-19 outbreak has important consequences for their ability to manage the strains from the epidemic, for supporting consumption, and savings lives.

3.6 Summary of Health, Economic, and Debt Crises

We now summarize our findings on the health, economic, and debt crises, under different lockdown scenarios and across different initial debt levels. We evaluate these scenarios with summary statistics for the crises.

We consider two summary measures for the health crisis: the eventual measure of deaths and the peak number of infections, both as a percentage of the total population. For the economic crisis, we report cumulative output losses in per capita terms, as well as the length and intensity of lockdowns. The cumulative output loss is the present discounted value of the output path relative to the analogous



(a) Lockdown Intensity

(b) Deceased

(c) Infected

(d) Susceptible

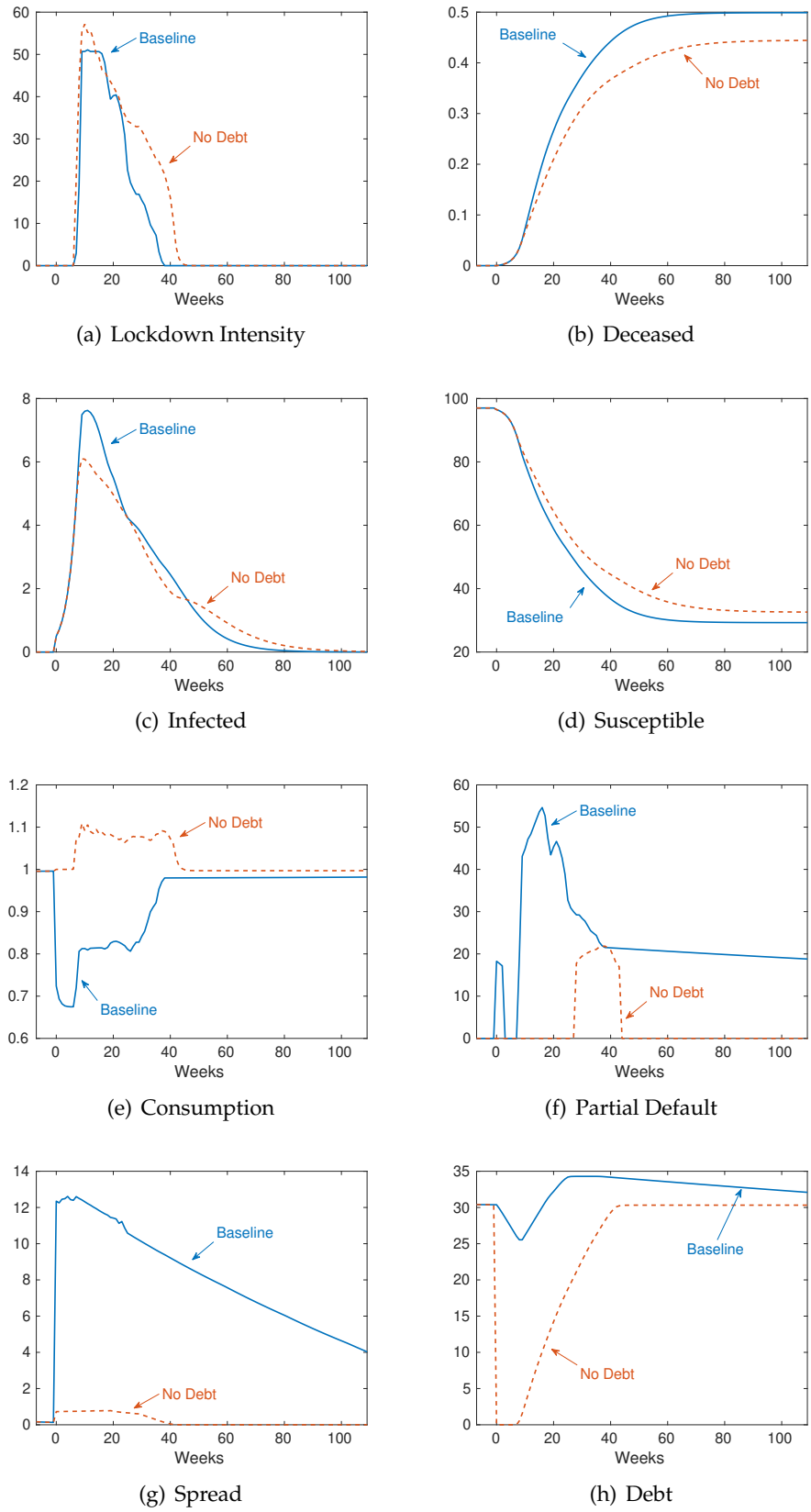
(e) Consumption

(f) Partial Default

(g) Spread

(h) Debt

Figure 5: Dynamics under Exogenous Lockdown



(a) Lockdown Intensity

(b) Deceased

(c) Infected

(d) Susceptible

(e) Consumption

(f) Partial Default

(g) Spread

(h) Debt

Figure 6: Dynamics under a Lower Debt Burden

output path pre-epidemic, discounted at the risk-free rate $r = 1\%$ and expressed in terms of average pre-epidemic annual output $\overline{Y}$. In each comparison, the pre-epidemic paths are constructed for the same initial debt level as for the paths during the epidemic. The length of the lockdown is the number of months with positive lockdown intensity $L_t > 0$. We asses the intensity of lockdowns with both the mean and maximum of their intensity, conditional on it being positive. For the debt crisis we focus on measures of partial default. We provide three summary measures for the debt crisis, akin to our statistics for lockdowns: the length of the crisis, measured by the number of months with positive partial default $d_t > 0$, and default intensity, using both the mean and maximum of partial default conditional on positive values.

