Historical

How to Use Econometric Models to Forecast

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I. Introduction

The purpose of this paper is to describe the procedures followed by the Research Department of the Federal Reserve Bank of Minneapolis in producing a forecast of national economic activity with the aid of a large econometric model.[1/] We produce such a forecast once a month, and this forecast forms the foundation for monetary policy discussion within the Research Department. This paper should be viewed as a "how-to-do-it" manual, and it is not a theoretical discussion of econometric models. It is intended to be a documentation of our procedures so that our staff can produce our monthly forecast in an efficient and systematic manner. Furthermore, since reproducibility is one desirable characteristic of scientific experiments, it is hoped that this documentation will bring us closer to the point where a forecast made by one of our staff members can be reproduced by any other staff member.

Reproducibility of experiments is a valid scientific criterion per se because it allows learning from past experience. When a correct forecast is made, was it due to good luck or did the forecaster really know what he was doing, i.e., under similar conditions could the forecaster duplicate the quality of that forecast? Similarly, when an incorrect forecast is made, was the error due to something completely beyond the forecaster's control or state of knowledge, or does the error help the forecaster isolate a problem in his procedure so that better forecasts

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[1/] For an excellent discussion of econometric models, see Introduction to the Use of Econometric Models in Economic Policy Making, Pres[illegible] Miller and Ronald Kaatz, Federal Reserve Bank of Minneapolis (May, 1974).

can be made in the future? In other words, without a systematic forecasting procedure, there is little information that can be extracted from forecasting errors--there is almost no learning by doing.

The body of the paper will focus primarily on procedures which are applicable to any econometric model so that it may be of some interest to forecasters outside our own staff. The Appendix contains some examples of procedures which we followed in modeling certain kinds of experiments where the details relate to the particular econometric model that we use, viz, a version of the Federal Reserve-MIT-PENN (FMP) Model.

At the most abstract level, an econometric model should not require any judgmental intervention on the part of the forecaster. The model is designed to reproduce economic history in a reasonable way, and the forecaster's role should largely be one of observing the outcome produced by the model when it is subjected to the stimuli which are expected to prevail in the future. However, as every economist is well aware, the present state of econometrics and economic theory is such that our models are incapable of reproducing the real world in an entirely acceptable manner. Therefore, users are faced with the need to manage their econometric models when they are used for forecasting.

Aside from theoretical inadequacies, there are additional reasons why econometric forecasting models must be managed in the sense of direct intervention, which causes the model to produce a result that is different from that which is implied by historical statistical experience alone. These considerations fall into two broad categories. First of all, there frequently occur events of major economic significance which represent a fundamental change from the conditions under which data were generated during the historical period. Since such events did not occur in the period of estimation of the econometric model, there is little reason to believe that the model will respond correctly to this new stimuli. Thus, the forecaster is faced with the task of making the model respond in what he believes to be the correct way. Recent examples of this kind of event include the oil embargo, the Nixon price freeze, a major public employment program, and the income tax rebate of 1975. The Appendix contains examples of how we managed the FMP Model to deal with some of these issues.

A second reason for managing an econometric model derives from the fact that, to the author's knowledge, all of the econometric models currently being used to regularly forecast the national economy are based on quarterly data. This means that the forecaster is faced with the task of somehow using the vast amount of subquarterly data that becomes available on virtually a continuous basis. Incorporating this information flow into the quarterly model forecast is the primary focus of the body of this paper.

II. First-Period Adds

The standard way of incorporating current information flows into a quarterly model forecast is to make additive adjustments (adds) to the structural equations of the model. If the equation

(i) $y_t = a + bx_t$

represents an actual structural equation of the model, then, for forecasting purposes, the equation will be coded into the computer in the form

(ii) $y_t = a + bx_t + A_t$

where the A_t are the add factors which must be supplied to the model and the computer by the forecaster or manager of the model. Thus, it is clear that additive adjustments amount to changing the intercept term of the equation while leaving the slope coefficients unaltered. By leaving the slope coefficients unchanged, the forecaster is not changing the implications of the estimated model with regard to alternative policy assumptions (this statement is exactly true only for strictly linear models). The adds, then, are the primary way that the informed judgment of the forecaster gets quantified into the forecast of the econometric model.

The forecaster's judgment with respect to the current level of the adds is influenced by a vast amount of actual data and opinion generated by the media. But the users of econometric models frequently begin with a set of data generated by the model itself, which is known as a <u>residuals check</u>. The residuals check is a listing of the past errors of each structural equation in the model. In the context of equation (ii), the residuals are computed according to the formula

$$e_t = y_t - a - bx_t$$

where both y and x take on their actual values in all the historical periods. In other words, the e_t's measure the errors that the particular equation has made when all the actual data were known.

By examining the residuals check, which is the listing of all the e_t's, the forecaster can see the recent historical performance of each equation and come to some decision on the likely value that e_t will take on in the first forecast period. For example, if y represents consumer purchases and the past six e_t's have all been around $4 billion, then the forecaster might infer that the estimated equation is consistently underpredicting consumer purchases by \$4 billion and, therefore, make the add in the first forecast period \$4 billion.

