

Federal Reserve Bank of Minneapolis
Research Department Working Paper

TRADE AND EXCHANGE RATES IN A
DYNAMIC COMPETITIVE ECONOMY

David K. Backus and Patrick J. Kehoe*

Working Paper 348

January 1989

NOT FOR DISTRIBUTION
WITHOUT AUTHOR APPROVAL

ABSTRACT

We derive the empirical implications of a popular class of international macroeconomic models. The real economy is a stochastic exchange model with complete markets. A standard result is that cross-country risk sharing implies perfect correlation between consumption paths across countries. With mild restrictions on the endowment process it also implies a positive correlation between net exports and output in every country. We introduce money using cash-in-advance constraints and show that the implications for real variables carry over into the monetary economy. These dichotomy and neutrality propositions generalize those in the literature to stochastic environments with heterogeneous agents, and do not require the cash-in-advance constraint to bind in every state. They imply that any correlation between the nominal exchange rate and the balance of trade can be made consistent with the theory.

Keywords: risk-sharing; government finance; cash-in-advance; monetary policy; exchange rates.

JEL Classification Numbers: 431, 023, 131.

*Backus, Federal Reserve Bank of Minneapolis and Queen's University; Kehoe, Federal Reserve Bank of Minneapolis and University of Minnesota.

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. The material contained is of a preliminary nature, is circulated to stimulate discussion, and is not to be quoted without the authors' permission.

1. Introduction

Dynamic competitive models have had considerable success recently explaining fluctuations in, and correlations between, aggregate variables in closed economies. In open economy macroeconomics Stockman (1980), Helpman (1981), Obstfeld (1981), Lucas (1982), Helpman and Razin (1982,1984,1985), Aschauer and Greenwood (1983), Persson (1984), Svensson (1985), Frenkel and Razin (1985), and Stockman and Svensson (1985) have used similar tools to advance our understanding of fluctuations in domestic and world output, trade, and exchange rates. We follow the Heckscher–Ohlin tradition in basing trade on differences among countries, and extend this line of research to general stochastic environments with heterogeneity in both endowments and preferences.

Our goal is to draw out the empirical content of this class of models, although we also extend the theory in several ways. We argue, to put it simply, that theory places strong restrictions on correlations between real quantities but fairly weak restrictions on correlations with nominal variables like the exchange rate. The suggestion is that the intensive study of exchange–rate behavior surveyed by Obstfeld and Stockman (1985) should be supplemented by a systematic investigation of quantity comovements. This change in emphasis follows directly from using dynamic competitive analysis, an approach that we think has a number of features that recommend it. First it builds on a long tradition of work in general equilibrium theory and, particularly in matters of government policy, enforces a degree of internal consistency that is absent from some models using other frameworks. Second, with some additional structure these models are capable of generating very strong predictions about the behavior of world and country–specific variables. These predictions serve to focus empirical work and provide a starting point for models with less restrictive structure.

We proceed by specifying a dynamic stochastic economy with many agents, associated with countries, who trade a single good each period. Intertemporal trade is generated by differences across countries in preferences, endowments, and government spending. In Section 2 we describe the real environment and review two interpretations of

equilibrium. In the first all trades are made at the beginning of time, "date 0," subject to present-value budget constraints. In the second trades are executed sequentially, and borrowing and lending takes place using two-period assets. The first is more convenient mathematically, but the second is needed to make sense of some kinds of government policies. It is essential to the cash-in-advance approach to money used in Sections 5 and 6.

We characterize equilibria in the real economy in Section 3. Consumers maximize expected discounted utility, but both the utility functions and endowments may differ across countries. With time-separable preferences and equal discount factors, consumption in each country is a deterministic function of world consumption: consumption is perfectly correlated across countries. With mild restrictions on the endowment processes consumption is less variable than output and net exports are positively correlated with output. Heterogeneity is handled easily by following Negishi (1960) and Mantel (1971) in computing competitive equilibria as solutions to social planning problems. We go on to consider ways of weakening the correlation between consumption across countries, including different and/or stochastic discount factors.

In Section 4 we extend the model to include government spending, taxation, and nontraded goods. We show that movements in government spending alter the correlation between net exports and output, but suggest that they are unlikely to change its sign. However, any correlation between government deficits and trade deficits can be made consistent with the theory. The reason is the Ricardian theorem, which allows us to alter the timing of taxes, and hence of deficits, without affecting consumption or the balance of trade. We then turn to nontraded goods, and show that the risk-sharing result from the previous section applies only to consumption of the traded good. With sufficient freedom in choosing the stochastic behavior of nontraded goods we can generate any correlation we like between total consumption in different countries.

In Sections 5 and 6 we introduce money using cash-in-advance constraints and examine the behavior of exchange rates. We generalize earlier work by Helpman (1981),

Lucas (1982), and Sargent (1987a) and prove a general irrelevance proposition: real variables in the monetary economy are identical to those of an analogous real economy. We then present two neutrality propositions to the effect that real variables are independent of both the currency of denomination of government debt and of government financial policy, including open market operations. As a result, the correlation between the exchange rate and the trade balance can have any sign. We contrast this strong separation between real and financial variables with contrary ones from other models in international finance, and discuss the reasons behind the irrelevance of the denomination of government debt.

We conclude with a summary of the model's predictions and suggestions for future empirical work.

2. The Real Economy

Our world is a dynamic stochastic Arrow–Debreu economy with a finite commodity space. In this section we review two definitions of competitive equilibrium and demonstrate the Ricardian theorem for this economy. In the "date–0" equilibrium, consumers and governments face lifetime budget constraints: the present value of their expenditures is constrained by the present value of their contingent claims. In the "sequence" equilibrium agents face a sequence of two–period budget constraints plus boundary conditions on initial and final wealth. The latter requires considerably more bookkeeping, but the extra detail is essential when we examine government financial and monetary policies. The Ricardian theorem amounts to noticing that the date–0 equilibrium depends on the present value, but not the timing, of taxes.

The environment and notation extend Lucas (1984) to allow for multiple agents. The economy has a single good each period whose quantity varies stochastically over time and across consumers. Each period t , for $t = 0, 1, \dots, T$, the economy experiences an event, s_t . We denote by $s^t = (s_1, \dots, s_t)$ the history of events up through and including period t and by S^t the finite set of all possible histories of length t . The probability, given s_0 , of

observing any particular history is known to all and denoted $f^t(s^t)$. This suggests a natural commodity space in which goods are differentiated by date and history. We use the abbreviation "commodity s^t " for the good at date t given history s^t . Its price will be denoted $q_t(s^t)$.

The economy contains I countries, each of which is represented by a government and a consumer. The government of country i , for $i = 1, 2, \dots, I$, consumes $g_t^i(s^t)$ and collects lump-sum taxes $\tau_t^i(s^t)$ in period t if history s^t occurs. The endowment of consumer i in period t is denoted $y_t^i(s^t)$; the aggregate endowment is $y_t(s^t) = \sum_i y_t^i(s^t)$. Consumption allocations are likewise denoted $c_t^i(s^t)$ individually and $c_t(s^t)$ in aggregate. In addition to their endowments, we also permit consumers to start with initial wealth, $x_0^i(s_0)$, governments initial debt, $b_0^i(s_0)$, both measured in units of the commodity at date 0. Consistency requires each asset to be matched by a debt:

$$\sum_i x_0^i(s_0) = \sum_i b_0^i(s_0).$$

The preferences of consumer i are represented by the expected utility function,

$$U_i = \sum_{t=0}^T \beta_i^t \sum_{s^t \in S^t} f^t(s^t) u_i[c_t^i(s^t)], \quad 0 < \beta_i < 1.$$

Each period-utility function, u_i , is increasing, concave, and differentiable and satisfies the Inada conditions,

$$\lim_{c \rightarrow 0} u_i'(c) = \infty, \quad \lim_{c \rightarrow \infty} u_i'(c) = 0.$$

Now consider two definitions of equilibrium, which we state using a notational convention from Debreu (1954). Let c^i denote the collection of elements $c_t^i(s^t)$, one for

each s^t , and let (c^i) denote the set $\{c^1, c^2, \dots, c^I\}$. Define similar objects for other variables.