We also report welfare losses from the epidemic for both the country and international lenders. The country suffers from the epidemic because of the loss of life, the loss of output from lockdowns, and costs associated with the prolonged debt crisis. To evaluate the country's welfare loss, we consider a consumption equivalence measure $c^{\text{eq}}(\mu_0, B_0)$, at the outbreak of the epidemic, implicitly defined by

$$\frac{1}{1-\beta} u(c^{\text{eq}}(\mu_0, B_0)) = V_0(\mu_0, B_0), \tag{18}$$

where $V_0(\mu_0, B_0)$ is the value function at time 0. The value function V_0 reflects both the stream of consumption and the stream of deaths. Our consumption equivalence measure summarizes these two streams into one quantity, which is the constant per capita consumption flow that equates a value absent any mortality risk. We express the welfare loss in two ways. The first measure, *CE flow*, is the percentage deviation from the pre-epidemic consumption equivalence, $c^{\text{eq}}(\mu_0, B_0)/c^{\text{pre,eq}}(B_0) - 1$, where $c^{\text{pre,eq}}(B_0)$ is the pre-epidemic consumption equivalence when initial debt is B_0. The second measure is the present value of the losses using the discount of the country β given by

$$\text{CE present value} = \frac{c^{\text{eq}}(\mu_0, B_0) - c^{\text{pre,eq}}(B_0)}{1-\beta}. \tag{19}$$

Lenders also suffer losses because the epidemic triggers a debt crisis that they did not forecast, a drop in the market value of the bonds they hold, a capital loss. We report welfare losses for lenders as the change in the market value of debt B_0

$$\text{Lenders' loss} = q_0(\mu_0, B_0)B_0 - q^{\text{pre}}(B_0)B_0. \tag{20}$$

where $q^{\text{pre}}(B_0)$ is the bond price before the epidemic.

Table 2 reports the health, economic, and debt crises summary measures for our baseline (with optimal

lockdowns), exogenous lockdowns, and no lockdowns. Without any mitigation policies, the epidemic eventually kills about one percent of the population, with 20% of the population simultaneously infected at the peak. The epidemic itself need not lead to economic or debt crises. The loss of lives reduces welfare, with an associated consumption equivalence loss of 2.65% relative to the pre-epidemic level. In the baseline, optimal lockdowns reduce the loss of life by half, with 0.5% of the population eventually deceased. Lockdowns save lives but with costs in terms of reductions in consumption and a prolonged and deep debt crisis. In the baseline the cumulative output decline is about 19% of pre-epidemic output; about 17.4% of the losses are due to lockdowns, while the rest are due to default costs. A lengthy debt crisis follows the lockdown, 43 months with a mean intensity of 22% and a maximum of 55%. Overall the epidemic is very costly for the economy. The welfare loss in the baseline economy corresponds to 1.80% of consumption equivalence every period, which equals 87% of pre-epidemic annual output in present value. Lenders are also slightly worse off from the epidemic, by about 1% of pre-epidemic annual output, via unexpected capital losses. The epidemic is an order of magnitude worse for the country than for its lenders.

The second column of Table 2 reports the case of exogenous lockdowns, 4 months at 50% intensity. As explained above, the exogenous lockdown starts earlier and is shorter than our baseline optimal lockdown. This policy saves fewer lives but results in similar economic crisis and a more severe debt crises. The present value loss of output with this lockdown policy is similar to the loss of in the optimal lockdown. The 4 month lockdown of intensity 50% induces a 5 year long debt crisis with intense defaults. The higher death toll and more severe debt crisis leads to a consumption equivalence loss higher than in the baseline, with a loss of 2.33% of consumption equivalence every period, or 113% in present value. Lenders lose about 2% of annual output in this scenario.

As shown above, debt levels matter for the health and economic outcomes. To highlight the role of debt we report our summary measures as we vary the initial debt-to-output ratio, ranging from 0 to 50% in 10% steps. We view this exercise as shedding light on how the indebtedness of countries at the outbreak can affect the outcomes of the epidemic. Table 3 reports the health, economic, and debt crises measures for each of these debt levels.

A higher initial debt reduces consumption, as the economy allocates more of its output to debt repayment. The higher marginal utility of consumption increases the effective price of lockdowns, and the government shortens the lockdown from 9.3 months for zero debt to 7.5 months for 50% debt-to-output. The mean intensity of lockdown is also curtailed from 35% to 31% over the same debt-to-output range.[10] Less mitigation arising from higher initial debt comes with a cost in lives, the fraction of deceased increases

[10]Lockdowns are not strictly monotonic with the level of debt because of default; more debt generates more default, which mitigates the reduction in consumption and supports tighter lockdowns. This non-monotonicity, however, is very minor.

Table 2: Health, Economic, and Debt Crisis: Lockdown Policies

	Baseline: Optimal Lockdown	Exogenous Lockdown	No lockdown
Health Crisis			
Deceased (% Pop)	0.50	0.72	0.99
Peak Infections (% Pop)	7.6	13.2	20.4
Economic Crisis			
Output loss (%)	−18.9	−17.8	0.0
Lockdown			
Length (months)	7.8	4	0
Intensity, max (%)	51	50	–
Intensity, mean (%)	29	50	–
Debt crisis			
Default			
Length (months)	43	66	–
Intensity, max (%)	55	78	–
Intensity, mean (%)	22	24	–
Welfare losses			
Country CE flow (%)	−1.80	−2.33	−2.65
Country CE present value (% output)	−87	−113	−129
Lender (% output)	−1.2	−1.9	0.0

Notes: Deceased are eventual deaths. Deceased and peak infections are reported in terms of population. Output losses are present discounted per capita cumulative losses during the epidemic in terms of annual output relative to the pre-epidemic paths. Lockdown and default length are the number of months with positive values, mean and max intensities are their respective statistics conditional on positive values. Welfare losses for the country are reported as consumption equivalence (CE) measures (equation (18)) relative to pre-epidemic level, both as percent change in flow and as present value changes in units of output. Welfare loses for the lender are market value losses on the outstanding debt relative to pre-epidemic values in units of output (equation (20)).

from 0.44% to 0.50%.