In addition to the residuals check, the forecaster will also [illegible] the subquarterly data which has become available since the last forecast was made. Monthly data such as unemployment, inventories, and prices will all influence the forecaster's judgment of the likely outcome of the quarterly variables, and, hence, his judgment about the proper setting of the current quarter adds. Similarly, daily movements in the stock market and interest rates may affect the forecaster's judgment not only of the current quarter, but also about future quarters as well.[2/] Thus, the subquarterly data may help the forecaster decide if the apparent errors in the model predictions are purely random errors or represent a structural change in the model.

Each forecaster may have his own way of quantifying the subquarterly data into the quarterly forecast. However, we have found the employment data produced by the household and establishment surveys to be especially useful and amenable to quantification because of the particular construction of the FMP Model. The following example shows the procedure which we follow to force the FMP Model to produce a forecast of the labor sector which is consistent with the known monthly labor market data.

[2/] For a more detailed procedure for using monthly data to predict current quarter residuals see: Paul Anderson and Tom Supel, "Augmenting Quarterly Econometric Forecasts by the Use of Within Quarter Data," Federal Reserve Bank of Minneapolis, Working Paper #39 (October 1975).

Model Adjustments to the Labor Sector

The labor sector of the FMP Model may be viewed as consisting of three behavioral relations, for LMHT, LH, and LF + LA, and seven identities. Focusing on the variables for which we receive monthly information via the household and establishment surveys, and suppressing those variables for which the feedback due to additive adjustments is minor, we may write the system as:

(1) $\ln LMHT = C_1 + f_1(XBNF) + g_1(ULU) + A_1$

(2) $\ln LH = C_2 + f_2(LMHT) + g_2(LE) + A_2$

(3) $LF + LA = C_3 + f_3(LE) + A_3$

(4) $\ln LMHT = \ln LEB - \ln LH$

(5) $LEB = LE - LG$

(6) $\ln ULU = C_4 + \ln LU - \ln(LF+LA)$

(7) $LU = (LF + LA) - (LE \,\&\, LA)$

(8) $(LF + LA) = LF + LA$

(9) $(LE \,\&\, LA) = LE + LA$

(10) $\ln UR = C_4 + \ln LU - \ln LF$

where the variables are standard FMP notation,[3/] the f_i's and g_i's denote functions, the C_i's are parameters, and A_1, A_2, and A_3 are the

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[3/] The descriptions of the variables are as follows:

LMHT = man-hours in private domestic nonfarm business sector, including proprietor and unpaid family workers.
LH = total hours per man in nonfarm private domestic business and household sectors.
LF = civilian labor force.
LA = armed forces.
LE = total civilian employment.
LU = total unemployment.
ULU = unemployment rate of total labor force (including armed forces).
UR = standard definition of unemployment rate.
LEB = employment in private domestic nonfarm business sector, including proprietor and unpaid family workers.
LG = government employment plus the correction for the difference between the household and payroll surveys.
XBNF = real nonfarm business product and product of households.

additive adjustments (adds) to the behavioral equations. Equations (1), (2), and (3) are the behavioral equations, and the remaining seven are identities. It is possible to solve this system containing sixteen variables by selecting six of them to be exogenous. For purposes of within-quarter adjustments, we typically take as exogenous: (a) LA because it is exogenous to the entire model, (b) LE and LF because of the data provided by the household survey, (c) LG because the feedbacks to this variable are generally small given the magnitude of the additive adjustments that are typically made, (d) LH because the structure is such that either LH or LMHT must be exogenous, and LH appears easier to predict via the establishment survey than LMHT, and (e) XBNF because it must be, i.e., the only way XBNF could be endogenous to this system is if A_1 is exogenous; but the point of this exercise is to determine a value for A_1.

Given the exogenous variable assumptions a through e, the identities (equations 4-10) determine a consistent set of values for all the model economic variables. Call these the "desired" values which we want to be solution values of the entire model. The adds may now be determined on a single-equation basis from equations 1-3. This is done as follows: Let a superscript " * " denote the desired value of a variable. Then the desired solution value for (1) may be written as

$$\ln \mathrm{LMHT}^* = C_1 + f_1(\mathrm{XBNF}^*) + g_1(\mathrm{ULU}^*) + A_1.$$

And the model solution without any adds may be represented as

$$\ln \mathrm{LMHT} = C_1 + f_1(\mathrm{XBNF}) + g_1(\mathrm{ULU}).$$

Therefore, in terms of the desired values and the solution values we may write A_1 as

$$A_1 = \ln \frac{\mathrm{LMHT}^*}{\mathrm{LMHT}} - [f_1(\mathrm{XBNF}^*) - f_1(\mathrm{XBNF})] - [g_1(\mathrm{ULU}^*) - g_1(\mathrm{ULU})].$$

