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TECHNICAL APPENDIX FOR LIQUIDITY EFFECTS, MONETARY POLICY, AND THE BUSINESS CYCLE

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This appendix describes, in detail, the computations done for the manuscript, "Liquidity Effects, Monetary Policy and the Business Cycle." The computations use that paper's first order conditions and market clearing conditions. The appendix starts by simply stating these. For a derivation and discussion, see the manuscript. Also, the MATLAB software used for the computations are available on request.

A. 14 Equations in 14 Unknowns

The first order conditions of the model are:

$$(N_t) \qquad E\{[\frac{u_{c,t}}{P_t} - \beta\frac{u_{c,t+1}}{P_{t+1}}R_t] \mid \Omega_t^0\} = 0$$

$$(L_{1t}) \qquad E\{[u_{L_1,t} + W_{1t}\frac{u_{c,t}}{P_t}] \mid \Omega_t^1\} = 0$$

$$(L_{2t}) \qquad u_{L_2,t} + W_{2,t}\frac{u_{c,t}}{P_t} = 0$$

$$(H_{1t}) \qquad E\{\frac{u_{c,t+1}}{P_{t+1}}[W_{1t}R_{1t} - P_t f_{H_1,t}] \mid \Omega_t^1\} = 0$$

$$(H_{2t}) \qquad W_{2t}R_{2t} - P_t f_{H_{2,t}} = 0$$

$$(K_{t+1}) \quad E\{[\frac{u_{c,t+1}}{P_{t+1}} - \beta\frac{u_{c,t+2}}{P_{t+2}}f_{K,t+1}\frac{P_{t+1}}{P_t}] \mid \Omega_t^1\} = 0$$

$$(N_{2t}) \qquad E\{[\frac{u_{c,t+1}}{P_{t+1}}(R_{1t} - R_{2t})] \mid \Omega_t^1\} = 0.$$

Here,

$$R_t = r + (1-r)[\omega(\Omega_t^1)R_{1t} + (1-\omega(\Omega_t^1))R_{2t}],$$

$$= r + (1-r)\omega(\Omega_t^1)(R_{1t} - R_{2t}) + (1-r)R_{2t}$$

is the return on the household's deposit with the financial intermediary. The parameter r denotes the reserve requirement, while $(1-r)\omega(\Omega_t^1)$ denotes the fraction of a deposit invested by the financial intermediary at return R_{1t}. That decision is assumed to be based on information set Ω_t^1.

Substituting,

$$E\{\frac{u_{c,t+1}}{P_{t+1}}R_t \mid \Omega_t^0\} = E\{\frac{u_{c,t+1}}{P_{t+1}}[r + (1-r)R_{2t}] \mid \Omega_t^0\}$$

$$+ (1-r)E\{\frac{u_{c,t+1}}{P_{t+1}}\omega(\Omega_t^1)(R_{1t}-R_{2t}) \mid \Omega_t^0\}.$$

From the law of iterated mathematical expectations,

$$E\{X \mid \Omega_t^0\} = E\{\, E[X \mid \Omega_t^1] \mid \Omega_t^0\}, \text{ since } \Omega_t^0 \subset \Omega_t^1.$$

Then,

$$E\{\frac{u_{c,t+1}}{P_{t+1}}\omega(\Omega_t^1)(R_{1t}-R_{2t}) \mid \Omega_t^0\} = E\{\omega(\Omega_t^1)E[\frac{u_{c,t+1}}{P_{t+1}}(R_{1t}-R_{2t}) \mid \Omega_t^1] \mid \Omega_t^0\} = 0,$$

by the N_{2t} FONC. Thus, $E\{\frac{u_{c,t+1}}{P_{t+1}}R_t \mid \Omega_t^0\} = E\{\frac{u_{c,t+1}}{P_{t+1}}[r + (1-r)R_{2t}] \mid \Omega_t^0\}$. Substituting this into the N_t FONC:

$$(N_t) \qquad E\{[\frac{u_{c,t}}{P_t} - \beta\frac{u_{c,t+1}}{P_{t+1}}(r + (1-r)R_{2t})] \mid \Omega_t^0\} = 0.$$

