Monopoly Markets and Monetary and Fiscal Policy

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by

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One can safely assert that, over the years, two-sector macro models have received a great deal of attention from the economic theorist. The literature contains numerous examples of static and dynamic two-sector macroeconomic models all entertaining, more or less, different sets of assumptions. However, one feature common to all these models is the characterization of all markets as perfectly competitive.[1/] Considering the fact that there exists a fairly well-developed theory of monopolies and other types of market imperfections, it is a relatively simple task to incorporate monopoly markets into two-sector macroeconomic models. It is to this simple task that the present paper addresses itself.

The model developed in this paper describes a closed economy in which two distinct and qualitatively different commodities are produced: (1) consumption goods and (2) capital goods. It is assumed that consumption goods are produced by perfectly competitive firms, which is the standard assumption. In contract to this, however, capital goods are assumed to be produced by a single firm which acts as a profit-maximizing monopoly. It is also assumed that capital stock is fixed at a point in time and that there is no market in existing stocks of physical capital.[2/]

Under these assumptions there emerges a model with the aid of which the short-run effects of monetary and fiscal policies can be studied. With time being a continuous variable and short run being defined as a time span during which capital stock is fixed, a meaningful short-run analysis can take place only by focusing at a point in time. Although such an analysis is static, it is not stationary because all the variables evolve over time.[3/]

In section I the model is formalized by describing the behavior of firms, households, and the government. In section II the solution of the model is presented. Short-run effects of monetary and fiscal policies are studied in section III and conclusions are given in section IV.

I. Behavior of Firms, Households, and the Government

Firms

A) Firms Producing Consumption Goods

Individual firms producing consumption goods use capital and labor as inputs with a linearly homogenous production function which is identical for all firms. Firms are assumed to be able to hire any amount of labor they want at a fixed wage rate of w at every instant. On the other hand, due to a lack of market in existing stocks of physical capital, firms are unable to alter their stocks of capital instantaneously. Instead, if it is profitable to do so, they change their capital stock at a finite rate per unit of time. It is assumed that capital does not depreciate; that all economic agents expect nominal wages and price of consumption goods to change at the rate π_c; and that government charges a corporate income tax at the rate t_c to the consumption goods-producing firms, where taxable income is defined as the net cash flow of the firm.

Let

$$C = C(K_c, N_c)$$

be the production function of the consumption goods-producing sector and let the following hold with respect to first and second-order partial derivatives of the production function:

$$C_k > 0,\ C_n > 0,\ C_{kk} < 0,\ C_{nn} < 0,\ C_{kn} > 0\ .$$

Net cash flow of a typical firm after taxes is equal to

$$(1-t_c)[P_c C(K_c,N_c)-WN_c] - P_i\dot{K}_c \qquad [1]$$

where P_c is the price of consumption goods and P_i is the price of capital goods.

The objective of the firm being to maximize the present value of its net cash flow, the maximand is given by

$$\int_0^\infty e^{-(r-\pi_c)t}\{(1-t_c)[P_c C(K_c,N_c)-WN_c] - P_i\dot{K}_c\}dt \ . \qquad [1a]$$

The Euler conditions for a maximum are

$$C_n = \frac{W}{P_c} \qquad [2]$$

$$C_k = \frac{P_i}{P_c}\left[\frac{r-\pi_c}{1-t_c}\right] \ . \qquad [3]$$

The ability of firms to alter the level of employment instantaneously implies that first equality always holds. However, the lack of a market in physical capital prevents the attainment of second equality every instant. I_k in equation [3] is evaluated at the fixed level of capital stock and the amount of employment implied by equation [2]. Consequently,

$$C_k \gtreqless P\left[\frac{r-\pi_c}{1-t_c}\right] \qquad [4]$$

where $P = \frac{P_i}{P_c}$ is the relative price of capital goods in terms of consumption goods. The implication of equations [3] and [4] is that had there been a market in physical capital, the firm would have been a buyer in that market if

$$C_k > P\left[\frac{r-\pi_c}{1-t_c}\right]$$

and a seller if

$$C_k < P[\frac{r-\pi_c}{1-t_c}] .$$

But due to the lack of a market in physical capital firm will do the next best thing: adjust its capital stock at a finite rate per unit of time by investing. The rate of investment, it is assumed, depends directly on the difference between the marginal product and real cost of capital.

$$\dot{K}_c = \phi(C_k - P[\frac{r-\pi_c}{1-t_c}]) \qquad [5]$$

with $\phi'(.) > 0$.

Firms have equities as their outstanding liabilities. Assuming that firms finance investment by issuing new equities, the flow of dividends can be written as

$$(1-t_c)[P_c C(K_c,N_c)-WN_c] .$$

As the value of equities is equal to the present value of the flow of dividends we can write

$$V_c = \int_0^{\infty} e^{-(r-\pi_c)t}\{(1-t_c)[P_c C(K_c,N_c)-WN_c]\}dt . \qquad [6]$$

The linear homogeneity of production function together with the profit-maximizing condition [2] implies

$$V_c = \frac{(1-t_c)P_c C_k K_c}{r-\pi_c} . \qquad [7]$$

Return on equities is defined as the ratio of dividends to the value of equities plus expected capital gains, and is equal to

$$v = \frac{(1-t_c)P_c C_k K_c}{V_c} + \pi_c = r .$$

B) Firm Producing Capital Goods

It will be assumed that the firm which produces capital goods behaves as a profit-maximizing monopolist. It produces capital goods with a linearly homogenous production function using capital and labor as inputs. Let the production function be

$$I = I(K_i, N_i)$$

with the following properties

$$I_k > 0, \; I_n > 0, \; I_{kk} < 0, \; I_{nn} < 0, \; I_{kn} > 0 \; .$$

The monopolist faces a demand curve which is given by [5] together with the following equilibrium condition

$$I = \dot{K}_c + \dot{K}_i$$

$$\dot{K}_c = I - \dot{K}_i \; .$$

Substitute this into [5] to get

$$I - \dot{K}_i = \phi(C_k - \frac{P_i}{P_c}[\frac{r-\pi_c}{1-t_c}]) \; . \qquad [8]$$