A bracketed term may be interpreted as the change in ln LMHT induced by a change in the solution value for that particular variable. For the most part, these terms may be computed exactly, but an approximation works quite well for the XBNF term, namely

$$f_1(XBNF^*) - f_1(XBNF) \approx 0.27 \frac{(XBNF^* - XBNF)}{XBNF} .$$

There remains the problem of establishing the change in output arising from the exogenous changes imposed on the labor sector. At this time we make only a crude approximation by assuming that the labor compensation rate (PL) is fixed, and infer the change in labor income (YL$) from the change in man-hours (LMHT) induced by the adds. Assume that the change in personal income is the same as the change in labor income. Since disposable income (YD$) is approximately 85 percent of personal income (at the margin), real disposable income, and hence the first round impact on consumption (CON) is easily computed. For small changes in man-hours, this usually serves as a decent approximation to the change in output (XBNF).[4/]

The other adds are derived in a similar way, and the computational formulae for all three adds are:

$$A_1 = \ln(\frac{LMHT^*}{LMHT}) - 0.6162 \ln(\frac{1 - .01\ ULU^*}{1 - .01\ ULU}) - 0.27(\frac{XBNF^* - XBNF}{XBNF})$$

$$A_2 = \ln(\frac{LH^*}{LH}) - 0.2829 \ln(\frac{LMHT^*}{LMHT}) + 0.0508 \ln(\frac{LE^*}{LE})$$

$$A_3 = (LF^* - LF) - 0.183(LE^* - LE).$$

[4/] Note that this procedure makes XBNF dependent on the adds, and, hence, violates our assumption that XBNF is exogenous. We could just as well have put the above words in equation form and added these equations to our system, but we felt that this was an unnecessary complication and addition of equations.

These adds give only the first forecast period impact--paths are discussed in the following section.

III. Path Adds

Once the forecaster has decided on the additive adjustments to the structural equations for the first forecast quarter, he is then faced with the decision of how to carry the adds forward to each quarter of the entire forecast horizon. Again, a wide variety of information will affect this decision; but if the forecaster can settle on a fixed criterion for setting the path of the adds, the procedure is fairly mechanical, and hence reproducible.

One criterion that we use at times is that the path adds should produce a forecast from the second forecast quarter and on into the future which is the same as the model itself would produce if the actual residuals for the first forecast quarter were exactly the same as the first period adds. This criterion assumes, in effect, that subquarterly data are useful for establishing the current quarter values of the model variables, but that the future forecast should be the model's and not the manager's.

A criterion which we use at other times is that the path adds should be consistent with a permanent structural shift in the equation. In terms of equation (i) this is equivalent to assuming that the true value of the parameter a has permanently changed from its historically estimated average. When this criterion is imposed, it usually follows from careful inspection of the residuals check data to see if they show that the recent errors of the particular equation have consistently been the same order of magnitude or at least the same sign.

Some users of econometric models implement path adds by assuming that the error of the last period of actual data gets carried forward by a setting of the serial correlation coefficient to a value of unity. We, for the most part, try to avoid this procedure.

The actual path adds implied by these various criteria depend on the particular form of the structural equation. What follows is an algebraic description of how to compute path adds for the various algebraic forms of equations which are common to the FMP Model.

We will consider the proper way to make adds to an equation of the form

$$y_t = a_t + by_{t-1} + \rho u_{t-1} + e_t$$

where y is the endogenous variable of interest, a represents all other variables, ρ is the serial correlation coefficient, e is a disturbance term, b is the coefficient of the one-period own lag, and u_{t-1} is the lagged error where $u_t = y_t - a_t - by_{t-1}$. We ignore feedbacks to y from the other endogenous variables contained in a. It is helpful to consider the various special cases:

A. $y_t = a_t + e_t$

Setting $e_t = 0$, the forecast at time t=0 is given by

$y_1 = a_1$

$y_2 = a_2$.

And, generally

$$y_t = a_t$$

where y_t without any superscript denotes the control solution.

With an observed e_1, the forecast at time t=1 is

$$y_1^* = a_1 + e_1$$

$$y_2^* = a_2 .$$

And, generally

$$y_t^* = a_t \qquad , t > 1 .$$

When adds are appended to the equation in each period, the forecast path is given by

$$y_1' = a_1 + add_1$$

$$y_2' = a_2 + add_2 .$$

And, generally

$$y_t' = a_t + add_t .$$

In order to make the adds path correspond to the desired path (i.e., the path that would be generated by the model when the data for time t=1 are known) set $add_1 = e_1$ and all other adds are zero. And if e_1 is interpreted as structural change, all adds should be equal to e_1.

