

THE ECONOMIC EFFECTS OF CORPORATE TAXES  
IN A STOCHASTIC GROWTH MODEL<sup>\*</sup>

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## I. Introduction

The Economic Recovery Tax Act of 1981 led to the largest postwar decline in effective tax rates on capital. The legislation also had its most significant effect on rates in 1982 due to the rapid decline in inflation. Although some of the tax cut was rescinded in 1982, effective corporate tax rates on plant and equipment, measured as the difference between before and after-tax rates of return to capital as a percentage of before-tax rates of return, remained at historically low values through 1986. Accompanying this tax cut is the current economic recovery which began in November, 1982. During this recovery we have witnessed relatively large increases in business fixed investment, a stock market boom, and a large rise in both the ex-post and ex-ante real interest rate. It is, therefore, natural to investigate the linkages between the tax cut and the increase in economic activity.

The effects of this particular tax cut have been consistent with a pattern of negative correlations between taxes and real interest rates, stock prices, investment, and output growth observed following previous business tax cuts. For example, using annual data on the effective tax rates on plant and equipment reported in Hulten and Robertson (1982) the correlation coefficients between tax rates and real gnp, real business fixed investment, and the New York Stock Exchange price index for the period 1952-1984 are  $-.81$ ,  $-.78$ , and  $-.87$ . These correlation coefficients all have a significance level of  $.0001$ . Further, using Livingston data on expected inflation over the period 1960-1984 the ex ante real rate of interest and the effective tax rate display a correlation coefficient of  $-.38$  while the coefficient with respect to the ex post real rate is  $-.50$ . These coefficients have significance levels of  $.0634$  and  $.0112$  respectively.

While the general effects of the recent tax cut are consistent with effects observed in other periods, the relative size of the 1981 tax cut is quite large. For instance, Hulten and Robertson calculate that the effective tax rate on capital in nonresidential business was reduced from roughly 33 percent in 1980 to approximately 1 percent in 1984. It is not surprising that a change of this magnitude has generated renewed interest in the interaction between taxes on capital and real economic variables.

This paper examines the effects of tax rate changes within the context of a stochastic growth model. Because taxes drive a wedge between the marginal product of capital and its after tax return the resulting equilibrium is suboptimal. Further, the effects of taxes are highlighted by the lump sum remittance of tax proceeds to individuals. The methodology for finding the solutions to an individuals dynamic programming problem relies on the envelope theorem and the construction of a policy function that simultaneously solves the individuals optimization problem and market clearing conditions. Attacking the problem in this way avoids explicit consideration of the value function and produces a tractable method of analysis for problems in which aggregate conditions affect an individuals value function. Essentially, the problem is reduced to solving one functional equation for policy.

Most of the work dealing with the effects of taxes has proceeded within the confines of standard nonstochastic growth models (e.g. Abel and Blanchard (1982)) or in models in which agents have perfect foresight regarding the path of tax rates (e.g. Goulder and Summers (1987)). Little effort seems to have been given to examining the effects of tax rate changes when taxes are explicitly depicted as following a particular stochastic process.

This paper takes the latter approach and investigates the effects of tax changes in a stochastic growth model in which only tastes, technology, and the stochastic process for taxes are exogenous. This procedure takes seriously the methodology advocated by Lucas (1976) and Sargent (1979) that a policy should be represented as a given outcome of some stochastic process. The analysis yields investment, output, and real interest rate behavior that depend explicitly on tastes and technology parameters as well as the underlying process generating tax rates.

The qualitative movements in endogenous variables that are generated by tax rate changes in this model are similar in some instances to results derived in the nonstochastic or perfect foresight models. For instance, when a high tax rate is generated (and the rate is expected to remain high), agents reduce the capital stock. This leads to lower real interest rates and lower transitional real output growth. The qualitative similarity of the results from the various models is a natural outcome of the behavior of agents, behavior which is characterized by analogous intertemporal optimization problems in all three types of models.

