

# *Working Paper*



**Research Department  
Federal Reserve Bank  
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**Social Insurance and Taxation  
Under Sequential Majority Voting and  
Utilitarian Regimes**

S. Rao Aiyagari and Dan Peled\*

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ABSTRACT

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Corresponding author: Dan Peled, Industrial Engineering and Management, Technion, Haifa 32000, Israel (fax: 972-4-235194; e-mail: peled@turbo.technion.ac.il)

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Keywords: Proportional taxes; sequential majority voting; utilitarian government

\*Aiyagari, Federal Reserve Bank of Minneapolis; Peled, Technion, Israel Institute of Technology and the University of Western Ontario. We thank Ed Green for helpful discussions. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

SOCIAL INSURANCE AND TAXATION UNDER  
SEQUENTIAL MAJORITY VOTING AND UTILITARIAN REGIMES

S. Rao Aiyagari<sup>1</sup> and Dan Peled<sup>2</sup>

June 1994

ABSTRACT<sup>3</sup>

It is often argued that with a positively skewed income distribution (median less than mean) a majority voting over proportional tax rates would result in higher tax rates than those that maximize average welfare, and will accordingly reduce aggregate savings. We reexamine this view in a capital accumulation model, in which distorting redistributive taxes provide insurance against idiosyncratic shocks, and income distributions evolve endogenously. We find small differences of either sign between the tax rates set by a majority voting and a utilitarian government, for reasonable parametric specifications. We show how these differences reflect a greater responsiveness of a utilitarian government to the average need for the insurance provided by the tax-redistribution scheme. These conclusions remain true despite the fact that the model simulations produce *positively skewed* distributions of total income across agents.

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<sup>1</sup>Federal Reserve Bank of Minneapolis

<sup>2</sup>Technion - Israel Institute of Technology and The University of Western Ontario.

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## 1. INTRODUCTION

The conventional wisdom appears to be that the democratic nature of political institutions combined with income inequality create strong pressures for income redistributions through fiscal policies in general, and in particular, through the tax system. These redistributive measures may entail large inefficiency costs of various kinds. For instance, a recent paper by Person and Tabellini (1991) shows how inequality in income and wealth in combination with a determination of tax rates through majority voting can lead to redistributive taxation which inhibits savings and growth. Thus it would appear that distributional issues play a central role in determining tax and saving rates, and failing to incorporate these features in economic models can be very misleading.

This paper examines the difference in taxation and savings under alternative political mechanisms by solving a reasonably parameterized dynamic capital accumulation model with heterogeneous agents. In our model, heterogeneity, and a consequent welfare role for redistributive taxation, arise from uninsurable idiosyncratic shocks to agents' labor incomes. Taxes on total income, however, affect the incentives to accumulate capital, and, therefore, the total income distribution in subsequent periods. We solve the model, accounting for these two crucial impacts of proportional taxes, under two political regimes: majority voting, and a utilitarian government acting to maximize the weighted average of agents' welfare. We find that there are no significant differences between the tax rates chosen by a sequence of utilitarian governments, and those chosen under majority voting for

empirically plausible parameter values. Moreover, those small differences can go either way for alternative reasonable specifications of the agents' preferences and the distribution of their individual characteristics. In fact, we find that the greater is the need for the insurance provided by the tax-redistribution scheme - the more likely it is that taxes under majority voting will be *smaller* than those under a utilitarian government.

We then try to sort out the contributions to saving rates of the missing insurance markets and the redistributive tax scheme, respectively, under the two regimes. We find very small differences between the saving rates under majority voting and a utilitarian government, in accord with the small tax rate differences. There are also small differences between the full insurance and the no-insurance versions of the model, holding tax rates fixed, with slightly higher saving rates when insurance markets are missing. However, our computations suggest that increased social insurance, which reduces the severity of idiosyncratic risks, can account for a relatively large reduction in the saving rates.

We describe a model in which heterogeneous and infinitely lived agents vote at each date on a tax rate for the following period, taking into account that future tax rates will be voted on again in the future, and also taking into account the interrelationship between the tax rates chosen and the income distribution across agents. In particular, tax rates chosen in the future will impact on behavior and welfare of agents in the present, and hence, on current choices of tax rates.<sup>4</sup>

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<sup>4</sup>The model can also be thought of as a model of the optimal provision of a

In our model, heterogeneity arises due to missing insurance markets and the consequent inability of ex-ante identical individuals to fully diversify away idiosyncratic risks. We focus on this particular source of heterogeneity for the following reasons. Carroll (1991) reports that 43% of the respondents to the FRB's 1983 *Survey of Consumer Finances* said that the most important reason for saving was to be prepared for emergencies, while only 15% mentioned retirement as the primary reason for saving. There is evidence that individual wealth holdings are highly volatile - about 60% of households were in different wealth decile in 1985 than in 1982. About 30% moved up, and 30% moved down. Only people in the top and bottom decile were more likely to stay put than to move to another decile. Incomes are also volatile - about 66% of households were in different income deciles in 1985 compared to their position in 1982. Such large movements of households across the wealth and income distributions over fairly short periods of time suggest that recurrent temporary idiosyncratic shocks may be quite important. Moreover, such large volatilities in individual incomes and wealth can explain the observed increase in the provision of various kinds of social insurance programs, financed by distorting taxes. Increased social insurance decreases the severity of the missing insurance markets, with a consequent reduction in precautionary savings.

