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Monetary Independence and Rollover Crises *

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Abstract

This paper shows that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis. We study a sovereign default model with self-fulfilling rollover crises, foreign currency debt, and nominal rigidities. When the government lacks monetary autonomy, lenders anticipate that the government will face a severe recession in the event of a liquidity crisis, and are therefore more prone to run on government bonds. By contrast, a government with monetary autonomy can stabilize the economy and can easily remain immune to a rollover crisis. In a quantitative application, we find that the lack of monetary autonomy played a central role in making the Eurozone vulnerable to a rollover crisis. A lender of last resort can help ease the costs from giving up monetary independence.

Keywords: Sovereign Debt Crises, Rollover Risk, Monetary unions.
JEL Codes: E4, E5, F34, G15

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1 Introduction

A prominent concern of policy makers during the Eurozone crisis was the risk of a rollover crisis. The fear was that an adverse shift in market expectations would restrict governments' ability to roll over their debt, creating liquidity problems that would feed back into investors' expectations and ultimately lead governments to default. At the same time, the premise was that the lack of monetary independence was aggravating the sovereign debt crisis in Southern Europe and, as a result, there was an increased risk of a breakup of the monetary union.[1]

The goal of this paper is to investigate how the lack of monetary independence affects the vulnerability to a rollover crisis. The key question we tackle is whether a country becomes more exposed to a rollover crisis after joining a monetary union. We propose a theory of rollover crises in which monetary policy plays a macroeconomic stabilization role. Using a sovereign default model featuring foreign currency debt and nominal rigidities, we show that the lack of monetary independence increases the vulnerability to a rollover crisis. A key insight that emerges is that when the government lacks monetary independence, a sudden panic by investors who refuse to roll over the debt creates a severe recession, which in turn makes the option to default more attractive to the government and rationalizes the initial panic. In a quantitative application to the Eurozone, we find that giving up monetary independence results in the significant cost of a higher exposure to rollover crises.[2]

The model we consider is a workhorse model of sovereign default (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008), which we extend with rollover crises (Cole and Kehoe, 2000) and downward nominal wage rigidity (Schmitt-Grohé and Uribe, 2016). In this setup, the government experiences income shocks and borrows externally in foreign currency to smooth fluctuations in consumption over time. Absent commitment problems, the government would increase debt issuances in bad times and repay debt in good times, following the basic arguments in Barro (1979). The government lacks commitment to repay, however, and hence sovereign bonds are traded at a discount, which depends on future probabilities of default that are endogenous to government borrowing decisions. We consider a model with both tradable and non-tradable goods. Firms produce non-tradable goods using labor, and the presence of downward nominal wage rigidity can cause involuntary unemployment. In particular, a shock leading to a contraction in aggregate demand reduces the price of non-tradables in equilibrium, generating a decline in labor demand. When the decline in wages necessary to clear the market is too large, unemployment arises.

[1] On September 6th, 2012, Mario Draghi, the president of the European Central Bank, expressed that "the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a 'bad equilibrium,' namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." Preceding these remarks, Draghi famously pledged to do "whatever it takes to preserve the euro."

[2] Notice that by considering foreign currency debt we are abstracting from the idea that monetary policy allows a government to inflate away its debt and mitigate a rollover crisis, as studied, for example, by Aguiar, Amador, Farhi, and Gopinath (2013).

To understand how downward wage rigidity and monetary policy affect the emergence of a rollover crisis, consider first the following situation in the canonical environment without nominal rigidities. Suppose that a government facing debt repayments is trying to roll over its debt. In this context, consider first what happens if investors are willing to lend to the government. Assuming that outstanding debt is not too high, the government will find it optimal to repay the debt. Consider next what happens when investors refuse to roll over the government's debt. Given that the government is unable to obtain new loans, it needs to engage in a drastic fiscal adjustment to meet its debt payments. As a result, the cost of meeting the debt payments increases, making repayment relatively less attractive. If the value of repayment is still larger than the value of default, the government repays the debt despite the liquidity problem. In this situation, investors will not have incentives to run on the government bonds, and in equilibrium the government would still be able to borrow. A rollover crisis does not emerge.

Consider instead what happens in the event of a run in the presence of nominal rigidities. When investors refuse to roll over the government's bonds, the extra fiscal effort the government incurs to repay the debt now has macroeconomic implications. As the government engages in large cuts in expenditures or increases in tax revenues, the economy will experience a contraction in aggregate demand, which will generate involuntary unemployment, absent monetary independence. The government's value of repayment now faces a much larger decline compared to the flexible wage economy. As a result, incentives to repay are weaker, and, if the increase in unemployment is large enough, the government defaults. In this case, investors' anticipation of government default is indeed validated, and a second rational expectations equilibrium emerges. That is, in addition to the good equilibrium in which investors lend, there is a rollover crisis equilibrium in which the government defaults, only because investors are unwilling to lend. Interestingly, for this second equilibrium to emerge, unemployment does not have to be realized along the equilibrium path. In fact, it is the off-equilibrium outcome of a recession that triggers the rollover crisis.

Our quantitative findings suggest that a significant cost from joining a monetary union is a higher exposure to a rollover crisis. We start by considering a calibration of the model for two different monetary policy regimes: a flexible exchange rate regime and a fixed exchange rate regime. In the flexible exchange rate regime, the government can implement the full-employment allocations by depreciating the currency, in line with the traditional argument for flexible exchange rates. The government, however, cannot alter the value of the debt since it is denominated in foreign currency. We show that in this economy, rollover crises plays a limited role, with less than 1% of default episodes being driven by rollover crises. Almost all defaults occur because of fundamental factors. We then consider an economy in which the government follows an exchange rate peg. Equivalently, this is a single economy within a monetary union in which wages are denominated in the foreign currency and the government of the small open economy cannot alter the exchange rate vis à vis the rest of the world. Our findings show that the zone that displays multiplicity expands significantly. As a result, the econ-

omy spends more time exposed to a rollover crisis and the number of defaults due to rollover crises increases by seven-fold.

Using the calibrated model, we then simulate the path of the Spanish economy from the adoption of the euro and conduct a counterfactual. Under a fixed exchange rate regime, we find that the economy hits the "crisis zone" around 2012, in line with the turmoil in sovereign debt markets that occurred at the time. We then consider what the outcome would have been if Spain had exited the Eurozone in 2012. According to our model, the government would have remained immune to a rollover crisis, thanks to the ability to use monetary policy for macroeconomic stabilization.

Related literature. Our paper contributes to a vast literature on monetary unions, pioneered by Mundell (1961). The main theme in this literature is a trade-off between the benefits of credibility (Alesina and Barro, 2002) and lower transaction costs for international trade (Frankel and Rose, 2002) and the costs of higher macroeconomic fluctuations due to the loss of monetary independence (Mundell, 1961).[3] Our paper adds a new dimension to the costs from joining a monetary union: a higher exposure to rollover crises. In this respect, our results shed some light on the interventions by the European Central Bank (ECB) as a lender of last resort, following Mario Draghi's July 2012 speech pledging to do "whatever it takes to preserve the euro." Through the lens of our model, one possible interpretation is that the ECB, by acting as a lender of last resort, was contributing to reducing the costs for individual members to remain in the monetary union.

Our paper belongs to the literature on rollover crises in sovereign debt markets, starting with Sachs (1984), Alesina, Prati, and Tabellini (1990), and Cole and Kehoe (2000). Our formulation more closely follows Cole-Kehoe, as do other recent quantitative studies that allow for fundamental and nonfundamental shocks as well as long-term debt (e.g., Chatterjee and Eyigungor, 2012, Bocola and Dovis, 2016, Roch and Uhlig, 2018, Conesa and Kehoe, 2017, and Aguiar, Chatterjee, Cole, and Stangebye, 2016). Different from these contributions, we consider an economy with production and nominal rigidities, and establish how the exchange rate regime is central for the exposure to rollover crises. With a flexible exchange rate regime, we find the exposure to a rollover crisis to be minimal, which is in line with Chatterjee and Eyigungor, who showed that in a canonical endowment economy model with long-term debt calibrated to the data, the presence of rollover crises has a negligible effect on debt and spreads.[4] By contrast, we show that with a fixed exchange rate regime, the multiplicity region expands significantly, and the government is heavily exposed to a rollover crisis.

[3]For a comprehensive discussion of these issues, see Alesina, Barro, and Tenreyro (2002) and De Grauwe (2018). For more recent work on aspects of credibility, see Chari, Dovis, and Kehoe (2018).

[4]With one-year maturity, as in Cole and Kehoe (1996), the exposure to a rollover crisis is typically large because the government has to roll over a large amount of debt relative to output every period. The typical maturity for sovereign bonds, however, is much larger, averaging around six years for the Eurozone. In a model featuring a subsistence level of consumption, Bocola and Dovis (2016) obtain a more moderate role for rollover crises. In a novel quantitative application with maturity choice, they find that about 14% of the increase in spreads during the Italian debt crisis can be explained by non-fundamentals.

A related literature studies the role of monetary policy and sovereign debt crises. The paper that is perhaps most closely related to ours is Aguiar, Amador, Farhi, and Gopinath (2013), who address the question of whether the government's ability to inflate away its debt reduces its exposure to rollover crises, an argument notably raised by De Grauwe (2013) and Krugman (2011).[5] They consider an endowment economy with domestic currency debt and show that when commitment to low inflation is weak, an independent monetary policy can actually increase the vulnerability to a rollover crisis, in contrast with De Grauwe and Krugman's view.[6] Our paper also studies how monetary policy matters for the exposure to a rollover crisis but considers instead a model with nominal rigidities and foreign currency debt. Our results show that the lack of monetary autonomy does increase the vulnerability to a rollover crisis and provides a new perspective that ascribes a role for monetary policy to deal with rollover crises, even when debt is denominated in foreign currency.

Our paper is also related to an emerging literature, which incorporates nominal rigidities into workhorse sovereign default models. Na, Schmitt-Grohé, Uribe, and Yue (2018) study a sovereign default model with downward nominal wage rigidity and show that it can account for the joint occurrence of large nominal devaluations and defaults, a phenomenon they dub the "twin Ds." Moreover, they show that an economy with a fixed exchange rate accumulates less debt than the flexible exchange rate regime. Bianchi, Ottonello, and Presno (2018) examine the trade-off between the expansionary effects of government spending and the increase in sovereign risk. Arellano, Mihalache, and Bai (2018) study the comovements of sovereign spreads with domestic nominal rates and inflation. In contrast to these papers, we consider an economy with rollover crises and examine how monetary policy affects the vulnerability to these episodes. Another contribution of our paper is to show analytically how the exchange rate policy alters the incentives to default.

Layout. Section 2 presents the model, and Section 3 presents the theoretical analysis. Section 4 presents the quantitative analysis, and Section 5 concludes. The proofs are listed in Appendix A.

2 Model

We study a small open economy (SOE) model of endogenous sovereign default subject to rollover crises. The SOE is populated by households, firms, and a government. In the international financial

[5]The arguments in De Grauwe (2013) and Krugman (2011) were partly motivated by the observation that Spain and Portugal had higher levels of sovereign spreads compared to the UK, despite having lower levels of debt.

[6]Corsetti and Dedola (2016) also qualifies that view for related reasons, but in a model featuring self-fulfilling crises a là Calvo (1988), in which multiplicity arises because the government can end up in the downward-sloping side of the Laffer curve. Building on a dynamic version of Calvo by Lorenzoni and Werning (2013), Bacchetta, Perazzi, and Van Wincoop (2018) found large costs from eliminating the bad equilibrium by inflating away the debt. The role of inflation as partial default also plays a key role in recent work by Araujo, Leon, and Santos (2013), Du and Schreger (2016), Aguiar, Amador, Farhi, and Gopinath (2015), Bassetto and Galli (2017), Nuño and Thomas (2017), Camous and Cooper (2018) and Farhi and Maggiori (2017).

markets, risk-neutral lenders buy the long-term government bonds of the SOE denominated in foreign currency. A single tradable good can be traded without any frictions, and as a result, the law of one price holds. In addition, a non-tradable good in the SOE is produced using labor, and downward nominal wage rigidity creates the possibility of involuntary unemployment. We describe next the decision problems of households, firms, lenders, and the government.

2.1 Households

There is a unit measure of households with preferences over consumption given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, U(c_t), \tag{1}$$

with

$$c_t = C(c_t^T, c_t^N) = [\omega (c_t^T)^{-\mu} + (1-\omega)(c_t^N)^{-\mu}]^{-1/\mu}, \quad \omega \in (0,1), \; \mu > -1.$$

The utility function $U(c)$ is of the constant relative risk aversion (CRRA) form, where c is a composite of tradable (c^T) and non-tradable goods (c^N), with constant elasticity of substitution (CES) equal to $1/(1+\mu)$.

Each period, households receive y_t^T units of tradable endowment, which is stochastic and follows a stationary first-order Markov process. Assuming a constant unit price of tradable goods in terms of foreign currency, the value of the tradable good endowment in domestic currency is given by $e_t y^T$, where e_t denotes the exchange rate measured as domestic currency per foreign currency (an increase in e_t denotes a depreciation of the domestic currency). Households also receive firms' profits, which we denote by ϕ_t^N, and labor income, $W_t h_t$, where W is the wage expressed in domestic currency and h is the amount of hours worked. Households inelastically supply $\overline{h}$ hours of work to the labor markets, but due to the presence of downward wage rigidity, they will work a strictly lower amount of hours when wage rigidity is binding. As we will discuss below, when wage rigidity is binding, the actual hours worked will be determined by firms' labor demand given prices and wages.

As is standard in the sovereign debt literature, we assume that households do not have direct access to external credit markets, although the government can borrow abroad and distribute the net proceedings to the households using lump-sum taxes or transfers. The households' budget constraint, expressed in domestic currency, is therefore given by

$$e_t c_t^T + P_t^N c_t^N = e_t y_t^T + \phi_t^N + W_t h_t - T_t, \tag{2}$$

where P_t^N denotes the price of non-tradables in domestic currency, and T_t denotes lump-sum taxes/transfers in units of domestic currency.

The households' problem consists of choosing c_t^T and c_t^N to maximize (1) given the sequence of

prices for non-tradables $\{P_t^N\}$, labor income $\{W_t h_t\}$, profits $\{\phi_t^N\}$, and taxes $\{T_t\}$. The static optimality condition equates the relative price of non-tradables to the marginal rate of substitution between tradables and non-tradables.

$$\frac{P_t^N}{e_t} = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\mu}. \tag{3}$$

Because preferences are homothetic, as a result of the assumptions of the CRRA utility function and the CES consumption aggregator, the relative consumption of tradable to non-tradable consumption is only a function of the relative price.

2.2 Firms

Firms operate a production function $y^N = F(h)$ where y_t^N denotes the output of non-tradable goods, and h_t denotes employment, the sole input. The production function $F(\cdot)$ is a differentiable, increasing, and concave function. In particular, we will consider a homogeneous production function $F(h) = h^\alpha$ where $\alpha \in (0, 1]$.