In addition to these 7 FONC's, we have 2 cash constraints and 5 market clearing conditions. The household's cash–in–advance constraint is:

(HC) $$P_tC_t = M_t - N_t + W_{1t}L_{1t} + W_{2t}L_{2t}.$$

The financial intermediary's reserve requirement (cash constraint) is:

(FC) $$N_{1t} + N_{2t} = (1-r)(N_t + X_t).$$

The goods market clearing condition is

(G) $$C_t + K_{t+1} = f(K_t, H_{1t}, H_{2t}, z_t).$$

The periods 1 and 2 loan market clearing conditions are:

(LO) $$W_{1t}H_{1t} = N_{1t},\ W_{2t}H_{2t} = N_{2t},$$

where $W_{1t}H_{1t}$ is period 1 loan demand, $W_{2t}H_{2t}$ is period 2 loan demand and N_{1t} and N_{2t} are the associated supplies.

Clearing in the periods 1 and 2 labor markets requires:

(LA) $$L_{1t} = H_{1t},\ L_{2t} = H_{2t}.$$

We seek 14 objects – 9 equilibrium decision rules: N_t, C_t, N_{1t}, N_{2t}, H_{1t}, H_{2t}, L_{1t}, L_{2t}, K_{t+1}, and 5 equilibrium price rules: P_t, W_{1t}, W_{2t}, R_{1t}, R_{2t}. We have 14 restrictions available for determining these objects: the 7 FONC's and the 2 cash constraints and 5 market clearing conditions. Not surprisingly, we have enough here to nail the objects we're after (a transversality condition will also turn out to be needed.)

Two things need to be done. Variables need to be scaled. Also, it is convenient to carry out some substitutions in order to whittle the problem down to finding four objects (equilibrium rules for N_t, L_{1t}, W_{1t}, K_{t+1}) subject to four constraints. First, the most straightforward simplification is obtained by using the two labor market clearing conditions to eliminate H_{1t} and H_{2t}. This gets us down to 12 restrictions and 12 unknowns.

Next, cut two more unknowns and restrictions by eliminating R_{1t} and R_{2t}, using the H_{1t} and H_{2t} FONC's. Use the H_{2t} FONC to substitute out for R_{2t} in the N_t and N_{2t} FONC's. Doing so, these FONC's become:

$$(N_t) \qquad E\{[\frac{u_{c,t}}{P_t} - \beta \frac{u_{c,t+1}}{P_{t+1}}(r + (1-r)\frac{P_t}{W_{2t}} f_{H_2,t})] \mid \Omega_t^0\} = 0$$

$$(N_{2t}) \qquad E\{[\frac{u_{c,t+1}}{P_{t+1}}(R_{1t} - \frac{P_t}{W_{2t}} f_{H_2,t})] \mid \Omega_t^1 \} = 0.$$

Substituting out for R_{1t} in the N_{2t} FONC from (H_{1t}),

$$(N_{2t}) \qquad E\{[\frac{u_{c,t+1}}{P_{t+1}} P_t (f_{H_1,t} - \frac{W_{1t}}{W_{2t}} f_{H_2,t})] \mid \Omega_t^1 \} = 0.$$

Here, we have used the fact that W_{1t} is nonstochastic, conditional on Ω_t^1.

Use the L_{2t} FONC to eliminate W_{2t} from the N_t and N_{2t} FONCs:

$$(N_t) \qquad E\{[\frac{u_{c,t}}{P_t} - \beta \frac{u_{c,t+1}}{P_{t+1}}(r - (1-r)\frac{u_{c,t}}{u_{L_2,t}} f_{H_2,t})] \mid \Omega_t^0\} = 0$$

$$(N_{2t}) \qquad E\{[\frac{u_{c,t+1}}{P_{t+1}} P_t (f_{H_1,t} + \frac{W_{1t}}{P_t}\frac{u_{c,t}}{u_{L_2,t}} f_{H_2,t})] \mid \Omega_t^1 \} = 0.$$

Combining the reserve requirement, (FC), and the loan market clearing condition, (LO):