As $\phi' > 0$, [8] can be inverted to yield

$$P_i = \psi(C_k, P_c, r-\pi, 1-t_c, I-\dot{K}_i) \qquad [9]$$

with

$$\left.\begin{aligned}
\psi_1 &= \frac{P_c(1-t_c)}{r-\pi_c} \\
\psi_2 &= \frac{P_i}{P_c} \\
\psi_3 &= -\frac{P_i}{r-\pi_c} \\
\psi_4 &= \frac{P_i}{1-t_c} \\
\psi_5 &= -\frac{P_c(1-t_c)}{\phi'(r-\pi_c)}
\end{aligned}\right\} \qquad [10]$$

Assuming that the monopoly profits are subject to a tax at the rate t_i, the net cash flow of the monopolist after taxes is equal to

$$(1-t_i)[\psi(C_k,P_c,r-\pi,1-t_c,I-\dot{K}_i)(I-\dot{K}_i)-WN_i] \ .$$

The maximand for the monopolist is the present value of its after-tax cash flow, which can be written as

$$Z = \int_0^{\infty} e^{-rt}\{(1-t_i)[\psi(C_k,P_c,r-\pi,1-t,I-\dot{K}_i)(I-\dot{K}_i)-WN_i]\}dt$$

Euler conditions for a maximum are

$$\frac{\partial Z}{\partial N_i} = 0 \qquad \frac{\partial Z}{\partial K_i} - \frac{\partial}{\partial_t}\frac{\partial Z}{\partial \dot{K}_i} = 0$$

$$\frac{\partial Z}{\partial N_i} = I_n(\psi+\psi_5(I-\dot{K}_i)) - W = 0 \ . \qquad [11]$$

But $\psi_5 = \dfrac{\partial P_i}{\partial(I-\dot{K}_i)} = \dfrac{\partial P_i}{\partial \dot{K}_c}$

and $I - \dot{K}_i = \dot{K}_c$, $\quad \psi = P_i$.

Then [11] can be written as

$$I_n\left(1 + \frac{\partial P_i}{\partial \dot{K}_c} \cdot \frac{\dot{K}_c}{P_i}\right) = \frac{W}{P_i}$$

$$I_n\left(1 + \frac{1}{\varepsilon}\right) = I_n\lambda = \frac{W}{P_i} \qquad [11a]$$

where ε is the price elasticity of demand.

On the other hand,

$$\frac{\partial Z}{\partial K_i} = e^{-rt}(1-t_i)I_k\{\psi_5(I-\dot{K}_i)+\psi\} \qquad [12a]$$

$$\frac{\partial Z}{\partial \dot{K}_i} = -e^{-rt}(1-t_i)\{\psi_5(I-\dot{K}_i)+\psi\} \qquad [12b]$$

$$\frac{\partial}{\partial t}\frac{\partial Z}{\partial \dot{K}_i} = e^{-rt}(1-t_i)\{r\lambda\psi-\dot{\lambda}\psi-\lambda\dot{\psi}\} . \qquad [12c]$$

Then

$$\frac{\partial Z}{\partial K_i} - \frac{\partial}{\partial t}\frac{\partial Z}{\partial \dot{K}_i} = e^{-rt}(1-t_i)\{I_k\lambda\psi-r\lambda\psi+\dot{\lambda}\psi+\lambda\dot{\psi}\} = 0$$

$$I_k = r - \frac{\dot{\lambda}}{\lambda} - \frac{\dot{\psi}}{\psi} . \qquad [13]$$

Notice that $\frac{\dot{\psi}}{\psi} = \frac{\dot{P}_i}{P_i}$, and, in terms of perceived quantities, it is the expected rate of inflation of the price of capital goods. Assuming that the expected rate of change of price elasticity of demand is zero, then the optimality condition for the monopolist with respect to capital becomes

$$I_k = r - \pi_i . \qquad [13a]$$

However, due to lack of a market in existing stocks of capital, equality in [13a] cannot be attained at every instant. Consequently, monopolist also invests at a rate which depends directly on the gap between the marginal product of capital and the real cost of capital

$$\dot{K}_i = \theta(I_k - r + \pi) \text{ with } \theta'(.) > 0 \,. \qquad [14]$$

It is assumed that monopoly firm has equities as outstanding liabilities the value of which is equal to

$$V_i = \frac{(1-t_i)P_i I_k K_i}{r-\pi_i} + \frac{I_n(1-\lambda)N_i}{r-\pi_i} \,.$$

This value implies that the return on equities is equal to r.

Households

The wealth of households is composed of money, bonds, and equities. In terms of consumption goods, this wealth can be written as:

$$\begin{aligned} W &= \frac{M+B+V_c+V_i}{P_c} \\ &= \frac{M+B}{P_c} + \frac{(1-t_c)C_k}{r-\pi_c} K_c + \frac{(1-t_i)P_i I_k}{P_c(r-\pi_i)} K_i \\ &\quad + \frac{(1-t_i)(1-\lambda)P_i I_n}{P_c(r-\pi_i)} N_i \,. \end{aligned} \qquad [15]$$

Real income of the public consists of wages, dividends, and interest income from bonds and is equal to

$$\frac{W}{P_c}(N_i+N_c) + (1-t_c)(C - \frac{W}{P_c}N_c) + (1-t_i)(\frac{P_i I - WN_i}{P_c}) + r\frac{B}{P_c}$$

which, in turn, implies that personal income is equal to:

$$Y + \frac{rB}{P_c} - T_1$$

where $Y = C + \frac{P_i}{P_c} I$ and T_1 is the total corporate income tax.