Firms operate in perfectly competitive markets, and each period they maximize profits that are given by

$$\phi_t^N = \max_{h_t} P_t^N F(h_t) - W_t h_t. \tag{4}$$

The optimal choice of labor employment h_t equates the value of the marginal product of labor to the nominal wage:

$$P_t^N F'(h_t) = W_t. \tag{5}$$

Given the price of non-tradables, a higher wage leads to lower employment. Likewise, given the wage, a lower price of non-tradables leads to lower employment. As we will see below, how the price of non-tradables reacts in general equilibrium will have important implications for debt crises.

2.2.1 Downward Nominal Wage Rigidity

We model downward nominal wage rigidity, following Schmitt-Grohé and Uribe (2016). In particular, we assume that wages in domestic currency cannot fall below $\overline{W}$:[7]

$$W_t \geq \overline{W} \tag{6}$$

for all t.

[7] For an economy within a monetary union, the lower bound is for the wage in foreign currency. As we will see, in a fixed exchange rate, the wage becomes effectively rigid in foreign currency.

The parameter $\overline{W}$ determines the severity of the wage rigidity.[8] There are two cases. If the nominal wage that clears the labor market is higher than $\overline{W}$, the economy is at full employment and (6) is not binding. If, however, the nominal wage that would clear the market is below $\overline{W}$, the economy experiences involuntary unemployment. In this case, the amount of employment in equilibrium is determined by the amount of labor demand (5), and households work strictly less than their endowment of hours. Formally, wages and employment need to satisfy the following slackness condition:

$$\left(W_t - \overline{W}\right)\left(\overline{h} - h_t\right) = 0. \tag{7}$$

2.3 Government

The government can issue a non-contingent long-term bond and can default at any point in time. As in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), a bond issued in period t promises an infinite stream of coupons that decrease at an exogenous constant rate $1-\delta$.[9] In particular, a bond issued in period t promises to pay $\delta(1-\delta)^{j-1}$ units of the tradable good (or foreign currency) in period $t+j$, for all $j \geq 1$. Hence, debt dynamics can be represented by the following law of motion:

$$b_{t+1} = (1-\delta)b_t + i_t, \tag{8}$$

where b_t is the stock of bonds due at the beginning of period t, and i_t is the stock of new bonds issued in period t.

Debt contracts cannot be enforced. If the government chooses to default, it faces two punishments. First, the government switches to financial autarky and cannot borrow for a stochastic number of periods. Second, there is a utility loss $\kappa(y_t^T)$, assumed to be increasing in tradable income. We think of this utility loss as capturing various default costs related to reputation, sanctions, or the misallocation of resources.[10]

The government's budget constraint in a period starting with good credit standing is

$$\delta e_t b_t(1-d_t) = T_t + e_t q_t i_t(1-d_t), \tag{9}$$

[8] In Schmitt-Grohé and Uribe (2016), $\overline{W}$ depends on the previous period wage and a parameter that controls the speed of wage adjustment. For numerical tractability, we take $\overline{W}$ as an exogenous (constant) value, as in Bianchi, Ottonello, and Presno (2018). Notice also that allowing for indexation to CPI inflation would not affect our theoretical mechanism because a nominal exchange rate depreciation in a state with unemployment would lead to a real exchange rate depreciation, and the price of non-tradables in domestic currency would rise by more than the increase in wages due to indexation.

[9] We take maturity as a primitive. There is an active literature studying maturity choices in sovereign default models (Arellano and Ramanarayanan, 2012; Bocola and Dovis, 2016; Sanchez, Sapriza, and Yurdagul, 2018).

[10] Utility losses from default in sovereign debt models are also used by Bianchi, Hatchondo, and Martinez (2018) and Roch and Uhlig (2018), among others. An alternative often used is the output costs from default. If the utility function is log over the composite consumption, and output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications. In any case, as it will become clear below, what will be important for our mechanism is the difference in the values of repayment when investors lend and when they refuse to lend. For this difference, the emergence of unemployment will be key.

where $d_t = 0(1)$ if the government repays (defaults) and $q(\cdot)$ denotes the price schedule, which we will characterize below. The budget constraint indicates that repayment of outstanding debt obligations is made by collecting lump-sum taxes and issuing new debt.[11]

The timing within each period follows Cole and Kehoe (2000). At the beginning of each period, the government has outstanding debt liabilities b_t and could be in good or bad credit standing. If the government is in good credit standing, it chooses new debt issuances at the price schedule offered by investors. At the end of each period, the government decides whether to default or repay the initial debt outstanding. The difference with respect to Eaton and Gersovitz (1981) that will give rise to multiplicity is that here the government does not have the ability to commit to repaying within the period.[12] As we will see, negative beliefs about the decision of the government to default can become self-fulfilling.

Monetary regimes. Regarding the policy for exchange rates, we will consider two regimes: a flexible exchange rate and a fixed exchange rate. In the flexible exchange rate regime, the government will choose the optimal exchange rate at all dates without commitment. In the fixed exchange rate regime, we assume that the government fixes the exchange rate to an exogenous level $e = \overline{e}$ at all times. One can also think about a fixed exchange rate as the policy of a single economy that enters a monetary union and gives up its currency, in which case wages would be directly denominated in the foreign currency. These cases are equivalent because the government is unable to alter the value of the currency is vis-à-vis the rest of the world.

2.4 International Lenders

Sovereign bonds are traded with atomistic, risk-neutral foreign lenders. In addition to investing through the defaultable bonds, lenders have access to a one-period risk-free security paying a net interest rate r. By a no-arbitrage condition, equilibrium bond prices when the government repays are then given by

$$q_t = \frac{1}{1+r}\mathbb{E}_t[(1-d_{t+1})(\delta + (1-\delta)q_{t+1})]. \tag{10}$$

Equation (10) indicates that in equilibrium, an investor has to be indifferent between investing

[11] We consider only lump-sum taxes and transfers and abstract from fiscal instruments such as specific taxes on consumption or payroll subsidies that could mimic a nominal depreciation, as studied in Farhi, Gopinath, and Itskhoki (2013) and Schmitt-Grohé and Uribe (2016). As long as there are some limits (either political or economic) to the use of these fiscal instruments that prevent the government from reaching full employment, our main results will continue to hold.

[12] A different source of multiplicity following Calvo (1988) arises if the government has to issue a fixed amount of debt revenues. In this case, the fact that bond prices decrease with debt generates a Laffer curve, which leads, directly through the budget constraint, to a high debt/spreads equilibrium and a low debt/spreads equilibrium. Lorenzoni and Werning (2013) explore this kind of multiplicity in a dynamic context with fiscal rules and long-term maturity and show how this gives rise to "slow moving debt crises" (see also Ayres, Navarro, Nicolini, and Teles, 2016).

in a risk-free security and buying a government bond at price q_t, bearing the risk of default. In case of repayment next period, the payoff is given by the coupon δ plus the market value q_{t+1} of the nonmaturing fraction of the bonds. Since we assume no recovery, the bond price is zero in the event of default.

2.5 Equilibrium

In equilibrium, the market for non-tradable goods clears:

$$c_t^N = F(h_t). \tag{11}$$

In addition, using the households' and government budget constraint (2) and the definition of the firms' profits and market clearing condition (11), we obtain the resource constraint for tradable goods in the economy:

$$c_t^T = y_t^T + (1 - d_t)[\delta b_t - q_t(b_{t+1} - (1 - \delta)b_t)]. \tag{12}$$

Before proceeding to study a Markov equilibrium in which the government chooses policies optimally without commitment, let us examine equilibrium for given government policies. A competitive equilibrium given government policies in our economy is defined as follows:

Definition 1 (Competitive Equilibrium)**.** Given an initial debt b_0, an initial credit standing, government policies $\{T_t, b_{t+1}, d_t, e_t\}_{t=0}^{\infty}$, and an exogenous process for the tradable endowment $\{y_t^T\}_{t=0}^{\infty}$ and for reentry after default, a *competitive equilibrium* is a sequence of allocations $\{c_t^T, c_t^N, h_t\}_{t=0}^{\infty}$ and prices $\{P_t^N, W_t, q_t\}_{t=0}^{\infty}$ such that:

1. Households and firms solve their optimization problems.
2. Government policies satisfy the government budget constraint (9).
3. The bond pricing equation (10) holds.
4. The market for non-tradable goods clears (11), and the resource constraint for tradables (12) holds.
5. The labor market satisfies conditions (6), (7), and $h \leq \overline{h}$.

Employment, Consumption, and Wages Using market clearing for non-tradable goods (11), together with the optimality conditions for households (3) and firms (5), we can obtain a useful (partial) characterization of equilibrium in a system of these three static equations, together with three variables (c_t^T, h_t, w_t). (The whole equilibrium is, of course, dynamic, as can directly be seen from the fact that b_{t+1} and c_t^T follow from dynamic equations.) Using this system of equations, we can then derive in every period a real equilibrium wage solely as a function of (c_t^T, h_t).

Lemma 1. *In any equilibrium, the real wage in terms of tradable goods is a function of tradable consumption and employment,*

$$\mathcal{W}(c_t^T, h_t) \equiv \frac{1-\omega}{\omega}\left(\frac{c_t^T}{F(h_t)}\right)^{1+\mu} F'(h_t). \tag{13}$$

Moreover, $\mathcal{W}(c_t^T, h_t)$ is increasing with respect to c_t^T and decreasing with respect to h_t.

One implication of this lemma is that an increase in the amount of tradable consumption is associated with a higher equilibrium wage. This occurs because higher tradable consumption is associated in equilibrium with a higher relative price of non-tradables, which in turn leads to a larger demand for labor and an increase in the real wage for a given level of employment. In addition, an increase in employment is associated in equilibrium with a lower real wage, to be consistent with firms' labor demand.

In equilibrium, we then have that downward nominal wage rigidity can be expressed as

$$\mathcal{W}(c_t^T, h_t)e_t \geq \overline{W}. \tag{14}$$

According to this lemma, we then have that if (14) is binding, a reduction in the amount of tradable consumption is associated with low employment in equilibrium. This result has important implications for the general equilibrium effects in the full dynamic system. If a shock reduces the demand for total consumption, we must have that for a given level of non-tradable output, the price of non-tradables needs to decline so that households switch consumption from tradables toward non-tradables and the market for non-tradable goods clears. Absent wage rigidity, we would have that the wage falls, and the only implication for the real economy is the reduction in tradable consumption. However, if wages are downwardly rigid, the decline in the relative price of non-tradables will lead to a decline in employment.

Based on Lemma 1, we can also analogously construct an equilibrium employment that is a function of c_t^T and $\overline{w_t} \equiv \overline{W}/e_t$.

Lemma 2. *In any equilibrium, employment is a piecewise linear function of tradable consumption for any $\overline{w_t}$,*

$$\mathcal{H}(c_t^T, \overline{w_t}) = \begin{cases} \left[\frac{1-\omega}{\omega}\left(\frac{\alpha}{\overline{w_t}}\right)\right]^{\frac{1}{1+\alpha\mu}} \left(c_t^T\right)^{\frac{1+\mu}{1+\alpha\mu}} & \text{if } c_t^T \leq \overline{c}_{\overline{w_t}}^T, \\ \overline{h} & \text{if } c_t^T > \overline{c}_{\overline{w_t}}^T \end{cases}, \tag{15}$$

where

$$\overline{c}_{\overline{w_t}}^T = \left[\frac{\omega}{1-\omega}\left(\frac{\overline{w_t}}{\alpha}\right)\right]^{\frac{1}{1+\mu}} \left(\overline{h}\right)^{\frac{1+\alpha\mu}{1+\mu}}.$$

This condition implies that when the wage rigidity is binding and there is unemployment, the government will realize that repaying debt and cutting back on consumption will create more unemployment. We will see below how the implied increase in the cost of repayment affects the vulnerability to a rollover crisis.

2.6 Recursive Government Problem

We consider the optimal policy of a benevolent government with no commitment, which chooses consumption and external borrowing to maximize households' welfare, subject to the implementability conditions. We focus on the Markov equilibria.

Every period in which the government enters with access to financial markets, it evaluates the lifetime utility of households if debt contracts are honored against the lifetime utility of households if they are repudiated. We use $\mathbf{s} = (y^T, \zeta)$ to denote the vector of exogenous states in every period. The variable ζ is a sunspot variable to index for the possibility of multiplicity of equilibria, as in Cole and Kehoe (2000), which we will describe below. Different from the equilibrium according to the timing in Eaton and Gersovitz (1981), the possibility of a rollover crisis implies that the bond price is a function of the initial debt position and the sunspot, in addition to the debt choice and current income shock.

Regarding the policy for exchange rates, we will start with the case in which the government is under a fixed exchange rate regime. That is, the exchange rate is fixed at an exogenous level $e = \overline{e}$ for every period. We can define a *real wage rigidity* constraint as $w \geq \overline{w}$ where $\overline{w} \equiv \overline{W}/\overline{e}$ and $w \equiv W/e$. We can then rewrite (14) as $\mathcal{W}(c^T, h) \geq \overline{w}$. Later on, we will study the case in which we allow the government to depreciate its currency. As should be clear from (14), an exchange rate depreciation will be able to undo the wage rigidity, and this will be the optimal policy for the government.

The government problem with access to financial markets can be formulated in recursive form as follows:

$$V(b, \mathbf{s}) = \max_{d \in \{0,1\}} \{(1-d)V_R(b, \mathbf{s}) + dV_D(y^T)\}, \tag{16}$$

where $V_R(b, \mathbf{s})$ and $V_D(y^T)$ denote, respectively, the values of repayment and default.

The value of repayment is given by the following Bellman equation:

$$V_R(b, \mathbf{s}) = \max_{b', c^T, h \leq \overline{h}} \left\{u(c^T, F(h)) + \beta \mathbb{E} V(b', \mathbf{s}')\right\} \tag{17}$$

subject to

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b)$$

$$\mathcal{W}(c^T, h) \geq \overline{w},$$

where $q(b', b, \mathbf{s})$ denotes the debt price schedule, taken as given by the government, and $\mathcal{W}$ is defined

in (13).[13] Meanwhile, the value of default is given by

$$V_D(y^T) = \max_{c^T, h \leq \overline{h}} \left\{ u\left(c^T, F(h)\right) - \kappa(y^T) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi) V_D(y^{T\prime})\right] \right\} \tag{18}$$

subject to

$$c^T = y^T$$

$$\mathcal{W}(c^T, h) \geq \overline{w},$$

where $\psi \in [0,1]$ is the probability of reentering financial markets after a default.

Let $\left\{\hat{d}(b,\mathbf{s}), \hat{c}^T(b,\mathbf{s}), \hat{b}(b,\mathbf{s}), \hat{h}(b,\mathbf{s})\right\}$ be the optimal policy rules associated with the government problem. A Markov-perfect equilibrium is then defined as follows.