B. $$y_t = a_t + \rho u_{t-1} + e_t$$

When all data are known through time t=0, the forecast path of y is produced by setting $e_t \equiv 0$. The model "control" solution is then given by

$$y_1 = a_1 + \rho u_0 \qquad \text{where } u_0 = y_0 - a_0$$

$$y_2 = a_2 + \rho u_1 \qquad \text{where } u_1 = y_1 - a_1 = \rho u_0$$

$$= a_2 + \rho^2 u_0 .$$

And, generally

$$y_t = a_t + \rho^t u_0 .$$

Now suppose that, for reasons exogenous to the model, the value of e_1 is known. With the data at t=1 known, the model would produce a new sequence of forecast values (denoted by *) given by

$$y_1^* = a_1 + \rho u_0 + e_1$$

$$= y_1 + e_1$$

$$y_2^* = a_2 + \rho u_1^* \qquad \text{where } u_1^* = y_1^* - a_1 = \rho u_0 + e_1$$

$$= a_2 + \rho^2 u_0 + \rho e_1$$

$$= y_2 + \rho e_1 \; .$$

And, generally

$$y_t^* = a_t + \rho u_{t-1}^*$$

$$= y_t + \rho^{t-1} e_1 \; .$$

When forecasting from time t=0, knowledge of the data at time t=1 may be incorporated into the forecast by a sequence of constant adjustments (or "adds") to the control solution. This new sequence of forecast values (denoted by ') is given by

$$y_1' = a_1 + \rho u_0 + add_1$$

$$= y_1 + add_1$$

$$y_2' = a_2 + \rho u_1' + add_2 \qquad \text{where } u_1' = u_1 = \rho u_0$$

$$= a_2 + \rho^2 u_0 + add_2$$

$$= y_2 + add_2 \; .$$

At this point in the analysis, the way that the model solution is programmed becomes important. In our version, the u_i's are calculated for all future periods at the beginning of the simulation, and are not calculated anew for each different set of adds. In other words, u_1' is not calculated as $u_1' = y_1' - a_1 - \rho u_0 + add_1$

And, generally

$$y_t' = y_t + add_t .$$

In order to make the forecast sequence correspond to the desired sequence which incorporates the knowledge of the first period disturbance, pick the adds so that $y_t' = y_t^*$, i.e.,

$$y_t + add_t = y_t + \rho^{t-1} e_1$$

or

$$add_t = \rho^{t-1} e_1 .$$

At times e_1 may be interpreted as a structural shift (in the intercept) of the equation, i.e., $a_t^* = a_t + e_1$. Beginning at t=1, the desired forecast sequence is then

$$\begin{aligned} y_1^* &= a_1^* + \rho u_0 \\ &= a_1 + e_1 + \rho u_0 \\ &= y_1 + e_1 \end{aligned}$$

$$\begin{aligned} y_2^* &= a_2^* + \rho u_1^* \qquad \text{where } u_1^* = y_1^* - a_1^* = \rho u_0 \\ &= a_2 + e_1 + \rho^2 u_0 \\ &= y_2 + e_1 . \end{aligned}$$

And, generally

$$y_t^* = y_t + e_1 .$$

Using the above sequence of y's with adds, it is clear that the sequence of adds which represents a structural shift in the equation is given by

$$add_t = e_1.$$

On occasion the model is run with a ρ value set equal to one. This produces the sequence

$$y_1' = a_1 + u_0$$

$$= y_1 + (1-\rho_a)u_0 \quad \text{since } y_1 = a_1 + \rho_a u_0$$

where ρ_a denotes the actual rho used in the control solution.

$$y_2' = a_2 + u_1 \qquad \text{where } u_1 = u_0$$

$$= a_2 + u_0 + \rho_a^2 u_0 - \rho_a^2 u_0$$

$$= y_2 + (1-\rho_a^2)u_0 .$$

And, generally

$$y_t' = y_t + (1-\rho_a^t)u_0 .$$

Note that setting ρ=1 is equivalent to an adds path where the adds are given by

$$add_t = (1-\rho_a^t)u_0 .$$

In general, setting ρ=1 is not equivalent to the assumption of a structural change.

If one assumes that the new path must be parallel to the old one, i.e., $y_t' - y_t = y_1' - y_1$, then the adds are determined by

$$add_t = y_1' - y_1 = add_1 = e_1$$

which is equivalent to the structural change adds.

C. $y_t = a_t + by_{t-1} + e_t$

When all data are known through time $t=0$, and $e_t = 0$, the control solution forecast is given by

$$y_1 = a_1 + by_0$$

$$y_2 = a_2 + by_1 .$$

And, generally

$$y_t = a_t + by_{t-1} .$$

When there is reason to believe that e_1 is some known nonzero number, then starting from $t=1$, the forecast path becomes

$$y_1^* = a_1 + by_0 + e_1$$
$$= y_1 + e_1$$

$$y_2^* = a_2 + by_1^*$$
$$= a_2 + by_1 + be_1$$
$$= y_2 + be_1$$

$$y_3^* = a_3 + by_2^*$$
$$= a_3 + by_2 + b^2e_1$$
$$= y_3 + b^2e_1 .$$

And, generally

$$y_t^* = y_t + b^{t-1}e_1 .$$

The sequence of y's with adds is given by

$$y_1' = a_1 + by_0 + add_1$$
$$= y_1 + add_1$$

$$y_2' = a_2 + by_1' + add_2$$
$$= a_2 + by_1 + b\ add_1 + add_2$$
$$= y_2 + b\ add_1 + add_2$$

$$y_3' = a_3 + by_2' + add_3$$
$$= a_3 + by_2 + b^2\ add_1 + b\ add_2 + add_3$$
$$= y_3 + b^2\ add_1 + b\ add_2 + add_3 .$$

And, generally

$$y_t' = y_t + \sum_{i=1}^{t} b^{t-i}\ add_i .$$

To make the adds sequence consistent with the known data sequence, i.e., $y_t' = y_t^*$, set adds according to

$$y_1 + add_1 = y_1 + e_1$$

or $$add_1 = e_1 .$$

And

$$y_2 + b\ add_1 + add_2 = y_2 + be_1$$

or $$add_2 = 0 .$$

Similarly, all other adds are zero.