The tax liability of the public consists of a lump-sum amount to and on income tax at the rate t_y. Consequently, after-tax income of the public is equal to

$$Y + \frac{rB}{P_c} - T \qquad [16]$$

where T is the total tax collections. Equation [16] is the disposable income of the public according to national income accounts definition.[4/] Based on this income, public makes consumption decisions as follows:

$$C_p = \gamma(Y + \frac{rB}{P_c} - T) \text{ with } 0 < \gamma'(.) < 1.$$

Finally, public makes portfolio decisions. The fact that bonds and equities are regarded as perfect substitutes when their yields are equal implies that public's portfolio decision consists of dividing their wealth between money on one hand and paper earning assets on the other. The portfolio balance of the public is satisfied when demand for real balances is equal to supply,

$$\frac{M}{P_c} = m(1-t_y)r, Y) \quad m_r < 0 \quad m_y > 0 \qquad [17]$$

where $(1-t_y)r$ is the after-tax return on bonds.

Government

Government is assumed to have money and bonds as outstanding liabilities. The nominal rate of return on money is fixed at zero, whereas the nominal rate of return on bonds, r, is market determined given the portfolio of the government. Government is assumed to be able to change its portfolio via open market operations subject to:

$$dM + dB = 0 .$$

Government's receipts consist of total tax collections, T, whereas the payments consist of expenditures on consumption goods, G, and transfer payments $\frac{rB}{P_c}$.

II. Solution of the Model

For convenience let us rewrite the behavioral equations together with identities and equilibrium conditions:

$$C = C(K_c, N_c) \qquad [18]$$

$$I = I(K_i, N_i) \qquad [19]$$

$$C_n = \frac{W}{P_c} \qquad [20]$$

$$I_n \lambda = \frac{W}{P_i} \qquad [21]$$

$$I = \phi(C_k - \frac{P_i}{P_c}(\frac{r-\pi_c}{1-t_c})) + \theta(I_k - r + \pi_i) \qquad [22]$$

$$C = \gamma(Y + \frac{rB}{P_c} - T) + G \qquad [23]$$

$$\frac{M}{P_c} = m((1-t_y)r, Y) \qquad [24]$$

$$Y = C + \frac{P_i}{P_c} I \qquad [25]$$

$$\lambda = 1 + \frac{1}{\varepsilon} \qquad [26]$$

$$\varepsilon = \frac{1}{\psi_5} \cdot \frac{\psi}{\dot{K}_c} = - \frac{P_i(r-\pi_c)}{P_c(1-t_c)} \cdot \frac{\phi'}{\phi} . \qquad [27]$$

The model has ten unknowns: C, I, N_c, N_i, P_c, P_i, Y, r, λ, and ε. Parameters of the model are: K_c, K_i, W, π_c, π_i, t_c, t_y, T, G, B, and M. It will be assumed that variables of the model adjust instantaneously in response to changes in exogenous variables. My strategy for solving the model will be to reduce it into three equations in P_i, P_c, and r. One of these equations will be the equilibrium condition in investment goods market; the second one will be the equilibrium condition in consumption goods market; and the third one will be the equilibrium condition in money market.

First of all, consider equations [20] and [21]. If $C_{nn} \neq 0$ and $I_{nn} \neq 0$, then they can be inverted to yield

$$N_c = N_c(W, P_c, K_c) \tag{28a}$$

$$N_i = N_i(W, P_i, \lambda, K_i) \tag{28b}$$

with $N_{cw} = \frac{1}{P_c C_{nn}} < 0$ $\qquad N_{cp} = -\frac{W}{P_c} N_{cw} > 0$

$N_{iw} = \frac{1}{\lambda P_i I_{nn}} < 0$ $\qquad N_{ip} = -\frac{W}{P_i} N_{iw} > 0$

$N_{i\lambda} = -\frac{W}{\lambda} N_{iw} > 0$.

If we substitute [28a] and [28b] into [18] and [19], respectively, we get

$$C = C(W, P_c, K_c) \tag{29a}$$

$$I = I(W, P_i, \lambda, K_i) \tag{29b}$$

with $C_w = C_n N_{cw} < 0$ $\qquad C_p = C_n N_{cp} > 0$

$I_w = I_n N_{iw} < 0$ $\qquad I_p = I_n N_{ip} > 0$

$I_\lambda = I_n N_{i\lambda} > 0$.