Definition 2 (Markov-perfect equilibrium)**.** A Markov-perfect equilibrium is defined by value functions $\{V(b,\mathbf{s}), V_R(b,\mathbf{s}), V_D(y^T)\}$, policy functions $\{\hat{d}(b,\mathbf{s}), \hat{c}^T(b,\mathbf{s}), \hat{b}(b,\mathbf{s}), \hat{h}(b,\mathbf{s})\}$, and a bond price schedule $q(b', b, \mathbf{s})$ such that

1. Given the bond price schedule, policy functions solve problems (16), (17), and (18),

2. The debt price schedule satisfies

$$q(b', b, \mathbf{s}) = \begin{cases} \frac{1}{1+r}\mathbb{E}[(1-d')(\delta + (1-\delta)q(b'', b', \mathbf{s}'))] & \text{if } \hat{d}(b,\mathbf{s}) = 0, \\ 0 & \text{if } \hat{d}(b,\mathbf{s}) = 1 \end{cases},$$

where

$$\begin{aligned} b'' &= \hat{b}(b', \mathbf{s}') \\ d' &= d(b', \mathbf{s}'). \end{aligned}$$

For the economy with a flexible exchange rate, the only difference would be that the government chooses e in addition to the prices and allocations that are chosen under the fixed exchange rate regime.

2.7 Multiplicity of Equilibrium

As in Cole and Kehoe (2000), the government is subject to self-fulfilling rollover crises. Let us define the debt price schedule, assuming there will be no default and the break-even condition of lenders is

[13] An equivalent representation uses equilibrium employment (15) rather than the explicit wage rigidity constraint.

satisfied. We will call this the *fundamental* debt price schedule:

$$\tilde{q}(b', y^T) \equiv \frac{1}{1+r}\mathbb{E}[(1-d')(\delta + (1-\delta)q(b'', b', \mathbf{s}'))], \tag{19}$$

where $b'' = \hat{b}(b', \mathbf{s}')$ and $d' = d(b', \mathbf{s}')$. This debt price schedule does not depend on the sunspot nor on the current amount of debt held by the government. Using this price schedule, we can construct the value of repayment when international lenders believe that the government will honor its debt commitments at the end of the period and therefore extend credit to the government. This value is as follows:

$$V_R^+(b, y^T) = \max_{b', c^T, h \leq \bar{h}} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V(b', \mathbf{s}') \right\} \tag{20}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b + \tilde{q}(b', y^T)[b' - (1-\delta)b]$$

$$\mathcal{W}(c^T, h) \geq \overline{w}.$$

Denote by $\hat{b}^+(b, y^T)$ the solution to the previous problem. Divide the state space where the government finds it optimal to issue strictly positive amounts of debt:

$$\mathcal{B} \equiv \left\{ (b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad \hat{b}^+(b, y^T) > (1-\delta)b \right\}.$$

Consider now the case in which investors are unwilling to lend to the government. Denote by V_R^- the value function in this case, when the government decides to repay. If $(b, y^T) \notin \mathcal{B}$, we have that $V_R^-(b, y^T) = V_R^+(b, y^T)$, as the government is not issuing debt even when investors are willing to lend to the government. Then, if $(b, y^T) \in \mathcal{B}$, the value is given by

$$V_R^-(b, y^T) = \max_{c^T, h \leq \bar{h}} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V((1-\delta)b, \sim') \right\} \tag{21}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b$$

$$\mathcal{W}(c^T, h) \geq \overline{w}.$$

Lemma 3. *For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b \in \mathbb{R}$, we have that $V_R^+(b, y^T) \geq V_R^-(b, y^T)$.*

Lemma 3 tells us that the value of repayment when lenders refuse to roll over government bonds is never greater than the value when lenders are willing to roll over. This must be the case since the

government can always choose not to borrow when lenders are willing to roll over.[14]

Three zones. Let us separate the state space (b, y^T) into three zones: the safe zone, default zone, and crisis zone. The *safe zone* will denote those states in which the government finds it optimal to repay its debt even if international lenders are not willing to issue more debt to the government. That is,

$$\mathcal{S} \equiv \left\{(b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^-(b, y^T)\right\}.$$

The *default zone* defines those states in which the government finds it optimal to default even if international lenders are willing to lend at the fundamental debt price schedule. That is,

$$\mathcal{D} \equiv \left\{(b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) > V_R^+(b, y^T)\right\}.$$

Finally, the *crisis zone* will correspond to those states in which the government finds it optimal to repay if investors are willing to lend at the fundamental debt price schedule, but finds it optimal to default if investors are not willing to lend. That is,

$$\mathcal{C} \equiv \left\{(b, y^T) \in \mathbb{R} \times \mathbb{R}_+ : \quad V_D(y^T) \leq V_R^+(b, y^T) \quad \& \quad V_D(y^T) > V_R^-(b, y^T)\right\}.$$

In this zone, the outcome is undetermined and depends on investors' beliefs. If investors believe the government will repay, the government will find it optimal to repay whereas if they believe that the government will default, the government will default. To select an equilibrium, we will use a sunspot $\zeta \in \{0, 1\}$. If $\zeta = 0$, we will say there is a "good sunspot", in which case the equilibrium with repayment is selected. If $\zeta = 1$, we will say there is a "bad sunspot", in which case the equilibrium with default is selected. We assume that ζ follows an *i.i.d.* process with probability π of selecting $\zeta = 1$.

Following these definitions, the optimal binary default decision and the optimal debt price sched-

[14] One element implicit here is that if the government were to try to repurchase debt when investors are unwilling to lend, the price of bonds would rise to the fundamental price, and hence the budget constraint when $(b, y^T) \in \mathcal{B}$ would be $c^T = y^T - \delta b$, as reflected in (21). See Aguiar and Amador (2013) and Bocola and Dovis (2016) for an elaboration of this point.

ule will satisfy

$$d(b,\mathbf{s}) = \begin{cases} 1 & \text{if } (b,y^T) \in \mathcal{D} \\ 1 & \text{if } (b,y^T) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ 0 & \text{if } (b,y^T) \in \mathcal{C} \quad \& \quad \zeta = 0 \\ 0 & \text{if } (b,y^T) \in \mathcal{S} \end{cases} \tag{22}$$

$$q(b',b,\mathbf{s}) = \begin{cases} 0 & \text{if } (b,y^T) \in \mathcal{D} \\ 0 & \text{if } (b,y^T) \in \mathcal{C} \quad \& \quad \zeta = 1 \\ \tilde{q}(b',y^T) & \textit{in every other case} \end{cases} . \tag{23}$$

3 Theoretical Analysis

In this section, we provide an analytical characterization of how monetary policy and downward nominal wage rigidity affect the government's vulnerability to a rollover crisis. The central point we will show is that fixing the exchange rate leaves a government more vulnerable to a rollover crisis. Simply put, the crisis zone will be larger for an economy with a fixed exchange rate.

3.1 Flexible Exchange Rate Regime

The flexible exchange rate regime allows the government to pick any nominal exchange rate every period. The following proposition characterizes the optimal exchange rate policy.

Proposition 1 (Optimal Exchange Rate Policy)**.** *Under a flexible exchange rate regime, the government chooses an exchange rate that delivers full employment in all states.*

This proposition establishes that the government finds it optimal to choose an exchange rate that delivers full employment, a result that can be seen from the value functions (17) and (18). If there was unemployment in the economy, the government could always relax the wage constraint by sufficiently depreciating the nominal exchange rate without bearing any other costs. This basic result is of course in line with the traditional benefit of having a flexible exchange rate in the presence of nominal rigidities, emphasized by Friedman (1953) and Mundell (1961) (see also Na, Schmitt-Grohé, Uribe, and Yue, 2018). One difference here is that to ensure full employment, the government needs to depreciate the currency not only on-equilibrium but also off-equilibrium.

It is worth pointing out that while we do not explicitly model why the government would deviate from this policy (i.e., why the government would fix the exchange rate or join a monetary union), doing so, in practice, offers a number of well-studied benefits. The gains could arise, for example, from

lower inflationary bias (Alesina and Barro, 2002, Barro and Gordon, 1983) or from improvements in trade from lower volatility and transaction costs (Mundell, 1961, Frankel and Rose, 2002). To focus squarely on the costs, we do not explicitly model these benefits and leave for future research a study of the trade-offs involved.

3.2 The Role of Rigidities

In this section, we study how wage rigidity and the exchange rate regime shape default decisions and the exposure to a rollover crisis. As a starting point, consider an environment in which the government is under a flexible exchange rate and, for only *one period*, assume that the government is subject to a fixed exogenous exchange rate $\overline{e} > 0$ while the economy remains under a flexible exchange rate in future periods. We will later study the consequences of permanent changes in the exchange rate regime, but introducing this comparative statics type of exercise is useful now because it allows us to isolate current changes in monetary policy while leaving constant future policies. Moreover, one implication of assuming that the change in the exchange rate regime is only for one period is that the *fundamental* price schedule remains the same. This is because continuation values do not change, and hence future default decisions also remain unchanged. Using this comparative statics exercise, we will be able to obtain a sharp analytical characterization.

Let us denote by $\left\{V^{flex}(b,\mathbf{s}), V_R^{flex}(b,\mathbf{s}), V_D^{flex}(y^T)\right\}$ the continuation values and by $\tilde{q}^{flex}(b,y^T)$ the price schedule in this environment in which the future Markov equilibrium has flexible wages. Let us denote the current value functions with one-period wage rigidity $\overline{w}$ as $\tilde{V}_D(y^T;\overline{w})$, $\tilde{V}_R^+(b,y^T;\overline{w})$, $\tilde{V}_R^-(b,y^T;\overline{w})$. These values are given by

$$\tilde{V}_D(y^T;\overline{w}) = \max_{c^T, F(h)\leq \overline{h}} \left\{u\left(c^T,h\right) - \kappa(y^T) + \beta\mathbb{E}\left[\psi V^{flex}(0,\mathbf{s}') + (1-\psi)V_D^{flex}(y^{T\prime})\right]\right\} \tag{24}$$

$$\text{subject to}$$

$$c^T = y^T,$$

$$\mathcal{W}(c^T,h) \geq \overline{w},$$

$$\tilde{V}_R^+(b,y^T;\overline{w}) = \max_{b',c^T,h\leq\overline{h}} \left\{u(c^T,F(h)) + \beta\mathbb{E}V^{flex}(b',\mathbf{s}')\right\} \tag{25}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b + \tilde{q}^{flex}(b',y^T)(b' - (1-\delta)b),$$

$$\mathcal{W}(c^T,h) \geq \overline{w},$$

and

$$\tilde{V}_R^-(b, y^T; \overline{w}) = \max_{c^T, F(h) \leq \bar{h}} \left\{ u(c^T, h) + \beta \mathbb{E} V^{flex}((1-\delta)b, \mathbf{s}') \right\} \tag{26}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b,$$

$$\mathcal{W}(c^T, h) \geq \overline{w}.$$

In order to show how the three zones (safe, default, and crisis) are affected by the nominal rigidity, we first present some useful properties.

Lemma 4. *The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to debt b.*

Lemma 5 (Debt Thresholds)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exist levels of debt $\bar{b}^+, \bar{b}^- \in \mathbb{R}_+$ such that $\tilde{V}_D(y^T) = V_R^+\left(\bar{b}^+, y^T\right)$ and $\tilde{V}_D(y^T) = V_R^-\left(\bar{b}^-, y^T\right)$. Furthermore, $\bar{b}^+ \geq \bar{b}^-$.*

The previous lemmas help us to construct the three zones into intervals conditional on a given level of tradable endowment. Lemma 4 says that the repayment value functions are strictly decreasing with respect to current debt. Using this result and the fact that the value of default is independent of debt, Lemma 5 states that the threshold at which the government is indifferent between repaying and defaulting depends on whether investors are willing to lend or not. In particular, the amount of debt in which the government is indifferent between repaying or defaulting when investors are willing to lend is lower than the amount of debt in which the government is indifferent between repaying or defaulting when investors refuse to lend.

Using these results, we can construct a safe region, a crisis region and a default region for every level of income:

$$\tilde{S}(y^T) \equiv \left(-\infty, \bar{b}^-\right], \qquad \tilde{C}(y^T) \equiv \left(\bar{b}^-, \bar{b}^+\right], \qquad \text{and} \qquad \tilde{D}(y^T) \equiv \left(\bar{b}^+, \infty\right).$$

Next we study now how these regions expand or contract as the real wage rigidity increases.[15]

Proposition 2 (Default Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{D}(y^T; \overline{w}_1) \subseteq \tilde{D}(y^T; \overline{w}_2)$.*

Proposition 3 (Safe Region Contraction)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{S}(y^T; \overline{w}_2) \subseteq \tilde{S}(y^T; \overline{w}_1)$.*

Proposition 2 tells us that if the government defaults for a given $\overline{w}$, it will also default for a higher wage rigidity. Likewise, Proposition 3 tells us that if the government is in the safe zone for a given

[15] Different from the "zones" constructed above which are in the (b, y^T) space, the "regions" fix the level of tradable endowment.

$\overline{w}$, it will remain in the safe zone for a looser wage rigidity. The essence of these two propositions is that the value of repayment is decreasing in $\overline{w}$, whereas the value of default does not respond to $\overline{w}$ provided that wage rigidity is not too tight.

Movements in the crisis region are not as straightforward as they are in the other two regions. If the safe region shrinks, the crisis region increases and the economy arrives at the crisis region with lower levels of debt. At the same time, if the default region expands, then the crisis region decreases. Nevertheless, we are able to provide a sharp result that establishes the conditions under which the crisis region expands with higher rigidity.

Proposition 4 (Crisis Region Expansion)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_C \in \mathbb{R}_+$ such that if $\overline{w}_1, \overline{w}_2 < \overline{w}_C$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{C}(y^T; \overline{w}_1) \subseteq \tilde{C}(y^T; \overline{w}_2)$. Moreover, there exists $\overline{w}_S \in \mathbb{R}_+$ such that if $\overline{w}_2 > \overline{w}_S$, then $\tilde{C}(y^T; \overline{w}_1) \subset \tilde{C}(y^T; \overline{w}_1)$*

Proposition 4 establishes that starting from a low $\overline{w}$, an increase in the real wage rigidity makes the crisis region weakly increase. Key for this result is that starting from full employment, a small increase in wage rigidity first affects the safe zone, thereby increasing the crisis region, and only after a sufficiently large increase does the default region start to increase. Moreover, we are able to show that for a sufficiently high increase in rigidity up to some level, the crisis region increases unequivocally.

Extensions and Generalizations. It is worth pointing out that while we obtained these theoretical results in a model with a particular set of assumptions, they can be extended and generalized in a relatively straightforward manner in a number of directions. In particular, the results can easily be extended to a model with an arbitrary maturity structure or to a model featuring price stickiness instead of wage stickiness. Likewise, we also derived these results assuming that the economy from $t+1$ onward is in a flexible exchange rate regime. If we allow the government to follow any arbitrary monetary policy regime from $t+1$ onward, the same results can be derived.

On the technical side, rather than considering a one-period deviation for $\overline{w}$, we could allow for independent shocks over time on the exchange rate, or $\overline{W}$, and our results would continue to hold.[16] In Section 4, we will consider numerically the differences in exposure to rollover crises across two different permanent regimes: a flexible exchange rate regime that eliminates unemployment and a fixed exchange rate regime.