Fiscal policies involving distorting taxes can provide some insurance, albeit, at the cost of generating disincentives to work and savings, (Eaton

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public good, which is a perfect substitute for the private good, financed by (distorting) income tax.

and Rosen 1980, Varian 1980). The optimal trade-off between these two considerations obviously depends on the mechanism for policy selection. Thus, in principle there can be systematic and large differences between the levels of such fiscal policies under majority voting and utilitarian regimes. However, our computations suggest that these differences are insignificant for reasonable specifications of the economy, despite the fact that the model generates *positively skewed* total income distributions which are the relevant distributions for agents' preferred tax rates. Moreover, agents with lower incomes benefit more from redistributive taxation than agents with high income. Consequently, taxes set by a utilitarian regime, which weigh all agents equally, are more responsive to the need for social insurance, as measured by the degree of risk aversion in agents preferences or the variance of future income shocks. It turns out that even for moderate degrees of these measures of risk - this effect suffices to make the tax rates *higher* under a utilitarian regime than under majority voting. Our quantitative analysis can also shed some light on the size of the negative effect on saving arising from increased social insurance.

The early literature on majority voting on tax rates, focused on static models and examined the static distortions associated with proportional taxation (Sheshinski 1972, Romer 1975, Roberts 1977, and Hellwig 1986, among others). Persson and Tabellini (1991) were among the first to focus on *dynamic* implications of majority voting. They use an overlapping generation model with two period lived agents, and a single good in each date. This leads to the feature that neither past policies nor expectations of future policies have any effect on the current distribution of wealth, and hence on current choices

of tax rates. Further, their preference specification is linear in the agent specific characteristic which is relevant for voting over tax rates. Consequently, a utilitarian government acts to maximize the welfare of the *mean* voter, and chooses a zero (distorting) tax rate period by period. Under majority voting on the other hand, a positive tax rate is chosen to maximize the welfare of the *median* voter, who is motivated only by current income redistribution. As can be expected, that tax rate increases with the skewness of the wealth distribution. In contrast, the dependence of the wealth distribution on past and expected future tax rates is unavoidable within a capital accumulation model with heterogeneous infinitely lived agents. Our analysis takes into account the implied dynamic feedback effects between tax rates chosen in different periods, and allows us to isolate it from the impact of the specification of preferences on the chosen tax rate. The specific questions we focus on are:

- (1) Is the tax rate under majority voting necessarily higher than in a utilitarian regime, when the degree of inequality is endogenously determined?
- (2) How much do tax and saving rates differ under majority voting compared to a utilitarian regime, and what determines those differences?
- (3) How large is the effect of increased social insurance on the saving rate?

In section 2 we present a simple static model of majority voting on taxes, that captures the conventional views on the subject, and produces the result that majority voting tax rates will be higher than those chosen by a utilitarian government. In section 3 we describe a dynamic model, which allows

us to focus on feedback between tax rates chosen at different dates, via their impact on income distributions and individual choices. In section 4 we present our choice of some key specifications of the model, and in section 5 we present the results of computing the stochastic steady state equilibria for various choices of the remaining parameters.

## 2. A SIMPLE LINEAR INCOME TAX EXAMPLE

In order to appreciate the intricate dynamic relationships involved under majority voting with endogenous distribution of agents' types, it is instructive to consider first a one-period economy with a fixed distribution of heterogeneous agents, which has been used before to examine majority voting on distorting taxes (Sheshinski 1972, Romer 1975, Roberts 1977, and Hellwig 1986). In this example, a fixed *positively skewed* distribution of agents' types will result in a higher tax from a median voter regime than from a utilitarian one. This example emphasizes the crucial role of preferences specifications and the exogenous degree of income inequality in generating the conventional tax rate ranking under the two regimes.

Assume there is a continuum of agents of size unity. Labor productivities are distributed according to the density function  $f(w)$ , with support  $[w_1, w_2]$ , where  $0 < w_1 < w_2 \leq 1$ . We assume that  $E\{w^2\} > w_2^2/2$ , where  $E$  is the expectations operator. Individual productivities and labor supply cannot be observed; only labor income is observable. Agents are endowed with one unit of time, to be allocated between work and leisure, while the amount of effective labor supplied is the amount of work time multiplied by the agent's

productivity. The technology is simple - each unit of effective labor produces one unit of the consumption good. Labor income before tax,  $y$ , is thus the same as effective labor input,  $w\ell$ . Agents' preferences over consumption,  $c$ , and work time are given by:

$$(1) \quad U(c, \ell) = c - \ell^2/2.$$

The linear income tax schedule is given by:

$$(2) \quad t = -\alpha + \tau y, \quad 0 \leq \tau \leq 1,$$

where  $y$  is income,  $\tau$  is the tax rate, and  $\alpha$  is the lump sum redistributed subsidy. Consumption is then given by:

$$(3) \quad c = \alpha + (1-\tau)w\ell,$$

and individual utility maximizing choice of  $\ell$  implies:

$$(4a) \quad y(w, \tau) = (1-\tau)w^2$$

$$(4b) \quad c(w, \tau) = \alpha + [w(1-\tau)]^2$$

$$(4c) \quad \ell(w, \tau) = w(1-\tau)$$

and a utility level:

$$(4d) \quad V(w, \tau) = \alpha + [w(1-\tau)]^2/2$$

Government balanced budget with zero government consumption restricts  $\alpha$  and  $\tau$  according to:

$$(5) \quad \int [-\alpha + \tau y(w, \tau)] f(w) dw = 0,$$

so that given the tax rate  $\tau$ , the utility of an agent with productivity  $w$  is given by:

$$(6) \quad V(w, \tau) = \tau(1-\tau)E(w^2) + [w(1-\tau)]^2/2.$$

A natural measure for social welfare, to be denoted by  $W$ , is:

$$(7) \quad \begin{aligned} W(\tau) &= \int V(w, \tau) f(w) dw \\ &= E(w^2)(1-\tau^2)/2. \end{aligned}$$

From (7) it obvious that the utilitarian optimal linear income tax schedule, (which is also Pareto optimal), is given by  $\alpha = \tau = 0$ . Since all agents' marginal utility of consumption are identical, and all agents receive the same weight in the social welfare function - there is no redistributive motive, and efficiency is achieved by equating marginal rates of substitution between work and consumption to marginal rates of transformation, for each agent.

Majority voting can be easily analyzed in this example, since, from (6), preferences over marginal tax rates are quadratic and concave in  $\tau$ , and the most preferred value of  $\tau$  given  $w$  is:

$$(8) \quad \tau(w) = \text{Max} \left\{ \frac{E(w^2) - w^2}{2E(w^2) - w^2}, 0 \right\}$$

The most preferred tax is weakly decreasing in  $w$ , and with single peakedness there are no strategic voting considerations, (i.e., misrepresentation of preferences - Arrow 1963, Black 1958). For  $w^2 < E(w^2)$ , the peak occurs for  $\tau \in (0,1)$ , whereas for  $w^2 \geq E(w^2)$  the peak occurs at  $\tau = 0$ .

Under majority voting the tax rate will be the one most preferred by the median voter. If  $w^m$  is the median productivity, then  $(w^m)^2 < E(w^2)$  implies that the tax rate  $\tau$  is positive. From (4a), this will hold whenever  $y^m$  (median income) is less than  $E(y)$ , i.e., whenever the income distribution is positively skewed, an empirically reasonable hypothesis. It should be clear, however, that even for this static example, the tax could be higher under a utilitarian regime for a utility function,  $U(c,\ell)$ , which is sufficiently concave in  $c$ .

What we do next is to consider a dynamic version of linear income taxation in a capital accumulation model in which the degree of inequality is endogenously determined.

### 3. DYNAMIC LINEAR INCOME TAXATION UNDER SEQUENTIAL MAJORITY VOTING

The basic structure is the standard capital accumulation model of Brock and Mirman (1972), with a continuum of heterogeneous infinitely lived agents (of size unity) as in Bewley (1980). Heterogeneity arises because, by assumption, labor endowments or labor productivities are subject to individual

specific random and uninsurable shocks. We assume identical and independent distribution for these idiosyncratic shocks, over time and across agents, so that ex-ante all agents are identical. The i.i.d. assumption ensures that agents differ across only one characteristic in terms of their induced preferences over tax rates.

To avoid time consistency problems in the presence of capital, we assume one period commitment to the linear income tax schedule voted on each period, to be in effect in the subsequent period. We only consider a stochastic steady state equilibrium, which will be formally defined below. To simplify the analysis, we assume away the distorting effect of proportional income tax on labor by assuming inelastic labor supply.

The distribution of agent types and the tax rate are determined simultaneously in equilibrium. The degree of inequality, which will affect the tax rate is determined endogenously, rather than assumed at the outset. Since the tax is on income from all sources, it has a disincentive effect on savings and capital accumulation. However, it also provides partial insurance against idiosyncratic labor shocks. Consequently, equilibrium tax rates involve a trade-off between these two effects, both under a utilitarian regime, and under majority voting.

Each agent has a random amount  $\ell_t$  of inelastically supplied labor endowment in each period  $t$ .  $\ell_t$  is distributed identically and independently over time as well as across agents, with a cumulative distribution function denoted by  $F(\cdot)$ , normalized such that  $E(\ell) = 1$ . Denote by  $f(k, \ell)$  a standard

neoclassical aggregate production function, where  $k_t$  denotes per capita capital in period  $t$ , and  $\delta$  is the depreciation rate of capital. Individual specific quantities are denoted by a superscript, while per capita aggregates are not.