An explicit stochastic treatment of taxes allows one to examine the effects of uncertainty on the decisions of individual agents. This uncertainty should be an important consideration in determining behavior since tax law changes are fairly frequent and changes in inflation do result in significant movements in the effective tax rate on capital. Also, the exact nature of the uncertainty is related to the specific process that taxes are assumed to follow. For example, the degree of persistence of the process generating tax rates will have important implications for individual behavior. Therefore, agents will show quantitatively different behavior for any specific realizations of tax rates when the inherent randomness of taxes is modelled as

opposed to treating tax rate data as being known with certainty. If one wants to derive realistic decision rules, then the stochastic nature of the agents problem needs to be analyzed explicitly.

The results generated by the stochastic growth model derived in this paper produce correlations that are consistent with those mentioned above. The model, therefore, indicates that the 1981 tax change was a potentially important factor in the current economic recovery.

The paper is structured as follows. Section II contains a description of the model, while Section III characterizes the model's equilibrium. Of particular interest is the derivation of a closed form solution to the nonlinear stochastic difference equations that determine the equilibrium. Section IV presents the solution when taxes are assumed to follow either a simple two state Markov process or are white noise. A comparison between capital stocks generated by the model for particular realizations of these processes with those generated by a perfect foresight model is made in Section V, while a short summary is given in Section VI.

## II. The Model

The model is a one sector stochastic growth model consisting of three economic entities: firms, consumers, and the government. Individuals are infinitely lived and maximize the discounted stream of momentary utility while firms produce output according to a concave production function,  $f(k)$ , where output is total output that is available for production and consumption. The various economic entities interact in two markets each period. First, there is a capital market in which firms purchase capital from individuals. Capital is carried over from the previous period and is therefore supplied inelastically as in Brock (1979). Next there is a combined goods and

securities market in which individuals allocate their wealth among goods and securities. Individuals also decide how much they will consume and how much capital to carry into the succeeding period. The government taxes away some of the firm's revenue and remits the proceeds lump sum to individuals. Tax rates are stochastic, and are announced at the beginning of each period so that there is no uncertainty over the current tax rate, but there is uncertainty over future tax rates.

a. Capital Market

At the beginning of period  $t$ , the representative individual has carried over  $k_t$  units of capital which is sold to the firm for  $p_t$  units of output per unit of capital. One can think of output as seeds which can either be eaten (consumed) or invested (carried into the next period and sold to firms). The firm maximizes its after tax profits

$$(1) \pi_t = (1-\tau_t)f(k_t) - p_t k_t$$

where  $\tau$  is the effective tax rate on capital. This optimization implies that the price of capital is equated to its after tax marginal product and therefore the price of capital and profits are determined as functions of the capital stock and the tax rate. Formally,  $p_t = (1-\tau_t)f'(k_t)$ . This formulation of tax on net output is chosen for analytical tractability and captures the tax wedge resulting from a given effective tax rate on capital. More elaborate attempts to capture the complexity of the tax system would not change the basic results of the paper. The distortionary tax rate is announced prior to trading in the capital market so that there is no uncertainty regarding current taxes. There is, however, uncertainty over future tax rates.

b. Goods and Securities Market

After selling capital to firms and receiving the distribution of profits and lump sum tax remissions, individuals choose their current consumption  $c_t$ , next periods holdings of capital,  $k_{t+1}$ , and their share of the firm  $s_{t+1}$  subject to the value of their current wealth,  $w_t$ , where

$$(2) w_t = (q_t + \pi_t)s_t + p_t k_t + T_t.$$

The first term after the equality represents the current value of the shares of the firms,  $q_t s_t$ , plus dividend payments  $\pi_t s_t$ . The second term is the payment for capital and the third term is the per capita lump sum transfer of tax proceeds. Since all firms have access to the same strictly concave production technology,  $f(k)$ , each firm will utilize the same amount of capital,  $\hat{k}$ . In general for  $m$  firms and  $n$  individuals (where  $m$  and  $n$  are large),  $T_t = \frac{m}{n} \tau_t f(\hat{k}_t)$ . For simplicity of exposition, let  $m=n$  and hence  $T_t = \tau_t f(K_t)$  where  $K_t$  is the aggregate per capita capital stock. The budget constraint facing the individual is