3.3 Graphical Illustration

Following the theoretical analysis above, this section provides a graphical illustration of how wage rigidity tends to increase the vulnerability to a rollover crisis. To construct the following figures, we use the calibrated version of our model, which we will explain in the quantitative section.

[16] Considering serially correlated shocks in $\overline{w}$ would make it harder to obtain analytical results because the fundamental price schedule would react to $\overline{w}$ in a way that is not possible to characterize analytically.

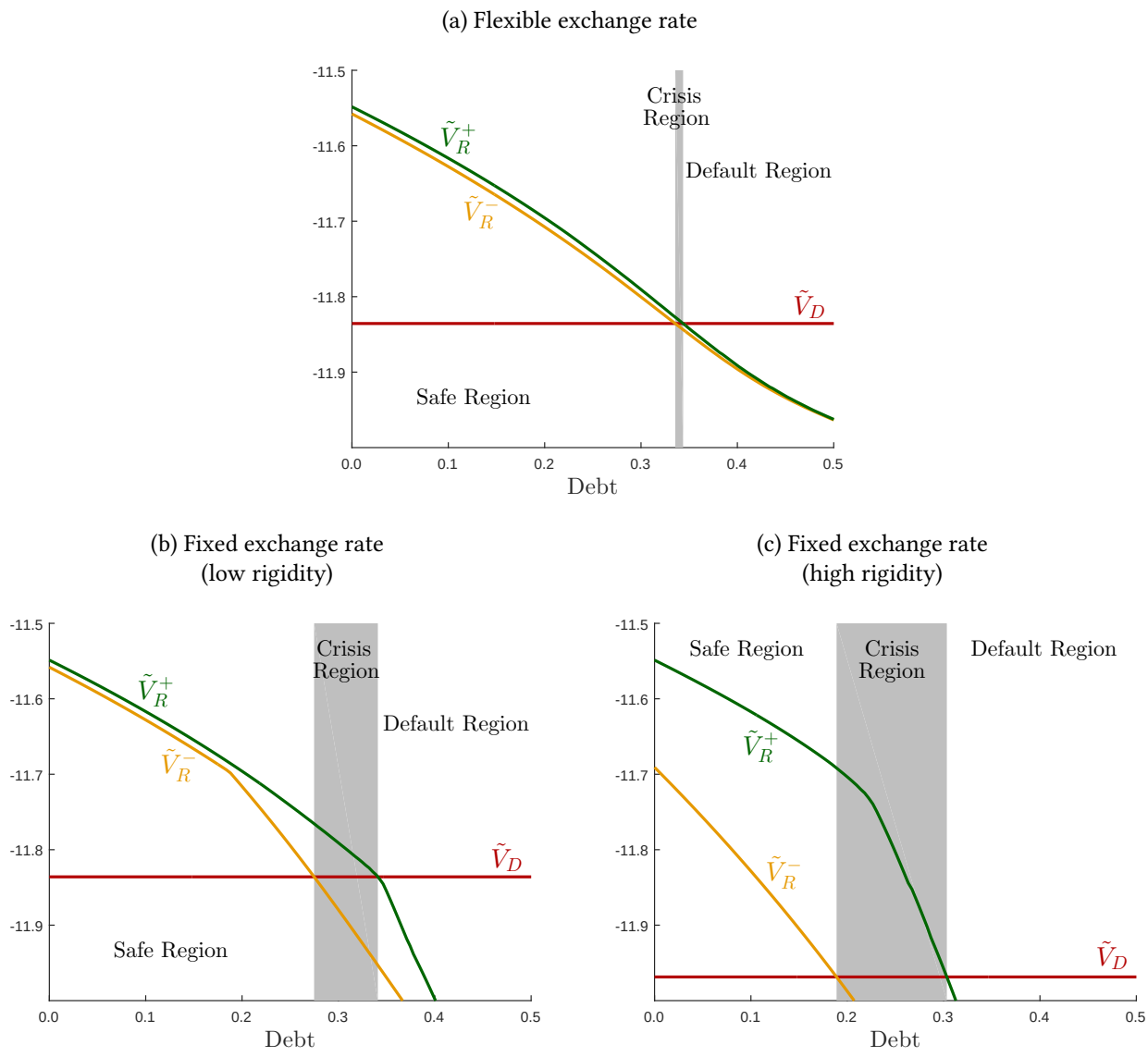

Figure 1: Value Functions and Crisis Regions

Notes: The income shock in the three panels is set to -4.3% below the mean, which is the average income shock before a default episode in the flexible exchange regime. Panel (a) uses parameter values from the calibrated flexible exchange rate economy. Panels (b) and (c) use the same parameters with the exception of the current level of wage rigidity $\overline{w}$. In panel (a), $\overline{w}$ is set to its highest value where full employment is achieved under a good sunspot. This is 1.33 times the real wage in the flexible exchange rate regime. Panel (c) increases the wage rigidity to 1.66 times higher than the wage in the flexible exchange rate regime.

In Figure 1 we present the values $\left\{\tilde{V}_D, \tilde{V}_R^+, \tilde{V}_R^-\right\}$ for different levels of debt. We fix the tradable endowment to the average value in default episodes in our simulation exercise for the flexible exchange rate regime (technically, the element in the grid that is closest to this point). This level is 4.3% below average. To facilitate the reading of the figures, we normalize debt by average GDP. Unless we specify otherwise, all numbers reported will be expressed in this way. Notice that in Figure 1, the

actual value function V, as defined in (16), is given by the upper envelope of $\tilde{V}_D$ and $\tilde{V}_R^+$ in the case of the good sunspot and by the upper envelope of $\tilde{V}_D$ and $\tilde{V}_R^-$ in the case of the bad sunspot. Panel (a) presents the values for the flexible exchange rate regime. It should be understood that when we refer to the flexible exchange rate, we mean the exchange rate policy that delivers the full employment case in all states. For the case of a fixed exchange rate, it will be useful to consider two values for $\overline{w}$. Panel (b) corresponds to the fixed exchange rate regime with "low" wage rigidity and panel (c) to a fixed exchange rate regime with "high" wage rigidity.

Using these value functions, it is straightforward to represent graphically the safe region, crisis region, and default region in Figure 1. The crisis region (i.e., the levels of debt in which a default would occur if there is a bad sunspot) appears shaded in the figure. The safe region (i.e., the levels of debt in which the government repays regardless of whether lenders extend credit or not) is to the left of the crisis region. The default region (i.e., the levels of debt in which the government defaults regardless of whether lenders extend credit or not) is to the right. It is apparent from these figures that vulnerability to a rollover crisis is higher in a fixed exchange rate regime than in a flexible one for both degrees of wage rigidity.

Crisis region for flexible exchange rate. Let us describe now how we arrive at the crisis region in the flexible exchange regime in panel (a) of Figure 1. The value of default $\tilde{V}_D$ is a constant because it does not depend on the amount of debt the government owes.[17] The value of repayment when rollover is feasible, $\tilde{V}_R^+$, and when no borrowing is allowed, $\tilde{V}_R^-$, is decreasing in debt in both cases because this means that the government owes more to international lenders and the resource constraint becomes tighter. The value function $\tilde{V}_R^+$ is uniformly above $\tilde{V}_R^-$. Moreover, the difference between these two values is higher for low levels of debt (when the government wants to issue more debt), and the values become identical for very high levels of debt (when the government does not issue debt even when it has access to financial markets).

At the debt level in which the curves $\tilde{V}_R^+$ and $\tilde{V}_D$ intersect, the government is indifferent between repaying when having access to credit markets and defaulting. For debt positions higher than this level, the government defaults regardless of the international lenders' beliefs. This is what we define as the default region. On the other hand, at the debt level in which the curves $\tilde{V}_R^-$ and $\tilde{V}_D$ intersect, the government is indifferent between defaulting and repaying when unable to roll over the debt. For debt positions lower than this level, the value of repayment is higher than the value of default, and the government repays its debt. This is what we define as the safe region, levels of debt in which the government repays even if investors are pessimistic. In between these two regions, there is an interval of debt positions in which the government will only default if international lenders are unwilling to roll over the debt. This is what we define as the crisis region: it is the range of debt levels in which the

[17] In the case of flexible wages, $\tilde{V} = V$. However, we keep the notation with $\tilde{V}$ to make it more uniform with the fixed exchange rate regime.

government only defaults because of pessimistic beliefs. This region, which appears shaded in panel (a) of Figure 1, is less than 1% of debt in terms of average GDP: the range is between 33.5% and 34.4%. The region in which the government is vulnerable to a rollover crisis is small for a flexible exchange rate regime.

Crisis region for fixed exchange rate. Panels (b) and (c) of Figure 1 consider the one-period fixed exchange rate regime. As described above, we consider a situation in which there is a fixed exchange rate regime for only the current period and the flexible regime prevails from next period onward. The impact of the fixed exchange rate regime depends, of course, on the level of nominal wages and the level of the exchange rate—in particular, a sufficient variable is $\overline{w}$, the lower bound on wages in foreign currency. We consider two values for this real wage rigidity $\overline{w}$. In panel (b), we consider the highest value of $\overline{w}$ such that the default region remains unchanged relative to the flexible wage. This case allows us to consider a situation in which only the left threshold of the crisis region changes while the right threshold remains the same.[18] One can also see that $\tilde{V}^D$ is at exactly the same level as in the flexible regime because the wage rigidity constraint is not binding for this income shock. In panel (c) we consider a higher degree of wage rigidity, in which case we also see a reduction in $\tilde{V}^D$ because the wage rigidity is also binding under default. In this case, we also see a decline in $\tilde{V}_R^+$ in the crisis region.

Panels (b) and (c) reveal that there is a bigger gap between $\tilde{V}_R^+$ and $\tilde{V}_R^-$ with a fixed exchange rate regime compared to the flexible exchange rate regime. In other words, both values drop, but $\tilde{V}_R^-$ is reduced by much more than V_R^+. Key for this result is the behavior of unemployment, as we will explain below. The consequence of the increase in the gap between these two curves is the increase in vulnerability to a rollover crisis, in line with Proposition 4.[19] In panel (b), the range of the crisis region is about 7% of GDP and goes from 27.1% to 34.4%. In panel (c), the crisis region increases to more than 12% percentage points of GDP and represents more than a third of the average debt-GDP. Moreover, the economy enters a rollover crisis with a level of debt that is 14 percentage points of GDP lower than the level it takes under a flexible exchange rate regime.

We have illustrated so far how the exchange rate regime shapes the crisis region for two values of $\overline{w}$. In Figure 2 we show how the safe, crisis, and default regions change for a whole range of $\overline{w}$, keeping the income level the same as before. The value of $\overline{w}$ is normalized by the highest $\overline{w}$ that is consistent with no changes in the three zones.[20] In this way, a value lower than unity in Figure 2 will

[18] To obtain the value of $\overline{w}$ that delivers this, we have to infer the highest value of $\overline{w}$ such that there is no unemployment when the economy faces a good sunspot. Intuitively, an increase in $\overline{w}$ beyond this point would reduce the value from repaying and lead the government to default when investors are willing to lend, which would imply an increase in the default region.

[19] Notice that the gap between $\tilde{V}_R^+$ and $\tilde{V}_R^-$ is not affected by the form of the current default cost because the government is not defaulting.

[20] This level can be computed by first obtaining b^- such that $V_R^-(b^-, y^T) = V_D(y^T)$ and then finding $\overline{w}$ such that $\tilde{V}_R^+(b^-, y^T; \overline{w}) = V^D(y^T)$.

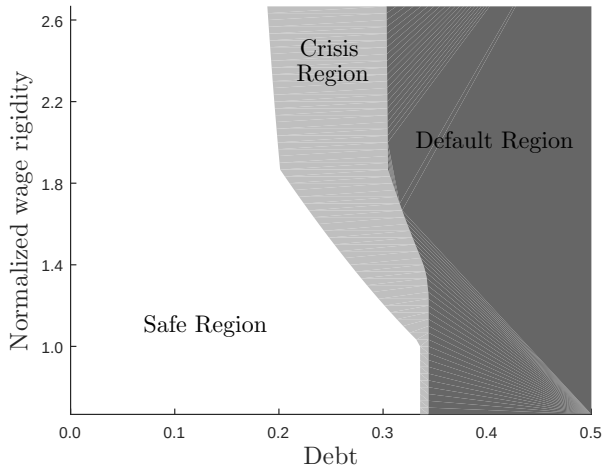

Figure 2: Safe, Crisis, and Default Regions under Different Wage Rigidities

correspond effectively to the flexible exchange rate regime. As soon as wage rigidity rises above one, by construction, it becomes binding and the crisis region starts to expand. For low values of wage rigidity, the intersection between $\tilde{V}_D$ and V_R^+ remains unaffected, and hence the crisis region expands at the expense of the safe region without changes in the default region. Once $\overline{w}$ reaches 1.33, which is the value used in panel (b) in Figure 1, the value of default starts to expand as well and it does so at the expense of the crisis region. However, we can see in Figure 2 that the crisis region continues to expand significantly because the safe region contracts by an amount greater than the default region expansion.

Crisis zone. We have shown the safe, crisis, and default regions for a given level of the tradable endowment. To have a more complete picture, we consider a whole range of combinations of debt and income. We show in Figure 3 the three zones in the (b, y^T) state space. For any given level of debt, the economy is in the default zone for a sufficiently low level of tradable endowment. As we increase the tradable endowment, the economy arrives in the crisis zone at some point. Finally, increasing it even further makes the economy reach the safe zone.