(1) $$W_{1t}L_{1t} + W_{2t}L_{2t} = (1-r)(N_t + X_t).$$

After substituting out for W_{2t} from the L_{2t} FONC, and rewriting,

(2) $$L_{2t} = [W_{1t}L_{1t} - (1-r)(N_t + X_t)]\frac{u_{c,t}}{u_{L_2,t}P_t}.$$

Combining the two cash constraints and (LO), we get,

(3) $$P_t = \frac{M_t - N_t + (1-r)(N_t + X_t)}{C_t}$$

Thus far, we have deleted the L_{2t}, H_{1t}, H_{2t} FONC's and the LA restriction. The cash and market clearing conditions we are left with are (2), (3), (G), and (LO). Note that N_{1t} and N_{2t} appear nowhere in this system except in (LO). Thus, we drop N_{1t} and N_{2t} as unknowns and drop the two (LO) equations too. Thus, we have dropped 3 FONC's and 4 market clearing conditions, and 7 unknowns.

B. Scaling 7 Equations in 7 Unknowns

We are left with 7 objects to be determined – decision rules for N_t, C_t, L_{1t}, L_{2t}, K_{t+1} and price rules for W_{1t} and P_t. We have 7 restrictions – the (N_t), (N_{2t}), (L_{1t}) and (K_{t+1}) FONC's and the constraints, (2), (3) and (G). We reproduce these here for convenience.

$$(N_t) \qquad E\{[\frac{u_{c,t}}{P_t} - \beta\frac{u_{c,t+1}}{P_{t+1}}(r-(1-r)\frac{u_{c,t}}{u_{L_2,t}}f_{H_2,t})] \mid \Omega_t^0\} = 0$$

$$(L_{1t}) \qquad E\{[u_{L_1,t} + W_{1t}\frac{u_{c,t}}{P_t}] \mid \Omega_t^1\} = 0$$

$$(N_{2t}) \qquad E\{[\frac{u_{c,t+1}}{P_{t+1}}P_t(f_{H_1,t} + \frac{W_{1t}}{P_t}\frac{u_{c,t}}{u_{L_2,t}}f_{H_2,t})] \mid \Omega_t^1\} = 0$$

$$(K_{t+1}) \qquad E\{[\frac{u_{c,t+1}}{P_{t+1}} - \beta\frac{u_{c,t+2}}{P_{t+2}}f_{K,t+1}\frac{P_{t+1}}{P_t}] \mid \Omega_t^1\} = 0$$

$$(2) \qquad L_{2t} = [W_{1t}L_{1t} - (1-r)(N_t + X_t)]\frac{u_{c,t}}{u_{L_2,t}P_t}.$$

$$(3) \qquad P_t = \frac{M_t - N_t + (1-r)(N_t + X_t)}{C_t}.$$

$$(G) \qquad C_t + K_{t+1} = f(K_t, H_{1t}, H_{2t}, z_t).$$

Consider the following scaling of variables:

$$k_{t+1} = \exp(-\mu t)K_{t+1},\ c_t = \exp(-\mu t)C_t,\ w_{1t} = W_{1t}/M_t,$$

$$n_t = N_t/M_t,\ p_t = P_t\exp(\mu t)/M_t.$$

The production function is:

$$f(K_t,H_{1t},H_{2t},z_t) = K_t^{\alpha}[z_tH_t]^{(1-\alpha)} + (1-\delta)K_t,\ z_t = \exp[\mu t+\theta_t],$$

$$H_t = [(1-\nu)H_{1t}^{1/\rho} + \nu H_{2t}^{1/\rho}]^{\rho}.$$

Here, θ_t is an AR(1) time series process. Then,

$$f^{*}(k_t,H_{1t},H_{2t},\theta_t) \equiv \exp(-\mu t)f(K_t,H_{1t},H_{2t},z_t)$$

$$\exp(-\alpha\mu)k_t^{\alpha}[\exp(\theta_t)H_t]^{(1-\alpha)} + (1-\delta^{*})k_t,$$

where $1-\delta^{*} \equiv (1-\delta)\exp(-\mu)$. Then, it is easily confirmed that,

$$f^{*}_{k,t} = \exp(-\mu)f_{K,t},\ f^{*}_{H_i,t} = \exp(-\mu t)f_{H_i,t},\ i = 1,2.$$