$$(3) c_t + k_{t+1} + q_t s_{t+1} \leq w_t.$$

c. The Individuals Maximization

The individual's problem is to maximize his discounted expected utility

$$(4) U = E_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_j)$$

subject to his budget constraint (3). The problem can be posed in terms of dynamic programming and is formally stated as:

$$(5) \quad V(k, K, s, \tau) = \max_{c, k', s'} \{u(c) + \beta EV(k', K', s', \tau')\}$$

$$\text{s.t.} \quad c + k' + qs' \leq w$$

$$\text{and} \quad w' = (q' + \pi')s' + p'k' + T'$$

where per capita capital is determined by  $K' = \Psi(K, \tau)$ . The price of capital,  $p(K, \tau)$ , and profits,  $\pi(K, \tau)$  are given by the solution to the firms problem. Tax rebates are given by  $T_t = \tau_t f(K_t)$  and asset prices are determined by the function  $q(K, \tau)$ . In solving their optimization problem individuals take as given  $q(K, \tau)$ ,  $\pi(K, \tau)$ , and  $p(K, \tau)$  as well as the policy function that describes the transition path of the aggregate per capita capital stock  $K' = \Psi(K, \tau)$ . Further, the notation  $E$  is used to denote the conditional expectations operator where expectations are conditioned on all current information including the realization of this periods tax rate.

The solution to this problem involves finding the individuals policy function  $k' = \psi(k, K, s, \tau)$  that simultaneously satisfies his first order conditions, is consistent with market clearing in the goods and asset markets, and yields  $k=K$  in equilibrium. Because individuals are too small to affect the aggregate capital stock and taxes are distortionary, the solution to the problem is suboptimal. The claim is that the solution to (5) can be shown to be a standard competitive equilibrium in the Arrow-Debreu sense (see Dotsey and Romer (1988)).

The first order conditions are given by

$$(6a) \quad Du(c) = \lambda$$

$$(6b) \quad \beta ED_1 V(k', K', s', \tau') = \lambda$$

$$(6c) \quad \beta ED_3 V(k', K', s', \tau') = \lambda$$



and the budget constraint (3). The notation  $D_i$  represents the partial derivative operator with respect to a functions  $i^{\text{th}}$  argument and  $\lambda$  is the Lagrange multiplier associated with the individuals budget constraint.

Equation (6a) implies that the marginal utility of consumption equals the marginal utility of wealth. That is, individuals are indifferent between consuming or holding an extra unit of wealth. Equation (6b) states that the discounted value of next period's marginal utility of wealth times the after tax marginal productivity of capital equals the marginal utility of wealth. This means that at an optimum the individual is indifferent between investing in an extra unit of capital and consuming less today. Equation (6c) is the difference equation determining the price of equity. It implies an indifference at the margin of selling equity today and holding the equity and selling it next period.

Making use of the envelope theorem one obtains  $D_1 V(k, K, s, \tau) = \lambda p$  and  $D_3 V(k, K, s, \tau) = \lambda (q + \pi)$ . Equation (6b) can be rewritten as

$$(7a) \quad \beta E D u(c') p' = D u(c).$$

Employing the equilibrium conditions  $k=K$ ,  $s=1$ ,  $c=f(k)-k'$ , and  $\psi(k, k, 1, \tau) = \psi(k, \tau) = h(k, \tau)$  the solution to the maximization problem involves finding a function  $h$  that satisfies

$$(7b) \quad \beta E D u[f(h(k, \tau) - h(h(k, \tau), \tau')) (1 - \tau') f(h(k, \tau))] = D u[f(k) - h(k, \tau)].$$

Finally the solution for equity prices is given by

$$(7c) \quad \beta E [\lambda' (q' + \pi')] = \lambda q.$$

### III. EQUILIBRIUM

#### a. A Particular Solution

Since the main concern of the analysis is to examine how various realizations of tax rates affect economic activity, the remainder of the paper will examine a particular functional form for which a closed form solution exists. In particular the utility function  $u(c) = \ln(c)$  and the production function  $f(k) = k^\alpha$ ,  $0 < \alpha < 1$ , will be emphasized. Since neither substitution nor income effects predominate with log utility, log utility represents a natural intermediate choice from the class of constant elasticity utility functions. The production function,  $Ak$ , that allows for steady state growth will also be examined. The closed form solution to this problem is nontrivial and involves the solution of a set of nonlinear difference equations.