Definition 1. A real date-0 equilibrium in this economy is a collection of allocations (c^i) , prices q , and government policies $[(g^i), (\tau^i)]$ satisfying:

- market clearing: for each date and history

$$(2.2) \quad \sum_i [c_t^i(s^t) + g_t^i(s^t)] = \sum_i y_t^i(s^t).$$

- consumer maximization: for each consumer i , quantities c^i maximize $U_i[c^i]$ subject to the budget constraint

$$(2.3) \quad \sum_{t=0}^T \sum_{s^t \in S} q_t(s^t) c_t^i(s^t) = \sum_{t=0}^T \sum_{s^t \in S} q_t(s^t) [y_t^i(s^t) - \tau_t^i(s^t)] + q_0(s_0) x_0^i(s_0).$$

- government budget constraints: for each government i , policies obey

$$(2.4) \quad \sum_{t=0}^T \sum_{s^t \in S} q_t(s^t) g_t^i(s^t) = \sum_{t=0}^T \sum_{s^t \in S} q_t(s^t) \tau_t^i(s^t) - q_0(s_0) b_0^i(s_0).$$

In a date-0 equilibrium all trades are executed at the beginning of time and there is no need to follow agents' balance sheets through time. To do just that, we introduce two-period assets and their associated prices. Let $v_t(s^t) = v_t(s^{t-1}, s_t)$ be the price of commodity s^t in units of commodity s^{t-1} . Let $x_t^i(s^t)$ denote ownership by consumer i of bonds paying one unit of commodity s^t and $b_t^i(s^t)$ government debt in like bonds. We can now state:

Definition 2. A real sequence equilibrium in this economy is a collection of allocations (c^i) , asset positions (x^i) , prices v , and government policies $[(g^i), (\tau^i), (b^i)]$ satisfying

- market clearing: for each date and history (2.2) holds.
- consumer maximization: for each consumer i , quantities c^i maximize $U_i[c^i]$ subject to the sequence of budget constraints

$$(2.6) \quad c_t^i(s^t) + \sum_{s_{t+1}} v_{t+1}(s^t, s_{t+1}) x_{t+1}^i(s^t, s_{t+1}) = y_t^i(s^t) - \tau_t^i(s^t) + x_t^i(s^t),$$

for all dates and histories, and the terminal condition, $x_{T+1}^i = 0$.

- government budget constraints: for each government i , policies obey the sequence of budget constraints,

$$(2.7) \quad \sum_{s_{t+1}} v_{t+1}(s^t, s_{t+1}) b_{t+1}^i(s^t, s_{t+1}) = g_t^i(s^t) - \tau_t^i(s^t) + b_t^i(s^t),$$

for all dates and histories, and the terminal condition $b_{T+1}^i = 0$. Equilibrium in bond markets is implied.

We now demonstrate that the two definitions are equivalent.

Proposition 1. (Equivalence of date-0 and sequence equilibria).

Any sequence equilibrium is also a date-0 equilibrium with prices $q_t(s^t) = \prod_{j=1}^t v_j(s^j)$ and $q_0(s_0) = 1$. Conversely, any date-0 equilibrium is a sequence equilibrium with prices $v_t(s^t) = q_t(s^t)/q_{t-1}(s^{t-1})$ for all dates $t = 1, 2, \dots, T$, and asset $t-1$ positions and bond supplies defined recursively by

$$(2.8) \quad x_t^i(s^t) = c_t^i(s^t) + \sum_{s_{t+1}} v_{t+1}(s^t, s_{t+1}) x_{t+1}^i(s^t, s_{t+1}) - y_t^i(s^t) + \tau_t^i(s^t)$$

and

$$(2.9) \quad b_t^i(s^t) = \sum_{s_{t+1}} v_{t+1}(s^t, s_{t+1}) b_{t+1}^i(s^t, s_{t+1}) - g_t^i(s^t) + \tau_t^i(s^t),$$

for all dates t and histories s^t , starting with $x_{T+1}^i = b_{T+1}^i = 0$.

Proof. Since market-clearing, consumer preferences, and initial wealth positions are identical in the two formulations, the proof consists of demonstrating equivalence of sequence and present-value budget constraints. We show first that the sequence of budget constraints is equivalent to the present-value constraint. Multiply each constraint (2.6) by $q_t(s^t)$, sum over t and s^t , and note that all of the asset terms cancel but $x_0^i(s_0)$ and $x_{T+1}^i(s^{T+1})$. The latter is zero by assumption, and we are left with the date-0 constraint, equation (2.3). The government budget constraints are handled analogously. Conversely we can derive the sequence constraints from (2.8) and (2.9) starting with $t = T$. \diamond

Proposition 1 contains the ingredients of a many-country version of the Ricardian theorem, which we state in two parts. The first part is: If $[(c^i), (x^i), v, (g^i), (\tau^i), (b^i)]$ is a sequence equilibrium for the economy with endowments (y^i) , then $[(c^i), q]$ is a date-0 equilibrium for the economy with endowments (y^i, g^i) , where q is given by (2.10). The timing of taxes, in other words, is irrelevant. The second part is: If $[(c^i), q]$ is a real date-0 equilibrium for the economy with endowments (y^i, g^i) , then $[(c^i), (x^i), v, (g^i), (\tau^i), (b^i)]$ is a sequence equilibrium for the economy with endowments (y^i) for any set of financing policies $[(\tau^i), (b^i)]$ satisfying (2.7); v and (x^i) are defined in Proposition 1. We return to this result in Sections 4 through 6.

3. Comovements in the Real Economy

We proceed to describe equilibrium in the model without governments ($g^i = \tau^i = 0$) and comment on its empirical content. We show that with identical discount factors the model implies a deterministic time-invariant relation between consumption in different countries. With additional restrictions on the endowment process it also implies that the covariance between net exports and domestic income is positive. Different discount factors induce trends in consumption, and therefore in net exports. We refer to these as "low-frequency" models and suggest that trends in the data be removed by filtering. The

implications for the detrended "high-frequency" data are the same as the equal discount factor case.

Consider then:

Proposition 2. (Equilibrium with equal discount factors).

In the exchange economy with equal discount factors ($\beta_i = \beta$) and no governments ($g^i = t^i = 0$), competitive equilibria are characterized by allocation functions, a_i , one for each consumer, and a pricing function, b , such that

$$c_t^i(s^t) = a_i[c_t(s^t)],$$

and

$$q_t(s^t) = \beta^t f^t(s^t) b[c_t(s^t)],$$

where $c_t(s^t)$ is aggregate consumption. Furthermore, the allocation functions are increasing, and the pricing function decreasing. With no governments $c_t(s^t) = y_t(s^t)$ and we can replace aggregate consumption with the aggregate endowment.

Proof. We exploit the equivalence between Pareto optima and competitive equilibria in Arrow-Debreu economies. Since the utility functions are concave, any optimum can be computed as the solution to a planning problem of the form

$$(3.1) \quad \max_{(c^i)} \sum_i \lambda_i U_i[c^i] = \max_{(c^i)} \sum_i \lambda_i \sum_t \beta^t \sum_{s^t \in S} f^t(s^t) u_i[c_t^i(s^t)]$$

subject to the resource constraints,

$$\sum_i c_t^i(s^t) \leq \sum_i y_t^i(s^t), \text{ for all } t, s^t,$$

for some choice of nonnegative welfare weights, λ .

If we reverse the order of summation in the welfare function, writing it as

$$\sum_{t, s^t} \beta^t f^t(s^t) \sum_i \lambda_i u_i[c_t^i(s^t)],$$

then the planning problem divides naturally into two steps. The first is

$$\max_{\{c_t^1(s^t), \dots, c_t^I(s^t)\}} \sum_i \lambda_i u_i[c_t^i(s^t)]$$

subject to $\sum_i c_t^i(s^t) = c_t(s^t)$, for each date and history. Concavity of the period utility functions, plus the Inada conditions, guarantees that consumption by each consumer is an increasing function of aggregate consumption. Since this problem has the same form for each date and history, the function does not depend on either, although it does depend on the choice of welfare weights.