If e_1 is deemed a structural change so that $a_t^* = a_t + e_1$, then the equation generates the forecast sequence

$$y_1^* = a_1^* + by_0$$
$$= a_1 + e_1 + by_0$$
$$= y_1 + e_1$$

$$y_2^* = a_2^* + by_1^*$$
$$= a_2 + e_1 + by_1 + be_1$$
$$= y_2 + e_1 + be_1 .$$

and, generally

$$y_t^* = y_t + e_1 \sum_{i=0}^{t-1} b^i$$

The adds sequence is determined by

$$y_1 + add_1 = y_1 + e_1$$

or $$add_1 = e_1$$

And

$$y_2 + b\ add_1 + add_2 = y_2 + e_1 + be_1$$

or $$add_2 = e_1 .$$

Similarly, all adds are equal to e_1 .

If one applies the criterion that y' should be parallel to y, i.e., $y_t' - y_t = y_1' - y_1$ then the add sequence becomes

$$add_1 = y_1' - y_1$$

$$b\ add_1 + add_2 = y_2' - y_2 = add_1$$

$$add_2 = (1-b)\ add_1$$

$$b^2\ add_1 + b\ add_2 + add_3 = y_3' - y_3 = add_1$$

$$b^2\ add_1 + b(1-b)\ add_1 + add_3 = add_1$$

$$add_3 = (1-b)\ add_1 .$$

And, generally

$$add_t = (1-b)\ add_1 .$$

The general statement follows from the fact that since

$$y_t' - y_t = \sum_{i=1}^{t} b^{t-i}\ add_i$$

and $\quad y_{t-1}' - y_{t-1} = \sum_{i=1}^{t-1} b^{t-1-i} \, add_i$

we may write

$$y_t' - y_t = \sum_{i=1}^{t-1} b^{t-i} \, add_i + add_t$$

$$= b \sum_{i=1}^{t-1} b^{t-1-i} \, add_i + add_t$$

$$= b(y_{t-1}' - y_{t-1}) + add_t$$

$$= b \, add_1 + add_t \; .$$

Therefore, to have

$$b \, add_1 + add_t = add_1$$

we must have $\qquad add_t = (1-b) \, add_1 \; .$

D. $\qquad y_t = a_t + by_{t-1} + \rho u_{t-1} + e_t$

When all data are known through time t=0, the forecast path of y is generated by setting $e_t \equiv 0$. The model "control" solution is then given by

$$y_1 = a_1 + by_0 + \rho u_0 \qquad \text{where } u_0 = y_0 - a_0 - by_{-1}$$

$$y_2 = a_2 + by_1 + \rho u_1 \qquad \text{where } u_1 = y_1 - a_1 - by_0 = \rho u_0$$

$$= a_2 + by_1 + \rho^2 u_0 \; .$$

And, generally

$$y_t = a_t + by_{t-1} + \rho^t u_0 \; .$$

Now suppose that, because of intraquarter information flows, the value of e_1 is known. With this information, the model would produce a new sequence of forecast values as if the data were known through time t=1 (of course, this assumes that a_1 is known or, equivalently, that $_1$ is identified). Thus

$$y_1^* = a_1 + by_0 + \rho u_0 + e_1$$
$$= y_1 + e_1$$

$$y_2^* = a_2 + by_1^* + \rho u_1^* \qquad \text{where } u_1^* = y_1^* - a_1 - by_0 = \rho u_0 + e_1$$
$$= a_2 + by_1 + be_1 + \rho^2 u_0 + \rho e_1$$
$$= y_2 + be_1 + \rho e_1$$

$$y_3^* = a_3 + by_2^* + \rho u_2^* \qquad \text{where } u_2^* = y_2^* - a_2 - by_1^* = \rho^2 u_0 + \rho e_1$$
$$= a_3 + by_2 + b^2 e_1 + b\ e_1 + \rho^3 u_0 + \rho^2 e_1$$
$$= y_3 + b^2 e_1 + b\rho e_1 + \rho^2 e_1 \ .$$

And, generally

$$y_t^* = y_t + e_1 \sum_{i=0}^{t-1} b^{t-1-i} \rho^i \ .$$

When forecasting at t=0, a new sequence of y's may be generated which incorporates the intraquarter information via constant adjustments (or "adds"). This sequence is

$$y_1' = a_1 + by_0 + \rho u_0 + add_1$$
$$= y_1 + add_1$$

$$y_2' = a_2 + by_1' + \rho u_1' + add_2 \qquad \text{where } u_1' = u_1 = \rho u_0$$
$$= a_2 + by_1 + b\ add_1 + \rho^2 u_0 + add_2$$
$$= y_2 + b\ add_1 + add_2$$

$$y_3' = a_3 + by_2' + \rho u_2' + add_3 \qquad \text{where } u_2' = u_2 = \rho^2 u_0$$
$$= a_3 + by_2 + b^2 add_1 + b\ add_2 + \rho^3 u_0 + add_3$$
$$= y_3 + b^2\ add_1 + b\ add_2 + add_3 \ .$$

And, generally

$$y_t' = y_t + \sum_{i=1}^{t} b^{t-i} \text{ add}_i .$$

Note, as mentioned above, that the definition of u_i is determined by the particular solution routine of the model. In our particular instance, the only way that adds are carried forward is through own lags.