Again, we can clearly see how vulnerability to a rollover crisis is lower in a flexible exchange rate regime compared to a fixed regime, and this occurs for all income levels.[21] An implication of this, as we will see below, is that the economy with a fixed exchange rate regime will spend more time in the crisis region than will the flexible exchange rate regime. As a result, the economy with a fixed

[21] Along the horizontal y-axis with $y^T = 0.957$ for all panels in Figure 3, we recover exactly the same thresholds that separate the three regions in Figure 1. Notice also that for any income level different from $y^T = 0.957$, the default region will change in panel (b).

exchange rate will experience more defaults due to rollover crises. [22]

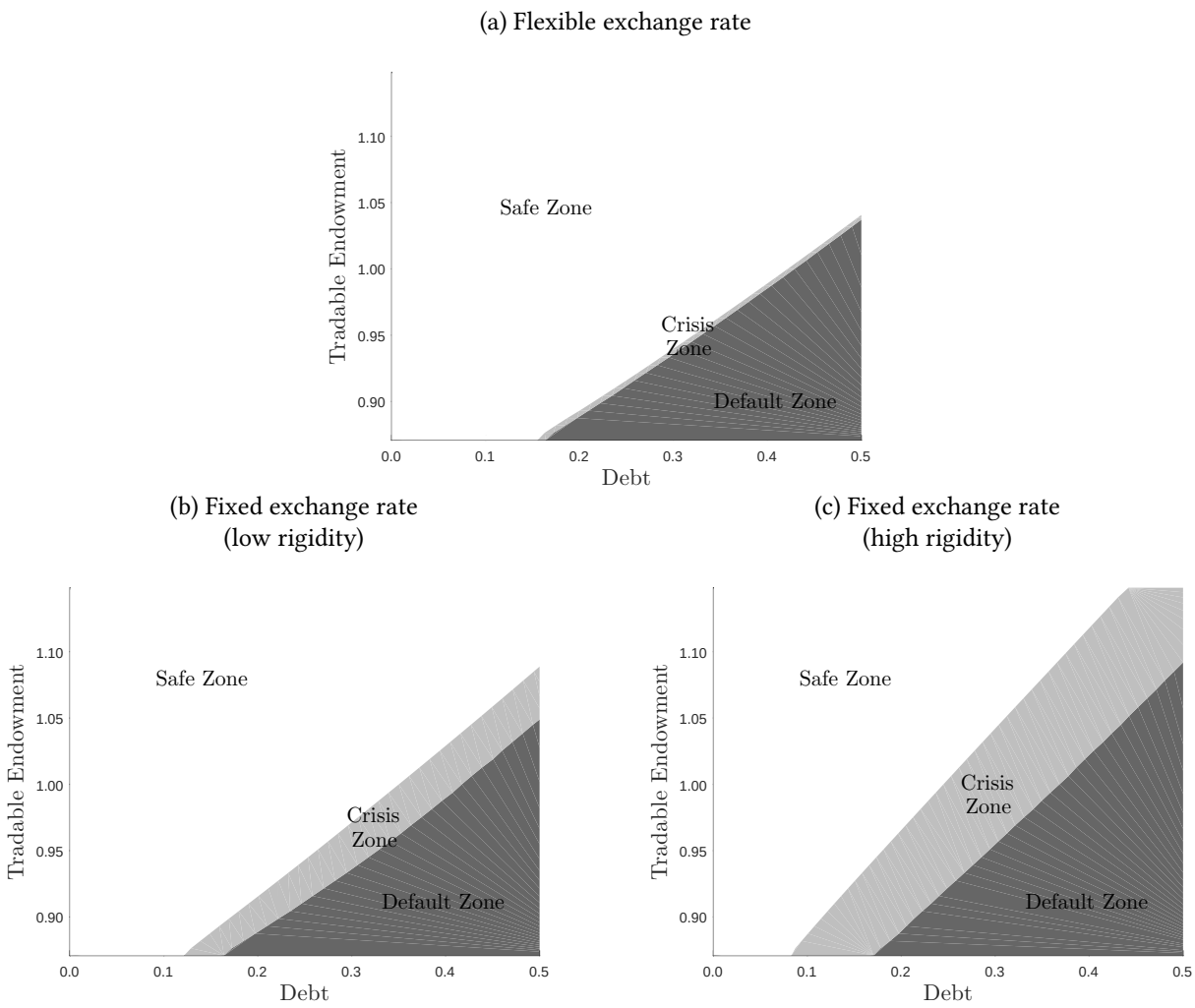

Figure 3: Safe, Crisis, and Default Zones under Different Wage Rigidities

3.4 Inspecting the Mechanism

We have established that an economy in a fixed exchange rate regime faces a large crisis region and hence a greater exposure to a rollover crisis. This section delves deeper into this result and underscores the importance of the response of unemployment for incentives of the government to repay.

Figure 4 shows the behavior of unemployment associated with the two different levels of wage rigidities considered earlier. For each panel, there are three lines: u_D denotes the unemployment rate if the government chooses to default: u_R^+ is the unemployment rate if the government chooses

[22] Notice that in Cole and Kehoe (2000), with no government impatience relative to international lenders ($\beta R = 1$), the government always eventually escapes the crisis region if bad sunspots do not trigger default. This is not the case in our model, given that we will consider $\beta R < 1$. Even in the case of $\beta R = 1$, our results from a larger crisis region for a fixed exchange rate regime suggest that the government will take more time to exit the crisis region with a fixed exchange rate regime.

to repay when investors lend, and u_R^- is the unemployment rate if the government chooses to repay when investors refuse to lend. The on-equilibrium unemployment rate depends on which region the debt level is in. In the crisis region, which again appears shaded in the figures, unemployment rate can take two values, u_D or u_R^+, depending on the realization of the sunspot. In the safe region, the on-equilibrium unemployment rate is u_R^+, while in the default region it is u_D.

When the government repays, unemployment is increasing in the current amount of debt both when the government can access the debt market and when it cannot. This is because a higher debt level reduces aggregate demand, which in turn generates a decline in the price of non-tradables in terms of foreign currency.[23] Under a fixed exchange rate, the wage rigidity in terms of domestic currency becomes a wage rigidity in foreign currency. Because of the downward rigidity in wages, the decline in the price of non-tradables leads to a rise in unemployment. When the government defaults, the unemployment rate is, of course, constant (and zero for the low-rigidity case). Following the way we set $\overline{w}$ in the low-rigidity case, unemployment only starts to become strictly positive at levels of debt in which the government, under a flexible regime, would be indifferent between defaulting and repaying when investors are willing to lend. Interestingly, this increase in unemployment that we see in the fixed regime is innocuous since the government would default anyway.

When investors refuse to lend, unemployment starts rising earlier (i.e., for lower levels of debt), and it is always higher than when investors are willing to lend—conditional, of course, on the government repaying in both cases. The reason is that when investors refuse to roll over the debt, the government is forced to raise tax revenues, which generate a decline in aggregate demand. In turn, this leads to deflationary pressures on the price of non-tradable goods, which cause a decline in labor demand. Because wages are downwardly rigid, the rollover crisis generates involuntary unemployment.

It is interesting to realize that in panel (a) of Figure 4, no unemployment equilibrium arises on the equilibrium path. In other words, what leads to default is the desire to avoid the large unemployment that would emerge if the government were to repay when it is cut off from the credit markets. In panel (b), because the wage rigidity is tighter, we do observe unemployment on the equilibrium path depending on the initial debt and the realization of the sunspot. Still, the large levels of unemployment that we observe in the case that investors refuse to lend are not observed in equilibrium. If the government finds it optimal to default in this scenario, unemployment falls to u^D, whereas if the government finds it optimal to repay, unemployment falls to u_R^+ because the run would not occur in equilibrium.

This increase in unemployment that emerges from fluctuations in labor demand from the non-tradable sector is at the heart of the mechanism to generate a larger exposure to a rollover crisis. It

[23] To understand this, recall that the price of tradables in terms of domestic currency is constant in a fixed exchange rate regime because the price of tradables in foreign currency is constant. A decline in aggregate demand therefore requires a decline in the price of non-tradables to clear the market for non-tradables.

is useful to point out that having production in the tradable sector would not affect the differences in employment when investors lend vis-à-vis when investors refuse to lend. The level of the exchange rate would affect employment in the tradable sector, but this would be independent of investors' beliefs. The key idea is that for tradable goods, the relevant demand is the international one. On the other hand, in the non-tradable sector, the availability of domestic resources is critical to determine the domestic price of tradables and firms' labor demand.

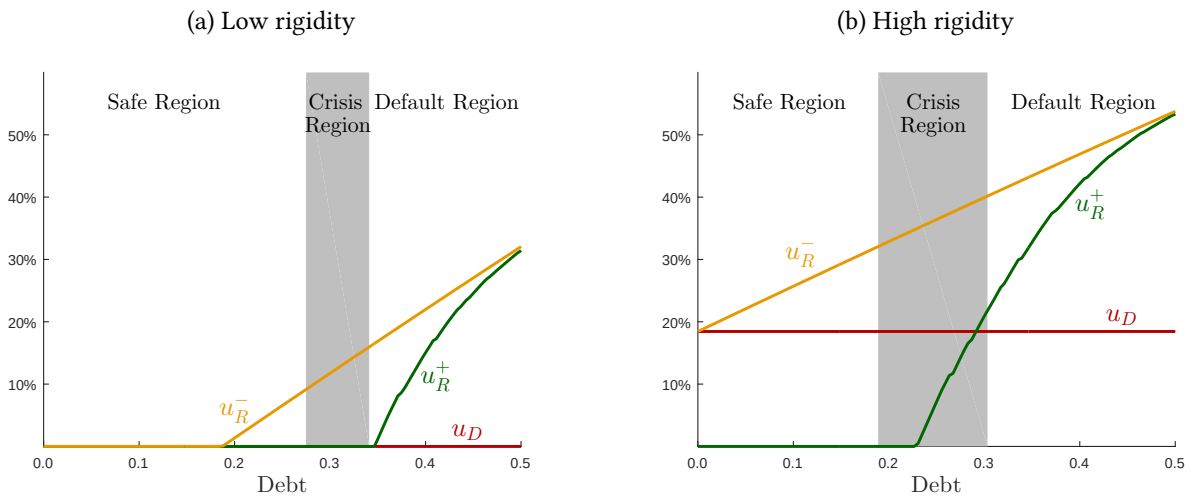

Figure 4: Unemployment Rates with Fixed Exchange Rate Regime

Notes: The unemployment rate if the government chooses to default is denoted by u_D. When the government repays, the unemployment rate is denoted by u_R^+ in the good sunspot and by u_R^- in the bad sunspot.

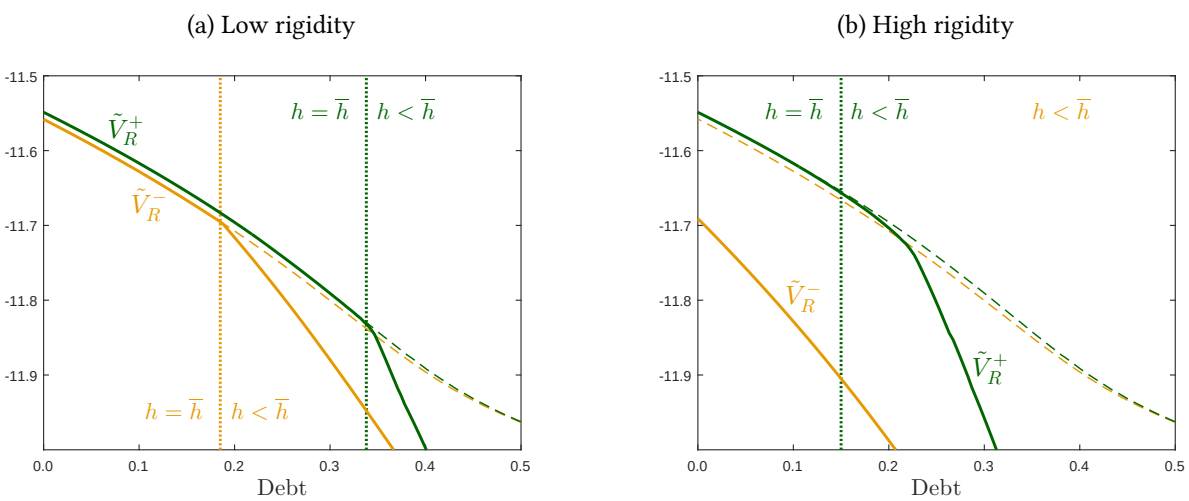

Figure 5: Values of Repayment in Flexible vs. Fixed Exchange Rate Regime.

Notes: Dashed lines correspond to the flexible exchange rate regime and straight lines correspond to the fixed exchange rate regime. Green (dark) lines correspond to $\tilde{V}_R^+$, and yellow (light) lines correspond to $\tilde{V}_R^-$.

These differences in unemployment that arise depending on whether investors are willing to lend or not translate into differences in the value functions. Figure 5 shows how $\tilde{V}_R^+$ and $\tilde{V}_R^-$ change when we introduce rigidities. These are the same value functions from Figure 1, but now we put them together to better appreciate the differences and mark the thresholds at which unemployment emerges. Consider first the repayment value functions under a flexible exchange rate regime, which are denoted with dashed lines. We can see that the gap between the two is very small: there is zero unemployment regardless of whether investors lend or not. Moreover, the gap is relatively wider at very low levels of debt (because the government wants to issue more debt). However, at those levels of debt, the government has a value of repayment that is far larger than the value of default, and hence this gap between $\tilde{V}^+$ and $\tilde{V}^-$ is innocuous. As debt increases and we approach the value of default, the gap becomes smaller (because the government does not want to issue as much debt). The outcome is a narrow crisis region.

Figure 5 shows that when the exchange rate is fixed, all value functions drop relative to the flexible case, and there is a strict decline at the debt threshold in which unemployment emerges. Most importantly, however, is that $\tilde{V}^-$ is reduced by more than $\tilde{V}^+$, and hence there is a bigger gap between the two compared to the flexible exchange rate. This arises because of the substantially different unemployment levels that arise depending on whether investors lend or not. Moreover, the widening of the gap occurs precisely at debt levels at which lenders' beliefs matter for the repayment decision. The outcome is a wide crisis region.

4 Quantitative Analysis

The goal of the quantitative analysis is twofold. The first goal is to establish how important rollover crises are in the model and how this importance depends on the exchange rate regime. For this purpose, we will choose the values for the parameters in the two versions of the model (with stickiness and without stickiness) that best match moments of the data.[24] Using this calibration, we will evaluate the fraction of default episodes that are triggered by non-fundamentals in both economies. The second goal is to perform counterfactual experiments. The approach in this case will be to consider a calibration for the fixed exchange rate regime and change the monetary regime to a flexible exchange rate keeping all parameters the same.

[24] One could also examine an "intermediate" regime in which monetary policy stabilizes the economy but not perfectly (for example, an inflation-targeting regime).

4.1 Calibration

We calibrate the model at an annual frequency using Spain as a case study.[25]

Functional forms. We use a CRRA utility function,

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \text{with} \quad \sigma > 0.$$

We parameterize the default utility cost as follows:

$$\kappa(y^T) = \max\left\{0, \kappa_0 + \kappa_1 \ln\left(y^T\right)\right\}.$$

As shown in Arellano (2008) and Chatterjee and Eyigungor (2012), a non-linear specification of the cost of default is important to allow the model to match the levels of debt and spreads in the data. In particular, we follow Bianchi, Hatchondo, and Martinez (2018) in specifying this default cost function in terms of utility.

The tradable endowment process follows a lognormal AR(1) process,

$$\ln(y_t^T) = \rho \ln(y_{t-1}^T) + \sigma_y \varepsilon_t,$$

where $|\rho| < 1$ and the shock ε_t is $i.i.d.$ and normally distributed, $\varepsilon \sim N(0,1)$. To estimate the tradable endowment stochastic process, we use the value-added series in the manufacturing and agricultural sectors in Spain. After we log-quadratically detrend the series, we estimate a persistence parameter of $\rho = 0.777$ and a standard deviation of $\sigma_y = 2.9\%$.

Model Parameters. Table 1 shows all the baseline calibration values for the parameters of the model. A first subset of parameters is specified directly. These are parameters that can be calibrated straight from the data or are relatively standard in the literature. We then pick a second subset of parameters to match key moments in the data under two different regimes: flexible and fixed exchange rates.