Equilibrium factor prices are then given by:

$$(9a) \quad r_t = f_1(k_t, 1) - \delta = \text{pre-tax return to capital,}$$

$$(9b) \quad w_t = f_2(k_t, 1) = \text{pre-tax wage.}$$

In period  $t$  there is a proportional income tax, denoted  $\tau_t$ , whose proceeds are rebated lump sum in equal per capita amounts. Each agent of type "a" maximizes the following expected discounted sum of utilities of consumption:

$$(10) \quad W^a = E \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t^a) \right\}$$

subject to:

$$(11a) \quad c_t^a + k_{t+1}^a = y_t^a; \quad c_t^a \geq 0, \quad k_{t+1}^a \geq 0, \quad (\text{no borrowing})$$

$$(11b) \quad y_{t+1}^a = k_{t+1}^a + (1-\tau_{t+1})(w_{t+1} \ell_{t+1}^a + r_{t+1} k_{t+1}^a) + \tau_{t+1}(w_{t+1} + r_{t+1} k_{t+1}^a),$$

where:

$y_t^a$  = total resources of agent a at date  $t$  after taxes and transfers,

$\tau_{t+1}$  = tax rate on income applicable in period  $t+1$ , chosen and announced in period  $t$ , before saving decisions for period  $t+1$  are made.

Let  $H_t(y)$  denote the distribution of total resources among the agents in period  $t$ .  $H_0$  is a given initial condition. Note that

$$(12) \quad \int y dH_t(y) = f(k_t, 1) + (1-\delta)k_t,$$

so that knowledge of  $H_t$  determines  $k_t$ , and, hence,  $w_t$  and  $r_t$ .

Let  $V(y_t^a, H_t)$  be the optimal value function for agent "a" with total resources  $y_t^a$  in period  $t$  with the current distribution of total resources  $H_t$ . The agent's beliefs about the future evolution of resource distributions and tax rates are assumed to be:

$$(13a) \quad H_{t+1} = \Phi(H_t, \tau_{t+1}),$$

$$(13b) \quad \tau_{t+1} = T(H_t).$$

Given (13), an agent's optimization problem can be expressed as follows.

$$(14a) \quad W(y_t^a, H_{t+1}, \tau_{t+1}) = \text{Max} \left\{ U(y_t^a - k_{t+1}^a) + \beta \int V(y_{t+1}^a, H_{t+1}) dF(\ell_{t+1}^a) \right\}$$

over  $k_{t+1}^a$  subject to (13b),

$$(14b) \quad V(y^a, H) = W(y^a, \Phi(H, T(H)), T(H)).$$

The optimal asset demand for agent "a" is obtained by solving the maximization on the right side of (14a) to yield:

$$(15) \quad k_{t+1}^a = A(y_t^a, H_{t+1}, \tau_{t+1}).$$

**Definition 1:** A dynamic sequential median voter equilibrium consists of:

- (i) a law of motion  $\Phi(\cdot)$  for  $H$ , and
- (ii) a policy rule for the tax rate  $T(\cdot)$ ,

such that (13) - (15) hold, and in addition:

$$(16) \quad T(H_t) = \underset{\tau_{t+1}}{\text{Argmax}} W(y_t^m, \Phi(H_t, \tau_{t+1}), \tau_{t+1}),$$

where  $y_t^m$  is the median total resources according to  $H_t$ ,

$$(17) \quad \int F \left( \frac{[y' - \tau_{t+1}(w_{t+1} + r_{t+1}k_{t+1}) - (1 + (1 - \tau_{t+1})r_{t+1})A(y, H_{t+1}, \tau_{t+1})]}{w_{t+1}(1 - \tau_{t+1})} \right) dH_t(y)$$

$$\equiv H_{t+1}(y'), \text{ for all } y',$$

where  $H_{t+1}$  is given by (13a).

Condition (16) says that  $\tau_{t+1} = T(H_t)$  is the optimal choice for the median voter at time  $t$ , given the belief that future tax rates and resource distributions evolve according to (13). Note that the median voter takes into account the effect of variations in  $\tau_{t+1}$  on  $H_{t+1}$ . Condition (17) requires that updating the distribution of total resources from period  $t$  to  $t+1$ , respecting individual asset demand rules and the distribution of total resources at date  $t$ , agrees with the law of motion  $\Phi$ .

Under a sequence of utilitarian governments the tax policy must satisfy the following condition which differs from (16) in that the tax rate is chosen to maximize an equally weighted utilitarian social welfare function.

$$(16') \quad T(H_t) = \underset{\tau_{t+1}}{\text{Argmax}} \int W(y_t, \Phi(H_t, \tau_{t+1}), \tau_{t+1}) dH_t(y_t)$$

The above definitions of equilibria are very hard to compute because one of the state variables is a distribution function. Therefore, we adopt below versions of the above definitions, which approximate the stochastic steady state equilibrium, and can be computed for any specification of the economy. In a stochastic steady state the equilibrium sequence of tax rates and per capita capital (and, hence, prices), will be constant over time. The constant values of the tax rate and the per capita capital stock will be denoted by  $\tau$  and  $k$ , respectively. At time  $t$ , imagine that  $k_{t+2}, k_{t+3}, \dots = k$ , and  $\tau_{t+2}, \tau_{t+3}, \dots = \tau$ . Let  $V(y_{t+1}^a, k, \tau)$  be the optimal value function for an agent of type "a" with total resources  $y_{t+1}^a$  in period  $t+1$ , who sees the constant sequences  $k$  and  $\tau$  for per capita capital and tax rates from period  $t+2$  onwards. The agent's optimization problem at date  $t$  can be stated as follows.