In order to make the forecast sequence correspond to the sequence that the model would produce if the data were known, set the adds so that $y_t' = y_t^*$, i.e.,

$$y_1 + \text{add}_1 = y_1 + e_1$$

or $$\text{add}_1 = e_1 .$$

And $$y_2 + b \text{ add}_1 + \text{add}_2 = y_2 + be_1 + \rho e_1$$

or $$\text{add}_2 = \rho e_1 .$$

And $$y_3 + b^2 \text{ add}_1 + b \text{ add}_2 + \text{add}_3 = y_3 + b^2 e_1 + b \rho e_1 + \rho^2 e_1$$

or $$\text{add}_3 = \rho^2 e_1$$

And, generally

$$\text{add}_t = \rho^{t-1} e_1 .$$

If e_1 is intepreted as a structural shift (in the intercept) of the equation so that $a_t^* = a_t + e_1$, then the model produces a new sequence as if it had a new intercept, i.e.,

$$y_1^* = a_1^* + by_0 + \rho u_0$$

$$= a_1 + e_1 + by_0 + \rho u_0$$

$$= y_1 + e_1$$

$$y_2^* = a_2^* + by_1^* + \rho u_1^* \qquad \text{where } u_1^* = y_1^* - a_1^* - by_0 = \rho u_0$$

$$= a_2 + e_1 + by_1 + be_1 + \rho^2 u_0$$

$$= y_2 + e_1 + be_1 \, .$$

And, generally

$$y_t^* = y_t + e_1 \sum_{i=0}^{t-1} b^i \, .$$

In order to make the adds consistent with the structural shift interpretation, they should be set according to

$$y_1 + add_1 = y_1 + e_1$$

or $$add_1 = e_1 \, .$$

And $$y_2 + b\,add_1 + add_2 = y_2 + e_1 + be_1$$

or $$add_2 = e_1 \, .$$

And, generally

$$add_t = e_1 \, .$$

If $\rho=1$ then

$$y_1' = a_1 + by_0 + u_0 + \rho_a u_0 - \rho_a u_0$$

$$= y_1 + (1-\rho_a)u_0$$

$$y_2' = a_2 + by_1' + u_1' \qquad \text{where } u_1' = u_0$$

$$= a_2 + by_1 + b(1-\rho_a)u_0 + u_0 + \rho_a^2 u_0 - \rho_a^2 u_0$$

$$= y_2 + b\bar{\rho}u_0 + (1-\rho_a^2)u_0 \qquad \text{where } \bar{\rho} = 1-\rho_a$$

$$y_3' = a_3 + by_2' + u_0$$

$$= a_3 + by_2 + b^2\bar{\rho}u_0 + b(1-\rho_a^2)u_0 + u_0 + \rho_a^3 u_0 - \rho_a^3 u_0$$

$$= y_3 + b^2\bar{\rho}u_0 + b(1-\rho_a^2)u_0 + (1-\rho_a^3)u_0 \, .$$

And, generally

$$y_t' = y_t + b^{t-1}\rho u_0 + u_0 \sum_{i=2}^{t} b^{t-i} (1-\rho_a^{\,i})$$

where ρ_a is the actual value of rho used in the control solution. Note that this forecast path is the same as the one that would be derived from a sequence of path adds given by

$$add_t = (1-\rho_a^{\,t})u_0 .$$

If the add criterion is to make $y_t' - y_t = y_1' - y_1 = add_1$, then

$$add_1 = y_1' - y_1$$

$$b\ add_1 + add_2 = y_2' - y_2 = add_1$$

$$add_2 = (1-b)\ add_1$$

$$b^2\ add_1 + b\ add_2 + add_3 = add_1$$

$$add_3 = (1-b)\ add_1 .$$

And, generally

$$add_t = (1-b)\ add_1 \qquad , t > 1 .$$

Table 1 summarizes the path adds. Note that all the adds are relative to the control run. The line denoted "$\rho=1$" should be interpreted as the adds to the control solution which are equivalent to setting $\rho=1$.