As mentioned the equilibrium conditions that characterize the model are

$$(8a) \quad s = 1$$

$$(8b) \quad c + k' = f(k)$$

Given these conditions, the first order conditions 6(a), (7b), (7c), the firms profit maximization condition, the budget constraint (3), and the assumption that taxes follow a first order Markov process the equilibrium stochastic processes for  $\{c_{t+j}\}_{j=0}^{\infty}$ ,  $\{k_{t+j+1}\}_{j=0}^{\infty}$ ,  $\{p_{t+j}\}_{j=0}^{\infty}$ , and  $\{q_{t+j}\}_{j=0}^{\infty}$  can be constructed. These processes simultaneously satisfy both profit and utility maximization as well as market clearing in the capital, output, and securities markets.

Without the remittance of taxes (i.e. if tax proceeds were destroyed) this problem would be equivalent to the one solved by Brock (1979) where the production function was subject to multiplicative productivity shocks. In that case the fraction of output allocated to investment would equal  $\alpha\beta$  and be independent of the stochastic process followed by taxes. The assumption that all tax proceeds are remitted lump sum is, to some extent, like assuming that government spending is valued exactly like consumption. This allows one to ignore the effects of government spending and to isolate the effect of taxes. Also, the remittance of these proceeds implies that it is the compensated effects of consumption that are being analyzed, and that even in the presence of log utility expectations of future taxes will be an important determinant of current decisions.<sup>1/</sup>

The procedure used to calculate the competitive equilibrium is to find a policy function that satisfies the particular form of the difference equation given in (7b) that is generated in this example. Alternatively, one could try to find the value function, but this latter procedure does not appear promising. In problems involving suboptimal equilibria where aggregates state variables appear in the individuals value function, finding a policy function seems to be a precondition to calculating the value function. (see Dotsey and Romer (1988)).

An intuitive guess regarding the decision rules governing consumption and investment is that each is a fraction of output and that these fractions are functions of the current realization of the tax rate (past realizations would be important for Markov processes of higher order). In particular  $k' = \gamma(\tau)f(k)$  and  $c = (1-\gamma(\tau))f(k)$  where

$$(9) \quad \gamma(\tau) = \frac{\alpha\beta g(1-\tau)}{1+\alpha\beta g(1-\tau)}$$

and  $g(1-\tau)$  is given by the recursive relationship

$$(10) \quad g(1-\tau) = E[(1-\tau') [1+\alpha\beta g(1-\tau')]]$$

That this is a solution to economy wide equilibrium is shown in the appendix.

The closed form solutions for  $c$  and  $k'$  indicate that the mixture between consumption and investment is based on the conditional expectation of the entire path of future taxes. The expected path of future taxes is relevant since it affects the value of future capital and the amount of consumption that can be purchased from the sale of capital to firms.

#### IV. Solutions for Specific Stochastic Processes.

Further intuition regarding the economic effects of tax rate change can be gained by looking at the results obtained when taxes follow a particular stochastic process. To highlight the difference between permanent and transitory tax rate movements both a white noise and simple two state first order Markov process will be used. The behavior of investment, equity prices, and the real rate of interest differ quite markedly under the two assumed distributions. These examples, therefore, clearly illustrate the importance of correctly specifying the stochastic process for taxes if one is to have confidence in the derived consequences of tax rate changes.