Denote the solution of the first step by $w[c_t(s^t)]$ and note that w inherits concavity and Inada conditions from the u_i 's. Then step two consists of maximizing

$$\sum_{t, s^t} \beta^t f^t(s^t) w[c_t(s^t)]$$

subject to the resource constraints, $c_t(s^t) \leq y_t(s^t)$. If we denote the Lagrange multipliers on the constraints by $q_t(s^t)$ then the first-order conditions imply

$$w'[c_t(s^t)] = q_t(s^t) / \beta^t f^t(s^t).$$

Since w is concave this establishes the form of the pricing function and the proof is complete. \diamond

Proposition 2 describes the well-known risk-sharing properties of competitive equilibria in Arrow-Debreu economies. Similar results appear in Breeden and Litzenberger (1978), Scheinkman (1984), and Townsend (1986). Its usefulness here lies in its strong implications for time-series data on consumption. In our economy we would observe a perfect, possibly nonlinear relation between consumption by one individual and consumption by any other, since both are increasing functions of world consumption. Since sums of increasing functions are also increasing, we can apply the same reasoning to aggregates of consumers, like states or countries. The proposition generalizes to economies with governments and some kinds of production; see section 4 and Backus and Kehoe (1986), respectively.

The proposition also tells us something about comovements between net exports and output. The consumption smoothing motive that underlies proposition 2 suggests that countries will import when their endowments are low, and export when they are high. We might expect, therefore, a positive correlation between net exports and domestic output. We can state this more precisely if we impose a small amount of structure on the endowment processes. In our pure-exchange economy, net exports is the difference between the endowment and consumption, $nx_t^i = y_t^i - c_t^i$, and the covariance of net exports with output is

$$\text{cov}(nx^i, y^i) = \text{var}(y^i) - \text{cov}(c^i, y^i).$$

Now since consumption is a function of the world endowment, net exports and output will be positively correlated if the covariance between domestic and world output is not too large; that is, if there is a large enough idiosyncratic component to domestic output fluctuations. A second implication of this assumption is that consumption has a smaller variance than output. Put another way: the covariance between net exports and output in a pure exchange economy is positive if the variance of output exceeds that of consumption. Strictly speaking this holds only for exchange economies without

government; we discuss government spending in section 4 and capital accumulation in Backus and Kehoe (1986).

Two examples illustrate these features of exchange economies. In the first agents have identical homothetic preferences, so the model could be solved more easily for prices and aggregate quantities by combining them into a single representative consumer. As a result of this aggregation possibility the pricing function does not depend on the welfare weights. We also show how the weights associated with a particular competitive equilibrium depend on the distribution of wealth. Preferences in the second example are neither homothetic nor identical, and both prices and aggregate quantities depend, in equilibrium, on the distribution of wealth.

Example 1. Let $u_i(c) = \log c$, all i . Then equilibrium quantities and prices for a particular distribution of wealth are

$$c_t^i(s^t) = \lambda_i^* y_t^i(s^t)$$

and

$$q_t(s^t) = b^{t-1} f^t(s^t) / y_t(s^t),$$

where $\lambda_i^* = \lambda_i / \sum \lambda_j$. Note that equilibrium prices do not depend on λ .

We generate the complete set of equilibria and optima by varying the welfare weights. For a particular distribution of wealth we find the appropriate weights (that is, the appropriate optimum) by imposing the budget constraints. With logarithmic preferences the weight on country i is just its share of the value of the world endowment, corrected for initial wealth:

$$\lambda_i^* = \frac{\sum_{t,s^t} q_t(s^t) y_t^i(s^t) + q_0(s_0) x_0(s_0)}{\sum_{t,s^t} q_t(s^t) y_t(s^t)},$$

with prices defined above. This recursive solution technique (find prices, then use the budget constraints to find the weights) works because equilibrium prices do not depend on λ .

We can also check comovements between consumption, net exports, and output if we specialize the model further. Suppose that events s_t are distributed independently and identically over time. Then y , c , and nx are iid random variables and we can be concrete about what we mean by covariances among them. In this case,

$$\text{cov}(nx^i, y^i) = \text{var}(y^i) - \lambda_i^* \text{cov}(y, y^i),$$

which, for any λ_i^* is positive if the covariance between domestic and world output is not too large. To see how this might fail to hold, let $y^i = \alpha_0 + \alpha_i y$ so that y and y^i are perfectly correlated. Then $\text{var } y^i = \alpha_i^2 \text{var } y$ and $\text{cov}(y, y^i) = \alpha_i \text{var } y$, and $\text{cov}(nx^i, y^i) > 0$ if $\alpha_i > h_i^*$. If α_i is small enough then domestic output will be smoother than world output and consumption, and the inequality goes the other way.

Example 2. Let $u_i(c) = -\alpha_i^{-1} \exp(-\alpha_i c)$, $\alpha_i > 0$, all i . Then equilibrium allocations and prices are

$$c_t^i(s^t) = \alpha_i^{-1} (\alpha^* y_i(s^t) + \log \lambda_i - \alpha^* \sum_j \alpha_j^{-1} \log \lambda_j)$$

and

$$q_t(s^t) = \beta^t f^t(s^t) \left[\prod_j \lambda_j^{1/\alpha_j} \alpha^* \exp(-\alpha^* y_t(s^t)) \right],$$

where $\alpha^* = [\sum \alpha_j^{-1}]^{-1}$. Note that the pricing function depends on the distribution of wealth through so that finding the weights associated with a particular distribution of wealth involves simultaneous determination of welfare weights and equilibrium prices. Mantel (1971) describes a fixed-point algorithm for doing this.

The empirical content of this model is its strength, but we suspect that the perfect correlation predicted between consumption across countries is too strong to be realistic. We therefore consider two extensions that weaken this link without losing the flavor of the original. The first allows consumers in different countries to have different discount factors, a device that figures prominently in Helpman and Razin (1982,1984,1985) and Frenkel and Razin (1985). We show that it introduces deterministic trends into consumption and net exports. The second introduces randomness on the demand side with state-contingent utility or discount factors.

Of course trends can be introduced into consumption paths even with identical discount factors if the aggregate endowment is nonstationary, as the following deterministic example demonstrates.

Example 3. Let there be two countries, with period utility functions $u_1(c) = c$ and $u_2(c) = \log c$, and let the aggregate endowment be $y_t = \gamma^t$, $\gamma > 0$. Then

$$c_t^1 = \gamma^t - \lambda_2/\lambda_1.$$

which clearly has a geometric trend. This motivates the boundedness condition in the following proposition. We assume here that T is infinity.

Proposition 3. (Equilibrium with different discount factors).

Let the process for endowments be bounded, so that $y_t(s^t) \leq Y < \infty$ for all t and s^t . Define I^* to be the set of countries with maximal discount factors:

$$I^* = \{i | \beta_i = \max(\beta_1, \beta_2, \dots, \beta_I)\}.$$

Then for i not in I^* $\lim_{t \rightarrow \infty} c_t^i(s^t) = 0$.

Proof. The first-order conditions for the planning problem (analogous to (3.1) but with different discount factors) are

$$\lambda_i \beta_i^t u_i^t [c_t^i(s^t)] = q_t(s^t) / f^t(s^t),$$

which implies

$$(\lambda_i / \lambda_j) u_i^t [c_t^i(s^t)] / u_j^t [c_t^j(s^t)] = (\beta_j / \beta_i)^t$$

for any two countries i and j . Let j be in I^* and i be any country not in I^* , so that $\beta_j > \beta_i$. Then as t increases the ratio of marginal utilities approaches infinity. Since the endowment is bounded so is consumption; the Inada conditions require $c_t^i(s^t)$ to approach zero, as stated. \diamond

Proposition 3, which has antecedents back to Ramsey (1928), shows that different discount factors introduce trends into consumption paths. If one is interested, as we are, in high-frequency fluctuations, the obvious solution is to filter the trend out of the data before computing cross-correlations. The point is illustrated with a specialization of example 1.

Example 4. Consider example 1 with two countries, and let events be iid with two equally probable outcomes. In "state" 1 country 1 has endowment $1 + a$, in state 2, $1 - a$. Country 2 has the reverse so that the aggregate endowment is 2 in each state. With equal discount factors the symmetric equilibrium involves consumption by both countries of 1 in each state. Net exports of country 1 are a in state 1, $-a$ in state 2, and the reverse for country 2; clearly net exports is positively correlated with domestic output in both countries.

