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VINTAGE HUMAN CAPITAL, GROWTH, AND STRUCTURAL UNEMPLOYMENT

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ABSTRACT. 'Structural unemployment' is said to occur in regions or 'sectors' of the economy as a consequence of technological changes. In this paper we present a model which provides an environment which gives rise to unemployment which could be labelled structural unemployment. There is exogenous technological change and vintage specific human capital. Unemployment arises as workers specialized in a particular technology within a vintage decide to search for a job within their vintage, so that their previously acquired special skills are used, instead of getting employed as unskilled workers in the newest vintage. As the rate of technological change increases, the incentives to reassign specialized workers to their same vintage, inccuring therefore in search costs, becomes less attractive, and in consequence the fraction of specialized workers doing search activities decreases. This provides some rationale for the negative correlation between rates of growth and unemployment observed in the data.

The Model.

Every N periods a new generation of 2N lived agents and a new set of production technologies appear. Agents are homogenous and have preferences over random streams of consumption $\{c_\tau\}_{\tau=1}^{2N}$ given by:

$$U(\{c_\tau\}_{\tau=1}^{2N})=E\sum_{\tau=1}^{2N}\beta^{\tau-1}c_\tau.$$

The set of technologies corresponding to generation or vintage t can be better described using the following, already familiar construct:

We will identify technologies within a vintage with 'islands' indexed by $s \epsilon S$, where S is the unit circle. Each vintage will be described by a uniform measure λ of islands on $(S, \mathcal{S})$ where $\mathcal{S}$ corresponds to the Borel σ-algebra on S. On each island the technology is given by a production function $\gamma^t f(N,Z)$, where N corresponds to the inputs of unskilled labor and Z to the total inputs of skilled labor where γ, the factor of technological change, is greater than one. We will assume that $f(N,Z)=N+\theta Z$ where $\theta>1$. After being N periods on an island, young people acquire a certain amount of skills that are specific to the vintage t and specialized in that type of island and hence can also be indexed by S.. These skills are useless on another vintage and costly adjustable to other type of island within the same vintage. More precisely, letting ρ be the euclidean distance in S, if a worker has acquired an amount m[1] of type s skills, his marginal productivity on an island of type s' is given by:

$$\gamma^t \theta z(\rho(s,s'),m),$$

where z is continous, strictly decreasing in its first argument, strictly increasing in the second argument, nonnegative and $z(0,m)=1$. The interpretation of z is that it transforms units of skilled labor of one type into units of another type. For notaational convenience, let

$$w(s,s',m)=\theta z(\rho(s,s',m)$$

The amount or quality of the skills acquired, denoted by m, will be assumed to take values in the unit interval and distributed according to

[1]The letter m stands for maleability and could be interpreted as the degree of maleability of the specialized skills acquired.

a fixed distribution υ, which will be denoted by the letter I. Hence if an island of type s' has a distribution μ of skilled workers, where μ is defined on the Borel space of S×I, then the total inputs of skilled workers Z are constrained by:

$$Z \leq \int w(s,s',m)\mu(ds,dm)$$

At the end of period N the existing islands recieve a shock $\varphi \in \{0,1\}$, assumed to be i.i.d. across islands. If the shock is 0 the island is no more productive and if the shock is 1 it will have the technology described above. This gives rise to the problem of how to allocate the experienced workers of the dissapearing islands. The skills acquired are not equally adaptable to the reamining islands. We will assume that the type of an island and of a worker, as well his level of skills are only mutually revealed when a worker 'visits' the island, and cannot be communicated in any other way[2]. This assumption, albeit extreme, reflects the fact that certain characteristics of specialized resources are costly to evaluate. We will assume that moving from one island to another takes one period. With this last assumption we have completed a 'search' environment.

-Insert a paragraph with reference to the literature on search and matching with emphasis on equilibrium search.

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[2]This is the spirit of the job matching models (see Jovanovic,).

Optimal allocation problem

The allocation problem for each generation can be divided in two parts. At period 1, young people have to be allocated to islands. Since the new technology is more productive than the old one, they will be assigned to the new vintage. Given the symmetry built into the environment -types indexed by the unit circle, islands uniformly distributed across types and productivity of match only depending on the distance of types- young people will be uniformly distributed across islands.

The second problem is the allocation of experienced workers. The surviving islands will keep their skilled labor force, so that the only problem is the allocation of workers from the ones that disappeared. The alternatives, at the beggining of period N+1, are to make the worker search for a new island within the current vintage or to transfer him to the newborn vintage as unskilled labor. In case the decision is to search, in the following period the alternatives will be to stay in the island visited, to continue doing search or go to the new vintage as unskilled. In case the decision is to continue the search, the same problem will be faced next period. For convenience we will assume that there is no recall in the search, e.g. that there is no memory.