On the other hand, if we substitute [28a] and [28b] into the equations defining C_k and I_k we can get

$$C_k = C^k(W, P_c K_c) \qquad [30a]$$

$$I_k = I^k(W, P_i, \lambda, K_i) \qquad [30b]$$

with $C^k_w = C_{kn} N_{cw} < 0 \qquad C^k_p = C_{kn} N_{cp} > 0$

$I^k_w = I_{kn} N_{iw} < 0 \qquad I^k_p = I_{kn} N_{ip} > 0$

$I^k_\lambda = I_{kn} N_{i\lambda} > 0$

Finally, we must express λ as a function of P_i, P_c, and r.

$$\lambda = 1 - \frac{P_c(1-t_c)\phi(C^k(W,P_c,K_c) - \frac{P_i}{P_c}\frac{r-\pi_c}{(1-t_c)})}{P_i(r-\pi_c)\phi'} \qquad [34]$$

$$\lambda = \lambda(P_i, P_c, r; tc, w, \pi_c) \qquad [34a]$$

with

$$\lambda_{P_i} = \frac{(r-\pi_c)\phi' P_i + 1}{(r-\pi_c)\phi' P_i^2} > 0$$

$$\lambda_{P_c} = - \frac{(1-t_c)\ P_c + (r-\pi_c)\phi' P_i + (1-t_c)\phi' P_c^2 C^k_p}{(r-\pi_c)\phi' P_i P_c} < 0$$

$$\lambda_r = \frac{P_i}{r-\pi_c}\lambda_{P_i} > 0$$

$$\lambda_{\pi_c} = -\lambda_r \qquad < 0$$

$$\lambda_{t_c} = \frac{(1-t_c)P_c\phi + (r-\pi_c)P_i\phi'}{(r-\pi_c)(1-t_c)P_i\phi'} > 0$$

$$\lambda_w = -\frac{(1-t_c)P_c}{(r-\pi_c)P_i} C^k_w \qquad < 0 \ .$$

Substitute [29b], [30a], and [30b] into [22] to get:

$$I(W,P_i,\lambda,K_i) = \phi\{C^k(W,P_cK_c) - \frac{P_i}{P_c}(\frac{r-\pi_c}{1-t_c})\} +$$

$$\theta\{I^k(W,P_i,\lambda,K_i) - r + \pi_i\} \ .$$

Then the excess demand function for investment goods can be written as

$$A(P_i,P_c,r,\lambda;w,\pi_c,\pi_i,t_c) = -I(W,P_i,\lambda,K_i)$$

$$+\theta\{C^k(W,P_c,K_c) - \frac{P_i}{P_c}(\frac{r-\pi_c}{1-t_c})\}+ \theta\{I^k(W,P_i,\lambda,K_i)-r+\pi_i\} = 0 \qquad [31]$$

with the following partial derivatives:

$$A_{P_i} = -\frac{\theta'}{P_c}(\frac{r-\pi_c}{1-t_c}) + (N_{ip}+N_{i\lambda}\lambda_{P_i})(\theta' I_{kn}-I_n) < 0$$

$$A_{P_c} = \phi'[C^k_p + \frac{P_i}{P_c}(\frac{r-\pi_c}{1-t_c})] + N_{i\lambda}\lambda_{P_c}(\theta' I_{kn}-I_n) > 0$$

$$A_r = -[\theta' + \frac{\phi' P_i}{P_c^2(1-t_c)}] + N_{i\lambda}\lambda_r(\theta' I_{kn}-I_n) \quad < 0$$

$$A_w = \phi' C^k_w + (N_{iw}+N_{i\lambda}\lambda_w)(\theta' I_{kn}-I_n)$$

$$A_{\pi_c} = [\theta' + \frac{\phi' P_i}{P_c(1-t_c)}] + N_{i\lambda}\lambda_{\pi_c}(\theta' I_{kn}-I_n) \quad > 0$$

$$A_{\pi_i} = \theta' > 0$$

$$A_{t_c} = -\phi'\frac{P_i}{P_c}\frac{(r-\pi_c)}{(1-t_c)^2} + N_{i\lambda}\lambda_{t_c}(\theta' I_{kn}-I_n) \quad < 0$$

In order to obtain the excess demand function for consumption goods, substitute [25], [29a], and [29b] into [23] to get

$$C(W,P_c,K_c) = \gamma\{C(W,P_c,K_c) + \frac{P_i}{P_c} I(W,P_i,\lambda,K_i) + \frac{rB}{P_c} - T\} + G$$

which, in turn, will imply the following excess demand function:

$$\mathcal{B}(P_i,P_c,r,\lambda;W,B,T,G) = -C(W,P_c,K_c) + G$$

$$+ \gamma\{C(W,P_c,K_c) + \frac{P_i}{P_c} I(W,P_i,\lambda,K_i) + \frac{rB}{P_c} - T\} = 0 \qquad [32]$$

whose partial derivatives are:

$$\mathcal{B}_{P_i} = \gamma'\{\frac{I}{P_c} + \frac{P_i}{P_c} I_n(N_{ip}+N_{i\lambda}\lambda_{p_i})\} > 0$$

$$\mathcal{B}_{P_c} = \gamma'\{\frac{P_i}{P_c}(\frac{I}{P_c}) + I_n N_{i\lambda}\lambda_{p_c}) + \frac{rB}{P_c^2} - (1-\gamma')C_n N_{cp} > 0$$

$$\mathcal{B}_r = \gamma'\{\frac{P_i}{P_c} I_n N_{i\lambda}\lambda_r + \frac{B}{P_c}\} > 0$$

$$\mathcal{B}_w = \gamma' \frac{P_i}{P_c} I_n(N_{iw}+N_{i\lambda}\lambda_w) - (1-\gamma')C_n N_{cw}$$

$$\mathcal{B}_{\pi_c} = \gamma' \frac{P_i}{P_c} I_n N_{i\lambda}\lambda_{\pi_c} < 0$$

$$\mathcal{B}_{t_c} = \gamma' \frac{P_i}{P_c} I_n N_{i\lambda}\lambda_{t_c} > 0$$

$$\mathcal{B}_b = \gamma' \frac{r}{P_c} > 0$$

$$\mathcal{B}_t = -\gamma' < 0$$

$$\mathcal{B}_c = 1 > 0$$

Thirdly, the excess demand function for real balances can be obtained by substituting [25], [29a], and [29b] into [24] to get:

$$\frac{M}{P_c} = m\{(1-t_y)r,\ C(W,P_c,K_i) + \frac{P_i}{P_c}\, I(W,P_i,\lambda,K_i)\}$$

which can also be written as follows:

$$C(P_i,P_c,r,\lambda;W,t_y,M) =$$

$$m\{(1-t_y)r,\ C(W,P_c,K_c) + \frac{P_i}{P_c}\, I(W,P_i,\lambda,K_i)\} - \frac{M}{P_c} = 0. \qquad [33]$$