With different discount factors the solution to the planning problem is

$$c_t^i = \lambda_i^*(t) \cdot 2$$

in each state for each country i , where $\lambda_i^*(t) = \lambda_i \beta_i^t / \sum \lambda_j \beta_j^t$. The point, which is clear from the definition of $\lambda_i^*(t)$, is that there is a trend in the consumption path of each country. Net exports fluctuates randomly around a similar trend.

We conclude this section with an example in which temporary random fluctuations in preferences, which we can interpret in this case as random discount factors, breaks the perfect correlation in consumption across countries. Sargent (1987b) describes this as a way of introducing transitory consumption into the permanent income hypothesis, and similar specifications are commonly used by econometricians in applied work.

Example 5. Consider example 4 with equal discount factors, but with state-dependent period utility functions,

$$u_i[c_t^i(1), c_t^i(2)] = \theta_1^i \log c_t^i(1) + \theta_2^i \log c_t^i(2),$$

for each country i , where θ_j^i is a random variable that affects country i 's utility of consuming in state j . A particularly simple parametric example is the following:

Event	Probability	y^1	y^2	θ^1	θ^2
1	1/2	1 + a	1 - a	1 + b	1 - b
2	1/2	1 - a	1 + a	1 - b	1 + b

The symmetric solution is then consumption by country 1 of 1 + b in state 1, 1 - b in state 2, and the reverse for country 2. Net exports for country 1 is then a - b and a + b

in the two states. The covariance between net exports and output in each country is $a(a-b)$, which is negative if $b > a$. With this condition the variance of consumption exceeds that of income, as must be the case. We think this might be a reasonable way of weakening the tight link predicted between consumption in countries 1 and 2, but too much of this medicine destroys the spirit of the analysis.

4. Government Finance and Nontraded Goods

We now extend the analysis in two directions. The first direction is government finance, and we show that the risk-sharing result of section 3 applies here to output net of government spending. Comovements between output and the trade balance now depend on the stochastic behavior of g . Taxes, on the other hand, are restricted only by government budget constraints and have no effect on either consumption or net exports. As a result the model places no restrictions on the correlation between trade deficits and government deficits. The second direction is nontraded goods, and we show that the risk-sharing argument applies only to traded goods. If traded and nontraded goods are not observable separately, apparent deviations from complete risk-sharing may in fact be the result of fluctuations in quantities of nontraded goods.

We start with government finance. With regard to government spending, we note that proposition 2 still holds. In the proof the resource constraint is now

$$\sum_i c_t^i(s^t) + \sum_i g_t^i(s^t) \leq \sum_i y_t^i(s^t), \text{ all } t, s^t,$$

and in the second step we simply substitute $y - g$, rather than y , for c . Consumption by each country is still a function of aggregate consumption so they are still perfectly correlated.

We must also modify the conditions that led us to predict a positive correlation between net exports and output. With nonzero government spending the covariance can be written

$$\text{cov}(nx^i, y^i) = \text{var } y^i - \text{cov}(c^i, y^i) - \text{cov}(g^i, y^i),$$

since $nx^i = y^i - c^i - g^i$ in this economy. The point is that a large positive covariance between g and y could reverse the predicted positive covariance between nx and y . During the postwar period government purchases of goods and services in the United States have been only slightly procyclical, so in practice this correction will probably not make much difference.

It has been popular recently to relate government deficits to trade deficits, so we consider the model's implications for comovements between these variables. We show, in essence, that it has none. Any correlation between the two deficits can be made consistent with the theory.

Proposition 4. (Comovements between government deficits and trade deficits are arbitrary).

Given stochastic processes for endowments and government spending that induce a nontrivial process for net exports, a process for taxes can be chosen to attain any correlation between the government deficit and net exports.

The proof follows directly from the Ricardian theorem. Its logic is illustrated by the following example.

Example 6. Consider once more the symmetric two-country model of example 4 with logarithmic preferences, equal discount factors, and equally-probable iid events:

Event	Probability	y^1	y^2	$g^1 = g^2$	$c^1 = c^2$	$nx^1 = -nx^2$
1	1/2	$1 + a$	$1 - a$	g_0	$1 - g_0$	a
2	1/2	$1 - a$	$1 + a$	g_0	$1 - g_0$	$-a$

Now consider taxes. We know from the Ricardian theorem that the timing of taxes can be chosen arbitrarily, without affecting consumption or trade, as long as their present value equals the present value of government spending (here zero) and the outstanding debt. We can therefore choose a tax policy, $\tau = \tau_0 + b$ in state 1, $\tau_0 - b$ in state 2, with appropriate choice of τ_0 (for example, $\tau_0 = g_0$). In this case government deficits and trade deficits are positively related if $b > 0$, and negatively related if $b < 0$, so the covariance is not restricted by the model.

A similar problem afflicts nontraded goods unless we can observe them separately. Let y and z denote endowments of traded and nontraded goods, respectively, and c and d consumption of the same. We extend the framework as follows. Assume for simplicity that the period utility functions are additively separable between the two goods, and can be written

$$u_i[c_t^i(s^t), d_t^i(s^t)] = u_{1i}[c_t^i(s^t)] + u_{2i}[d_t^i(s^t)].$$

Proposition 5. (With nontraded goods comovements between consumption across countries are arbitrary).

Given stochastic processes for endowments of traded goods, processes for nontraded goods can be chosen to attain any correlation between total consumption across countries.

Proof. The planning problem for the model with nontraded goods involves consumption of nontraded-goods endowments by all countries, since by assumption they cannot be traded, and risk-sharing as before over traded goods. (With nonseparable preferences there would be some interaction between the two.) Total consumption in each country, as measured in the national income accounts, is the sum of the two quantities, evaluated at a relative price from some base period. Since the nontraded goods component of this total does not depend on the quantity of traded goods consumed, it can be chosen to attain

literally any behavior for consumption. By choosing different processes for nontraded

goods in different countries we can generate any correlation between consumption across countries. \diamond

By similar means we can also generate arbitrary comovements between net exports and output within a country. Both results are illustrated in the next example.

Example 7. Consider example 6 with the addition of a nontraded goods endowment of $1 + b_i$ in state 1, $1 - b_i$ in state 2, for countries $i = 1, 2$. Preferences are $\log c + \log d$ in each country, and the relative price of the two goods in the base period is 1. Total consumption in country i is $2 + b_i$ in state 1, $2 - b_i$ in state 2; domestic output is $2 + a + b_i$ and $2 - a - b_i$, respectively. If b_1 and b_2 have opposite signs then the correlation between consumption in the two countries is not only less than one, it is negative. Moreover, the covariance between net exports and output in country i is positive if and only if $a + b_i > 0$.

Nontraded goods, therefore, provide a second way of weakening the predicted perfect correlation between consumption across countries. Clearly empirical work will want to try to separate these two goods.

5. The Monetary Economy

The exchange rate is without question the most intensively studied variable in international macroeconomics. We examine its behavior in a theoretical context by introducing money into the economy of Section 2 using cash-in-advance constraints. Our analysis builds on earlier work by Stockman (1980), Helpman (1981), Lucas (1982), and Sargent (1987a), and extends it in several ways. Perhaps the most interesting are that we deal with a general stochastic environment and do not require the cash-in-advance constraint to bind in all states. The logical structure is closest to Helpman (1981).

The cash-in-advance environment has been described clearly elsewhere in the literature so we will be brief. In each period t agents trade money, assets, and goods in

particular ways. At the start of the period, after observing the current event, s_t , agents trade currencies and assets in a centralized securities market. The assets are one-period state-contingent nominal claims and are available in all currencies. At this time claims incurred in the previous period's security market are settled and taxes are paid to the government.

Each household then splits into a worker and a shopper. The shopper travels to different countries and purchases goods from their workers. The worker stays in his own country and sells his endowment to shoppers for local currency. All markets close and the shopper returns home bringing goods and unspent cash. The household enters the next period holding the cash from both the shopper and the worker and claims accumulated from maturing assets.