$$(18a) \quad W(y_t^a, k_{t+1}, \tau_{t+1}, k, \tau) \equiv \max_{k_{t+1}^a} \left\{ U(y_t^a - k_{t+1}^a) + \beta \int V(y_{t+1}^a, k, \tau) dF(\ell_{t+1}) \right\}$$

subject to (11b),

$$(18b) \quad V(y^a, k, \tau) \equiv W(y^a, k, \tau, k, \tau).$$

The optimal asset demand for the agent is obtained by solving the maximization on the right side of (18a), to yield:

$$(19) \quad k_{t+1}^a = s(y_t^a, k_{t+1}, \tau_{t+1}, k, \tau), \text{ (saving function).}$$

**Definition 2:** A *stochastic steady state median voter equilibrium* consists of:

- (i) a distribution function  $H(y)$  of total resources across agents;
- (ii) a per capita stock of capital,  $k^*$ ;
- (iii) a tax rate  $\tau^*$ ;
- (iv) a function  $\phi(\tau', k^*, \tau^*)$  relating per capita stock of capital and the tax rate next period;

such that the following conditions hold:

$$(20) \quad k' = \phi(\tau', k^*, \tau^*), \text{ for any } \tau'$$

$$(21) \quad k' = \int s(x, k', \tau', k^*, \tau^*) H'(x) dx,$$

$$(22) \quad \tau^* = \underset{\tau'}{\text{Argmax}} W(y^m, \phi(\tau', k^*, \tau^*), \tau', k^*, \tau^*)$$

where  $H(y^m) = 1/2$ , (i.e.  $y^m$  is the median total resources);

$$(23) \quad k^* = \phi(\tau^*, k^*, \tau^*)$$

$$(24) \quad w^* = f_2(k^*, 1)$$

$$(25) \quad r^* = f_1(k^*, 1) - \delta$$

$$(26) \quad \int F([y' - \tau^*(w^* + r^*k^*) - (1 + (1 - \tau^*)r^*)s(y, k^*, \tau^*, k^*, \tau^*)] / [w^*(1 - \tau^*)]) H'(y) dy \\ \equiv H(y'), \text{ for all } y'.$$

In this definition, the role of the total resources distribution and its law of motion are being replaced by the constant equilibrium values of the tax

rate and of mean capital holding under the time invariant equilibrium distribution. The time invariance property of that distribution is reflected in (26), which requires that  $H$  regenerates itself in equilibrium under agents' optimal saving rules.

Definition 2 of a stochastic steady state is somewhat unsatisfactory, but unavoidable for computational reasons. The basic problem is that agents' conjectures off the steady state path are needed to evaluate the effects of choosing a tax rate different from the steady state tax rate. The median voter takes into account the effect of  $\tau_{t+1}$  on  $k_{t+1}$  via  $\phi(\cdot)$ , assuming unchanged state variables from  $t+2$  onwards. It seems rather artificial to restrict beliefs about future relevant state variables to their time-invariant equilibrium values.

Note that we need to verify single peakedness of  $W(\cdot)$  in  $\tau'$ , to be free of strategic misrepresentation problems, and weak monotonicity of  $\tau^*$  in  $y^m$  in order to associate the equilibrium tax rate with the one most preferred by the median voter.

In order to contrast the median voter equilibrium with a utilitarian regime, we assume that the government at date  $t$  maximizes social welfare at date  $t$ , taking as given the (constant) sequence of tax rates chosen by future governments. This leads to a similar definition of a *stochastic steady state utilitarian equilibrium*, in which equation (22) is replaced by:

$$(27) \quad \tau^* = \underset{\tau'}{\operatorname{Argmax}} \int W(y, \phi(\tau', k^*, \tau^*), \tau', k^*, \tau^*) H'(y) dy.$$

Appendix A provides details of our computational procedure for the equilibrium in definition 2.

#### 4. MODEL PARAMETERIZATION AND COMPUTATION METHOD

We have fixed some of the fundamentals of the economy, and computed the median voter and the utilitarian equilibria for alternative values of the remaining parameters. We set the period length to 4 years, corresponding to the typical length between elections in most democratic countries. We set  $\beta$ , the utility discount factor to 0.85, corresponding to an annual time preference rate of 4.15 percent. The production function is a Cobb-Douglas constant returns to scale, with capital share of output, denoted by  $\alpha$ , set at 0.36. The depreciation rate of capital is set to 0.34 over the 4-year period, which is equivalent to 9.87 percent on an annual basis. These values would imply a saving rate of 0.2370 in a balanced path full insurance, or a representative agent equilibrium, since the saving rate equals  $\delta k/f(k,1) = \delta[kf_1/f]/f_1 = \delta\alpha/(r+\delta)$ , and  $r$  equals  $(\beta^{-1}-1)$ . The utility function is assumed to be of the constant relative risk aversion type,  $U(c) = [c^{1-\mu} - 1]/(1-\mu)$ .