The utility function is:

$$u(C_t,L_{1t},L_{2t}) = [C_t^{(1-\gamma)}v(L_{1t},L_{2t})^{\gamma}]^{\psi}/\psi,\ \ \psi \neq 0$$

$$= (1-\gamma)\log(C_t) + \gamma\log[v(L_{1t},L_{2t})],\ \psi = 0,$$

where,

$$v(L_{1t},L_{2t}) = 1 - L_{1t} - L_{2t}.$$

Then, it is readily confirmed that,

$$u_{c,t} \equiv u_c(C_t,L_{1t},L_{2t}) = u_c(c_t,L_{1t},L_{2t})\exp\{\mu t[(1-\gamma)\psi-1]\}$$

$$\equiv u^*_{c,t}\exp\{\mu t[(1-\gamma)\psi-1]\}$$

$$u_{L_i,t} \equiv u_{L_i}(C_t,L_{1t},L_{2t}) = u_{L_i}(c_t,L_{1t},L_{2t})\exp\{\mu t(1-\gamma)\psi\}$$

$$\equiv u^*_{L_i,t}\exp\{\mu t(1-\gamma)\psi\},\ i = 1,2.$$

Substituting the scaled variables into the FONC's:

$$(N_t) \qquad E\{[\frac{u^*_{c,t}}{p_t} - \beta^* \frac{u^*_{c,t+1}}{p_{t+1}(1+x_t)}(r-(1-r)\frac{u^*_{c,t}}{u^*_{L_2,t}} f^*_{H_2,t})] \mid \Omega^0_t\} = 0,$$

where $\beta^* \equiv \beta\exp\{\mu(1-\gamma)\psi\}$.

$$(L_{1t}) \qquad E\{[u^*_{L_1,t} + w_{1t}\frac{u^*_{c,t}}{p_t}] \mid \Omega^1_t\} = 0$$

$$(N_{2t}) \qquad E\{[\frac{u^*_{c,t+1}}{p_{t+1}(1+x_t)}p_t(f^*_{H_1,t} + \frac{w_{1t}}{p_t}\frac{u^*_{c,t}}{u^*_{L_2,t}} f^*_{H_2,t})] \mid \Omega^1_t\} = 0$$

$$(K_{t+1}) \quad E\{[\frac{u^*_{c,t+1}}{p_{t+1}(1+x_t)} - \beta^* \frac{u^*_{c,t+2}}{p_{t+2}(1+x_{t+1})} f^*_{k,t+1}\frac{p_{t+1}}{p_t}] \mid \Omega^1_t\} = 0$$

$$(G) \qquad c_t = f^*(k_t,H_{1t},H_{2t},\theta_t) - k_{t+1} \equiv c(k_t,k_{t+1},H_{1t},H_{2t},\theta_t).$$

(CIA) $$p_t = \frac{1 - n_t + (1-r)(n_t + x_t)}{c_t} = \frac{1 - n_t + (1-r)(n_t + x_t)}{c(k_t, k_{t+1}, H_{1t}, H_{2t}, \theta_t)}$$

$$\equiv p(k_t, k_{t+1}, H_{1t}, H_{2t}, n_t, x_t, \theta_t)$$

(2) $$L_{2t} = [w_{1t}L_{1t} - (1-r)(n_t+x_t)]\frac{u^*_{c,t}}{u^*_{L_2,t}p_t}$$

$$= -[w_{1t}L_{1t} - (1-r)(n_t+x_t)]\,\frac{1-\gamma}{\gamma}\frac{1 - L_{1t} - L_{2t}}{c_t p_t}$$

$$= \zeta_t[1 - L_{1t} - L_{2t}],$$

where,

$$\zeta_t \equiv -\frac{w_{1t}L_{1t} - (1-r)(n_t+x_t)}{1 - n_t + (1-r)(n_t+x_t)}\,\frac{1-\gamma}{\gamma}$$