The timing of the government's problem is similar. In the securities market each government trades assets and currencies of all denominations. Each government also collects taxes and issues or destroys units of its currency. In goods markets governments purchase goods from workers of the various countries with local currency. These purchases are financed through taxes, creation of money, and sale of assets.

This physical environment leads to the following constraints for the consumer of country i in state s^t . In the goods market for country j , the consumer of country i purchases $c_t^{ij}(s^t)$ units of goods using the $MD_t^{ij}(s^t)$ units of currency j acquired in the securities market subject to the cash-in-advance constraint,

$$(5.1) \quad p_t^j(s^t)c_t^{ij}(s^t) \leq MD_t^{ij}(s^t), \text{ for } j = 1, \dots, I,$$

where $p_t^j(s^t)$ is the currency- j price of good s^t .

In the securities market consumer i acquires $MD_t^{ij}(s^t)$ units of currency j and $X_{t+1}^{ij}(s^t, s_{t+1})$ shares of an asset paying one unit of currency j at date $t + 1$ if s_{t+1} occurs. These assets sell for $V_{t+1}^j(s^t, s_{t+1})$ units of currency j in state s^t . If $e_t^{lj}(s^t)$ is the exchange rate between currencies l and j (the currency- l price of one unit of currency j)

then consumer 2's budget constraint as he enters the securities market is, in units of currency 1,

$$(5.2) \quad \sum_j e_t^{1j}(s^t) MD_t^{ij}(s^t) + \sum_j \sum_{s_{t+1}} e_t^{1j}(s^t) V_{t+1}^j(s^{t+1}) X_{t+1}^{ij}(s^{t+1}) \leq W_t^j(s^t),$$

where $W_t^i(s^t)$ is the total value of his assets. We construct W as follows: The consumer enters the securities market with $p_{t+1}^i(s^{t+1}) y_{t-1}^i(s^{t-1})$ units of currency i collected from the sale of endowments at $t-1$ and $MD_{t-1}^{ij}(s^{t-1}) - p_{t-1}^j(s^{t-1}) c_{t-1}^{ij}(s^{t-1})$ units of unspent cash for each currency j . He then pays $T_t^i(s^t)$ units of currency i in taxes and collects $X_t^{ij}(s^t)$ units of each currency j . The net value of his portfolio, measured in units of currency 1, is

$$(5.3) \quad W_t^i(s^t) = e_t^{1i}(s^t) p_{t-1}^i(s^{t-1}) y_{t-1}^i(s^{t-1}) - e_t^{1i}(s^t) T_t^i(s^t) \\ + \sum_j e_t^{1j}(s^t) X_t^{ij}(s^t) + \sum_j e_t^{1j}(s^t) [MD_{t-1}^{ij}(s^{t-1}) - p_{t-1}^j(s^{t-1}) c_{t-1}^{ij}(s^{t-1})].$$

Consumer i , then, chooses consumption quantities c_t^{ij} , currency positions MD_t^{ij} , and asset positions X_t^{ij} to maximize U_i subject to the cash-in-advance constraint (5.1), the budget constraints (5.2) and (5.3), and the boundary conditions, $X_0^{ij} = 0$ and $X_{T+1}^{ij} \equiv 0$. It will be understood that utility depends only on the aggregate quantities,

$$c_t^i(s^t) = \sum_{j=1}^I c_t^{ij}(s^t),$$

since goods in different countries are identical.

The government of country i faces similar constraints. The goods-market constraints are

$$p_t^j(s^t) g_t^{ij}(s^t) = MG_t^{ij}(s^t), \text{ for } j = 1, \dots, I,$$

where $g_t^{ij}(s^t)$ and $MG_t^{ij}(s^t)$ are, respectively, the quantities of country j 's goods and currency purchased by the government of country i in state s^t . The asset-market constraints, expressed in units of currency 1, are

$$(5.6) \quad \sum_j \sum_{s^{t+1}} e_t^{lj}(s^t) V_{t+1}^j(s^{t+1}) B_{t+1}^{ij}(s^{t+1}) = \sum_j e_t^{lj}(s^t) B_t^{ij}(s^t) + \sum_j e_t^{lj}(s^t) MG_t^{ij}(s^t) - e_t^{li}(s^t) T_t^i(s^t) - e^{li}(s^t) [M_t^i(s^t) - M_{t-1}^i(s^{t-1})],$$

together with the terminal conditions, $B_{T+1}^{ij} \equiv 0$ for all j . $B_{t+1}^{ij}(s^{t+1})$ is the number of bonds of type s^{t+1} and denomination j issued by government i . Like consumers' assets each bond sells for $V_{t+1}^j(s^{t+1})$ units of currency j at date t and pays one unit of the same currency the following period if state s^{t+1} occurs.

We can now state:

Definition 3. A monetary sequence equilibrium is a collection of allocations (c^{ij}) , money and asset positions $[(MD^{ij})(X^{ij})]$, prices $[(p^i),(V^i),(e^{li})]$, and government policies $[(g^{ij}),(T^i),(B^{ij}),(MG^{ij}),(M^i)]$ satisfying:

- market clearing: for each country j , date t , and history s^t , markets for goods, assets, and money clear:

$$(5.7) \quad \sum_{i=1}^I [c_t^{ij}(s^t) + g_t^{ij}(s^t)] = y_t^j(s^t),$$

$$(5.8) \quad \sum_{i=1}^I X_t^{ij}(s^t) = \sum_{i=1}^I B_t^{ij}(s^t),$$

$$(5.9) \quad \sum_{i=1}^I [MD_t^{ij}(s^t) + MG_t^{ij}(s^t)] = M_t^j(s^t).$$

- consumer maximization: for each consumer i , the collection c^{ij} , X^{ij} , and MD^{ij} , for $j = 1, \dots, I$, maximize U_i subject to (5.1)–(5.3) and the boundary conditions, $X_0^{ij} = 0$ and $X_{T+1}^{ij} \equiv 0$.
- government budget constraints: for each government i , policies obey the goods and asset market constraints (5.5) and (5.6) for all dates, histories, and currencies, and the boundary conditions, $B_0^{ij} = 0$ and $B_{T+1}^{ij} \equiv 0$, for all j .

The rest of this section describes various features of monetary equilibria through a series of lemmas and propositions. We begin with two relationships that characterize equilibrium prices. First, since physical goods located in different countries are perfect substitutes, their prices are equal in all states. Second, with I currency and asset markets there are multiple sets of transactions that convert units of currency j , say, in state into currency j in state s^t into currency j in state s^{t+1} . Arbitrage guarantees that these transactions have equivalent value, which places restrictions on equilibrium prices.

We summarize this in:

Lemma 1. (Arbitrage restrictions on prices).

In any monetary sequence equilibrium prices obey

$$(5.10) \quad e_t^{1j}(s^t) = p_t^1(s^t)/p_t^j(s^t)$$

and

$$(5.11) \quad V_{t+1}^1(s^{t+1}) = e_t^{1j}(s^t)V_{t+1}^j(s^{t+1})/e_{t+1}^{1j}(s^{t+1})$$

for all dates, histories, and currencies.

We call (5.10) the law of one price for state-contingent prices and (5.11) interest rate parity for state-contingent prices. Notice that we have written down only the pairwise arbitrage restrictions between currency 1 and other currencies. These exhaust the

arbitrage restrictions on prices since any prices satisfying these conditions satisfy all possible arbitrage restrictions.

We now turn to the cash-in-advance constraints and present a condition that tells us whether the cash-in-advance constraints bind. In doing so we let $MH_t^{ij}(s^t)$ denote the amount of currency j that consumer i has left over from the date- t goods market:

$$(5.12) \quad MH_t^{ij}(s^t) = MD_t^{ij}(s^t) - p_t^j(s^t)c_t^{ij}(s^t).$$

Thus $MH_t^{ij}(s^t)$ is the amount of currency j that consumer i plans to hold from period t to period $t + 1$ while $[MD_t^{ij}(s^t) - MH_t^{ij}(s^t)]$ is the amount he plans to spend in the country- j goods market. Combining (5.12), the government's cash-in-advance constraint, and the equilibrium condition for goods, we find that the price level obeys the quantity-theory relation,

$$p_t^j(s^t) = M_t^j(s^t)/y_t^j(s^t),$$

when the constraint binds, and

$$p_t^j(s^t) = [M_t^j(s^t) - \sum_i MH_t^{ij}(s^t)]/y_t^j(s^t)$$

when it does not.