Output losses are non-monotonic with respect to the initial debt level. Starting debt-free, the output loss is high due to a lengthy and intense lockdown, about 24.6% of annual output, relative to pre-epidemic. When debt-to-output is at 20%, the associated output loss is close to 18%. When highly indebted, at 50% debt-to-output, output losses are higher and about 22% due to the prolonged and intense default spell, with its associated costs.

The severity of the debt crises increases exponentially with level of debt at the onset of the epidemic. The length of the partial default spell increases from four months, for zero initial debt, to about three and a half years (43 months) in the baseline, and to over 13 years with a 50% initial debt-to-output. The intensity of default also rises rapidly with debt, to the 100% maximum intensity, at 50% debt-to-output.

The table also reports welfare losses from the epidemic for the country and its lenders, evaluated relative to pre-epidemic levels for the same initial book value of debt. The epidemic induces welfare losses to both, although as before the country's losses are an order of magnitude higher. Welfare losses for both groups increase in the initial debt burden.

These comparisons suggest that the level of debt that emerging markets have at the outbreak of the pandemic can shape eventual outcomes, not only in terms of default and consumption but also epidemic mitigation and loss of life.

3.7 Debt Relief Counterfactuals During COVID-19

In our environment, international financial assistance can have a profound impact on the epidemic outcome, because the economy's debt burden weighs heavily on the government's ability to mitigate through lockdowns. The International Monetary Fund, the Inter-Development Bank, and other international organizations have rapidly established debt relief programs for countries during the COVID-19 epidemic, to support them in dealing with the crisis. We now use our model to measure the benefits from debt relief programs. We find that financial assistance programs can have a large positive social value because they shorten the debt crisis and allow for better mitigation policies that save lives.

The experiment we consider is a debt relief policy that lowers the face value of debt by 10% of pre-epidemic output. In this scenario, an external finance assistance entity reduces the face value of the debt by buying back from lenders part of the outstanding debt of the country. The buyback is conducted at market prices. Our analysis in this section relates to the work on debt buybacks by Bulow, Rogoff, and Dornbusch (1988) and Aguiar, Amador, Hopenhayn, and Werning (2019). We describe next, in more detail, the experiment and the associated gains for the country and its lenders, as well as the cost to the financial assistance entity.

Table 3: Health, Economic, and Debt Crisis: Debt Levels

Initial debt to output	0%	10%	20%	30% (baseline)	40%	50%
Health Crisis						
Deceased (% Pop)	0.44	0.47	0.49	0.50	0.50	0.50
Peak Infections (% Pop)	6.1	6.9	7.4	7.6	7.7	7.7
Economic Crisis						
Output loss (%)	-24.6	-21.4	-18.3	-18.7	-20.0	-21.7
Lockdown						
Length (months)	9.3	7.8	8.0	7.8	7.8	7.5
Intensity, max (%)	57	61	51	51	51	51
Intensity, mean (%)	35	36	30	29	29	31
Debt crisis						
Default						
Length (months)	4	6	7	43	112	161
Intensity, max (%)	22	28	36	55	94	100
Intensity, mean (%)	20	22	27	22	28	35
Welfare loss						
Country CE flow (%)	-1.56	-1.59	-1.67	-1.80	-1.79	-1.78
Country CE present value (% output)	−76	−78	−81	−87	−87	−86
Lender (% output)	−0	−0.1	−0.2	−1.2	−2.0	−3.4

Notes: The table reports outcomes for different initial debt to output ratios at the epidemic outbreak. Deceased are eventual deaths. Deceased and peak infections are reported in terms of population. Output losses are present discounted per capita cumulative losses during the epidemic in terms of annual output relative to the pre-epidemic paths. Lockdown and default length are the number of months with positive values, mean and max intensities are their respective statistics conditional on positive values. Welfare losses for the country are reported as consumption equivalence (CE) measures (equation (18)) relative to pre-epidemic level, both as percent change in flow and as present value changes in units of output. Welfare loses for the lender are market value losses on the outstanding debt relative to pre-epidemic values in units of output (equation (20).

Consider a model economy that is hit by the epidemic with an initial debt level B_0. The market value of this debt in secondary markets is equals to $q_0(\mu_0, B_0)B_0$ and depends on the bond price at time 0, where μ_0 is the initial epidemiological state. The financial assistance entity buys back debt in order to reduce the face value of outstanding debt by 10% of pre-epidemic output $\overline{Y}$. The resulting debt level for the country is $B_0^{\text{new}} = B_0 - 0.1 \times \overline{Y}$. This buyback is done at market prices, so that required spending is $q_0(\mu_0, B_0^{\text{new}})(B_0^{\text{new}} - B_0)$; the price for the buyback depends on the eventual debt level, which in general will be higher because $B_0^{\text{new}} < B_0$. This price change induces a capital gain for the lenders, who thus benefit from the debt reduction program. The change in payoff for the lenders is given by $[q_0(\mu_0, B_0^{\text{new}}) - q_0(\mu_0, B_0)]B_0$.

The country benefits from the buyback because it lowers its outstanding debt. We measure these gains with the difference in values $V_0(\mu_0, B^{\text{new}}) - V_0(\mu_0, B_0)$, expressed in terms of the difference in present value of consumption equivalence. This facilitates comparison with the gains and losses of the other parties, the lenders and the financial assistance entity. Our welfare-based measure of the country's gains reflect not only the increase in countries wealth, but also the fact that lower indebtedness changes behavior and impacts welfare through three channels. First, lowering debt reduces incentives to default and allows the country to save on default costs, which in our model are a social waste. Second, lower debt gives the government fiscal space to implement tighter lockdowns, that saves more lives. Third, the country is relatively impatient, $\beta(1+r) < 1$, and benefits from tilting consumption.