We start with the first subset of parameters. First, we specify the parameters governing preferences and technology, which will take standard values in the literature. The coefficient of risk aversion will be set to $\sigma = 2$. Meanwhile, the elasticity of substitution between tradable and non-tradable goods is set to $\frac{1}{1+\mu} = 0.5$, which is in the range of empirical estimates. The share of tradable goods in the consumption aggregator is set to $\omega = 0.197$, so it matches the ratio between tradable good and total

[25] The model is solved numerically using value function iteration with interpolation. Linear interpolation is used for the endowment and debt levels. We use 25 grid-points for the tradable endowment grid and 99 grid-points for debt. To compute expectations, we use 105 quadrature points for the endowment realizations.

Table 1: Parameter Values

Parameter	Value		Description
$\overline{h}$	1.000		Normalization
σ	2.000		Standard risk aversion
ω	0.197		Share of tradables
μ	1.000		Unitary elasticity of substitution between T-NT
ρ	0.777		Output persistence
σ_y	0.029		Standard deviation of tradable output shock
α	0.750		Labor share in non-tradable sector
r	0.020		German 6-year government bond yield
δ	0.141		Spanish bond maturity 6 years
ψ	0.240		Reentry to financial markets probability
π	0.030		Sunspot probability
Calibration	Flexible	Fixed	Target
β	0.914	0.908	Average external debt-GDP ratio 29.05%
κ_0	0.101	0.315	Average spread 2.01%
κ_1	0.759	3.273	Standard deviation interest rate spread 1.42%
$\overline{w}$	-	2.493	Δ unemployment rate 2.00%

output, which averages around 20% for Spain in the period considered.[26] Regarding the labor share in non-tradable production, we set $\alpha = 0.75$, an estimate from Uribe (1997) for the non-tradable sector. Last, we normalize the inelastic labor supply of households to $\overline{h} = 1$.

Next, we set the parameters from financial markets. We set the international risk-free interest rate to $r = 2\%$, which is the average annual gross yield on German 6-year government bonds over the period 2000 to 2015. We calculate a maturity parameter of $\delta = 0.141$ to reproduce an average bond duration of 6 years, in line with Spanish data.[27] We set the reentry to financial markets probability after default to $\psi = 0.24$ to capture an average autarky spell of 4 years. This value is consistent with Gelos, Sahay, and Sandleris (2011), who found that around 4.7 is the average amount of years before recovering financial access over the period 1980 to 2000 for 150 developing economies. Finally, we need to set the sunspot probability, which is a more difficult parameter to calibrate. In the literature, the probability of drawing bad sunspot is usually set to a relatively low value (e.g., Chatterjee and Eyigungor, 2012, study a range between [0,0.10]). Our baseline value is 3%, but we examine a wide

[26] In a nonstochastic version of the model with a mean value of debt $\bar{b}$ and average employment $\bar{h}$, the value of ω can be pinned down from $\frac{y^T}{y^T + \frac{1-\omega}{\omega}\left(\frac{y^T + r\bar{b}}{F(\bar{h})}\right)^{\mu+1}} = 20\%$.

[27] The Macaulay duration of a bond with price q and our coupon structure is given by

$$D = \sum_{t=1}^{\infty} t \frac{\delta}{q} \left(\frac{1-\delta}{1+i_b}\right)^t = \frac{1+i_b}{\delta + i_b},$$

where the constant per-period yield i_b is determined by $q = \sum_{t=1}^{\infty} \delta(\frac{1-\delta}{1+i_b})^t$.

range as well.

For the second subset of parameters $\{\beta, \kappa_0, \kappa_1, \overline{w}\}$, we will set these parameters so that the moments in the model match the counterparts in the data. Since we have two different exchange rate regimes, we have two sets of parameters. The difference in the two calibrations is that $\overline{w}$ is set to zero for the flexible exchange rate regime, whereas this value has to be calibrated for the fixed exchange rate regime. In particular, we calibrate $\overline{w}$ in the fixed exchange rate regime to be consistent with the increase in unemployment during episodes of high sovereign spreads. In the data for Spain, the increase in unemployment relative to the HP-filtered trend was 2% in 2011, the year prior to the EU and ECB's intervention.[28] We set $\overline{w} = 2.493$. With this value and given the rest of the calibrated parameters, the average increase in unemployment in the year prior to default is 2% in the model, matching the empirical counterpart.

For both regimes, we calibrate the parameters β, κ_0, and κ_1 to match three moments from the data, and we follow Hatchondo, Martinez, and Sosa-Padilla (2016) in considering the moments in the years following 2008 to concentrate on the period around the crisis. The three moments targeted are the average debt-GDP ratio, and the average and standard deviation of spreads. For the average debt-GDP ratio, we target an average external debt of 29%. For the average and the volatility of spreads, we target 2.0 and 1.4, respectively. [29] The resulting values for these parameters appear in Table 1.

4.2 Simulation Statistics

We now conduct simulations to investigate how the exchange rate regime determines what type of default, fundamental or rollover crisis, is more likely. To compute the statistics of the model, we perform a Monte Carlo simulation process. We collect 2,500 samples from 35 periods preceding a default episode. To eliminate the dependence from initial conditions, we only consider the samples that have at least 15 prior periods of access to international financial markets. After collecting the samples that fulfill this criteria, we compute a simple average of a set of statistics.

Baseline Statistics. In Table 2, the first block of statistics reported are those that are targets of our calibration, and the model does a fairly good job at matching those targets. The rest of the moments are not targeted. The second block shows that the model can also account for other standard features of the data involving the cyclicality of spreads as well as the cyclicality and variability of consumption.

[28] We use a smoothing parameter of 100 for the HP filtering. If we use a log-quadratic filter, we obtain a value closer to 3%.

[29] The debt level in the model is computed as the present value of future payment obligations discounted at the risk-free rate r. Given our coupon structure, we thus have that the debt level is $\frac{\delta}{1-(1-\delta)/(1+r)} b_t$.

Table 2: Simulation Statistics

Statistic	Data	Flexible	Fixed
Average spread (%)	2.01	2.46	1.43
Average debt-income (%)	29.05	29.73	31.33
Spread volatility (%)	1.42	1.33	1.60
Unemployment Increase (%)	2.00	0.00	1.83
$\rho(y, c)$	0.98	0.97	0.94
$\rho(y, spread)$	-0.38	-0.87	-0.77
$\sigma(\hat{c})/\sigma(\hat{y})$	1.10	1.55	1.33
Fraction of time in crisis region (%)	-	0.77	2.59
Fraction of defaults due to rollover crisis (%)	-	0.92	6.53

The main takeaway from this table are the two final statistics reported. In the flexible exchange rate regime, out of 100 default episodes, the share of defaults due to a rollover crisis is only 0.92. In line with this result, only 0.77% of the time, the economy is in the crisis zone and therefore vulnerable to a rollover crisis. On the other hand, under a fixed exchange rate regime, the number of defaults due to a rollover crisis increases to 6.5 out of 100 default episodes. Considering that the probability of drawing a sunspot that selects the bad equilibrium is only 3% in our baseline, this suggests that the economy under a fixed exchange rate is significantly exposed to a rollover crisis.

Degree of Wage Rigidity. To explore further how the degree of wage rigidity matters for the exposure to a rollover crisis, we conduct a related set of simulations in which we keep the same calibrated parameters for the flexible exchange rate economy and vary only the wage rigidity parameter $\overline{w}$. Different from our analysis in the comparative statics exercise of Section 3, the change in $\overline{w}$ is now permanent, and therefore the bond price schedule is affected.

In Figure 6 we present the fraction of defaults that are explained by rollover crises as a function of $\overline{w}$. Again, we can see here that the tighter is the wage rigidity, the larger the fraction of defaults that are explained by non-fundamentals. While we do not plot this in the figure, it is worth highlighting that in these simulations, we also obtain that the average debt level falls with $\overline{w}$. Two reasons explain this. First, the government faces borrowing terms that are more adverse, given that there are higher incentives to default in the future for both fundamental and non-fundamental reasons. Second, the government also attempts to stay further away from the crisis zone by reducing debt. Despite this attempt, the fact that the crisis region expands significantly implies that the government ends up being more heavily exposed to a rollover crisis.

Sunspot Probability. The fraction of defaults that are the outcome of a rollover crisis depends on two factors. One factor is the probability of a bad sunspot (i.e., the probability of selecting the bad

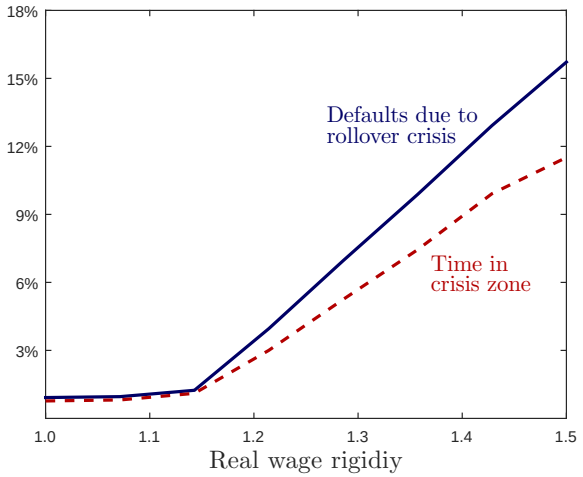

Figure 6: Defaults Due to Rollover Crises and Time in Crisis Region

equilibrium whenever the economy is conditional on being in the crisis zone). The second factor is the probability of ending up in the crisis zone in the first place, which is an endogenous outcome that depends critically on borrowing decisions and on the monetary policy regime. Next, we increase the probability of selecting the bad equilibrium while keeping the rest of the parameter values for fixed and flexible exchange rate regimes at their respective baseline values.

Table 3 shows how increasing the likelihood of a bad sunspot increases the fraction of defaults due to a rollover crisis for the two economies, and in particular for the economy under a fixed exchange rate regime. Specifically, when the probability of a bad sunspot is 20%, up from 3% in the baseline, about 1/5th of all defaults are for non-fundamental reasons. Moreover, one can see that the fraction of time spent in the crisis region decreases as the government reduces its exposure, but this duration is not enough to offset the higher likelihood of a bad sunspot.

Table 3: Sensitivity to Sunspot Probability

Sunspot probability	$\pi = 3\%$		$\pi = 10\%$		$\pi = 20\%$	
(percentage %)	Flexible	Fixed	Flexible	Fixed	Flexible	Fixed
Average spread	2.46	1.43	2.45	1.47	2.46	1.53
Average debt-income	29.73	31.33	29.58	29.29	29.37	28.53
Spread volatility	1.33	1.60	1.30	1.72	1.31	1.75
Unemployment increase	0.00	1.83	0.00	1.80	0.00	1.35
Fraction of time in crisis region	0.77	2.59	0.68	1.93	0.58	1.41
Fraction of defaults due to rollover crisis	0.92	6.53	3.70	11.80	6.20	19.80

Notes: All parameter values correspond to the benchmark calibrations for fixed and flexible exchange rate regimes. The benchmark calibration uses $\pi = 3\%$.

Lender of last resort. These statistics have important implications for lender of last resort policies. As is well understood, a third party with deep pockets can eliminate the coordination problem behind a rollover crisis. The basic argument is that by purchasing a sufficiently large amount of government bonds, either in the primary market or in the secondary market, this can induce the government to repay and therefore make investors willing to lend to the government.[30] Our results show that economies with a fixed exchange rate regime or that belong to a monetary union are more likely to experience a self-fulfilling debt crisis, and therefore a lender of last resort is highly valuable. On the contrary, an economy with a flexible exchange rate faces defaults that are almost exclusively for fundamental reasons. A lender of last resort is therefore less valuable.

4.3 The Path to Spain's Rollover Crisis

In this section, we evaluate the model with the Spanish experience after adopting the euro as its currency and show that the model predicts a default due to a rollover crisis in 2012.

The exercise is as follows. Starting at Spain's external debt-GDP ratio in 2000, we feed in the sequence of shocks to tradable output and simulate the model under a fixed exchange rate regime. In these simulations, we assume that the good sunspot hits throughout the period. Interestingly, the realization of the sunspot is irrelevant up to 2011 since the economy remains in the safe zone. In 2012, the economy is in the crisis zone, and a negative sunspot would trigger a rollover crisis and a default. While Spain did not actually default in 2012, a €100 billion assistance package by the European Union was channeled through the European Financial Stability Fund and the European Stability Mechanism, in addition to the announcement of the ECB's OMT bond purchasing program following the "whatever it takes" speech by Mario Draghi.

Panels (b) and (c) of Figure 7 show the dynamics of debt and spread in these simulations, while panel (a) shows the evolution of the model-implied probability of being in the crisis zone following the government's choices at each period. That is, using the level of debt, we compute the probability of receiving an income shock in the following period that would take the economy to the crisis zone. In early 2000, the government increases its debt, and this is driven by the low initial debt (recall that the calibrated mean debt is close to 30%) and by relatively good income shocks that allow for favorable borrowing terms. These dynamics are fairly similar to those in the data, except that the model overpredicts the initial increase. One can also see that the model is able to replicate the low and stable spreads before 2008 in the data. Finally, the evolution of the probability of being in the crisis zone in Figure 7 reveals interesting dynamics. After the debt accumulation that occurs initially and the negative income shocks that pile up after 2008, the economy's probability of a rollover crisis moves to the crisis zone. By 2012, the year in which the ECB intervened, the economy becomes significantly

[30] See Roch and Uhlig (2018) and Bocola and Dovis (2016) for an analysis of lender of last resort in the context of the Outright Monetary Transactions (OMT) program by the ECB.