Then,

(2) $$L_{2t} = L_2(w_{1t}, L_{1t}, n_t, x_t),$$

where

$$L_2(w_{1t}, L_{1t}, n_t, x_t) = [\zeta_t/(1 + \zeta_t)](1 - L_{1t}).$$

C. 4 Equations in 4 Unknowns.

After substituting out for c_t, p_t, L_{2t} in the scaled FONC's using c( ), p( ) and L_2( ),

$$q^N(k_t,k_{t+1},k_{t+2},H_{1t},H_{1t+1},w_{1t},w_{1t+1},n_t,n_{t+1},x_t,x_{t+1},\theta_t,\theta_{t+1})$$

$$\equiv [\frac{u^*_{c,t}}{p_t} - \beta^* \frac{u^*_{c,t+1}}{p_{t+1}(1+x_t)}(r-(1-r)\frac{u^*_{c,t}}{u^*_{L_2,t}} f^*_{H_2,t})]$$

$$q^{L_1}(k_t,k_{t+1},H_{1t},w_{1t},n_t,x_t,\theta_t) \equiv [u^*_{L_1,t} + \frac{w_{1t}}{p_t} u^*_{c,t}]$$

$$q^{N_2}(k_t,k_{t+1},k_{t+2},H_{1t},H_{1t+1},w_{1t},w_{1t+1},n_t,n_{t+1},x_t,x_{t+1},\theta_t,\theta_{t+1})$$

$$\equiv [\frac{u^*_{c,t+1}}{p_{t+1}(1+x_t)} p_t (f^*_{H_1,t} + \frac{w_{1t}}{p_t}\frac{u^*_{c,t}}{u^*_{L_2,t}} f^*_{H_2,t})]$$

$$q^k(k_t,k_{t+1},k_{t+2},k_{t+3},H_{1t},H_{1t+1},H_{1t+2},w_{1t},w_{1t+1},w_{1t+2},n_t,n_{t+1},n_{t+2},x_t,x_{t+1},x_{t+2},\theta_t,\theta_{t+1},\theta_{t+2})$$

$$\equiv [\frac{u^*_{c,t+1}}{p_{t+1}(1+x_t)} - \beta^* \frac{u^*_{c,t+2}}{p_{t+2}(1+x_{t+1})} f^*_{k,t+1} \frac{p_{t+1}}{p_t}]$$

Finally, the equations have been boiled down to the following four, which can be solved to find equilibrium rules for k_t, H_{1t}, w_{1t} and n_t:

$$E[q^{N}(k_t,k_{t+1},k_{t+2},H_{1t},H_{1t+1},w_{1t},w_{1t+1},n_t,n_{t+1},x_t,x_{t+1},\theta_t,\theta_{t+1})\,|\,\Omega_t^0] = 0$$

$$E[q^{L_1}(k_t,k_{t+1},H_{1t},w_{1t},n_t,x_t,\theta_t)\,|\,\Omega_t^1] = 0$$

$$E[q^{N_2}(k_t,k_{t+1},k_{t+2},H_{1t},H_{1t+1},w_{1t},w_{1t+1},n_t,n_{t+1},x_t,x_{t+1},\theta_t,\theta_{t+1})\,|\,\Omega_t^1]=0$$

$$E[q^{k}(k_t,k_{t+1},k_{t+2},k_{t+3},H_{1t},H_{1t+1},H_{1t+2},$$

$$w_{1t},w_{1t+1},w_{1t+2},n_t,n_{t+1},n_{t+2},x_t,x_{t+1},x_{t+2},\theta_t,\theta_{t+1},\theta_{t+2})\,|\,\Omega_t^1] = 0$$

To obtain an approximate solution to these equations, proceed in three steps.

(i) Replace q^N, q^{L_1},q^{N_2},q^k by their linear Taylor series expansion about the nonstochastic steady–state values of their arguments. The nonstochastic steady–state of θ_t and x_t is assumed to be the unconditional mean of these variables. Call these new functions Q^N, Q^{L_1},Q^{N_2},Q^k.