Let the transaction probabilities for the first order Markov process be given by

$$(11a) \quad \text{prob} (\tau_{t+1} = \tau_0 \mid \tau_t = \tau_0) = p_0$$

$$(11b) \quad \text{prob} (\tau_{t+1} = \tau_1 \mid \tau_t = \tau_1) = p_1$$

where  $0 < \tau_0 < \tau_1 < 1$ . Given the discussions in Barro (1979) and Lucas and Stokey (1983), concerning the use of government debt to smooth tax distortions over time, one would expect the tax rate to show a good deal of persistence with both  $p_0$  and  $p_1$  being greater than  $1/2$  and perhaps close to one. Further, the qualitative results generated using the simple process described in (11a) and (11b) are not altered by using a Markov process having more than two states. Therefore, the qualitative results yielded by this example are of general interest. Taking advantage of the recursive nature of  $g(1-\tau_t)$  yields

$$(12) \quad g(1-\tau_j) = \frac{(1-\tau_j)p_j + (1-\tau_i)(1-p_j) - \alpha\beta(1-\tau_i)(1-\tau_j)\delta}{1-\alpha\beta[(1-\tau_j)p_j + (1-\tau_i)p_i] + \alpha^2\beta^2[(1-\tau_i)(1-\tau_j)\delta]}$$

for  $i, j = 0, 1, i \neq j$ , and  $\delta = p_1 + p_0 - 1$ .

Now if taxes are distributed as white noise with mean  $\bar{\tau}$ , then

$g(1-\tau_t) = (1-\bar{\tau})/(1-\alpha\beta(1-\bar{\tau}))$ . In this case the function  $g$  is a constant.

#### a. Consumption and Investment

The behavior of consumption and investment are analyzed under the two different processes for tax rates. Regarding the Markov process one observes that

$$(13) \quad g(1-\tau_0) - g(1-\tau_1) = \frac{1}{\Delta}[(\tau_1 - \tau_0)(p_0 + p_1 - 1)]$$

where  $\Delta$  is the denominator in (12). Equation (13) implies that  $g(1-\tau_0) > g(1-\tau_1)$  if  $p_0 + p_1 > 1$ , that is if tax rates are likely to persist over time. From the definition of  $\gamma(\tau)$  this means  $\gamma(\tau_0) > \gamma(\tau_1)$ . Hence, a greater fraction of output will be invested in the low tax state, if the low tax state implies that taxes are more likely to be low in the future.

Alternatively, if tax rates were white noise with a mean of  $\bar{\tau}$ , then the fraction of output devoted to investment would be  $\alpha\beta(1-\bar{\tau})$ . This fraction would be independent of the current realization of taxes.

A comparison between these two process, a first order Markov process with  $p_0 + p_1 > 1$  and a white noise process, points out the problems that arise if one were to simply assume that agents have perfect foresight. The behavior of investment for some given realization of taxes depends crucially on the distribution that tax realizations were drawn from. As will be shown in more detail in Section V, arbitrarily forcing expectations to equal actual realizations may be misleading and certainly affects the quantitative results of the analysis.

#### b. Security Prices

Equation (7c) represents a first order difference equation determining security prices. Employing the assumptions regarding the utility and production functions as well as (1), (6b), and the envelope theorem yields

$$(14) \quad q_t = \beta c_t \sum_{j=0}^{\infty} \beta^j E_{t,t+j} \frac{\Pi_{t+j}}{c_{t+j}} = \beta(1-\alpha)c_t E_t \sum_{j=0}^{\infty} \beta^j g(1-\tau_{t+j})$$

and that security prices equal the stream of after tax profits discounted by the risk adjusted real rate. This equation turns out to be a complicated expression involving expectations of future tax rates.<sup>2/</sup> As in Brock (1979)

the asset price represents a return to the technology  $k^\alpha$  and is directly related to profits share of output,  $(1-\alpha)$ , as well as to the discount rate  $\beta$ .

For the case where taxes follow a simple first order two state Markov process described by (11a) and (11b) equation (14) becomes

$$(15) \quad q_t(1-\tau_i) = \beta(1-\alpha) c_t \left\{ \left[ \frac{1}{1-\beta\alpha} + (1-p_j) \sum_{n=1}^{\infty} \beta^n \frac{1-\delta^n}{1-\delta} \right] g(1-\tau_i) + \left[ (1-p_i) \sum_{n=1}^{\infty} \beta^n \frac{1-\delta^n}{1-\delta} \right] g(1-\tau_j) \right\}$$

for  $i, j = 0, 1$  and  $i \neq j$ .