With these specifications fixed, we have considered alternative choices of agents' degree of risk aversion,  $\mu$ , and different distributions of agents' productivity shocks,  $F(\ell)$ . In particular, we let  $\mu \in \{0.5, 0.9, 1.1, 1.5, 1.9, 6\}$ , ranging from low to high degrees of risk aversion. We used five different labor shock distributions, labeled DFL1, ..., DFL5, which were all defined over a labor grid of five intervals, each of length 0.5, with the grid points:  $\{1,$

1.5, 2, 2.5, 3, 3.5}. The different densities for the labor shocks that we used were all constant over each of the labor grid intervals, and are given below in Table 1.

**Table 1 - LABOR SHOCK DENSITY FUNCTIONS**

INTERVAL	DFL1	DFL2	DFL3	DFL4	DFL5
[1, 1.5)	0	0.2	0.5	0	0
[1.5, 2)	2	1.6	1.0	1.4	0
[2, 2.5)	0	0.2	0.5	0.6	$2/3 + 1/2$
[2.5, 3)	0	0	0	0	$2/3$
[3, 3.5]	0	0	0	0	$2/3 - 1/2$
Coefficient of Variation	0.08	0.15	0.22	0.14	0.14

These distributions are normalized in the computations such that  $E(\ell) = 1$ . Note that the first three distributions are symmetric. The last two are positively skewed, with median/mean ratios of 0.97, and 0.98, and with skewness coefficients of 0.64, and 0.43, respectively, for DFL4 and DFL5<sup>5</sup>. For comparison, a log-normal distribution with coefficient of variation of 0.14 has a skewness coefficient of 0.21, so that the last two distributions are highly skewed.

<sup>5</sup> According to Pearson's second coefficient of skewness,  $3(E(\ell) - \ell^m)/\sigma$ , where  $\ell^m$  is the median and  $\sigma$  is the standard deviation.

A coefficient of variation of 15-30 percent in earning at an annual rate seems reasonable. Kydland (1984) reports a standard deviation of hours worked from PSID data of 15 percent. Abowd and Card (1987) report from PSID and NLS data standard deviations of percent changes in real earnings and annual hours of 40 and 35 percent, respectively. This translates into a coefficient of variation of about 28 percent, assuming i.i.d. shocks. At a four year period length, this figure would be cut in half, again assuming i.i.d. shocks, as we do, so that the appropriate range for the percent variation in labor income is 8-15 percent.

## 5. RESULTS

First, we report the actual tax rates chosen under the two alternative regimes, the utilitarian government, and the median voter. Recall, that according to the conventional wisdom, the median voter is expected to prefer higher taxes with a positively skewed income distribution, for redistributive motives. All our runs resulted in positively skewed total income distributions, even when the underlying labor shocks were symmetrically distributed. Nevertheless, as the next table shows, the political mechanism for choosing equilibrium tax rates is not enough to predict which tax rates will be higher.

**Table 2 - UTILITARIAN GOVERNMENT/MEDIAN VOTER  
TAX RATE COMPARISON**

$\mu$	0.5	0.9	1.1	1.5	1.9	6
DFL1	.04/.1	.1/.15	.13/.17	.18/.20	.23/.23	.52/.37
DFL2	.1/.15	.19/.21	.23/.24	.30/.29	.36/.32	.62/.52
DFL3	.15/.18	.26/.27	.31/.30	.38/.35	.44/.40	.68/.61
DFL4	.09/.15	.18/.22	.22/.25	.29/.29	.34/.32	.61/.51
DFL5	.09/.16	.18/.22	.22/.25	.29/.29	.34/.32	.61/.51

Recall that DFL1, DFL2, and DFL3 are symmetric, with coefficient of variation of 0.08, 0.15, and 0.22, respectively. DFL4 and DFL5 are highly skewed, with skewness coefficient of 0.64, and 0.43, respectively, and the same coefficient of variation as DFL2.

Two conclusions can be drawn from the results in Table 2. First, there are very small differences between the tax rates chosen by the median voter and utilitarian governments for reasonable and moderate levels of risk aversion. Those differences can be large, however, for very high or very low risk aversion. Second, the tax rate chosen by the median voter can be either higher or lower than that chosen by utilitarian government within the range of reasonable parameter values.

Holding the distribution fixed, (at any of the five distributions that we have tried), low levels of risk aversion result in lower taxes under utilitarian regime than under median voter, while the reverse happens with high degrees of risk aversion. Holding the risk aversion fixed, increasing the

variation of the idiosyncratic shocks, (moving from DFL1 to DFL2 to DFL3), results in higher tax rates under both regimes, but the effect is stronger under the utilitarian regime. The conclusion we arrive at is as the uninsurable risks become more important, agents prefer more social insurance, provided here by the tax/transfer redistribution scheme, and these effects are stronger under the utilitarian regime.