An alternative way to measure the country's gains is based on the change in the market value of the debt $q_0(\mu_0, B_0)B_0 - q_0(\mu_0, B_0^{\text{new}})B_0^{\text{new}}$. This market based measure is used for example in Bulow, Rogoff, and Dornbusch (1988) when studying the buybacks of Bolivia in the 1980s. We focus on the consumption equivalence measures, as opposed to the market base measures, because in our model with costly equilibrium default and value for life, the market based measures miss the additional benefits that come about from debt relief that change default paths and mitigation policies.

In Table 4 we present gains and losses to the involved parties when the initial debt for the economy, before the debt relief program, ranges from 10% to 50% of output. As described above, the gains and losses for the three parties are

$$\text{Country gain CE present value} = [c^{\text{eq}}(\mu_0, B_0^{\text{new}}) - c^{\text{eq}}(\mu_0, B_0)]/(1-\beta) \tag{21}$$

$$\text{Lenders gain} = [q_0(\mu_0, B_0^{\text{new}}) - q_0(\mu_0, B_0)]B_0 \tag{22}$$

$$\text{Financial assistance cost} = q_0(\mu_0, B_0^{\text{new}})(B_0^{\text{new}} - B_0) \tag{23}$$

reported as percentages of pre-epidemic annual output $\overline{Y}$, where $c^{\text{eq}}(\mu_0, B)$ is computed according to

equation (18).

Consider the outcomes of the debt relief program for the baseline economy, which starts with a face value of debt to output of 30% that is reduced by 10% with the program. The country gains from this debt reduction by 13.7%, the lender gains by about 0.9%, while the cost to the financial assistance entity is 9.9%. Most of the gains accrue to the country, about 94% (13.7 out of a total of 13.7+0.9), because the change in bond prices from this program are modest. The gains from the debt relief program are large for the economy because debt matters for future default paths and mitigation policies. We found that the counterfactual gains from the same debt relief program in the pre-epidemic economy would be about half and equal to 7%. Absent the epidemic the economy does not experience a debt crisis nor requires fiscal space to save lives, which lowers the value of the debt relief program.

The table also reports outcomes for economies that start with other initial debt levels. The gains for lenders monotonically increase with debt because the capital gains from the debt relief program are the highest for the most indebted economy. For example, the gain for lenders is highest when the economy starts with 50%, where they capture 25% of total gains. The cost of financial assistance modestly decreases with debt, because the buyback price is lower when debt is high. The gains for the country from a 10% face value reduction are, however, non-monotonic with respect to debt, reaching a high for the economy with an average debt level. Debt relief benefits most these economies because it allows them to both dampen the debt crisis and implement more aggressive lockdowns, saving more lives. Economies with low debt levels have large fiscal space to manage the epidemic and financial assistance programs do not change their outcomes much. For highly indebted economies, modest debt relief programs also do not alter their lockdown choices much, as they continue to experience long debt crisis even with financial assistance.

The debt relief program has varying gains and losses for the three parties: country, lenders, and financial assistance entity. The social benefits from the program depend on the weight that society puts on the three parties. We consider two sets of weighting functions: a case that puts equal weight on the gains and losses of all three parties and a weighting function that puts equal weight on the country and the financial assistance entity and disregards the capital gains of the lenders. The second panel in Table 4 reports the social value for the program under the two weighting functions, reported as a percentage of the cost of financial assistance.

We find that the social value of debt relief is positive for economies that start with debt-to-output ratios above 20%. Under equal weighting of all three parties, the social value of debt relief increases with the country's debt and can be very large, reaching 75% for an economy with 50% debt-to-output. The social value of debt relief is negative when initial debt to output is 10% because of our choice to evaluate

the present value of consumption equivalence gains using the economy's discount factor β, which is lower than the inverse of the risk-free rate. Evaluating this present values using instead the risk-free rates increases estimated gains substantially and results in them always being positive independent on the initial level of debt.

Consider now the weighting function that weights only the country and the financial assistance entity, while excluding the lenders. The social value of debt relief continues to be large for the majority of the cases and is non-monotonic with respect to debt. This social value peaks for middle levels of debt, reaching here 38% of the cost of the program. These findings suggest debt relief policies which prioritize gains to countries rather than to their lenders should be targeted towards marginal countries, on the brink of a debt crisis. Debt relief is most useful for them because the program changes their behavior the most, towards avoiding deep debt crises and saving more lives during the epidemic.

Table 4: Gains from Debt Relief: 10% Debt-to-Output Reduction

Initial debt-to-output	10%	20%	30%	40%	50%
Welfare Gains (% output)					
Country gain CE present value	8.7	11.0	13.7	12.7	11.6
Lenders gain	0.0	0.1	0.9	3.1	3.9
Financial assistance cost	−10	−9.9	−9.9	−9.6	−8.8
Social Value of Debt Relief (% financial assistance cost)					
Weights country, lenders, financial assistance	−12	11	47	64	75
Weights country, financial assistance	−12	11	38	33	31

Note: Welfare gains for the country are consumption equivalence (CE) measures in present value as in equation (21)). Lender gains arise from capital gains induced by debt relief given by equation (21) and the cost for the financial assistance entity is market value given by (23)). The social values are the sum of gains and losses under and equal weighting function for the three parties and under a weighting function that excludes lenders reported as percentage of the cost of the financial assistance.

4 Discussion and Other Results

The recent active literature studying the impact of COVID-19 in the world has explored additional aspects of the epidemic. Here we relate how some of the findings from this literature apply to emerging markets that face debt crises.

Smart Mitigation In our work we have focused on nondiscriminatory lockdowns for controlling the disease. An important strategy for combating the epidemic is the use of smart mitigation strategies such as trace and test, which is explored in Chari, Kirpalani, and Phelan (2020) and Berger, Herkenhoff, and Mongey (2020), age specific lockdowns, which is explored in Glover, Heathcote, Krueger, and Rios-Rull (2020), Favero, Ichino, and Rustichini (2020), and Acemoglu, Chernozhukov, Werning, and Whinston

(2020), and sector specific lockdowns explored in more detail in Baqaee, Farhi, Mina, and Stock (2020) and Azzimonti, Fogli, Perri, and Ponder (2020). Implementing these smart mitigations are akin in our model to reducing the parameter π_{SI}. Such policies would be very specially useful in emerging markets because of the additional costs from nondiscriminatory lockdowns that generate costly debt crisis.