Notice that  $q_t(1-\tau_0) - q_t(1-\tau_1)$  equals  $\frac{1}{1-\beta\delta} [g(1-\tau_0) - g(1-\tau_1)] \beta(1-\alpha)c_t$  which is positive if taxes have some persistence. Therefore, stock prices are higher when a low tax state is realized. Also since effective tax rates are inversely related to inflation, the model is capable of generating the negative correlation between stock prices and inflation that is commonly observed in the U.S.

If, instead taxes were generated as a white noise process then stock prices would not vary with tax realizations. Thus in order to forecast stock prices one must know the process generating tax rates and not merely some assumed realizations.

### c. The Real Rate of Interest

Calculating the equilibrium real rate of interest,  $\rho$ , is accomplished by considering the price,  $p^R$ , of a tax free bond,  $R$ , that yields one unit of consumption next period. This is done by adding  $p_t^R R_{t+1}$  to the LHS of (3) and  $R_t$  to the definition of wealth. The resulting first order

condition with respect to  $R_{t+1}$  is

$$(16) \quad \beta E_t \lambda_{t+1}^R = \lambda_t^R p_t^R.$$

Equation (16) implies that an individual is indifferent at the margin between sacrificing  $p_t^R$  units of wealth today for one unit of wealth next period.

Using the expressions for consumption and investment and the fact that  $1/(1-\gamma(\tau_t)) = 1+\alpha\beta g(1-\tau_t)$  implies that

$$(17) \quad 1+\rho_t = \frac{1}{p_t^R} = \frac{1/c_t}{\beta E_t (1/c_{t+1})} = \frac{1+\alpha\beta g(1-\tau_t)}{\beta E_t (1+\alpha\beta g(1-\tau_{t+1}))} \gamma(\tau_t)^\alpha y_t^{\alpha-1}$$

For the case where  $\tau$  follows the process given by (11a) and (11b) and where taxes show persistence ( $p_0, p_1 > 1/2$ ) it can be shown that the real rate of interest is higher when a low tax state is realized.<sup>3/</sup> This occurs because a lowering of the tax rate indicates that capital will be more valuable in the future and that there will be more future output. Individuals will, therefore, value a unit of future wealth by less, causing  $p_t^R$  to fall and  $1+\rho_t$  to rise. Alternatively, individuals will wish to accumulate more capital. In order to induce lower consumption, the real rate of interest must rise. As in previous cases, when  $\tau$  is distributed as white noise the real interest rate does not respond to movements in tax rates.

#### d. Economic Growth

With production that admits steady state economic growth, for example  $f(k) = Ak$ , one can show that taxes will effect economic growth. The solution to the problem with  $f(k) = Ak$  is analogous to the solution derived in Section III, with  $k' = \delta(\tau)AK$  and  $\delta(\tau)$  is given by (9). Since output growth is proportional to growth in the capital stock, the gross growth rate is



$\delta(\tau)A$ . For a white noise stochastic process the growth rate will be  $\beta(1-\bar{\tau})A$  and is independent of any particular realization of taxes. For a Markov process in which high realizations imply a high probability of continually higher taxes, the growth rate will be inversely related to the tax rate.

## V. A Numerical Example

In this section, a direct comparison is made between capital stock accumulation under perfect foresight and situations where agents behave under uncertainty. To perform the experiment 20 realizations of tax rates were generated using a random number generator and the actual transition probabilities associated with the effective tax rate series given in Hulten and Robertson.<sup>4/</sup> In terms of (11a) and (11b),  $\tau_0 = 1/4$ ,  $\tau_1 = 1/2$ , and  $p_0 = p_1 = .9$ . The realizations and resulting levels of the capital stock are given in table 1. The starting value of the capital stock is the value that would result if  $\tau = .375$  (the average value of the tax rate) for all time. In calculating the values under the assumption of perfect foresight it was assumed that  $g(1-\tau_{21}) = .60$ . This assumption is important regarding the solution for the last period's capital stock but the terminal value has almost no effect on the numbers reported in table 1.