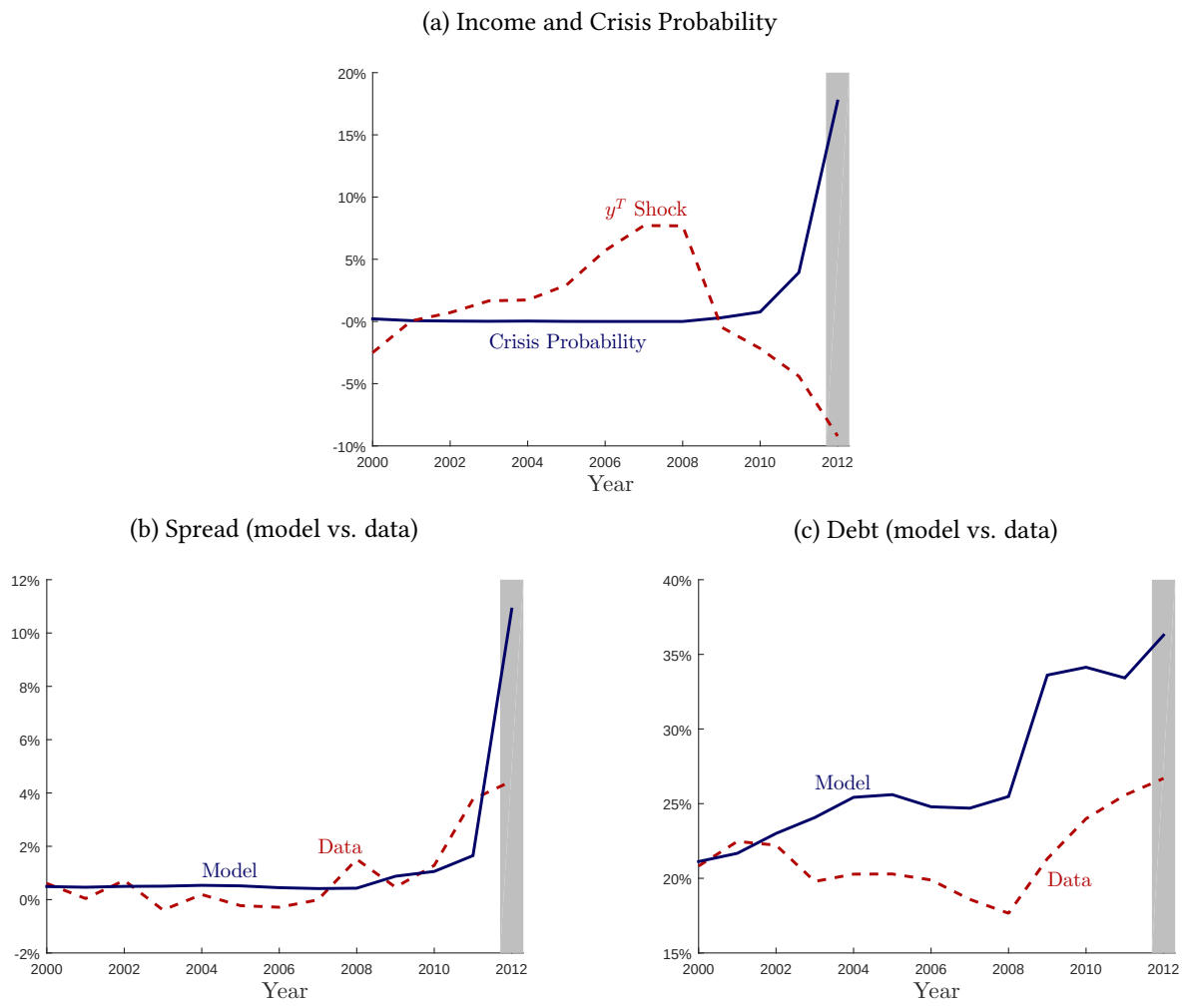

Figure 7: Path to a Rollover Crisis in 2012

Notes: Crisis probability denotes the probability that the economy would be in a crisis region in the following period given the current choices of debt and initial states. The tradable endowment shock was obtained using a log-quadratic filter to the Spanish tradable output from 1995 to 2017. Debt levels in the data correspond to Spain's external debt-GDP ratio. The shaded region denotes that the economy is in the crisis zone.

exposed to a rollover crisis, with a 20% probability of being in the crisis zone.

Counterfactual The final experiment we conduct is a counterfactual to examine what would have happened if Spain had exited the monetary union in 2012 when the government was in the crisis zone under a fixed exchange rate. We continue to assume that debt remains denominated in foreign currency, since a currency redenomination would be akin to a default. Meanwhile, we assume that wages start to be denominated in pesetas, so that effectively we are considering a transition from a fixed exchange rate to a flexible exchange rate regime. To conduct this counterfactual, we compute the equilibrium of the model keeping all parameters at the baseline except for $\overline{w}$, which we set to zero so that the wage rigidity never binds (i.e., the government adjusts the exchange rate to reduce the real wage and the level of unemployment). Given the levels of debt in the model in 2012 and the income

shock, we find that the government would not be in the crisis zone if it had monetary independence. Thanks to the ability to use monetary policy for stabilization, the model suggests that Spain would have remained immune to a rollover crisis.

Some remarks about the results of this counterfactual experiment are in order. First, we are keeping all parameters, except monetary policy, constant when we analyze the implications of exiting the Eurozone. We are therefore abstracting from any structural changes that Spain that could experience upon exiting a monetary union. To the extent that these structural changes would symmetrically affect ${V_R}^+$ and ${V_R}^-$, we expect that the large gap between these two values that arise because of the inability to depreciate the currency would remain intact, and hence these structural changes should not significantly alter the crisis region. Second, we do not suggest that Spain would have been better off by exiting the monetary union since there are benefits from being in a monetary union that we are not modeling. Our goal is to point out an additional cost of remaining in a monetary union, which arises from the higher exposure to rollover crises.

5 Conclusion

This paper showed that the inability to use monetary policy for macroeconomic stabilization leaves a government more vulnerable to a rollover crisis. In the presence of nominal rigidities, a run on government bonds can lead to a large recession absent monetary autonomy. In turn, anticipating that the government finds it more costly to repay during a recession, investors become more prone to run and the crisis becomes self-fulfilling. In a calibrated version of the model, we have found that the higher exposure to rollover crises is a significant cost from losing monetary independence.

Our analysis provides a new perspective on discussions about whether the lack of monetary autonomy in Southern European countries made them more vulnerable to a rollover crisis. According to a popular view, the fact that their debt was not denominated in domestic currency contributed to their vulnerability by preventing them from inflating away the debt. We argue instead that monetary policy has a role in preventing rollover crises that goes beyond the ability to inflate away the debt.

Extending beyond our current analysis, several implications and avenues remain for future work. In terms of debt maturity management, our model suggests that economies with more rigid labor markets or a less flexible monetary policy should seek longer debt maturities. Finally, from a policy perspective, our model suggests that an institutionalized lender of last resort facility is critical for the stability of a monetary union.

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A Proofs

Proof of Lemma 1

In any equilibrium, the real wage in terms of tradable goods is a function of tradable consumption and employment:

$$\mathcal{W}(c_t^T, h_t) \equiv \frac{1-\omega}{\omega}\left(\frac{c_t^T}{F(h_t)}\right)^{1+\mu} F'(h_t).$$

Moreover, $\mathcal{W}(c_t^T, h_t)$ is increasing with respect to c_t^T and decreasing with respect to h_t.

Proof. Using the firm's first order condition (5) and the equilibrium relative price, the equilibrium real wages in terms of tradable goods can be defined as a function of tradable consumption goods and employment:

$$\mathcal{W}(c_t^T, h_t) = p_t^N F'(h_t) = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{F(h_t)}\right)^{1+\mu} F'(h_t).$$

Using this, we can find that

$$\frac{\partial \mathcal{W}_t}{\partial c_t^T} = \frac{(1+\mu)p_t^N F'(h_t)}{c_t^T} \qquad \text{and}$$
$$\frac{\partial \mathcal{W}_t}{\partial h_t} = -(1+\mu)p_t^N F'(h_t)\left(\frac{F'(h_t)}{F(h_t)} + \left(\frac{1}{1+\mu}\right)\frac{-F''(h_t)}{F'(h_t)}\right).$$

Because $F(\cdot)$ is a nonnegative, strictly increasing, and decreasing returns to scale function, we know that F, $F' > 0$, and $F'' < 0$. Therefore, we can conclude that $\frac{\partial \mathcal{W}_t}{\partial c_t^T} > 0$ and $\frac{\partial \mathcal{W}_t}{\partial h_t} < 0$. □

Proof of Lemma 2

Under a fixed exchange rate regime, the employment function is a piecewise linear function:

$$\mathcal{H}(c^T) = \begin{cases} \left[\left(\frac{1-\omega}{\omega}\right)\left(\frac{\alpha}{\overline{w}}\right)\right]^{\frac{1}{1+\alpha\mu}} \left(c^T\right)^{\frac{1+\mu}{1+\alpha\mu}} & \text{if } c^T \leq \bar{c}^T_{\overline{w}} \\ \overline{h} & \text{if } c^T > \bar{c}^T_{\overline{w}} \end{cases},$$

where

$$\bar{c}^T_{\overline{w}} = \left[\left(\frac{\omega}{1-\omega}\right)\left(\frac{\overline{w}}{\alpha}\right)\right]^{\frac{1}{1+\mu}} \overline{h}^{\frac{1+\alpha\mu}{1+\mu}}.$$

Proof. When the real wage rigidity is binding,

$$\overline{w} = \mathcal{W}(c^T, h) = \frac{1-\omega}{\omega}\left(\frac{c^T}{F(h)}\right)^{1+\mu} F'(h) = \frac{1-\omega}{\omega}\left(\frac{\left(c^T\right)^{1+\mu}}{F(h)^{\mu}}\right)\left(\frac{F'(h)}{F(h)}\right).$$

Using the property that $F(\cdot)$ is a homogeneous function of degree $\alpha \in [0,1]$, we then know that $F'(h)$ is homogeneous of degree $\alpha - 1$. Moreover, because it is a unidimensional function, we can also assert that $F(h) = h^{\alpha}$ and $F'(h) = h^{\alpha-1}$. Finally, we also can say that $hF'(h) = \alpha F(h)$. Hence, we can say that

$$\overline{w} = \frac{1-\omega}{\omega}\left(\frac{\alpha\left(c^T\right)^{1+\mu}}{h^{1+\alpha\mu}}\right).$$

Hence, solving for h, we get

$$h_{\overline{w}}(c^T) = \left[\left(\frac{1-\omega}{\omega}\right)\left(\frac{\alpha}{\overline{w}}\right)\right]^{\frac{1}{1+\alpha\mu}} \left(c^T\right)^{\frac{1+\mu}{1+\alpha\mu}}.$$

Moreover, this labor function is increasing with respect to the consumption of nontradables. Knowing that labor cannot go above the household's labor endowment of $\overline{h}$, we can compute the consumption of tradables threshold in which employment reaches this cap. Hence, we solve for this level:

$$\overline{c}^T_{\overline{w}} = \left[\left(\frac{\omega}{1-\omega}\right)\left(\frac{\overline{w}}{\alpha}\right)\right]^{\frac{1}{1+\mu}} \overline{h}^{\frac{1+\alpha\mu}{1+\mu}}.$$

In levels of tradable consumption above this threshold, the supply of labor in the economy will be in full employment. □

Proof of Lemma 3

For every tradable endowment $y^T \in \mathbb{R}_+$ and debt level $b \in \mathbb{R}$, we have that $V_R^+(b, y^T) \geq V_R^-(b, y^T)$.

Proof. Realize that problem (21) is a particular case of (20). That is,

$$\begin{aligned} V_R^+(b, y^T) &= \max_{b', h \leq \overline{h}} \left\{u(y^T - \delta b + \tilde{q}(b', y^T)\left(b' - (1-\delta)b\right), h) + \beta \mathbb{E}\left[V(b', \mathbf{s}')\right]\right\} \\ &\geq \max_{h \leq \overline{h}} \left\{u(y^T - \delta b, h) + \beta \mathbb{E}\left[V((1-\delta)b, \mathbf{s}')\right]\right\} \\ &= V_R^-(b, y^T), \end{aligned}$$

where both problems satisfy the same labor and wage constraints. □

Proof of Proposition 1

Under a flexible exchange rate regime, the government chooses an exchange rate that delivers full employment in all states.

Proof. The value of repayment when the government can choose the exchange rate is given by the following Bellman equation:

$$V_R(b, \mathbf{s}) = \max_{b', c^T, h \leq \overline{h}, e} \left\{ u(c^T, F(h)) + \beta \mathbb{E} V(b', \mathbf{s}') \right\} \tag{27}$$

$$\text{subject to}$$

$$c^T = y^T - \delta b + q(b', b, \mathbf{s})(b' - (1-\delta)b)$$

$$\mathcal{W}(c^T, h)e \geq \overline{W}$$

Meanwhile, the value of default when the government can choose the exchange rate is given by the following Bellman equation:

$$V_D(y^T) = \max_{c^T, h \leq \overline{h}, e} \left\{ u\left(c^T, F(h)\right) - \kappa(y^T) + \beta \mathbb{E}\left[\psi V(0, \mathbf{s}') + (1-\psi) V_D(y^{T\prime})\right] \right\} \tag{28}$$

$$\text{subject to}$$

$$c^T = y^T$$

$$\mathcal{W}(c^T, h)e \geq \overline{W}$$

It is immediate from (27) and (28) that an increase in e relaxes the wage rigidity constraint without tightening any other constraint. Fully relaxing the wage rigidity constraint allows the government to achieve full employment.

□

Proof of Lemma 4

The value functions $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing with respect to debt b.

Proof. Suppose two different debt values $b_1, b_2 \in \mathbb{R}$ such that $b_1 > b_2$. First, analyze the tradable resource constraint of V_R^- for these two values of debt:

$$c_1^T = y^T - \delta b_1 < y^T - \delta b_2 = c_2^T.$$

Likewise, do the same for the V_R^+ tradable resource constraint:

$$\begin{aligned} c_1^T &= y^T + \tilde{q}(b', y^T)b' - \left(\delta + \tilde{q}(b', y^T)(1-\delta)\right) b_1 \\ &< y^T + \tilde{q}(b', y^T)b' - \left(\delta + \tilde{q}(b', y^T)(1-\delta)\right) b_2 = c_2^T. \end{aligned}$$

In other words, the budget constraint is tighter when debt is higher. Furthermore, for both problems, the wage rigidity constraint will imply that

$$\mathcal{W}(c_1^T, h_1) \leq \mathcal{W}(c_2^T, h_2),$$

where $h_1 \leq h_2$. Therefore, we can conclude that $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are decreasing. □

Proof of Lemma 5

For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exist levels of debt $\bar{b}^+, \bar{b}^- \in \mathbb{R}$ such that $\tilde{V}_D(y^T) = V_R^+\left(\bar{b}^+, y^T\right)$ and $\tilde{V}_D(y^T) = V_R^-\left(\bar{b}^-, y^T\right)$. Furthermore, it also satisfies $\bar{b}^+ \geq \bar{b}^-$.