External Shocks The world pandemic also brings additional shocks to emerging markets as reduced demand for their exports and disruptions in global supply chains. In the context of our model these shocks can be introduced by modifying the underlying productivity paths for the emerging economy upon the outbreak of the epidemic. We have experimented in our model with declines in productivity and find that for modest declines the results are unchanged. Using estimates for the IMF that forecasts a global recession of about 5% during 2020, we fed into the model a decline in productivity of 5% for one year followed by a slow recovery. The baseline results do not change substantially. The reason is that in the context of the lockdowns and the debt crisis a 5% decline in productivity is a small additional shock to already large epidemic response. Larger and more persistent declines in productivity do make the debt and health crisis more severe.

Externalities An emerging consensus is developing on the need of additional government imposed lockdowns during the epidemic because of negative externalities arising from consumers not being quarantined. Farboodi, Jarosch, and Shimer (2020) and Eichenbaum, Rebelo, and Trabandt (2020) show that although consumers would have incentives to self-quarantine, they would choose insufficient quarantines relative to a planner, because they don't internalize their behavior affect the well being of the economy as a whole. In our work, we have not directly studied the epidemic externality but instead have considered directly the government problem that internalizes these externalities. Nevertheless, debt crises bring additional negative externalities arising from lockdowns as consumers would not internalize that by self-quarantining they are causing a debt crisis. We leave for future work analyzing whether government controlled lockdowns are larger or smaller than what consumers would choose in the presence of debt crises.

Vaccines and Treatment Emerging markets face the additional hurdle of limited medical and scientific resources for managing the epidemic. As vaccines and treatments become available, accessing those resources could lead to additional expenses and constraints. Our model suggests, however, that if these medical resources become available far in the future, their cost does not matter. In our model we have assumed that the vaccine and/or treatment arrives free of cost 2 years after the outbreak of the epidemic. Given this timing, it turns out the vaccine/treatment is irrelevant for the emerging country because herd

immunity had already been reached by then.

5 Conclusion

In this paper we studied the COVID-19 epidemic in emerging markets. We developed a framework that combines an epidemiology model with a sovereign default model. Our results suggest that this epidemic threatens not only a large health and economic crisis, but also a prolonged debt crisis. We also show that default risk makes lockdowns more costly because they limit the fiscal capacity of governments to support consumption. These additional costs from default risk in turn result in a deeper health crisis, with more lives lost to the epidemic.

Through counterfactuals, we show that debt relief programs can have profound positive effects: debt relief supports consumption, can reduce the severity of the debt crisis, and can save lives. In this context, our work suggests that the recent debt relief policies promoted by the International Monetary Fund and other international organizations, are right on target to combat the costs associated with COVID-19. We hope that our work contributes to the discussion on the optimal domestic and international policy response to the COVID-19 pandemic in emerging markets.

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Online Appendix to "Deadly Debt Crises: COVID-19 in Emerging Markets"

by Cristina Arellano, Yan Bai, and Gabriel Mihalache

A Definition of Epidemic Equilibrium

The epidemic equilibrium consists of the sequence of functions of consumption $\mathbf{c_t}(\mu_t, B_t)$, the government's policy of borrowings $\mathbf{B_{t+1}}(\mu_t, B_t)$, default $\mathbf{d_t}(\mu_t, B_t)$, and lockdown $\mathbf{L_t}(\mu_t, B_t)$, the value function $V_t(\mu_t, B_t)$, the bond price schedule $q_t(\mu_t, B_{t+1})$, and the epidemiological state $\boldsymbol{\mu_{t+1}}(\mu_t, B_t)$ that summarizes the mass of susceptible, infected, and recovered for period $t = 0, 1, 2, ...$ such that given the initial state (μ_0, B_0) and the availability of the vaccine at period H,

(i) For period $t > H$, the epidemic is eliminated, $\mu_t^S = 0$, $\mu_t^I = 0$, and $\mu_t^R = \mu_H^R + \mu_H^S + (1 - \pi_D^0/\pi_I)\mu_H^I$ under the assumption that at period H a fraction of π_D^0/π_I fraction of infected dies and $1 - \pi_D^0/\pi_I$ fraction gets recovered. The optimal lockdown intensity is zero, $\mathbf{L_t}(\mu_t, B_t) = 0$. The government's borrowing and default policy, the value function, and the bond price schedule are the same as the pre-epidemic ones, $V_t(\mu_t, B_t) = V^{\text{pre}}(B_t)$, $\mathbf{d_t}(\mu_t, B_t) = \mathbf{d}^{\text{pre}}(B_t)$, $\mathbf{B_{t+1}}(\mu_t, B_t) = \mathbf{B}^{\text{pre}}(\mu_t, B_t)$, and $\mathbf{q_t}(\mu_t, B_{t+1}) = \mathbf{q}^{\text{pre}}(B_{t+1})$.

(ii) For period $t \leq H$, taking as given the value function and the bond price schedule at period $t+1$, the value function and the government's policy solves the following problem,

$$V_t(\mu_t, B_t) = \max_{B_{t+1},\, d_t \in [0,1],\, L_t \in [0,\overline{L}]} \left[u(c_t) - \chi\phi_{D,t}\right] + \beta V_{t+1}\left(\mu_{t+1}(\mu_t, L_t), B_{t+1}\right)$$

subject to the resource constraint

$$N_t c_t + (1 - d_t)(\delta + r)B_t = \tilde{z}\phi(d_t)N_t(1 - L_t) + q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1})(B_{t+1} - (1 - \delta)B_t),$$

the SIR dynamics (6)-(9), the deaths $\phi_{D,t} = \pi_D(\mu_t^I)\mu_t^I$ for $t < H$ and $\phi_{D,t} = \pi_D^0/\pi_I\mu_t^I$ for $t = H$, and $N_t = \mu_t^S + \mu_t^I + \mu_t^R$.