Some intuition for the latter conclusion can be provided by the following argument, which ignores the endogeneity of the income distribution. For a given income distribution,  $H(\cdot)$ , let  $\hat{\tau}$  be the tax rate chosen by a utilitarian regime, and let  $\hat{y}$  be the income level of an agent whose most preferred tax rate is  $\hat{\tau}$ ,

$$(28) \quad \underset{\tau}{\text{Argmax}} W(\hat{y}, \tau) = \underset{\tau}{\text{Argmax}} \int W(y, \tau) dH(y) = \hat{\tau}.$$

Let  $\tau^m$  be the tax rate chosen by the agent with the median income,  $y^m$ ,

$$(29) \quad \underset{\tau}{\text{Argmax}} W(y^m, \tau) = \tau^m.$$

The value function  $W(y, \tau)$  reflects the extent of the need for insurance against future income uncertainty, generated by both the risk aversion in the period utility,  $U(\cdot)$ , as well as the variability of future labor shocks. Due to the concavity of  $W$  in  $y$ ,  $\hat{y}$  in (28) *decreases* as the need for insurance increases, while  $y^m$  remains fixed. What our computations establish is that even for moderate need for insurance, and even when the impact on future income distributions and future tax rates are taken into account by the agents

in choosing the tax rate for the next period, this force is sufficient to make  $\hat{\tau}$  higher than  $\tau^m$ . If the position of each agent in the income distribution were *fixed* over time, there would have been no insurance motive for distorting taxes, which would then be determined solely on the basis of the trade off between efficiency and income redistribution.

In all our examples, it is always the case that the resulting total resources distribution (after taxes and transfers), which is the relevant distribution for voting and welfare weights, is *positively skewed*, (median < mean), and quite highly so for DFL4 and DFL5<sup>6</sup>. In spite of this, the tax rate under majority voting can be higher or lower than under utilitarian regime, depending on risk aversion and the the variability of uninsurable risks. This result holds also for the highly skewed distributions DFL4 and DFL5. We found it to be surprising that high levels of skewness in the distribution relevant for agents' preferences over tax rates do not necessarily generate higher taxes under majority voting. Another aspect of our results is that tax rates are almost the same regardless of the skewness of the distribution. This can be seen by comparing the tax rates for DFL2 and DFL5, (which have the same c.v.), in Table 2. Recall that in Persson and Tabellini (1991), the tax rate under majority voting increases with the distance between median and mean incomes, while the tax rate corresponding to our utilitarian regime remains at zero. If we were to introduce growth through some external effects - such as increasing social returns to capital - it is not obvious whether greater

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<sup>6</sup>The skewness coefficient of the after taxes resources distribution under DFL2, with moderate risk aversion coefficient  $\mu = 1.9$ , were 0.013 and 0.019 for utilitarian and median voter regimes, respectively. The corresponding values for DFL4 were 0.358 and 0.360.

inequality would lead to higher or lower growth.

The following table presents the equilibrium values of some aggregate variables for moderate risk aversion, ( $\mu = 1.9$ ), and different distributions of labor productivities. Although the tax rates differed considerably depending on risk aversion, their aggregate implications were not large, and moved in the expected direction. We report below the annual equilibrium interest rate before and after taxes, ( $r^*$ , and  $r^*(1-\tau^*)$ , respectively), and the saving rates. We also report the saving rates that would have prevailed had full insurance markets been allowed. This enables us to evaluate how well a representative agent model approximates our heterogeneous agents model, and to measure the contribution of the missing insurance markets to the saving rate. Finally, we also present the saving rate that would prevail in our economy with taxes being set to zero, in order to appreciate the full extent by which the tax/transfer policies reduce the need for precautionary savings.

Table 3 - TAX RATES AND SAVING RATES ( $\mu = 1.9$ )

	DFL1	DFL2	DFL3	
$\tau^*$	.23	.36	.44	UG
	.23	.32	.40	MV
$r^*$ (ANNUAL)	.052	.060	.066	UG
	.051	.057	.061	MV
$(1-\tau^*)r^*$	.040	.038	.037	UG
	.039	.039	.037	MV
SAVING RATE	.217	.203	.195	UG
	.218	.209	.201	MV
FULL INS. SAV RATE ( $\tau = \tau^*$ )	.215	.199	.187	UG
	.215	.204	.193	MV
NO INS. ( $\tau = 0$ ) SAV RATE	.239	.243	.247	

In all cases, increased insurance needs, be it due to increased variability of uninsurable income, or increased risk aversion, result in higher taxes, higher interest rates before taxes, lower interest net of taxes, and lower saving rates. These effects are contrary to what one would expect if taxation were exogenous. With a fixed tax rate  $\tau$ , increased insurance needs result in *increased* saving rate, which pushes interest rates *down*<sup>7</sup>. These opposite impacts on savings can be seen in Table 3, by comparing the saving rates with endogenously chosen taxes and those corresponding to taxes set at zero for different distributions.