IV. Use and Evaluation

By following the procedures described in this paper, it is possible to produce a reasonably systematic forecast of future economic events. The value of such a forecast to any particular organization

Table 1

Equation Form

Interpretation of e_1	$y_t = a_t + e_t$	$y_t = a_t + \rho u_{t-1} + e_t$	$y_t = a_t + by_{t-1} + e_t$	$y_t = a_t + by_{t-1} + \rho u_{t-1} + e_t$
Random Disturbance	$add_1 = e_1$ $add_t = 0,\ t>1$	$add_t = \rho^{t-1} e_1$	$add_1 = e_1$ $add_t = 0,\ t>1$	$add_t = \rho^{t-1} e_1$
Structural Change	$add_t = e_1$	$add_t = e_1$	$add_t = e_1$	$add_t = e_1$
$\rho=1$	$add_t = u_0$	$add_t = (1-\rho_a{}^t)u_0$	$add_t = u_0$	$add_t = (1-\rho_a{}^t)u_0$
Parallel Shift	$add_t = e_1$	$add_t = e_1$	$add_1 = e_1$ $add_t = (1-b)e_1,\ t>1$	$add_1 = e_1$ $add_t = (1-b)e_1,\ t>1$

depends on two factors: (a) the quality of the forecast, and (b) the nature of the organization. Assuming that the quality of the forecast is good enough (this is discussed below), the forecast of national measures of economic activity may be of little use to a particular firm unless it is able to systematically relate its own business to national economic developments. Our primary concern, of course, is with the impact that changes in economic policy will have on inflation, output, and employment. Thus, we typically produce more than one forecast--our base forecast embodies our best judgment about the likely paths of monetary and fiscal policy, but we then produce forecasts with alternative monetary and fiscal assumptions so that the policy maker has some idea of the quantitative range of various policy decisions. It is in the context of producing alternative forecasts that using a formal model becomes very important to the forecaster. For it is this formal structure which makes the production of an internally consistent alternative forecast possible in a minimum amount of time.

Production of some kinds of alternatives involves simply changing the forecast path of a single exogenous variable. This would be the case for a change in the money supply path or in the level of federal government purchases, for example. Other kinds of alternatives might require alterations to the structure of the model, and thus would require the user of the model to understand the particular mathematical equations that were being used to generate the forecast. Examples of this type of alteration to the model include modeling of public employment programs and income tax rebates--our alterations to the FMP Model for these experiments are shown in the Appendix.

On the issue of quality of the forecast, we, unfortunately, have little to offer in the way of quantitatively evaluating the procedure described in this paper. There are a number of factors which can contribute to producing a poor forecast. The most obvious source of error is the accuracy of the assumption about the set of exogenous and policy variables. The projected paths of the money supply and the federal budget will significantly affect the forecast values of inflation and unemployment in virtually all econometric models in use today. A glaring example of the importance of a reasonably correct federal budget projection occurred in 1969 when most forecasters underpredicted the strength of the economy because the budget implications of the Vietnam escalation were grossly underestimated by the administration. Because of this dependence of the forecast on the policy assumptions, we refer to our forecast as a conditional forecast. A conditional forecast carries with it the implication that any evaluator of our forecast should consider the accuracy of the policy assumptions before rendering a final verdict on the overall quality of the forecast.

A more subtle, but scientifically more important, factor affecting the quality of the forecast is the quality of the econometric model that the forecaster is using. Establishing the quality of a particular model is a difficult task; however, the evidence that does exist does not speak well for existing models. A great deal of work has been done which indicates that naive models of a very simple mathematical form (e.g., the percent change next quarter will be the same as last quarter) can predict better than large econometric models. We tend to discount this sort of evidence a great deal on the grounds that the naive models tell us nothing about the process that determines the value of a particular variable, and it is this process which is important to the policy maker, especially when there is reason to be concerned about alternative policy choices.

Similarly, small models, i.e., models with few equations, are subject to the same criticism; but in addition, small models are conditional on a different set of information than a large model. That is, small models generally require a set of exogenous variables which encompass more information than the set of exogenous variables in a large econometric model.

The crucial test of whether or not a given econometric model captures the way the world really works is to test the predictions of the entire system of equations as a whole against the data that is not included in the estimation sample.[5/] The idea behind this kind of statistical test is that there exists a certain amount of randomness that is inherent in the world, and a model which properly quantifies this randomness may be a very good representation of the real world. The crucial test, then, is whether or not the prediction errors are consistent with the errors implied by the randomness quantified in the estimated model. Unfortunately, the implications of the systems tests also provide negative evidence about the quality of large econometric models as we know them today.

[5/] The only system test that the author is aware of is reported in "Tests For Structural Change and Prediction Intervals for the Reduced Forms of Two Structural Models of the U.S.: The FRB-MIT and Michigan Quarterly Models" by T. Muench, A. Rolnick, N. Wallace, and W. Weiler, Annals of Economic and Social Measurement (July 1974), pp. 491-519.

It is this kind of evidence which leads us to the conclusion that large econometric models need to be managed. We believe that following the procedures described in this paper can significantly improve the forecast accuracy of the models, and thereby render them useful devices not only for prediction but also for examining the implications of alternative policies and structural changes. The evidence in support of this assertion is as yet meager, but our experience has clearly shown that a managed econometric model can predict better than an unmanaged one. Whether or not this is good enough is the question that we will continue to explore.