(iii) For period $t \leq H$, taking as given the government's policy in period $t+1$ and the epidemiological state, the bond price schedule satisfies

$$q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1}) = \frac{1}{1+r}\left\{(\delta + r)(1 - \mathbf{d_{t+1}}) + \left[1 - \delta + \kappa(\delta + r)\mathbf{d_{t+1}}\right] q_{t+1}(\boldsymbol{\mu_{t+2}}, \mathbf{B_{t+2}})\right\}.$$

(iv) The evolution of the epidemiological state $\boldsymbol{\mu_{t+1}}(\mu_t, B_t)$ is consistent with the SIR dynamics (6)-(9) and $L_t = \mathbf{L_t}(\mu_t, B_t)$.

B Computational Algorithm

In this appendix, we first augment the original problem with taste shocks on borrowing B', following Dvorkin, Sanchez, Sapriza, and Yurdagul (2018) and Gordon (2019). We then describe our computation algorithm, which works backwards for value function and policy functions.

The taste shock on B' helps stability and convergence of our model with long-term defaultable debt. The set of B is a discrete set and each element in the set is associated with an i.i.d taste shock distributed Gumbel (Extreme Value Type I). Let the parameter control taste shocks be ρ_B.

Model with taste shocks In this appendix, we switch the state from $(\mu_t^S, \mu_t^I, \mu_t^R)$ in the main text to $(\mu_t^S, \mu_t^I, \mu_t^D)$. Given that the sum of the four group is always 1, $\mu_t^S + \mu_t^I + \mu_t^R + \mu_t^D = 1$, the two states are equivalent. By abuse of notation, we use $\mu_t = (\mu_t^S, \mu_t^I, \mu_t^D)$ as the SIR state. The government's problem becomes,

$$W_t\left(\mu_t, B_t, B_{t+1}\right) = \max_{L_t, d_t} \; u(c_t) - \pi_D(\mu_t^I)\mu_t^I + \beta V_{t+1}\left(\mu_{t+1}(\mu_t, L_t), B_{t+1}\right) \tag{24}$$

subject to the resource constraint and the SIR dynamics

$$c_t = \tilde{z}\phi(d_t)(1 - L_t) + \frac{\left\{B_{t+1} - \left[1 - \delta + \kappa(\delta + r)d_t\right] B_t\right\} q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1}) - (\delta + r)(1 - d_t)B_t}{1 - \mu_t^D} \tag{25}$$

$$\mu_t^x = \pi_{SI}(1 - \theta L_t)^2 \mu_t^S \mu_t^I \tag{26}$$

$$\mu_{t+1}^S = \mu_t^S - \mu_t^x \tag{27}$$

$$\mu_{t+1}^I = \pi_I \mu_t^I + \mu_t^x \tag{28}$$

$$\mu_{t+1}^D = \mu_t^D + \pi_D(\mu_t^I)\mu_t^I \tag{29}$$

Let the optimal default and lockdown choice be $d_t(\mu_t, B_t, B_{t+1})$ and $L_t(\mu_t, B_t, B_{t+1})$. The choice probabilities of B_{t+1} are given by

$$Pr\left(B_{t+1}|\mu_t, B_t\right) = \frac{\exp((W_t(\mu_t, B_t, B_{t+1}) - \overline{W}_t(\mu_t, B_t))/\rho_B)}{\sum_{\tilde{B}_{t+1}} \exp((W_t(\mu_t, B_t, \tilde{B}_{t+1}) - \overline{W}_t(\mu_t, B_t))/\rho_B)}$$

where $\overline{W}$ is the maximum value $\overline{W}_t(\mu_t, B_t) = \max_{B_{t+1}} W_t(\mu_t, B_t, B_{t+1})$. The expected value V satisfies

$$V_t(\mu_t, B_t) = \overline{W}_t(\mu_t, B_t) + \rho_B \log \left\{ \sum_{B_{t+1}} \frac{W_t(\mu_t, B_t, B_{t+1}) - \overline{W}_t(\mu_t, B_t)}{\rho_B} \right\}.$$

We can write the bond price schedule for any choice of (L_t, B_{t+1}) under state μ_t as,

$$\begin{aligned} q_t\left(\mu_{t+1}, B_{t+1}\right) = \frac{1}{1+r} \sum_{B_{t+2}} & Pr\left(B_{t+2} | \mu_{t+1}, B_{t+1}\right) \times \\ & \left\{(\delta + r)(1 - d^*_{t+1}) + \left[1 - \delta + \kappa(\delta + r) d^*_{t+1}\right] q_{t+1}\left(\mu_{t+2}(\mu_{t+1}, L^*_{t+1}), B_{t+2}\right)\right\} \end{aligned}$$

where $d^*_{t+1} = d_{t+1}(\mu_{t+1}, B_{t+1}, B_{t+2})$ and $L^*_{t+1} = L_{t+1}(\mu_{t+1}, B_{t+1}, B_{t+2})$.

Computational algorithm Note that at period t, the state of the deceased μ^D_t only affects the period t equilibrium through the per-capita debt burden $B_t/(1 - \mu^D_t)$ and per-capita borrowings $B_{t+1}/(1 - \mu^D_t)$. In particular, μ^D_t does not affect current or future losses of death. At our benchmark SIR parameters, the eventual death is less than 1% without lockdowns. Hence its impact on per-capita debt is very small. Therefore, we approximate the original problem with a simplified problem with SIR state only includes (μ^S_t, μ^I_t) and rewrite the resource constraint as

$$c_t = \tilde{z}\phi(d_t)(1 - L_t) + \left\{B_{t+1} - \left[1 - \delta + \kappa(\delta + r) d_t\right] B_t\right\} q_t(\mu_{t+1}(\mu_t, L_t), B_{t+1}) - (\delta + r)(1 - d_t) B_t. \quad (30)$$

We verified that the simplified problem approximates well the original one. This simplification, however, reduces the state space by one variable and saves computation time dramatically.