Given the linearity of the technology, the allocation problem of each skilled worker can be treated separately. Hence consider a worker of generation t and type (s,m) from a disappearing island. If he had done search up to the next to the last period and the island currently visited where of type s', then the value of his alternatives would be:

i) If he stays in that island for the rest of his life he would produce $\gamma^t w(s,s',m)$ for two periods, giving a discounted total value of $\gamma^t(1+\beta)w(s,s',m)$.

ii) If he decides to work as unskilled worker in the new vintage for the rest of his life he would produce γ^{t+1} each period, giving a discounted total value of $\gamma^{t+1}(1+\beta)$.

iii) If he continues searching, and next period the island visited is of type s'' he would produce $\max\{\gamma^{t+1},\gamma^t w(s,s'',m)\}$, thus the expected discounted value of continue searching would be:

$$\gamma^t\beta\int\max\{w(s,s'',m),\gamma\}\lambda(ds'').$$

Denote by $\gamma^t V_{N-1}(s,s',m)$ the maximum value that could be achieved if a generation t type (s,m) agent were on an island of type s' at the next to the last period of his life, i.e. at period 2N-1. Then

$$\begin{aligned}\gamma^t V_{N-1}(s,s',m) &= \max\{\gamma^{t+1}(1+\beta),\gamma^t(1+\beta)w(s,s',m),\\ &\qquad \gamma^t\beta\int\max\{w(s,s'',m),\gamma\}\lambda(ds'')\}\\ &= \gamma^t\max\{\gamma(1+\beta),(1+\beta)w(s,s',m),\\ &\qquad \beta\int\max\{w(s,s'',m),\gamma\}\lambda(ds'')\}\end{aligned}$$

Let $\gamma^t V_{\tau+1}(s,s'',m)$ be the maximum total expected output of an agent of generation t and type (s,m) if he is at an island of type s'' at the beggining of period $N+\tau+1$ of his life. If an agent of type (s,m) is at an island of type s' at period $N+\tau$ of his life, then the value of his relevant alternatives are:

i) If he is assigned until the end of this life to the new vintage, the value of his output is $\gamma^{t+1}\sum_{j=0}^{N-\tau}\beta^j$.

ii) If he remains for the rest of his life in that island, the value is $\gamma^t w(s,s',m)\sum_{j=0}^{N-\tau}\beta^j$.

iii) If he continues searching the value is $\beta\int\gamma^t V_{t+1}(s,s'',m)\lambda(ds'')$.

Naturally, the agent will be assigned to the alternative of highest expected value. In consequence, the value of this agent will be

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\begin{aligned}
\gamma^t V_\tau(s,s',m) &= \max\{\gamma^{t+1}\sum_{j=0}^{N-\tau}\beta^j\ ,\ \gamma^t w(s,s',m)\sum_{j=0}^{N-\tau}\beta^j\ , \\
&\qquad \beta\int\gamma^t V_{t+1}(s,s'',m)\lambda(ds'')\} \\
&= \gamma^t\max\{\gamma\sum_{j=0}^{N-\tau}\beta^j\ ,\ w(s,s',m)\sum_{j=0}^{N-\tau}\beta^j\ , \\
&\qquad \beta\int V_{t+1}(s,s'',m)\lambda(ds'')\}
\end{aligned}
$$

The above defines recursively V_τ for $\tau=2,\ldots,N-1$ and the initial date is 2 since it takes at least one period for an agent of a disappeared island to visit a new island.

We can now state the allocation problem of an agent of generation t and type (s,m) of a disappeared island at the beggining of his 'skilled' life. The alternatives are:

i) Assign to the new vintage as unskilled worker, with a total discounted value of $\gamma^{t+1}\sum_{j=0}^{N-1}\beta^{j}$.

ii) Assign him to search activities with a total expected discounted value of $\beta\gamma^{t}\int V_2(s,s',m)\lambda(ds')$.

Again, this agent will be assigned to the alternative with higher value. His total value will then be

$$\begin{aligned}\gamma^{t}V_1(s,m) &= \max\ \{\gamma^{t+1}\sum_{j=0}^{N-1}\beta^{j},\ \beta\gamma^{t}\int V_2(s,s',m)\lambda(ds')\}\\ &= \gamma^{t}\max\ \{\gamma\sum_{j=0}^{N-1}\beta^{j},\ \beta\int V_2(s,s',m)\lambda(ds')\}.\end{aligned}$$

It can easily be seen that all the V functions are strictly increasing in m. Hence the optimal allocation problem will have the a solution of the following form:

In period N+1 the workers from disappearing islands with values of m above some cutoff point m^* will be assigned to search and the rest to the new vintage. In subsequent periods, the assignment decisions will depend on the outcome of the search process. Since the value of continuing the search is strictly increasing in m, at any τ and given the decision of rejecting an offer there will be a cutoff point $m^*(\tau)$ such that those with $m>m^*(\tau)$ will continue searching and the rest will be assigned to the new industry. The connection between the value of m and the acceptance or not of an offer depends on the particular form of the function w. It is straightforward to show that these decision rules will be independent of the generation considered.

Finally the cutoff points can be shown to be increasing in γ, the rate of technological change and also the rate of growth in this economy. In consequence, higher rates of growth would imply that more 'laid off' skilled workers are assigned to the new vintage and hence the fraction that remain in search activities decreases. Hence, this model is consistent with the observation of a negative correlation between rates of growth and rates of unemployment.