V. Summary

In this document we have described the procedures currently being followed by the Research Department of the Federal Reserve Bank of Minneapolis to produce forecasts of national economic activity. While we recognize the fact that a reasonably accurate forecast of single aggregate variable, such as real gross national product, can probably be generated with a much simpler statistical device than a large econometric model, we believe that properly managed large models are extremely useful as the foundation for prediction. The state of economics as a science is really quite primitive, and we are a long way from being able to write down a mathematical description of the world which will generate a useful forecast of the future simply at the push of a button.

The models which exist today must be managed. And in managing and using these statistical descriptions of the world, we will hopefully discover ways of improving our understanding of how the economic agents in our economy react to various kinds of stimuli.

Finally, we note that while the purpose of this paper was to systematize the forecasting procedure, it by no means reduces forecasting to a mechanical process. We have given no criteria which lead to a clear decision when an add should be interpreted as a random disturbance or as a structural change. We have given no evidence that large econometric models such as the FMP Model are useful for analysis of alternative monetary and fiscal policies. These are among the issues which economists will continue to debate; meanwhile, there remains a large element of art in producing a quantitative forecast of economic activity.

Appendix

In this appendix we will describe, via specific examples, some of the ways that we manage the FMP Model. For readers who are using some other econometric model, the examples may at least suggest an approach to adapting any particular model to the phenomena considered.

1. The simplest type of change that we can make to the FMP Model is to change the path of a single exogenous variable. For forecasting and policy simulation purposes, we run the FMP Model with the money supply as an exogenous variable. (It should be noted, however, that the money demand function is estimated with the stock of money as the dependent variable.) Thus to simulate any money supply policy, we need only specify the desired quarterly growth rates (at annual rates), and the model will automatically translate this into a consistent set of output and interest rate variables.

2. In the FMP Model, the expenditure side of the federal budget is divided into seven distinct variables, all of which are exogenous except one, the level of payments for unemployment compensation. The projections of the six exogenous variables represent our best judgment of the likely budget based on the official administration budget and the actions of the Congress, and the level of unemployment compensation payments are determined by the level of unemployment which the model forecasts. Thus modeling any specific assumption about the level of federal expenditures is simply a matter of specifying a path of the six relevant variables so as to achieve the desired target of total expenditures. Producing the

desired level of total expenditures frequently involves more than one trial because of the feedback which the model generates on the level of unemployment compensation payments.

i. Modeling a federally enacted public employment program requires a number of model alterations because the legislation specifies both a level of expenditure as well as a number of persons to be employed in the program. For ease of calculation, suppose the program, such as the one enacted in December 1974, called for an expenditure of \$3.0 billion to employ 300,000 persons. This implies an average annual wage of \$10,000.

The first adjustment is to increase the grants component of federal expenditure by \$3.0 billion, assuming that this is entirely a net addition to the expenditure level in the base forecast. The increase in federal grants to state and local governments will automatically increase state and local expenditures because of the construction of the model equations in the state and local sector. However, there are three equations which define expenditures for the state and local sector—one for expenditures on wages and salaries, and two others. In order to channel all of the increase in grants into public sector employment, it is necessary to make an add to the wages and salaries equation to force the change in this variable to be exactly \$3.0 billion and to offset this add in the other equations so that their levels do not change even though grants have increased. Once the new level of state and local government expenditure on wages and salaries is established, the employment target is reached by adjusting the average wage of state and local government employees—in the FMP Model it is actually the reciprocal of the average wage that is an exogenous variable.

The adds to the structural equations describing the state and local sector must be calculated for each quarter of the forecast because the equations represent the historical experience where the expenditures of state and local governments do not adapt immediately to a change in federal grants. The equations also indicate that total state and local expenditures will not expand by the full amount of the change in federal grants, and this fact must be recognized in calculating the adds if it is assumed that the legislation will truly be effective, i.e., if it is assumed that state and local governments will make this net addition in employment and not simply substitute federal monies for local monies to pay their wage bills.

4. An income tax rebate, such as that of the second quarter of 1975, is an example of a model adjustment which doesn't require computation of path adds since it is simply a one-shot change in income. (Note that consumption expenditures over time will be affected by this income change, but that is not a problem which necessarily requires structural adds. The behavioral equations for consumption expenditures are estimated to automatically distribute the change in income over a long period of time.) Since the tax rebate of 1975 was distributed via checks from the government to persons, we chose to model the rebate as an increase in federal transfer payments. (Note that this is contrary to the official National Income Accounts which record the rebate as a negative receipt of personal income taxes.) Thus, our first adjustment was to increase

federal transfer payments in the second quarter of 1975 by $32.8 billion ($8.2 billion actual rebate at an annual rate). Because of the particular structure of the FMP Model, this adjustment increased personal [illegible] $32.8 billion; but disposable income was increased by only about 85 percent of this amount. We therefore made an add to the equation which computes personal tax liability from personal income in order to force all of the rebate into disposable income.

The increase in personal income caused receipts in three other tax equations to rise--OASI taxes, unemployment insurance taxes, and state and local taxes. Since we assumed that the entire rebate was nontaxable, we made offsetting adds to these three equations.