The following lemma tells us when the quantity theory is appropriate.

Lemma 2. (Restrictions across prices and cash-in-advance constraints).

In a given monetary equilibrium, if all sure nominal interest rates are positive in the sense that

$$\sum_{s_{t+1}} V_{t+1}^j(s^t, s_{t+1}) < 1,$$

for all currencies j and histories s^t , then all cash-in-advance constraints at date t bind with equality. If the cash-in-advance constraint on currency j of consumer i in state s^t does not bind, then

$$\sum_{s_{t+1}} V_{t+1}^i(s^t, s_{t+1}) = 1.$$

Proof. Consider the substitution possibilities between money and bonds. If a consumer holds one unit of currency j from state s^t into the following period, he "receives" one unit of currency j for all possible events s_{t+1} . Using asset markets, the currency- j price in state s^t of a bundle that pays one unit in all states is

$$\sum_{s_{t+1}} V_{t+1}^j(s^t, s_{t+1}).$$

If the price of this bundle is less than one, then assets dominate cash as a means of savings, money-to-hold is zero, and the cash-in-advance constraints bind. If money-to-hold is nonzero then cash and assets must bear the same return and

$$\sum_{s_{t+1}} V_{t+1}^j(s^t) = 1. \diamond$$

We now demonstrate that a monetary equilibrium divides naturally into a "real part" and a "monetary part," where the real part is a real sequence equilibrium (definition 2 of Section 2).

Proposition 5. (Classical dichotomy: reduction of a monetary equilibrium to a equilibrium.)

If consumer decisions $[(c^{ij}), (MD^{ij}), (X^{ij})]$, prices $[(p^i), (V^i), (e^{1i})]$ and government policies $[(g^{ij}), (T^i), (B^{ij}), (MG^{ij}), (M^i)]$ constitute a monetary sequence equilibrium then $[(c^i), (x^i), v, (g^i), (\tau^i), (b^i)]$ is a real sequence equilibrium, where for all consumers i , dates t , and histories s^t real variables are defined by

$$(5.13) \quad c_t^i(s^t) = \sum_j c_t^{ij}(s^t), \quad g_t^i(s^t) = \sum_j g_t^{ij}(s^t), \quad b_t^i(s^t) = \sum_j B_t^{ij}(s^t)/p_t^j(s^t),$$

$$x_t^i(s^t) = \sum_j X_t^{ij}(s^t)/p_t^j(s^t) + \sum_j MH_{t-1}^{ij}(s^{t-1})/p_{t-1}^j(s^{t-1}),$$

$$\tau_t^i(s^t) = T_t^i(s^t)/p_t^i(s^t) + [p_t^i(s^t)y_t^i(s^t) - p_{t-1}^i(s^{t-1})y_{t-1}^i(s^{t-1})]/p_t^i(s^t),$$

and

$$v_t(s^t) = p_t^1(s^t)V_t^1(s^t)/p_{t-1}^1(s^{t-1}).$$

Proof. Given a monetary sequence equilibrium we show that its real part, as defined in (5.13), is a real sequence equilibrium. To prove this we use lemmas 1 and 2 to convert the monetary equilibrium into a real equilibrium and then simply compare the two definitions. First, if $[(c^{ij}), (g^{ij})]$ satisfy the equilibrium conditions (5.7), then summing over j we have that $[(c^i), (g^i)]$ satisfy condition (2.2) for a real equilibrium. Next we claim that if c^{ij} , X^{ij} , MD^{ij} , for $j = 1, \dots, I$, maximize utility, (2.1), subject to constraints (5.1)–(5.3) and the terminal conditions given prices $[(V^i), (p^i), (e^{1i})]$ and tax policy T^i , then c^i and x^i maximize (2.1) subject to (2.6) and the boundary conditions given prices v and tax policy τ^i . To demonstrate this we reduce the monetary budget constraints to real budget constraints. The asset–market constraints (5.2) and (5.3) are reduced to (2.6) as follows. Add and subtract

$$\sum_j e_t^{1j}(s^t) p_t^j(s^t) c_t^{ij}(s^t)$$

from (5.2) and use (5.10) and (5.13) to write (5.2) as

$$(5.14) \quad p_t^1(s^t) \sum_j c_t^{ij}(s^t) + \sum_j e_t^{1j}(s^t) M H_t^{ij}(s^t) \\ + \sum_j \sum_{s_{t+1}} e_t^{1j}(s^t) V_{t+1}^j(s^{t+1}) X_{t+1}^{ij}(s^{t+1}) \leq W_t^i(s^t).$$

Lemma 2 implies that if $M H_t^{ij}(s^t)$ is nonzero then

$$\sum_{s_{t+1}} V_{t+1}^j(s^{t+1}) = 1,$$

which enables us to aggregate the second and third terms in (5.14) into

$$(5.15) \quad \sum_j \sum_{s_{t+1}} e_t^{1j}(s^t) V_{t+1}^j(s^{t+1}) [M H_t^{ij}(s^t) + X_{t+1}^{ij}(s^{t+1})].$$

From lemma 1 we have

$$(5.16) \quad e_t^{1j}(s^t) V_{t+1}^j(s^{t+1}) = p_{t+1}^1(s^{t+1}) V_{t+1}^1(s^{t+1}) / p_{t+1}^j(s^{t+1}),$$

which we can use to rewrite (5.15) as

$$(5.17) \quad \sum_{s_{t+1}} \sum_j [p_{t+1}^1(s^{t+1}) V_{t+1}^1(s^{t+1})] [M H_t^{ij}(s^t) + X_{t+1}^{ij}(s^{t+1})] / p_{t+1}^j(s^{t+1}).$$

Now substitute (5.17) into (5.14), divide by $p_t^1(s^t)$, and use definitions (5.13) in the proposition to obtain

$$(5.18) \quad c_t^i(s^t) + \sum_{s_{t+1}} v_{t+1}(s^{t+1}) X_{t+1}^i(s^{t+1}) \leq W_t^i(s^t)/P_t^1(s^t).$$

Using (5.3) and (5.10) we can express the right-hand side of (5.18) as

$$(5.19) \quad [p_{t-1}^i(s^{t-1})y_{t-1}^i(s^{t-1}) - T_t^i(s^t)]/p_t^i(s^t) + \sum_j [MH_{t-1}^{ij}(s^{t-1}) + X_t^{ij}(s^t)]/p_t^j(s^t),$$

which, by (5.13), reduces to $y_t^i(s^t) - \tau_t^i(s^t) + x_t^i(s^t)$. Then (5.18) becomes

$$(5.20) \quad c_t^i(s^t) + \sum_{s_{t+1}} v_{t+1}(s^{t+1}) x_{t+1}^i(s^{t+1}) \leq y_t^i(s^t) - \tau_t^i(s^t) + x_t^i(s^t),$$

the real sequence constraint (2.6).

Finally we need to show that if the monetary government budget constraints hold then the real government budget constraints hold. The argument is nearly identical to the one used for the consumer. \diamond

There are two ideas behind this proof that give insight into the structure of the monetary economy. First, in states in which the sure nominal interest rate is positive, bonds dominate money as a means of saving and the cash-in-advance constraints bind. In states where it is zero, money and "sure bonds" are equivalent from a consumer's point of view and we must include money-to-hold in our definition of real bonds. Second, the real value of taxes in a monetary economy is the sum of the real value of nominal taxes and the "inflation tax," which in nominal terms for country i is

$$[p_t^i(s^t)y_t^i(s^t) - p_{t-1}^i(s^{t-1})y_{t-1}^i(s^{t-1})].$$

The dichotomy result depends on this tax being nondistortionary, which holds here because output is exogenous. If, as in Aschauer and Greenwood (1983), the inflation tax affected real decisions, proposition 5 would not apply.