Proof. First, realize that for every level of tradable endowment $y^T \in \mathbb{R}_+$, if $b = 0$ then $V_D(y^T) \leq \tilde{V}_R^-(0, y^T) \leq \tilde{V}_R^+(0, y^T)$. Now, pick a level of debt outrageously high $b >> 0$, then $\tilde{V}_D(y^T) > \tilde{V}_R^+(b, y^T) \geq \tilde{V}_R^-(b, y^T)$. Because $\tilde{V}_R^+$ and $\tilde{V}_R^-$ are continuous functions, then there exist levels of debt $\bar{b}^+, \bar{b}^- \in \mathbb{R}$ such that $\tilde{V}_D(y^T) = \tilde{V}_R^+\left(\bar{b}^+, y^T\right)$ and $\tilde{V}_D(y^T) = \tilde{V}_R^-\left(\bar{b}^-, y^T\right)$. Acknowledge that for every level of endowment $y^T \in \mathbb{R}_+$

$$\tilde{V}_R^-\left(\bar{b}^-, y^T\right) = \tilde{V}_D(y^T) = \tilde{V}_R^+\left(\bar{b}^+, y^T\right) \geq \tilde{V}_R^-\left(\bar{b}^+, y^T\right).$$

Using that V_R^- is decreasing, we can conclude that $\bar{b}^+ \geq \bar{b}^-$. □

Auxiliary Lemmas for Propositions 2, 3, 4

Lemma A1 (Default Real Wage Rigidity Neutrality)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists a real wage rigidity $\overline{w}_D \in \mathbb{R}_+$ such that for $\overline{w}_1, \overline{w}_2 \leq \overline{w}_D$ the value of default is the same, $\tilde{V}_D\left(y^T; \overline{w}_1\right) = \tilde{V}_D\left(y^T; \overline{w}_2\right)$.*

Proof. Pick an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$. Using the real wage function, define the real wage rigidity $\overline{w}_D \in \mathbb{R}_+$ under full employment as

$$\overline{w}_D \equiv \mathcal{W}(y^T, \overline{h}) = \frac{1-\omega}{\omega}\left(\frac{y^T}{F(\overline{h})}\right)^{1+\mu} F'(\overline{h}).$$

This level $\overline{w}_D$ is the highest level of wage rigidity where full employment can be achieved. This means that if we pick two arbitrary real wage rigidities such that $\overline{w}_1, \overline{w}_2 \leq \overline{w}_D$, then the default state is in

full employment because the real wage constraint is not binding. Therefore, the optimal allocations of full employment are achieved and are the same, $\tilde{V}_D\left(y^T;\overline{w}_1\right)=\tilde{V}_D\left(y^T;\overline{w}_2\right)$. □

Lemma A2 (Repayment Real Wages Ordering). *For every level of tradable endowment $y^T\in\mathbb{R}_+$ and level of debt $b\in\mathbb{R}_+$, under a flexible exchange rate environment the real wage when lenders are willing to lend is higher than the real wage when borrowing is not allowed.*

Proof. Pick an arbitrary level of tradable endowment $y^T\in\mathbb{R}_+$ and debt level $b\in\mathbb{R}_+$. Using Proposition 1, we can guarantee that full employment is always achieved. Call $\hat{b}$ the optimal solution for the problem $\tilde{V}_R^+$. Define the consumption level of tradables $\hat{c}_R^+$ and $\hat{c}_R^-$ for the problems $\tilde{V}_R^+$ and $\tilde{V}_R^-$, respectively. These are

$$\hat{c}_R^+=y^T-\delta b+\tilde{q}(b,y^T)\left(\hat{b}'-(1-\delta)b\right)\qquad\text{and}$$
$$\hat{c}_R^-=\begin{cases}y^T-\delta b+\tilde{q}(b,y^T)\left(\hat{b}-(1-\delta)b\right) & \text{if }\hat{b}<(1-\delta)b\\ y^T-\delta b & \text{if }\hat{b}\geq(1-\delta)b\end{cases}.$$

In other words, by construction we know that $\hat{c}_R^+\geq\hat{c}_R^-$. Using Lemma 1, we can conclude that $\mathcal{W}\left(\hat{c}_R^+,\overline{h}\right)\geq\mathcal{W}\left(\hat{c}_R^-,\overline{h}\right)$. □

Lemma A3 (Default Region Neutrality). *For every level of tradable endowment $y^T\in\mathbb{R}_+$, there exists a real wage rigidity $\overline{w}_C\in\mathbb{R}_+$ such that, for any $\overline{w}_1,\overline{w}_2\leq\overline{w}_C$, the default region is unchanged $\tilde{\mathcal{D}}\left(y^T;\overline{w}_1\right)=\tilde{\mathcal{D}}\left(y^T;\overline{w}_2\right)$.*

Proof. Pick an arbitrary level of tradable endowment $y^T\in\mathbb{R}_+$. Set for a moment $\overline{w}=0$ in (24) and (25) and call $\overline{b}\in\mathbb{R}_+$ the level of debt that matches $\tilde{V}_R^+\left(\overline{b},y^T;0\right)=\tilde{V}_D\left(y^T;0\right)$. Also, call $\hat{b}\in\mathbb{R}_+$ the optimal level of debt that solves (25). Now, define the real wage floor $\overline{w}_R\in\mathbb{R}_+$ such that

$$\overline{w}_R=\mathcal{W}\left(\hat{c}_T^+,\overline{h}\right)=\left(\frac{1-\omega}{\omega}\right)\left(\frac{y^T-\delta\overline{b}+\tilde{q}\left(\hat{b},y^T\right)\left(\hat{b}-(1-\delta)\overline{b}\right)}{F\left(\overline{h}\right)}\right)^{1+\mu}F'\left(\overline{h}\right).$$

Using Lemma A3, call $\overline{w}_C\equiv\min\left\{\overline{w}_D,\overline{w}_R\right\}$ and pick two arbitrary real wage rigidities $\overline{w}_1,\overline{w}_2\leq\overline{w}_C$. Using Lemma 5, call the thresholds $\overline{b}_1^+,\overline{b}_2^+\in\mathbb{R}_+$ for the problems under real wage floors $\overline{w}_1$ and $\overline{w}_2$. Acknowledge that with these real wage floors, full employment is achieved. Then it follows that

$$\tilde{V}_R^+\left(\overline{b}_1,y^T;\overline{w}_1\right)=\tilde{V}_D\left(y^T;\overline{w}_1\right)=\tilde{V}_D\left(y^T;\overline{w}_2\right)=\tilde{V}_R^+\left(\overline{b}_2,y^T;\overline{w}_2\right).$$

This implies that $\overline{b}_1=\overline{b}_2$, leading to the conclusion that $\tilde{\mathcal{D}}\left(y^T;\overline{w}_1\right)=\tilde{\mathcal{D}}\left(y^T;\overline{w}_2\right)$. □

Lemma A4 (Safe Region Neutrality). *For every level of tradable endowment $y^T\in\mathbb{R}_+$, there exists a real wage rigidity $\overline{w}_S\in\mathbb{R}_+$ such that, for any $\overline{w}_1,\overline{w}_2\leq\overline{w}_S$, the default region is unchanged $\tilde{\mathcal{S}}\left(y^T;\overline{w}_1\right)=\tilde{\mathcal{S}}\left(y^T;\overline{w}_2\right)$.*

Proof. Pick an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$. Set for a moment $\overline{w} = 0$ in (24) and (26) and call $\overline{b} \in \mathbb{R}_+$ the level of debt that matches $\tilde{V}_R^- \left(\overline{b}, y^T; 0\right) = \tilde{V}_D \left(y^T; 0\right)$. Now define the real wage floor $\overline{w}_S \in \mathbb{R}_+$ such that

$$\overline{w}_R^- = \mathcal{W}\left(\hat{c}_T^-, \overline{h}\right) = \left(\frac{1-\omega}{\omega}\right)\left(\frac{y^T - \delta\overline{b}}{F\left(\overline{h}\right)}\right)^{1+\mu} F'\left(\overline{h}\right).$$

Now, pick two arbitrary real wage rigidities $\overline{w}_1, \overline{w}_2 \leq \overline{w}_S$. Using Lemma 5, call the thresholds $\overline{b}_1^-, \overline{b}_2^- \in \mathbb{R}_+$ for the problems under real wage floors $\overline{w}_1$ and $\overline{w}_2$, respectively. Acknowledge that with these real wage floors, full employment is achieved. Then it follows that

$$\tilde{V}_R^- \left(\overline{b}_1, y^T; \overline{w}_1\right) = \tilde{V}_D \left(y^T; \overline{w}_1\right) = \tilde{V}_D \left(y^T; \overline{w}_2\right) = \tilde{V}_R^- \left(\overline{b}_2, y^T; \overline{w}_2\right).$$

This implies that $\overline{b}_1 = \overline{b}_2$, leading to the conclusion that $\tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right) = \tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right)$. □

Lemma A5 (Safe Region Contraction)**.** *For every level of tradable endowment $y^T \in \mathbb{R}_+$, for any $\overline{w}_1, \overline{w}_2 \in \mathbb{R}_+$ such that $\overline{w}_2 > \overline{w}_S$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right) \subset \tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right)$.*

Proof. Pick an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$ and real wage rigidities $\overline{w}_1, \overline{w}_2 \in \mathbb{R}_+$ such that $\overline{w}_2 > \overline{w}_S$ and $\overline{w}_1 < \overline{w}_2$. Using Lemma 5, call the thresholds $\overline{b}_1^-, \overline{b}_2^- \in \mathbb{R}_+$ for the problems under real wage rigidities $\overline{w}_1$ and $\overline{w}_2$, respectively. Call $h_1, h_2 \in \mathbb{R}_+$ the labor in the economy under real wage rigidities $\overline{w}_1$ and $\overline{w}_2$, respectively. Using Lemma 1, it follows that $h_2 < h_1 \leq \overline{h}$. Hence, it follows that $\tilde{V}_R^- \left(\overline{b}_2^-, y^T; \overline{w}_2\right) < \tilde{V}_R^- \left(\overline{b}_2^-, y^T; \overline{w}_1\right)$. Thus,

$$\tilde{V}_R^- \left(\overline{b}_2^-, y^T; \overline{w}_1\right) > \tilde{V}_R^- \left(\overline{b}_2^-, y^T; \overline{w}_2\right) = \tilde{V}_D \left(y^T; \overline{w}_2\right) = \tilde{V}_D \left(y^T; \overline{w}_1\right) = \tilde{V}_R^- \left(\overline{b}_1^-; \overline{w}_1\right).$$

Using Lemma 4, we arrive at $\overline{b}_1^- > \overline{b}_2^-$. Finally, this tells us that $\tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right) \subset \tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right)$. □

Proof of Proposition 2

For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{D}(y^T; \overline{w}_1) \subseteq \tilde{D}(y^T; \overline{w}_2)$.

Proof. Pick arbitrary levels of tradable endowment $y^T \in \mathbb{R}_+$, and real wage rigidities $\overline{w}_1, \overline{w}_2 \leq \overline{w}_D$ where $\overline{w}_1 > \overline{w}_2$. Using Lemma A1, we know that there exists $\overline{w}_D \in \mathbb{R}_+$ such that $\tilde{V}_D \left(y^T; \overline{w}_1\right) = \tilde{V}_D \left(y^T; \overline{w}_2\right)$. Using Lemma 5, define $\overline{b}_1^+$ and $\overline{b}_2^+$ as the debt thresholds that limit the default region under real wage rigidites $\overline{w}_1$ and $\overline{w}_2$, respectively. Acknowledging that a higher real wage rigidity makes the problem of repayment when new debt contracts are allowed more constrained, we know

that $\tilde{V}_R^+\left(b, y^T; \overline{w}_2\right) \geq \tilde{V}_R^+\left(b, y^T; \overline{w}_1\right)$ for any amount of debt $b \in \mathbb{R}$. Thus,

$$\tilde{V}_R^+\left(\overline{b}_1^+, y^T; \overline{w}_1\right) = \tilde{V}_D\left(y^T, ; \overline{w}_1\right) = \tilde{V}_D\left(y^T, ; \overline{w}_2\right) = \tilde{V}_R^+\left(\overline{b}_2^+, y^T; \overline{w}_2\right) \geq \tilde{V}_R^+\left(\overline{b}_2^+, y^T; \overline{w}_1\right).$$

Using Lemma 4, it follows that $\overline{b}_2^+ \geq \overline{b}_1^+$. This implies that $\tilde{D}(y^T; \overline{w}_2) \subseteq \tilde{D}(y^T; \overline{w}_1)$. □

Proof of Proposition 3

For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_D \in \mathbb{R}_+$ such that if $\overline{w}_1 < \overline{w}_2 \leq \overline{w}_D$, then $\tilde{S}(y^T; \overline{w}_2) \subseteq \tilde{S}(y^T; \overline{w}_1)$.

Proof. Pick arbitrary levels of tradable endowment $y^T \in \mathbb{R}_+$ and real wage rigidities $\overline{w}_1, \overline{w}_2 \leq \overline{w}_D$ where $\overline{w}_1 > \overline{w}_2$. Using Lemma A1, we know that there exists $\overline{w}_D \in \mathbb{R}_+$ such that $\tilde{V}_D\left(y^T; \overline{w}_1\right) = \tilde{V}_D\left(y^T; \overline{w}_2\right)$. Using Lemma 5, define $\overline{b}_1^-$ and $\overline{b}_2^-$ as the debt thresholds that limit the safe region under real wage rigidites $\overline{w}_1$ and $\overline{w}_2$, respectively. Acknowledging that a higher real wage rigidity makes the value of repayment under no borrowing more likely to bind and result in possibly more unemployment, we know that $V_R^-\left(b, y^T; \overline{w}_2\right) \geq V_R^-\left(b, y^T; \overline{w}_1\right)$ for any amount of debt $b \in \mathbb{R}$. Thus,

$$\tilde{V}_R^-\left(\overline{b}_1^-, y^T; \overline{w}_1\right) = \tilde{V}_D\left(y^T; \overline{w}_1\right) = \tilde{V}_D\left(y^T; \overline{w}_2\right) = \tilde{V}_R^-\left(\overline{b}_2^-, y^T; \overline{w}_2\right) \geq \tilde{V}_R^-\left(\overline{b}_2^-, y^T; \overline{w}_1\right).$$

Using Lemma 4, it follows that $\overline{b}_2^- \geq \overline{b}_1^-$. This implies that $\tilde{S}(y^T; \overline{w}_1) \subseteq \tilde{S}(y^T; \overline{w}_2)$. □

Proof of Proposition 4

For every level of tradable endowment $y^T \in \mathbb{R}_+$, there exists $\overline{w}_C \in \mathbb{R}_+$ such that if $\overline{w}_1, \overline{w}_2 < \overline{w}_C$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{C}(y^T; \overline{w}_1) \subseteq \tilde{C}(y^T; \overline{w}_2)$. Moreover, there exists $\overline{w}_S \in \mathbb{R}_+$ such that if $\overline{w}_2 > \overline{w}_S$ then $\tilde{C}(y^T; \overline{w}_1) \subset \tilde{C}(y^T; \overline{w}_2)$.

Proof. Pick an arbitrary level of tradable endowment $y^T \in \mathbb{R}_+$. Using Lemma A3, there exists $\overline{w}_C \in \mathbb{R}_+$ such that if $\overline{w}_1, \overline{w}_2 < \overline{w}_C$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{\mathcal{D}}\left(y^T; \overline{w}_2\right) = \tilde{\mathcal{D}}\left(y^T; \overline{w}_1\right)$. Using Lemma A4, there exists $\overline{w}_S$ such that if $\overline{w}_1, \overline{w}_2 < \overline{w}_S$ and $\overline{w}_1 < \overline{w}_2$, then $\tilde{\mathcal{S}}\left(y^T; \overline{w}_2\right) = \tilde{\mathcal{S}}\left(y^T; \overline{w}_1\right)$. Using Lemma A5, we can arrive at the conclusion that $\tilde{C}(y^T; \overline{w}_1) \subseteq \tilde{C}(y^T; \overline{w}_2)$. Furthermore, if $\overline{w}_2 > \overline{w}_S$, then $\tilde{C}(y^T; \overline{w}_1) \subset \tilde{C}(y^T; \overline{w}_2)$. □