We first solve the stationary equilibrium, which is both the solution for the pre-epidemic equilibrium and the solution after introducing of vaccine. We then solve backwards the equilibrium under SIR.

1. Stationary equilibrium

   (a) Guess value function $V^s(B)$ and bond price schedule $q^s(B')$.

   (b) For each (B, B'), solve for the optimal default decision. We find the solution d^* of the following first order condition of partial default,

   $$-\tilde{z}\phi'(d) = (\delta + r) B \left[1 - \kappa q^s(B')\right].$$

   Evaluate the consumption per capita c from the resource constraint

   $$c = \tilde{z}\phi(d) + \left\{B' - \left[1 - \delta + \kappa(\delta + r) d\right] B\right\} q^s(B') - (\delta + r)(1 - d) B$$

at three default choices, $d = \{d^*, 0, 1\}$, and pick the default value that gives the highest consumption per capita under (B, B'). Let the optimal default be $d^s(B, B')$ and the corresponding consumption be $c^s(B, B')$.

(c) Calculate the value under (B, B') and the optimal default $d^s(B, B')$. We then evaluate the value of $W^s(B, B') = u(c^s(B, B')) + \beta V^s(B')$.

(d) Calculate the probability of each B'

$$Pr\left(B'|B\right) = \frac{\exp((W^s(B, B') - \overline{W}^s(B))/\rho_B)}{\sum_{\tilde{B}'} \exp((W^s(B, \tilde{B}') - \overline{W}^s(B))/\rho_B)},$$

and the maximum of W^s for each b, $\overline{W}^s(B) = \max_{B'} W^s(B, B')$.

(e) Update $V^s(B)$, $V^s(B) = \overline{W}^s(B) + \rho_B \log\left\{\sum_{B'} \frac{W^s(B,B') - \overline{W}^s(B)}{\rho_B}\right\}$.

(f) Update the bond price schedule $q^s(B')$

$$q^s\left(B'\right) = \frac{1}{1+r} \sum_{B''} Pr\left(B''|B'\right) \left\{(\delta + r)(1 - d(B', B'')) + \left[1 - \delta + \kappa(\delta + r) d^s(B', B'')\right] q^s\left(B''\right)\right\}$$

(g) Check whether V^s and q^s converge. If so, we are done. Otherwise go back to step 1(a).

2. Period H problem (with vaccine)

In period H a vaccine is available. All susceptible individuals recover, a fraction π_D^0/π_I of the infected dies, while the rest recover.

$$W_H\left(\mu_H, B_H, B_{H+1}\right) = \max_{d_H} \left\{u(c_H) - \left[\frac{\pi_D^0}{\pi_I}\right] \mu_H^I \chi + \beta V^s\left(B_{H+1}\right)\right\} \tag{31}$$

subject to the resource constraint (30). Let the solution be $V_H^{\{0\}}(\mu_H, B_H)$, $d_H^{\{0\}}(\mu_H, B_H, B_{H+1})$, $Pr^{\{0\}}\left(B_{H+1}|\mu_H, B_H\right)$, $q_H^{\{0\}}(\mu_{H+1}, B_{H+1})$, and $L_H^{\{0\}}(\mu_H, B_H, B_{H+1}) = 0$.

3. Period $t < H$ problem

(a) Start with $q_{t+1}^{\{0\}}(\mu_{t+2}, B_{t+2})$, $d_{t+1}^0(\mu_{t+1}, B_{t+1}, B_{t+2})$, $L_{t+1}^0(\mu_{t+1}, B_{t+1}, B_{t+2})$, $Pr^{\{0\}}\left(B_{t+2}|\mu_{t+1}, B_{t+1}\right)$, and $V_{t+1}^{\{0\}}(\mu_{t+1}, B_{t+1})$.

(b) Construct bond price $q_t^{\{1\}}\left(\mu_{t+1}, B_{t+1}\right)$

$$\begin{aligned} q_t^{\{1\}}\left(\mu_{t+1}, B_{t+1}\right) = \frac{1}{1+r} \sum_{B_{t+2}} Pr^{\{0\}}\left(B_{t+2}|\mu_{t+1}, B_{t+1}\right) \Big\{&(\delta + r)(1 - d_{t+1}^{\{0\}}) \\ &+ \left[1 - \delta + \kappa(\delta + r) d_{t+1}^{\{0\}}\right] q_{t+1}^{\{0\}}\left(\mu_{t+2}(\mu_{t+1}, L_{t+1}^{\{0\}}), B_{t+2}\right)\Big\} \end{aligned}$$

where $d_{t+1}^{\{0\}} = d_{t+1}^{\{0\}}(\mu_{t+1}, B_{t+1}, B_{t+2})$ and $L_{t+1}^{\{0\}} = L_{t+1}^{\{0\}}(\mu_{t+1}, B_{t+1}, B_{t+2})$.

(c) Solve for the optimal default and lockdown policy for each B_t and B_{t+1}

$$W_t\left(\mu_t, B_t, B_{t+1}\right) = \max_{L_t, d_t} u(c_t) - \pi_D(\mu_t^I)\mu_t^I\chi + \beta V_{t+1}^{\{0\}}\left(\mu_{t+1}(\mu_t, L_t), B_{t+1}\right)$$

subject to the resource constraint (30) and the SIR dynamics. Specifically, we search over the grid of L_t to find the maximum value. For each $(\mu_t, B_t, B_{t+1}, L_t)$, we find the solution d^* to the following equation,

$$-\tilde{z}\phi'(d)(1 - L_t)(1 - \mu_t^D) = (\delta + r)B_t\left[1 - \kappa q_t^{\{1\}}(\mu_{t+1}(\mu_t, L_t), B_{t+1})\right]$$

We pick the default value of $\{d^*, 0, 1\}$, which has the highest consumption per capita under $(\mu_t, B_t, B_{t+1}, L_t)$. Let the optimal default and lockdown choice be $d_t^{\{1\}}(\mu_t, B_t, B_{t+1})$ and $L_t^{\{1\}}(\mu_t, B_t, B_{t+1})$.