It is clear from proposition 5 that a monetary equilibrium has a trivial type of indeterminacy. Since goods in different countries are perfect substitutes the equilibrium only pins down total consumption by each consumer and government. It also has a less trivial indeterminacy. Since the real returns generated from holding one currency can be duplicated by holding another currency (or currencies) there is also a redundancy in assets: the equilibrium only pins down the real values of asset holdings by consumers and governments. More formally we have:

Proposition 6. (Irrelevance of assets' currency of denomination).

If consumer decisions $[(c^{ij}), (X^{ij}), (MD^{ij})]$, prices $[(V^i), (p^i), (e^{1i})]$, and government policies $[(g^{ij}), (T^i), (B^{ij}), (MG^{ij}), (M^i)]$ constitute a monetary equilibrium, then so do any $[(\hat{c}^{ij}), (\hat{X}^{ij}), (\hat{MD}^{ij})]$, $[(V^i), (p^i), (e^{1i})]$, and $[(\hat{g}^{ij}), (T^i), (\hat{B}^{ij}), (\hat{MG}^{ij}), (M^i)]$ satisfying the market clearing conditions (5.7)–(5.9), the cash-in-advance constraints (5.1) and (5.5), and

$$(5.21) \quad \sum_j c_t^{ij}(s^t) = \sum_j \hat{c}_t^{ij}(s^t), \quad \sum_j g_t^{ij}(s^t) = \sum_j \hat{g}_t^{ij}(s^t),$$

$$\sum_j B_t^{ij}(s^t)/p_t^j(s^t) = \sum_j \hat{B}_t^{ij}(s^t)/p_t^j(s^t),$$

$$\sum_j X_t^{ij}(s^t)/p_t^j(s^t) = \sum_j \hat{X}_t^{ij}(s^t)/p_t^j(s^t).$$

Proof. Given proposition 5, the only part of this proposition that is not obvious is why nominal prices are the same in the two equilibria. From Lemmas 1 and 2 it is clear that prices in the two equilibria are equal if and only if

$$(5.22) \quad \sum_i MH_t^{ij}(s^t) = \sum_i \hat{MH}_t^{ij}(s^t) \text{ all } j, t, s^t.$$

This follows directly from the market-clearing conditions and governments' cash-in-advance constraints. \diamond

An implication of this proposition is that the denomination of government debt is irrelevant: changing the composition of the debt affects neither real variables nor nominal prices. The result is probably obvious in this context, but it is in stark contrast to the results obtained in either portfolio balance models or international capital asset pricing models [see Branson and Henderson (1985) for a survey]. In both of these classes of models the exchange rate depends on the supplies of government bonds of different denominations.

The question is why our model rules this out. The proof simply amounts to observing that the real claims on a government that arise from issuing currency- j -denominated debt can be matched by issuing a suitable bundle of debt denominated in any other currency. The proposition, therefore, seems to depend on governments' ability to issue state-contingent debt. A similar result holds, however, even if governments only issue uncontingent debt. This latter result is more subtle and makes use of the fact that a Ricardian theorem holds for this environment; see Backus and Kehoe (1987) for details. We argue there that the experiments considered in the portfolio balance and international CAPM literatures are not well-posed because they ignore government budget constraints.

The proposition tells us that the monetary equilibrium has several dimensions along which there are trivial indeterminacies. To make this clear we will record in a monetary equilibrium only the real value of consumption and assets for consumers and governments together with nominal prices, taxes, and money supplies. Using proposition 6 we are justified in writing a monetary equilibrium more compactly as consumer decisions

$[(c^i), (x^i)]$, prices $[(V^i), (p^i), (e^i)]$, and government policies $[(g^i), (b^i), (M^i), (T^i)]$. In this representation it will prove useful to write government budget constraints as

$$(5.23) \quad \sum_{s_{t+1}^i} v_{t+1}(s^{t+1}) b_{t+1}^i(s^{t+1}) = b_t^i(s^t) + g_t^i(s^t) - T_t^i(s^t)/p_t^i(s^t) \\ - [M_t^i(s^t) - M_{t-1}^i(s^{t-1})]/p_t^i(s^t),$$

and consumer budget constraints as

$$(5.24) \quad c_t^i(s^t) + \sum_{s_{t+1}^i} v_{t+1}(s^{t+1}) x_{t+1}^i(s^{t+1}) = [p_{t-1}^i(s^{t-1}) y_{t-1}^i(s^{t-1}) - T_t^i(s^t)]/p_t^i(s^t) \\ + x_t^i(s^t),$$

where in both (5.23) and (5.24) $v_{t+1}(s^{t+1})$ is defined to be $p_{t+1}^1(s^{t+1}) V_{t+1}^1(s^{t+1})/p_t^1(s^t)$. We use this compact notation from now on.

In the next section we uncover classes of financing strategies for the government that are consistent with the same real equilibrium. In doing so we require all monetary equilibria in the class to be associated with the same real equilibrium. This stronger version of the dichotomy theorem follows.

Proposition 7. (Classical dichotomy: reduction of a monetary equilibrium to a real equilibrium without governments).

If consumer decisions $[(c^i), (x^i)]$, prices $[V, p, e]$, and government policies $[(g^i), (b^i), (M^i), (T^i)]$ constitute a monetary equilibrium, then $[(c^i), q]$ is a real date-0 equilibrium for an economy with endowments $(y^i - g^i)$.

The proof follows directly from propositions 1, 5, and 6 and the Ricardian theorem.

6. Comovements in the Monetary Economy

We now turn to the empirical implications of the monetary economy. We start by reversing proposition 7 to construct multiple monetary equilibria consistent with a single real equilibrium. This allows us to construct economies with arbitrary comovements between real and nominal variables. We show, for example, that any pattern of comovements between the exchange rate and trade deficits, or between the exchange rate and government deficits, can be reconciled with the theory for some choice of monetary policy.

Proposition 8. (Construction of a monetary equilibrium from a real equilibrium).

If $[(c^i), q]$ is a real date-0 equilibrium in an economy with endowments $(y^i - g^i)$, then we can construct a monetary sequence equilibrium in an economy with endowments (y^i) for any set of financing strategies $[(b^i), (T^i), (M^i)]$ that satisfy the government budget constraints (5.23). Prices in $[(p^i), (V^i), (e^{li})]$ the monetary equilibrium are constructed as follows. In each country i set $p_T^i(s^T) = M_T^i(s^T)/y_T^i(s^T)$, since in the final period money-to-hold is zero. For any period t , starting with $t = T - 1$ and working backwards to $t = 0$, compute

$$m_t^i(s^t) = [M_t^i(s^t)/y_t^i(s^t)] \sum_{s_{t+1}} v_{t+1}(s^{t+1})/p_{t+1}^i(s^{t+1})$$

for each history s^t . If $m_t^i(s^t) \leq 1$ then set $p_t^i(s^t) = M_t^i(s^t)/y_t^i(s^t)$. Otherwise, choose $p_t^i(s^t)$ so that

$$p_t^i(s^t) \sum_{s_{t+1}} v_{t+1}(s^{t+1})/p_{t+1}^i(s^{t+1}) = 1.$$

Using

$$V_{t+1}^i(s^{t+1}) = p_t^i(s^t) v_{t+1}(s^{t+1})/p_{t+1}^i(s^{t+1})$$

and

$$e_t^{li}(s^t) = p_t^l(s^t)/p_t^i(s^t),$$

we compute bond prices and exchange rates and complete our construction of equilibrium.

This proposition is proved by reversing the steps in the proof of proposition 5, using lemmas 1 and 2 and the Ricardian theorem. The condition on $m_t^i(s^t)$ is simply a check to see whether the cash-in-advance constraint binds. When it does we compute the price level from the quantity theory relation. Otherwise, we choose it to maintain a zero nominal rate of interest on bonds.

The proposition is essentially an algorithm for constructing monetary equilibria in stochastic cash-in-advance models. Specify first a real economy, including stochastic processes for endowments and government spending, and compute real allocations and bond prices. Then specify stochastic processes for money supplies and compute price levels, bond prices, and exchange rates as indicated.