The partial derivatives of the excess demand function for money are:

$$C_{P_i} = m_y\{\frac{I}{P_c} + \frac{P_i}{P_c}\, I_h(N_{ip}+N_{i\lambda}\lambda_{P_i})\} \qquad > 0$$

$$C_{P_c} = m_y\{C_n N_{cp} + \frac{P_i}{P_c}\,(-\frac{I}{P_c} + I_n N_{i\lambda}\lambda_{P_c})\} + \frac{M}{P_c^2} \qquad > 0$$

$$C_r = M_r(1-t_y) + M_y\, \frac{P_i}{P_c}\, I_n N_{i\lambda}\lambda_r \qquad < 0$$

$$C_w = M_y[C_n N_{cw} + \frac{P_i}{P_c}\, I_n(N_{iw}+N_{i\lambda}\lambda_w)] \qquad < 0$$

$$C_{t_y} = -m_r r \qquad > 0$$

$$C_{\pi_c} = m_y\, \frac{P_i}{P_c}\, I_n N_{i\lambda}\lambda_{\pi_c} \qquad < 0$$

$$C_{t_c} = m_y\, \frac{P_i}{P_c}\, I_n N_{i\lambda}\lambda_{t_c} \qquad > 0$$

$$C_m = -\frac{1}{P_c} \qquad < 0$$

The inequalities show those partial derivatives that can be determined unambiguously with the assumption that

$$\theta' I_{kn} - I_n < 0$$

in addition to the assumptions that were made so far. Notice that the term $\theta' I_{kn}$ shows the effect of a rise in N_i upon the demand for capital goods. A rise in N_i induces a rise in the marginal product of capital and hence increases the demand for capital goods. On the other hand, a rise in N_i also increases the supply of capital goods, which is represented by the term I_n. The assumption that

$$\theta' I_{kn} - I_n < 0$$

then implies that a rise in N_i tends to reduce the excess demand for capital goods.

Now consider B_{p_c}; it shows the effect of a rise in the price of consumption goods upon excess demand for consumption goods. A rise in price of consumption goods tends to increase the output of consumption goods. This rise in output, on the one hand, tends to reduce excess demand through increased output and, on the other hand, tends to increase excess demand by raising gross national product. It will be assumed that supply effect is stronger implying

$$B_{p_c} < 0 \ .$$

Likewise, C_{p_c} shows the effects of a rise in P_c on excess demand for real balances. A rise in P_c, on the one hand, reduces the real supply of money measured in terms of consumption goods and increases the demand by increasing Y. However, a rise in P_c tends to reduce λ and hence the output of capital goods and hence Y. It will be assumed that

$$C_{p_c} > 0 \ .$$

Lastly, consider C_r which gives the effect of a rise in interest rate on excess demand for real balances. On one hand, a rise in r tends to reduce the excess demand for money by increasing the opportunity cost of holding money. On the other hand, a rise in r tends to increase λ and hence the output of capital goods. This will tend to increase excess demand for real balances by increasing Y. It will be assumed that

$$C_r < 0 .$$

Stability of Equilibrium

The general equilibrium of the model is defined as

$$\left.\begin{aligned} A(.) &= 0 \\ B(.) &= 0 \\ C(.) &= 0 \end{aligned}\right\} \qquad [34]$$

In order to study the stability of equilibrium defined by [34] it will be assumed that in response to excess demand in every market, the price of that market goes up. Consequently,

$$\dot{P}_i = \alpha_1[A(.)] \qquad \alpha_1 > 0$$

$$\dot{P}_c = \alpha_2[B(.)] \qquad \alpha_2 > 0$$

$$\dot{r} = \alpha_3[C(.)] \qquad \alpha_3 > 0$$

If we linearize around equilibrium and cancel out second and higher-order terms we get

$$\begin{bmatrix} \dot{P}_i \\ \dot{P}_c \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix} \begin{bmatrix} \mathcal{A}_{p_i} & \mathcal{A}_{p_c} & \mathcal{A}_r \\ \mathcal{B}_{p_i} & \mathcal{B}_{p_c} & \mathcal{B}_r \\ \mathcal{C}_{p_i} & \mathcal{C}_{p_c} & \mathcal{C}_r \end{bmatrix} \begin{bmatrix} dP_i \\ dP_c \\ dr \end{bmatrix} \qquad [35]$$

It is well known that for a system

$$\dot{X} = A\ dx$$

to be stable, the matrix A must have eigen values with negative real parts. Hence, for the system defined by [35] to be stable it is sufficient to have the matrix

$$A_1 = \begin{bmatrix} \mathcal{A}_{p_i} & \mathcal{A}_{p_c} & \mathcal{A}_r \\ \mathcal{B}_{p_i} & \mathcal{B}_{p_c} & \mathcal{B}_r \\ \mathcal{C}_{p_i} & \mathcal{C}_{p_c} & \mathcal{C}_r \end{bmatrix} \leqq 0 \qquad [36]$$