Another aspect of our results is the impact on saving rate of increased

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<sup>7</sup>See Aiyagari (1992) for a more comprehensive discussion of aggregate savings with uninsurable idiosyncratic risks.

social insurance. It can be argued that increases in social insurance over the last several decades - via unemployment benefits, medicare, medicaid, maternity leaves, etc. - financed by higher taxes on personal income, may have played a role in the observed decline in U.S. saving rates. As can be seen in the Table 3, for medium levels of risk aversion, the saving rate drops by 3.4 percentage points as we move from no insurance, (SAV. RATE = 0.243) to a median voter equilibrium tax rate, (SAV. RATE = 0.209), and the difference is larger for higher degrees of risk aversion, or for more variability in labor income. It also appears from Table 3 that the representative agent model yields a good approximation as far as aggregate saving goes. This can be seen by the fact that with taxes set at their endogenously chosen levels, there are only small differences between the saving rates with full insurance markets and without them<sup>8</sup>.

## 6. SUMMARY

There are no large differences between majority voting equilibrium and utilitarian government equilibrium for moderate levels of insurance needs as measured by risk aversion in agents' preferences and variability of the uninsurable idiosyncratic risks. For relatively high or low levels of insurance needs - the difference in tax rates can be as large as 10 percentage points. The tax rates are higher under utilitarian government than under a majority voting when the need for insurance is greater, and vice versa. These

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<sup>8</sup>This is probably due to the assumption that the idiosyncratic shocks are i.i.d. over time. Aiyagari [1992] shows that if the shocks are persistent then there can be significant differences in aggregate saving rates between the no insurance and the full insurance (representative agent) versions.

conclusions continue to hold even when the distribution of labor shocks is highly positively skewed.

Increased social insurance may be able to explain a significant part of the decline in the saving rate. - maybe 4% points or more. The model does generate empirically plausible tax rates, (tax rates may be smaller with elastic labor supply since there is an additional disincentive effect). Aggregate characteristics of the economy are not much different from those of a representative agent (full insurance) model. Aggregate saving rates are about the same assuming same tax rates.

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## Appendix A: Computational Algorithm of Stochastic Steady State Equilibrium

### Computation

**Tax grid:** initial run with a tax grid of 21 points, going from 0 to 1 in steps of 0.05. After convergence, say to  $\tau^*$ , we used a second run with a refined tax grid of 21 points from  $\max(0, \tau^* - 0.1)$  to  $\min(1, \tau^* + 0.1)$  in steps of 0.01.

### Computing the saving function:

Combining (18a) and (18b) we have

$$(A1) \quad V(y_t^a, k, \tau) \equiv \max_{k_{t+1}^a} \left\{ U(y_t^a - k_{t+1}^a) + \beta \int V(y_{t+1}^a, k, \tau) dF(\ell_{t+1}) \right\}$$

subject to:

$$(A2) \quad y_{t+1}^a = k_{t+1}^a + (1-\tau)(w\ell_{t+1}^a + rk_{t+1}^a) + \tau(w + rk).$$

For given  $(k, \tau)$ , (A1) and (A2) are used to calculate the value function  $V(\cdot)$ . This value function is then used in (18a) and (18b) to calculate the saving function in (19). We approximate the saving function as a function of total resources of the household by a piece-wise linear function with a grid of 11 points.

We approximate the density function of total resources by subdividing each of the 10 intervals above into 25 subintervals and taking the density to be uniform over each subinterval. Thus, we have 250 intervals for calculating the probability distribution of total income.

### The Algorithm

We start with some initial guesses for  $(k, \tau, H)$  denoted  $(k_0, \tau_0, H_0)$ . The initial guesses  $(k_0, \tau_0)$  are used as described above to calculate the saving function  $s(y_t^a, k_{t+1}, \tau_{t+1}, k_0, \tau_0)$ . The initial density  $H_0$  is used in the following version of (21) together with the saving function to calculate the function  $\phi$  in (20).

$$(A3) \quad k' = \int s(x, k', \tau', k_0, \tau_0) H_0'(x) dx,$$

$$(A4) \quad k' = \phi(\tau', k_0, \tau_0).$$

The function  $\phi$  is then used to find an updated value of the tax rate  $\tau$ , denoted  $\tau_1$  using the following version of (22).

$$(A5) \quad \tau_1 = \underset{\tau'}{\text{Argmax}} W(y_0^m, \phi(\tau', k_0, \tau_0), \tau', k_0, \tau_0)$$

where  $H_0(y_0^m) = 1/2$ , (i.e.  $y_0^m$  is the median total resources);

The updated value  $\tau_1$  is used to calculate the updated value of the per capita capital, denoted  $k_1$  using the following version of (23).

$$(A6) \quad k_1 = \phi(\tau_1, k_0, \tau_0)$$

The value  $k_1$  is used to calculate the updated values of  $w$  and  $r$ , denoted  $w_1$  and  $r_1$  using the following versions of (24) and (25).

$$(A7) \quad w_1 = f_2(k_1, 1)$$

$$(A8) \quad r_1 = f_1(k_1, 1) - \delta.$$

Lastly, the updated distribution  $H_1$  is calculated using the following version of (26).

$$(A9) \quad \int F([y' - \tau_1(w_1 + r_1 k_1) - (1 + (1 - \tau_1)r_1)s(y, k_1, \tau_1, k_0, \tau_0)] / [w_1(1 - \tau)]) H_0'(y) dy \\ \equiv H_1(y'), \text{ for all } y'.$$

This procedure is repeated until  $(k, \tau, H)$  converge.