From table 1, it is clear that the capital stock under uncertainty behaves in a quantitatively different manner than it does under perfect foresight. The movements in capital tend to be smoothed out by uncertainty since there is always some positive probability that next periods tax rate will be different from today's. Also, under perfect foresight agents respond one period sooner to tax rate changes than do agents who are unsure of the value of next periods tax rate.

This difference in behavior would also occur if taxes were assumed to follow a white noise process. Under uncertainty the level of the capital stock would remain at .095 independent of the actual realizations of taxes, while with perfect foresight the capital stock would respond considerably. Therefore, if one is to accurately predict how agents will respond to tax rate changes, one must carefully consider the forecasting problem facing agents. In order to do this requires an explicit stochastic treatment of the problem.

## VI. Summary

This article analyzes the effects of corporate taxes in a simple stochastic growth model. The model is able to produce responses that are consistent with many of the observed correlations between economic activity and effective tax rates on capital. The paper also represents an advance since it treats tax rates as inherently stochastic. As shown, the actual process generating taxes is an important determinant in understanding how the economy will behave with respect to particular realizations of tax rates.

One might also wish to explore the interaction between tax changes and nominal magnitudes such as inflation and the nominal interest rate. Extending the model to incorporate money via a cash in advance constraint is not a problem. The basic solution governing the real side of the economy is unchanged if the cash in advance constraint is only on consumption and if one rules out precautionary demands for cash. This is easily done so long as monetary growth is not overly deflationary. The solution is only slightly changed if the cash in advance constraint includes capital as well. In these cases a decrease in taxes causes capital accumulation and an increase in output. For given money growth the price level falls and the economy

experiences lower inflation. Since little is to be gained through these additions the paper concentrates on real economic variables.

Table 1

time period	tax rate	$\gamma(\tau_t)$ under	$k_{t+1}$ under	$\gamma(\tau_t)$ under	$k_{t+1}$ under
		perfect foresight	perfect foresight	uncertainty	uncertainty
0			.095		.095
1	.50	.167	.076	.18	.082
2	.50	.167	.071	.18	.078
3	.50	.167	.069	.18	.077
4	.50	.167	.068	.18	.077
5	.50	.167	.068	.18	.076
6	.50	.167	.068	.18	.076
7	.50	.169	.069	.18	.076
8	.50	.180	.074	.18	.076
9	.50	.250	.105	.18	.076
10	.25	.250	.118	.24	.102
11	.25	.250	.123	.24	.112
12	.25	.250	.124	.24	.116
13	.25	.250	.125	.24	.117
14	.25	.250	.125	.24	.117
15	.25	.245	.123	.24	.118
16	.25	.231	.115	.24	.118
17	.50	.167	.081	.18	.089
18	.50	.167	.072	.18	.080
19	.50	.167	.069	.18	.078
20	.50	.167	.068	.18	.077

# FOOTNOTES

1/ With a log utility function income and substitution effects can be shown to be exactly offsetting. The remittance of taxes implies that we are only interested in analyzing substitution effects. Since substitution effects are usually viewed as being of primary importance in most macroeconomic analysis the examination of the compensated effects of tax changes is more relevant under the log utility specification.

2/ The second equality in (14) is derived using (1), (6b), and the envelope theorem. Using the equation for profits (1) and the consumption function,

$$\pi_{t+1}/c_{t+1} = (1-\alpha) \frac{1-\tau_{t+1}}{1-\gamma(\tau_{t+1})}.$$

Equation (7b) leads to the difference equation

$$\alpha \beta E_t \frac{1-\tau_{t+1}}{1-\gamma(\tau_{t+1})} = \frac{\lambda(\tau_t)}{1-\lambda(\tau_t)}.$$

From the definition of  $g(1-\tau_t)$  the latter expression is also equal to  $\alpha \beta g(1-\tau_t)$ .

3/ The proof of this relies on the fact that if  $p_0, p_1 > 1/2$  then  $g(1-\tau_0) > g(1-\tau_1)$  and that

$$\frac{1+\alpha \beta g(1-\tau_0)}{\beta(1+\alpha \beta E g(1-\tau_{t+1}) | \tau_t = \tau_0)} > \frac{1+\alpha \beta g(1-\tau_1)}{\beta(1+\alpha \beta E g(1-\tau_{t+1}) | \tau_t = \tau_1)}.$$

The proof of the latter inequality involves some cumbersome algebra and is omitted.