This, in turn, implies

$$\mathcal{A}_{p_i} < 0 \qquad [36a]$$

$$\mathcal{A}_{p_i}\mathcal{B}_{p_c} - \mathcal{A}_{p_c}\mathcal{B}_{p_i} > 0 \qquad [36b]$$

$$\begin{vmatrix} \mathcal{A}_{p_i} & \mathcal{A}_{p_c} & \mathcal{A}_r \\ \mathcal{B}_{p_i} & \mathcal{B}_{p_c} & \mathcal{B}_r \\ \mathcal{C}_{p_i} & \mathcal{C}_{p_c} & \mathcal{C}_r \end{vmatrix} = \Delta < 0 \qquad [36c]$$

Notice that [36b] can be written as

$$\frac{\mathcal{A}_{p_c}}{\mathcal{A}_{p_i}} > \frac{\mathcal{B}_{p_c}}{\mathcal{B}_{p_i}}$$

or alternatively

$$\left.\frac{\partial P_i}{\partial P_c}\right|_{\mathcal{A}} > \left.\frac{\partial P_i}{\partial P_c}\right|_{\mathcal{B}}$$

which means that in $P_c - P_i$ plane the excess demand for consumption goods should be steeper than the excess demand for capital goods for a given rate of interest.

Now consider the inverse of the matrix in [36] which can be written as:

$$A_1^{-1} = \frac{1}{\Delta}\begin{bmatrix} \mathcal{B}_{p_c}\mathcal{C}_r-\mathcal{B}_r\mathcal{C}_{p_c} & -(\mathcal{A}_{p_c}\mathcal{C}_r-\mathcal{A}_r\mathcal{C}_{p_c}) & \mathcal{A}_{p_c}\mathcal{B}_r-\mathcal{A}_r\mathcal{B}_{p_c} \\ -(\mathcal{B}_{p_i}\mathcal{C}_r-\mathcal{B}_r\mathcal{C}_{p_i}) & \mathcal{A}_{p_i}\mathcal{C}_r-\mathcal{A}_r\mathcal{C}_{p_i} & -(\mathcal{A}_{p_i}\mathcal{B}_r-\mathcal{A}_r\mathcal{B}_{p_i}) \\ \mathcal{B}_{p_i}\mathcal{C}_{p_c}-\mathcal{B}_{p_c}\mathcal{C}_{p_i} & -(\mathcal{A}_{p_i}-\mathcal{C}_{p_c}-\mathcal{A}_{p_c}\mathcal{C}_{p_i}) & \mathcal{A}_{p_i}\mathcal{B}_{p_c}-\mathcal{A}_{p_c}\mathcal{B}_{p_i} \end{bmatrix}$$

$$A_1^{-1} = \frac{1}{\Delta}\Gamma$$

$$A_1 < 0 \Rightarrow A_1^{-1} < 0 .$$

As $\Delta < 0$ then $A_1^{-1} < 0 \Rightarrow \Gamma > 0$

$$\Gamma > 0 \Rightarrow |\Gamma_i| > 0 \qquad i=1,2,3 \qquad [37]$$

where Γ_i are the i^{th} order principal minors of Γ. Consider Γ_1

$$\Gamma_1 = \mathcal{B}_{p_c}\mathcal{C}_r - \mathcal{B}_r\mathcal{C}_{p_c} > 0$$

which can be rewritten as,

$$\frac{\mathcal{C}_{p_c}}{\mathcal{C}_r} > \frac{\mathcal{B}_{p_c}}{\mathcal{B}_r}$$

which in turn implies that

$$\left.\frac{\partial r}{\partial P_c}\right|_{\mathcal{C}} > \left.\frac{\partial r}{\partial P_c}\right|_{\mathcal{B}} .$$

Hence, in r-P_c plane the excess demand function for real balances should be steeper than the excess demand for consumption goods. Now let us concentrate on the elements of Γ

$\Gamma_{11} = \mathcal{B}_{p_c}\mathcal{C}_r - \mathcal{B}_r\mathcal{C}_{p_c} > 0$ by stability assumption [37]

$\Gamma_{22} = \mathcal{A}_{p_i}\mathcal{C}_r - \mathcal{A}_r\mathcal{C}_{p_i} > 0$

$\Gamma_{33} = \mathcal{A}_{p_i}\mathcal{B}_{p_c} - \mathcal{A}_{p_c}\mathcal{B}_{p_i} > 0$ by assumption [36b]

$\Gamma_{21} = -(\mathcal{B}_{p_i}\mathcal{C}_r - \mathcal{B}_r\mathcal{C}_{p_i}) > 0$

$\Gamma_{31} = \mathcal{B}_{p_i}\mathcal{C}_{p_c} - \mathcal{B}_{p_c}\mathcal{C}_{p_i} > 0$

$\Gamma_{32} = -(\mathcal{A}_{p_i}\mathcal{C}_{p_c} - \mathcal{A}_{p_c}\mathcal{C}_{p_i}) > 0$

$\Gamma_{12} = -(\mathcal{A}_{p_c}\mathcal{C}_r - \mathcal{A}_r\mathcal{C}_{p_c})$ ambiguous

$\Gamma_{13} = \mathcal{A}_{p_c}\mathcal{B}_r - \mathcal{A}_r\mathcal{B}_{p_c}$ ambiguous

$\Gamma_{23} = -(\mathcal{A}_{p_i}\mathcal{B}_r - \mathcal{A}_r\mathcal{B}_{p_i})$ ambiguous