An immediate corollary of proposition 8 is a neutrality result on government financing that generalizes earlier results of Helpman (1981), Lucas (1982), and Sargent (1987a). Helpman's environment is deterministic, but in other respects we follow him quite closely. Lucas and Sargent allow uncertainty, but consider only symmetric equilibria and assume that cash-in-advance constraints bind in all states. Proposition 8, rather than the policy-neutrality corollary, is more useful for our purposes because it describes how one might construct equilibria with different correlations between the exchange rate and real variables. Consider then:

Proposition 9. (Comovements between exchange rates and net exports are arbitrary).

Given stochastic processes for endowments and government spending that induce a nontrivial process for net exports, processes for money supplies and taxes can be chosen to yield any desired correlation between exchange rates and net exports.

Proof. The proof is similar in spirit to those of propositions 4 and 5. Given stochastic processes for endowments and government spending, we compute the implied stochastic process for net exports. Proposition 8 then tells us how to choose stochastic processes for money supplies to support any exchange-rate process. Finally, we specify nominal tax processes so that governments' budget constraints are satisfied. \diamond

The following example illustrates the procedure.

Example 8. We return to Example 6, in which real variables take on the values

Event	Probability	y^1	y^2	$c^1 = c^2$	$nx^1 = -nx^2$
1	1/2	$1 + a$	$1 - a$	1	a
2	1/2	$1 - a$	$1 + a$	1	$-a$

We now construct equilibria with arbitrary comovements between net exports and the exchange rate. To keep the analysis simple, let $g^1 = g^2 = 0$ and choose country 2's monetary policy so that p^2 is growing at a deterministic, geometric rate $\mu > 0$. The appropriate monetary process for country 2 is $M_t^2 = \mu^t(1-a)$ in state 1, $\mu^t(1+a)$ in state 2. We choose large enough that the cash-in-advance constraint always binds. From the Ricardian theorem we know we can choose nominal taxes T^2 to balance the government's budget without influencing equilibrium quantities.

Now consider country 1. Let $M_t^1 = \mu^t(1+b)$ in state 1, $\mu^t(1-b)$ in state 2, and choose taxes to balance the government's budget. Then $p^1 = \mu^t(1+b)/(1+a)$ in state 1, $\mu^t(1-b)/(1-a)$ in state 2, and $e = p^1/p^2 = (1+b)/(1+a)$ in state 1, $(1-b)/(1-a)$ in state 2. If $b > a$ then the covariance between nx^1 and e is positive, if it is negative.

A similar property applies to comovements between exchange rates and government deficits.

Proposition 10. (Comovements between exchange rates and government deficits are arbitrary).

Given stochastic processes for endowments and government spending that yield a nontrivial process for net exports, there exist processes for money supplies and taxes that yield any desired correlation between exchange rates and government deficits.

The logic should be clear from the next example.

Example 9. We continue example 8. Consider first a process for taxes that balances government 1's budget each period:

$$T_t^1 = -\Delta M_t^1 \text{ or } T_t^1/p_t^1 = -\Delta M_t^1/p_t^1.$$

Now consider a deviation from this such that the government's deficit, $(T_t^1 + \Delta M_t^1)/p_t^1$, is f in state 1, $-f$ in state 2. Since the states are equally likely and there is no aggregate risk, they have the same price and this new policy satisfies the government's budget constraint. Now note that the covariance between the deficit and the exchange rate is positive if $f > 0$, and negative if $f < 0$.

Propositions 9 and 10 are probably obvious in our environment, but they provide an interesting contrast to recent work relating exchange rates, real and nominal, to the two deficits. A lot of this work is essentially static, and focuses therefore on intratemporal relative prices. Our own working hypothesis, which is embedded in the assumption of a single good per period, is that aggregate trade dynamics are the result of intertemporal considerations.

7. Final Remarks

We have, to summarize briefly, taken a dynamic competitive world economy, with and without government finance and money, and derived its implications for comovements between aggregate variables. Unlike many theories in international

macroeconomics, ours has strong implications for comovements between real quantities like consumption and net exports, but weak implications for nominal variables like exchange rates. The former is the result of the risk-sharing properties of competitive equilibria. In our framework these lead to strong predictions about comovements between consumption paths in different countries and between output and the balance of trade within a country. The latter stems from the neutrality and dichotomy properties of monetary equilibria in cash-in-advance models, which allow us to associate arbitrary price-level and exchange-rate paths with any path of trade deficits.

The suggestion is that empirical work in international macroeconomics might be fruitfully shifted away from exchange-rate dynamics toward quantity comovements. The next step is to take a systematic look at international macroeconomic data to see where the model does well, and where it does not. Preliminary efforts along these lines are described by Leme (1984) and Backus And Kehoe (1986).

References

- Aschauer, D., and Greenwood J. 1983. A further exploration in the theory of exchange rate regimes. Journal of Political Economics 91 (October): 868–75.
- Backus, D., and Kehoe, P. 1986. Dynamics of the trade balance in theory and practice. Working Paper 323, Federal Reserve Bank of Minneapolis, November.
- Backus, D., and Kehoe, P. 1987. On the denomination of government debt. Work in progress.
- Branson, W., and Henderson, D. 1985. The specification and influence of asset markets. In Handbook of International Economics (New York: North Holland).
- Breeden, D., and Litzenberger, R. 1978. Prices of state-contingent claims implicit in option prices. Journal of Business 51 (October): 621–51.
- Debreu, G. 1954. Valuation equilibrium and Pareto optimum. Proceedings of the National Academy of Sciences 40, 588–92; reprinted in Mathematical Economics: Twenty Papers of Gerard Debreu, (New York: Cambridge University Press, 1983).
- Frenkel, J., and Razin, A. 1985. Government spending, debt, and international economic interdependence. Economic Journal 95 (September): 619–36.
- Helpman, E. 1981. An exploration in the theory of exchange-rate regimes. Journal of Political Economics 89 (October): 865–90.
- Helpman, E., and Razin, A. 1982. Dynamics of a floating exchange-rate regime. Journal of Political Economics 90 (August): 728–59.
- Helpman, E., and Razin, A. 1984. The role of saving and investment in exchange-rate determination. Journal of Monetary Economics 13 (May): 307–25.
- Helpman, E., and Razin, A. 1985. Floating exchange rates with liquidity constraints in financial markets. Journal of International Economics 19 (August): 99–117.
- Leme, P. 1984. Integration of international capital markets. University of Chicago Seminar Paper, February.

- Lucas, R. 1982. Interest rates and currency prices in a two-country world. Journal of Monetary Economics 10 (November): 335–60.
- Lucas, R. 1984. Money in a theory of finance. Carnegie-Rochester Conference Series on Public Policy 21, 9–46.
- Mantel, R. 1971. The welfare adjustment process: its stability properties. International Economic Review 12 (October): 415–30.
- Negishi, T. 1960. Welfare economics and existence of an equilibrium for a competitive economy. Metroeconomica 12 (August–December): 92–97.
- Obstfeld, M. 1981. Macroeconomic policy, exchange-rate dynamics, and optimal asset accumulation. Journal of Political Economics 89 (December): 1142–61.
- Obstfeld, M., and Stockman, A. 1985. Exchange-rate dynamics. In Handbook of International Economics (New York: North Holland).
- Persson, T. 1984. Real transfers in fixed exchange rate systems and the international adjustment mechanism. Journal of International Economics 13 (May), 349–69.
- Ramsey, F. 1928. A mathematical theory of saving. Economic Journal 38 (December): 543–59.
- Sargent, T. 1987a. Dynamic Macroeconomic Theory (Cambridge: Harvard University Press).
- Sargent, T. 1987b. Macroeconomic Theory, Second Edition (New York: Academic Press).
- Scheinkman, J. 1984. General equilibrium models of economic fluctuations: A survey of theory. Mimeo, University of Chicago, June.
- Stockman, A. 1980. A theory of exchange rate determination. Journal of Political Economics 88 (August): 673–98.
- Stockman, A., and Svensson, L. 1985. Capital flows, investment, and exchange rates. Working Paper 1598, National Bureau of Economic Research, April.

- Svensson, L. 1985. Currency prices, terms of trade, and interest rates: A general-equilibrium asset-pricing approach. Journal of International Economics 15 (February): 17-41.
- Townsend, R. 1986. Arrow-Debreu programs as microfoundations for macroeconomics. University of Chicago, ERC/NORC Discussion Paper 86-7, June.