(d) Calculate the probability of choosing each B_{t+1}

$$Pr^{\{1\}}\left(B_{t+1}|\mu_t, B_t\right) = \frac{\exp((W_t(\mu_t, B_t, B_{t+1}) - \overline{W}_t(\mu_t, B_t))/\rho_B)}{\sum_{\tilde{B}_{t+1}} \exp((W_t(\mu_t, B_t, \tilde{B}_{t+1}) - \overline{W}_t(\mu_t, B_t))/\rho_B)}$$

with the maximum value given by $\overline{W}_t(\mu_t, B_t) = \max_{B_{t+1}} W_t(\mu_t, B_t, B_{t+1})$.

(e) Calculate the period t's value and bond price function for $t - 1$

$$V_t^{\{1\}}(\mu_t, B_t) = \overline{W}_t(\mu_t, B_t) + \rho_B \log\left\{\sum_{B_{t+1}} \frac{W_t(\mu_t, B_t, B_{t+1}) - \overline{W}_t(\mu_t, B_t)}{\rho_B}\right\}$$

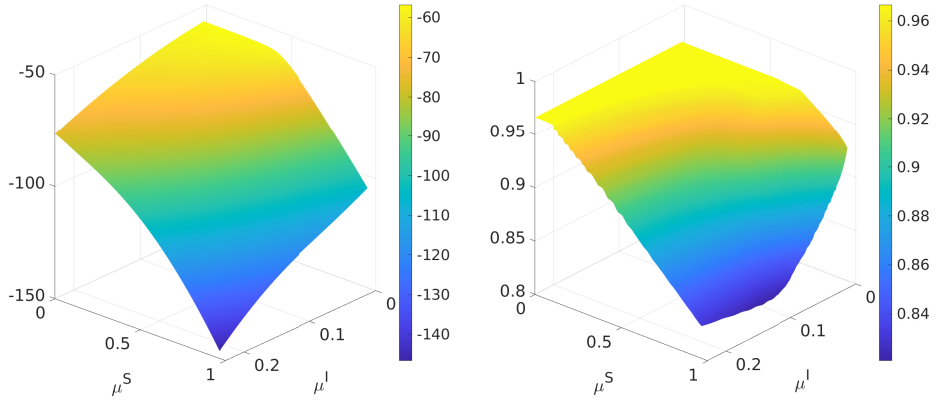
(f) Assign the functions with superscript $\{1\}$ to functions with superscript $\{0\}$. Go back to step 3(a) until $t = 0$.

C Value Function and Bond Price Schedule

We plot the period-0 value function and bond price schedule in Figure 7.

D Baseline Dynamics under Optimal Lockdown: extended weeks

This section reports extended time paths for optimal lockdowns, consumption and output per capita, partial default, spreads, and debt for the baseline economy. The epidemic gives rise to long lasting debt

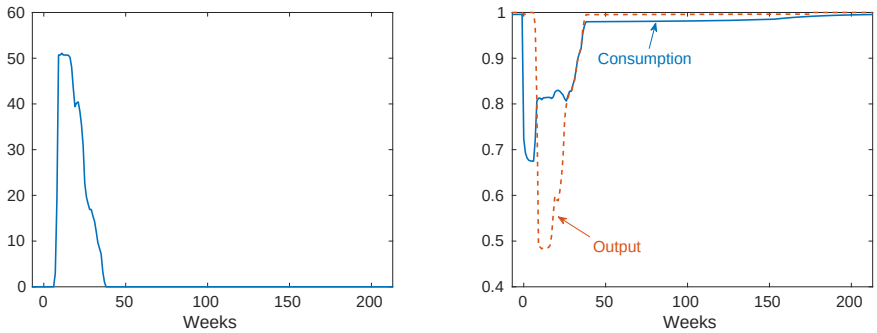


(a) Bond Price Schedule

(b) Value Function

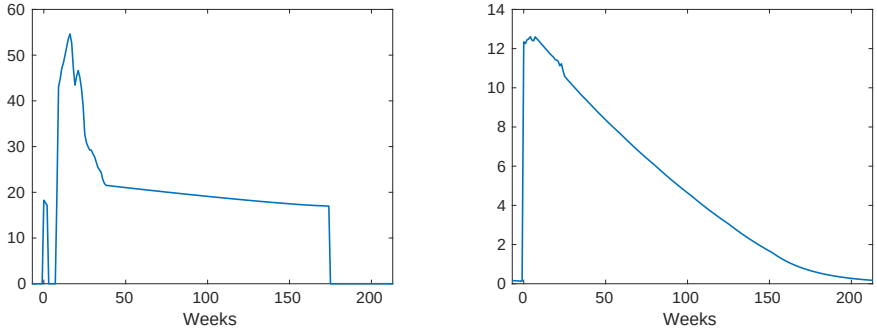
Figure 7: Bond Price Schedule and Value Function

crises with defaults happening well past the resolution of the health crisis. Lockdowns end 48 periods after the epidemic outbreak, while partial defaults end much later, 172 periods after.



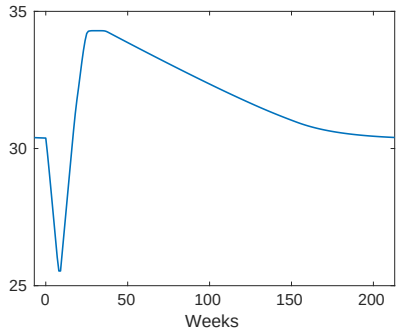
(a) Lockdown Intensity

(b) Consumption and Output



(c) Partial Default

(d) Spread



(e) Debt

Figure 8: Dynamics under Optimal Lockdown: Extended Weeks