However, equation [37] also implies that

$$\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21} > 0 \qquad [37a]$$

A sufficient condition for [37a] to hold is to have $\Gamma_{12} < 0$. If this is so, then

$$\Gamma_{11} > 0 \Rightarrow \frac{\mathcal{C}_{p_c}}{\mathcal{C}_r} > \frac{\mathcal{B}_{p_c}}{P_r}$$

$$\Gamma_{12} < 0 \Rightarrow \frac{A_{p_c}}{A_r} > \frac{\mathcal{C}_{p_c}}{\mathcal{C}_r} .$$

These two together will imply

$$\frac{A_{p_c}}{A_r} > \frac{\mathcal{B}_{p_c}}{\mathcal{B}_r} \Rightarrow \Gamma_{13} < 0 .$$

If $\Gamma_{13} < 0 \Rightarrow A_{p_c} < \dfrac{A_r \mathcal{B}_{p_c}}{\mathcal{B}_r}$ [38a]

but $\Gamma_{33} > 0 \Rightarrow A_{p_i} < \dfrac{A_{p_c} \mathcal{B}_{p_i}}{\mathcal{B}_{p_c}}$ [38b]

[38a] and [38b], taken together will imply

$$A_{p_i} < \frac{A_r \mathcal{B}_{p_i}}{\mathcal{B}_r} \Rightarrow A_{p_i}\mathcal{B}_r - A_r\mathcal{B}_{p_i} < 0 \Rightarrow \Gamma_{23} > 0 .$$

Hence the signs of the matrix Γ is equal to

$$\begin{bmatrix} + & - & - \\ + & + & + \\ + & + & + \end{bmatrix}$$

III. Short-Run Effects of Fiscal and Monetary Policies

The short-run effects of fiscal and monetary policies will be studied by performing some static exercises in which one of the exogenous variables will be changed and the response of the general equilibrium in terms of the variables P_i, P_c, and r will be analyzed. In terms of first-order differentials the equilibrium can be expressed with the following matrix equation

$$\begin{bmatrix} dP_i \\ dP_c \\ dr \end{bmatrix} = -\frac{1}{\Delta}\Gamma A_2 \begin{bmatrix} dw \\ d\pi_c \\ d\pi_i \\ dt_c \\ dt_y \\ dT \\ dG \\ dB \\ dM \end{bmatrix}$$

where Δ and Γ were defined previously and

$$A_2 = \begin{bmatrix} A_w & A_{\pi_c} & A_{\pi_i} & A_{t_c} & 0 & 0 & 0 & 0 & 0 \\ \mathcal{B}_w & \mathcal{B}_{\pi_c} & 0 & \mathcal{B}_{t_c} & 0 & \mathcal{B}_t & \mathcal{B}_g & \mathcal{B}_b & 0 \\ \mathcal{C}_w & \mathcal{C}_{\pi_c} & 0 & \mathcal{C}_{t_c} & \mathcal{C}_{t_y} & 0 & 0 & 0 & \mathcal{C}_m \end{bmatrix}$$

a) Rise in corporate tax rate applicable on consumption goods-producing firms.

$$\begin{bmatrix} dP_i \\ dP_c \\ dr \end{bmatrix} = \begin{bmatrix} \Gamma_{11}A_{t_c} + \Gamma_{12}\mathcal{B}_{t_c} + \Gamma_{13}\mathcal{C}_{t_c} \\ \Gamma_{21}A_{t_c} + \Gamma_{22}\mathcal{B}_{t_c} + \Gamma_{23}\mathcal{C}_{t_c} \\ \Gamma_{31}A_{t_c} + \Gamma_{32}\mathcal{B}_{t_c} + \Gamma_{33}\mathcal{C}_{t_c} \end{bmatrix} dt_c$$

The sign of dP_i is unambiguous

$$dP_i < 0 \ .$$

However, the signs of dP_c and dr are ambiguous. Consequently, we can say that a rise in t_c causes the price of capital goods to fall but its effects on the price of consumption goods and interest rate are ambiguous.

b) Rise in income tax rate

$$\begin{bmatrix} dP_i \\ dP_c \\ dr \end{bmatrix} = \begin{bmatrix} \Gamma_{13} \ C_{t_y} \\ \Gamma_{23} \ C_{t_y} \\ \Gamma_{33} \ C_{t_y} \end{bmatrix} dt_y$$

this implies the following signs

$$dP_i < 0 \ , \quad dP_c > 0, \quad dr > 0 \ .$$

Hence a rise in income tax rate results in a fall in the price of capital goods but a rise in the price of consumption goods and interest rate.

c) A rise in autonomous taxes

$$\begin{bmatrix} dP_i \\ dP_c \\ dr \end{bmatrix} = \begin{bmatrix} \Gamma_{12} \ \mathcal{B}_t \\ \Gamma_{22} \ \mathcal{B}_t \\ \Gamma_{32} \ \mathcal{B}_t \end{bmatrix} dT$$

The implied signs are

$$dP_i < 0, \quad dP_c > 0, \quad dr > 0 \ .$$

Consequently, the effects of a rise in autonomous taxes is to raise the price of capital goods and reduce the price of consumption goods and interest rate.

d) A rise in government expenditures

$$\begin{bmatrix} dP_i \\ dP_c \\ dr \end{bmatrix} = \begin{bmatrix} \Gamma_{12} & B_g \\ \Gamma_{22} & B_g \\ \Gamma_{32} & B_g \end{bmatrix} dG \qquad dG > 0 \ .$$

Signs of the changes in endogenous variables are

$$dP_i < 0, \quad dP_c > 0, \quad dr > 0 \ .$$

Then we can say that a rise in government expenditures tends to reduce the price of capital goods, whereas it tends to raise the price of consumption goods and interest rate.