(ii) Posit linear equilibrium rules, $k_{t+1} = k(\Omega_t^1)$, $H_{1t} = H_1(\Omega_t^1)$, $w_{1t} = w_1(\Omega_t^1)$, $n_t = n(\Omega_t^0)$. Compute the coefficients of the linear functions $\tilde{N}$, $\tilde{L}_1$, $\tilde{N}_2$, $\tilde{k}$:

$$E[Q_t^N\,|\,\Omega_t^0] = \tilde{N}(\Omega_t^0),$$

$$E[Q_t^{L_1}|\,\Omega_t^1] = \tilde{L}_1(\Omega_t^1)$$

$$E[Q_t^{N_2}|\,\Omega_t^1] = \tilde{N}_2(\Omega_t^1)$$

$$E[Q_t^k\,|\,\Omega_t^1] = \tilde{k}(\Omega_t^1).$$

Here, Q_t^N denotes $Q^N(\ )$ with decision variables substituted out using the linear rules, k, H_1, w_1, n. The objects $Q_t^{L_1}$, $Q_t^{N_2}$, Q_t^k are defined analogously.

(iii) Find values for the (as yet) undetermined parameters in the rules k, H_1, w_1, n which set all parameters in $\tilde{N}$, $\tilde{L}_1$, $\tilde{N}_2$, and $\tilde{k}$ equal to zero. This defines a system with an equal number of equations and unknowns.

Note that the decision rules, $k(\Omega_t^1)$, $H_1(\Omega_t^1)$, $w_1(\Omega_t^1)$, $n(\Omega_t^0)$, are only approximations to the exact rules, since they satisfy only an approximation to the restrictions. It would be straightforward to improve on these decision rules by applying a method like that discussed by Judd, or Bizer–Judd and Coleman. In the following sections, we discuss the details of solving the model under alternative specifications of Ω_t^0, Ω_t^1. All methods require first finding the nonstochastic steady–state of the nonstochastic version of the model, a task to which we turn to first.

D. Nonstochastic Steady State.

In steady state, all time subscripted variables are constant. From the scaled K_{t+1} FONC, $1 = \beta^* f_k^*$. Solving this equation for k/H:

$$\frac{k}{H} = \exp(\theta)\left[\frac{\alpha\exp(-\alpha\mu)}{(\beta^*)^{-1} + \delta^* - 1}\right]^{1/(1-\alpha)}.$$

Suppose

$$\nu = 1/2.$$

Because of the definition of $v(L_{1t},L_{2t})$, $u^*_{L_1,t} = u^*_{L_2,t}$. Then the L_{1t} and N_{2t} FONC's imply

$$H_1 = H_2 = H,$$

where

$$H = [(1-\nu)H_1^{1/\rho} + \nu H_2^{1/\rho}]^\rho.$$

The N_t FONC implies:

$$1 = \frac{\beta^*}{1+x}[r + (1-r)\,\frac{1-\gamma}{\gamma}\,\frac{1-2H}{c}f^*_{H_2}].$$

But,

$$f^*_{H_2} = \frac{1}{2}f^*_H,\ c = H\xi,$$

where,

$$f_H^* = (1-\alpha)\exp(-\alpha\mu)(k/H)^{\alpha}\exp[(1-\alpha)\theta]$$

$$\xi = \frac{f_H^*}{1-\alpha} - \delta^* \frac{k}{H}.$$

Then,

$$H = \frac{1}{(2+\eta)}, \quad \eta = \frac{2\xi\gamma[(1+x)/\beta^* - r]}{(1-\gamma)(1-r)f_H^*}.$$

Next, go after n. Let w denote the steady state value of W_{1t}/M_t and W_{2t}/M_t:

$$w = -\frac{u_{L_1}^* p}{u_c^*} = \frac{\gamma}{1-\gamma}\frac{pc}{1-2H}$$

$$= \frac{\gamma}{1-\gamma}\frac{1-n+(1-r)(n+x)}{1-2H}.$$

Substituting this into $2wH = (1-r)(n+x)$, get

$$n = \frac{2H[1+(1-r)x] - (1-r)(1-2H)x(1-\gamma)/\gamma}{(1-\gamma)(1-r)(1-2H)/\gamma + 2Hr}.$$