<sup>4/</sup> The exact procedure was to look at the Hulten and Robertson effective tax rate series as a realization of a two-state first order Markov process and use the calculated sample transition probabilities and sample means. Then, 20 random numbers between zero and one were generated. It was assumed that the initial tax rate was high and that a number between 0-.1 implied a change in the tax rate while a number between .1-1.0 implied that the tax rate remained at its previous value.

## Appendix

In this appendix the solutions for consumption,  $c = [1-\gamma(\tau)]f(k)$  and  $k' = \gamma(\tau)f(k)$ , where  $f(k) = k^\alpha$ ,  $u(c) = \ln(c)$ , and  $\tau$  is a first order Markov process are shown to be an equilibrium solution to the model.

### Proof

Substituting the postulated solution for  $k$  into 7B yields

$$\alpha\beta E \frac{1}{\gamma(\tau)} \frac{1-\tau'}{1-\gamma(\tau')} = \frac{1}{1-\gamma(\tau)}$$

The solution to this nonlinear first order difference equation is

$$\gamma(\tau) = \frac{\alpha\beta g(1-\tau)}{1+\alpha\beta g(1-\tau)}$$

Making this substitution and the fact that  $g(1-\tau)$  is known at time  $t$  results in

$$\frac{1}{g(1-\tau)} E (1-\tau')(1+\alpha\beta g(1-\tau')) = 1$$

one notes that  $g(1-\tau)$  satisfying  $g(1-\tau) = E(1-\tau')(1+\alpha\beta g(1-\tau'))$

satisfies this conditions. The function  $g(1-\tau_t)$  can also be written

$$\text{as } g(1-\tau_t) = E_t[(1-\tau_{t+1})[1+\alpha\beta E_{t+1}[(1-\tau_{t+2})[1+\alpha\beta E_{t+2}[\dots$$

Now given this solution it is easy to see that equilibrium in the goods market is satisfied.

The solution for  $q_t$  given in (14) satisfies (6c). The solution also satisfies the consumers budget constraint. Therefore, the constructed solution maximizes individual utility, clears both the goods and asset markets, and maximizes the firm's profits. It therefore represents a competitive equilibrium.



## REFERENCES

1. Abel, Andrew B., and Olivier Blanchard. "An Intertemporal Model of Saving and Investment." Econometrica 51 (May 1983), 675-92.
2. Barro, Robert J. "On the Determination of Public Debt." Journal of Political Economy 87 (October 1979), 940-71.
3. Brock, William A. "Asset Prices in a Production Economy," in The Economics of Information and Uncertainty, edited by John J. McCall, The University of Chicago Press, Chicago, 1979.
4. Dotsey, Michael and Paul Romer. "Solution Procedures for Calculating Suboptimal Equilibrium in a Dynamic Programming Context." Processed.
5. Goulder, Lawrence H., and Lawrence H. Summers. "Tax Policy, Asset Prices, and Growth: A General Equilibrium Analysis." National Bureau of Economic Research, Working Paper No. 2128, January 1987.
6. Hall, Robert E. "The Dynamic Effects of Fiscal Policy in an Economy with Foresight." Review of Economic Studies 38 (January 1971), 229-44.
7. Hulten, Charles R., and James W. Robertson. "Corporate Tax Policy and Economic Growth: An Analysis of the 1981 and 1982 Tax Acts." Discussion Paper, The Urban Institute, December 1982.
8. Lucas, Robert E., Jr. "Econometric Policy Evaluation: A Critique." The Phillips Curve and the Labor Market, edited by K. Brunner and A. Meltzer, Vol. 1 of Carnegie-Rochester Conference Series in Public Policy. Amsterdam: North Holland, 1976.
9. \_\_\_\_\_ and Nancy L. Stokey. "Optimal Fiscal and Monetary Policy in an Economy without Capital." Journal of Monetary Economics 12 (July 1983), 55-94.

10. Sargent, Thomas J. Macroeconomic Theory. New York: Academic Press, 1979.