e) A rise in money supply

A rise in money supply brought about by open market operations tends to shift both the excess demand curve for real balances and the excess demand curve for consumption goods, resulting in ambiguous expressions for endogenous variables. To overcome this ambiguity it will be assumed that whenever government performs on open market operation, the autonomous taxes will be changed simultaneously so as to cancel the effects of open market operations upon the excess demand for consumption goods. Consequently, we can write

$$\begin{bmatrix} dP_i \\ dP_c \\ dr \end{bmatrix} = \begin{bmatrix} \Gamma_{13} C_m \\ \Gamma_{23} C_m \\ \Gamma_{33} C_m \end{bmatrix} dM$$

The implied signs are

$$dP_i > 0, \quad dP_c < 0, \quad dr < 0 .$$

Hence, a rise in money supply tends to increase the price of capital goods and reduce the price of consumption goods and interest rate.

Implications Regarding Employment

In order to study the employment implications of monetary and fiscal policies, recall equations [28a] and [28b]

$$N_c = N_c(W,P_c,K_c)$$

$$N_i = N_i(W,P_i,\lambda,K_i) .$$

Notice that employment in consumption goods sector depends only on the price of consumption goods. Remembering that $N_{cp} > 0$ we can conclude that all those policies which result in a higher price of consumption. Therefore, a rise in government expenditures tends to increase the employment in consumption goods sector, whereas a rise in money supply tends to reduce employment in consumption goods sector.

If, on the other hand, we focus our attention on employment in capital goods sector, we must proceed as follows:

$$dN_i = N_{ip}dP_i + N_{i\lambda}\lambda_{p_i}dP_i + N_{i\lambda}\lambda_{p_c}dP_c + N_{i\lambda}\lambda rdr$$

$$= (N_{ip} + N_{i\lambda}\lambda_{pi})dP_i + N_{i\lambda}(\lambda_{p_c}dP_c + \lambda rdr)$$

$$= (N_{ip} + N_{ip}\frac{P_i}{\lambda}\lambda_{p_i})dP_i + N_{ip}\frac{P_i}{\lambda}(\lambda P_c dP_c + \lambda rdr)$$

$$= N_{ip}(1 + P_i\frac{P_i}{\lambda})dP_i + N_{ip}(P_1\frac{\lambda p_c}{\lambda}dP_c + P_1\frac{\lambda r}{\lambda}dr)$$

$$= N_{ip}[(1 + p_i\frac{\lambda P_i}{\lambda})dP_i + (P_i\frac{\lambda P_c}{\lambda}dP_c + P_i\frac{\lambda r}{\lambda}dr)] .$$

Hence, the change in the level of employment in capital goods sector is ambiguous. As $N_{ip} > 0$, then the employment will go up if the term in brackets is positive and down if negative. Consequently, the effects of both the fiscal policies and monetary policies upon employment in capital goods sector are unclear. Notice that as P_c and P_i move in opposite directions in response to monetary and fiscal policies, a sufficient condition for being able to assert the sign of dN_i is

$$\left|\frac{dP_i}{P_i}\right| > \left|\frac{dr}{r-\pi_c}\right| .$$

If this is so, then a rise in government expenditures results in a fall in employment in capital goods sector, whereas a rise in money supply tends to increase the employment in capital goods sector.

IV. Conclusions

This paper should be regarded as a first attempt to incorporate monopolies in product markets to a two-sector macroeconomic model in a meaningful way. Under a broad set of assumptions, previously stated, an economically meaningful two-sector macro model can be constructed whose results are significantly different from those of neoclassical and Keynesian two-sector models.

A very useful result of this paper is that it helps to distinguish between monetary and fiscal policies with respect to their structural effects. In particular, fiscal policy tends to reallocate resources, in the short run, in favor of consumption goods. This conclusion is based on the result that a rise in government expenditures causes the employment in consumption goods sector to go up and employment in capital goods sector to go down. On the other hand, a rise in money supply causes employment in consumption goods sector to go down and employment in capital goods sector to go up. In view of the fact that monopolies are realities of life, the model of this paper serves a useful task by magnifying this reallocational aspects of monetary and fiscal policies.

Comparing these results with those of Keynesian and Neoclassical models; it is well known that standard results of Keynesian models involve a rise in employment in both sectors of the economy in response to a rise in government expenditures. The results of Neoclassical models, on the other hand, involve a rise in employment in both sectors in response to a rise in money supply. The effect of a rise in money supply in Keynesian models upon employment is either ambiguous or tends to increase it. In a similar manner a standard result in Neoclassical models is the ambiguity of the effect of a rise in government expenditures upon employment.

Another conclusion that is in order is the fact that if one assumes that elasticity of demand for capital goods is a constant, then the resulting model becomes virtually identical to that of a two-sector Keynesian model. To see the basis of this conclusion, notice that if ε is constant, then equations [26] and [27] drop from the model. However, equations [18]-[25] constitute a complete Keynesian model with perfect competition in all markets. The only difference is the term λ in equation [21], which, in turn, falls upon differentiation.

Finally, I would like to express my belief that more research on macroeconomic models postulating market imperfections may prove to be extremely valuable in expanding our understanding of economics.

Footnotes

1/ For example, see Bailey [1], Foley and Sidrauski [2], Henderson and Sargent [3].

2/ For the relevance of stock markets in existing stocks of physical capital see Foley and Sidrauski [2], Sargent and Wallace [4].

3/ For a detailed discussion of this see Wallace [5].

References

[1] Martin J. Bailey, National Income and the Price Level: A Study in Macroeconomic Theory, 2nd edition, New York, 1971.

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