These calculations made use of:

$$p = [1-n+(1-r)(n+x)]/c.$$

The period 2 real wage is W_{2t}/P_t, which, after detrending, is

$$w_{2t} \equiv \exp(-\mu t)\frac{W_{2,t}}{P_t} = \frac{-u^*_{L_2,t}}{u^*_{c,t}},$$

$$= \frac{\gamma}{1-\gamma}\frac{c}{1-2H},$$

in steady state. Then,

$$R_1 = R_2 = (1/2)f^*_H/w_2,$$

and

$$R = r + (1-r)R_2.$$

An alternative way to find R and R_2 works with the scaled representation of

$$E\{[\frac{u_{c,t}}{P_t} - \beta\frac{u_{c,t+1}}{P_{t+1}}R_t] \mid \Omega^0_t\} = 0.$$

After scaling, the steady state version of this is just

$$\frac{1+x}{\beta^*} = R,$$

so that,

$$R_2 = \frac{R - r}{1 - r}.$$

Consider the following quarterly parameterization:

$\delta = .02$, $\beta = 1.03^{-.25}$, $\mu = .004$, $r = .01$, $\gamma = 2/3$, $\alpha = .36$, $\theta = 0$, $\psi = -2$, $x = \beta^{*} - 1 \cong -.007$, $\rho = 10/9$.

Then,

$$2H = .299919,\ k/H = 39.804,\ R = R_1 = R_2 = 1.0.$$

When $x = .012$:

$$2H = .295277,\ k/H = 39.804,\ R = 1.0222\ (\cong 9.2\%\ \mathrm{AR})$$
$$R_1 = R_2 = 1.0225\ (\cong 9.3\%\ \mathrm{AR}).$$

Then, .299919/.295277 = 1.01572. Thus, an 9 percentage point drop in the inflation rate (AR), induced by an equal drop in the money growth rate, induces a 1.6 percent increase in employment. Roughly, a one percent drop in inflation (AR) induces a .178 percent increase in employment. The corresponding figure in the Cooley–Hansen model is a .5 percent increase in employment, and in the Fuerst model it's 2 percent.

Following is a simple procedure for estimating the parameters. First, set $\psi = 0$, $\beta = 1.03^{-.25}$, $\theta = 0$, $r = .005$ a priori. The capital first–order condition boils down to

$$\frac{u^{*}_{c,t}}{\beta u^{*}_{c,t+1}} = f_{K,t+1},$$

or,

$$\frac{\exp(\mu)}{\beta} = \alpha\frac{Y}{K} + 1 - \delta,$$

where Y/K is the steady state gross output–capital ratio. Then,

$$\alpha = \frac{K}{Y}\{ \frac{\exp(\mu)}{\beta} - (1-\delta) \}.$$

The average rate of depreciation rate on capital is .0212. That is, this is the sample average of

$$1 - \frac{K_{t+1} - I_t}{K_t}.$$

Also, μ is set to the sample average of per capita output, .0041. Christiano (Table 1, 1988) reports 10.59 as the sample average of the capital output ratio, thus

$$\alpha = .346 = 10.59\{ \frac{\exp(.0041)}{1.03^{-.25}} - (1 - .0212)\},$$

after rounding.

The parameter x was set to .0119, the sample average growth in the monetary base. Next, consider the utility function parameter, γ. The period 2 intratemporal first order condition says:

$$\frac{f^*_{H_2}}{R_2} = w_2 = -\frac{u^*_{L_2}}{u^*_c}.$$

But, $f^*_{H_2} = (1/2)f^*_H + (1/2)(1-\alpha)Y/H$. Then,

$$\frac{(1-\alpha)Y/(2H)}{R_2} = \frac{\gamma}{1-\gamma}\frac{c}{1-2H}.$$

Rearranging,

$$\gamma = \{ \frac{c}{Y}\frac{2H}{1 - 2H}\frac{R_2}{1 - \alpha} + 1 \}^{-1},$$

where,

$$R_2 = \frac{R - r}{1 - r},\ R = (1+x)/\beta.$$

In the formula for R we have used the assumption that $\psi = 0$. Otherwise R would be a function of γ and our estimator of γ would have involved solving a fixed-point problem.

Christiano (Table 1, 1988) reports a sample average for c/Y of .72. Substituting this and the other parameters into the formula for γ, we get

$$\gamma = .761$$

E. Convergence Properties of Nonstochastic Model.

In this section we set $\theta_t = \theta$, $x_t = x$ for all t, and study the properties of the remaining variables. We assume $q^N(\cdot)$, $q^{L_1}(\cdot)$, $q^{N_2}(\cdot)$, and $q^k(\cdot)$ have been linearized around steady state. We call these linearized functions $Q^N(\cdot)$, $Q^{L_1}(\cdot)$, $Q^{N_2}(\cdot)$, and $Q^k(\cdot)$, respectively. We establish that there exists a unique transition path which converges to steady–state in the linearized system.

To do this, we first establish $q_5^N = q_5^{N_2} = 0$, $q_i^{L_1} = 0$, i = 1, 2, 3, $q_i^k = 0$, i = 5, 6, 7. Here, a subscript i means the partial derivative with respect to the i^{th} argument, evaluated in steady–state. It is easily verified that

$$\frac{dH_{2t}}{dH_{1t}} = -1$$

in steady state, so that

$$\begin{aligned}\frac{dc_t}{dH_{1t}} &= c_{H_{1,t}} + c_{H_{2,t}} \frac{dH_{2t}}{dH_{1t}} \\ &= f^*_{H_{1,t}} - f^*_{H_{2,t}} \\ &= 0\end{aligned}$$

is steady state. It follows that

$$\frac{df^*_{k,t}}{d_{H_{1t}}} = \frac{du^*_{L_{1,t}}}{dH_{1t}} = \frac{du^*_{c,t}}{dH_{1t}} = \frac{dp_t}{dH_{1t}} = 0,$$

in steady state. It is then easily confirmed that $q_5^N = q_3^{L_1} = q_5^{N_2} = q_i^k = 0$, i = 5, 6, 7.

Consider $q_1^{L_1}$:

$$q_{1,t}^{L_1} = \{u_{cL_1,t}^{*} + \frac{w_{1t}}{p_t} u_{cc,t}^{*} + \frac{w_{1t}}{p_t^2} u_{c,t}^{*} \frac{p_t}{c_t}\} \frac{dc_t}{dk_t}.$$

Here, we have used the fact, $dp_t/dc_t = -p_t/c_t$.

Then,

$$q_{1,t}^{L_1} = \{u_{cL_1,t}^{*} + \frac{w_{1t}}{p_t} [u_{cc,t}^{*} + \frac{u_{c,t}^{*}}{c_t}]\} \frac{dc_t}{dk_t}$$

$$= \{u_{cL_1,t}^{*} - \frac{u_{L_1,t}^{*}}{u_{c,t}^{*}} [u_{cc,t}^{*} - \frac{u_{c,t}^{*}}{c_t}]\} \frac{dc_t}{dk_t},$$

by the first order condition for L_{1t}. It is easily confirmed that

$$u_{cc,t}^{*} = [(1-\gamma)\psi - 1] u_{c,t}^{*}/c_t$$

$$u_{cL_1,t}^{*} = -\gamma\psi u_{c,t}^{*}/(1 - H_{1t} - H_{2t}).$$

Simple substitution can be used to show that the term in braces defining $q_{1,t}^{L_1}$ is zero. Thus, $q_1^{L_1} = 0$. The result, $q_2^{L_1} = 0$, is derived similarly.

These derivative results allow us to simplify the linearized first order conditions, $Q^N(\cdot)$, $Q^{L_1}(\cdot)$, $Q^{N_2}(\cdot)$, $Q^k(\cdot)$. Use $Q^N(\cdot) = 0$ to solve for $\tilde{H}_{1t}$:

$$q_k^N \tilde{k}_t + q_{k'}^N \tilde{k}_{t+1} + q_{k''}^N \tilde{k}_{t+2} + q_w^N \tilde{w}_{1t} + q_{w'}^N \tilde{w}_{1t+1}$$

$$+ q_n^N \tilde{n}_t + q_{n'}^N \tilde{n}_{t+1} = -q_H^N \tilde{H}_{1t}.$$

Here, a time–subscripted variable with a tilde over it denotes derivation from steady state. Thus, $\tilde{k}_t \equiv k_t - k$. Substitute this into the Q^{N_2} equation:

$$(q_k^N - q_H^N q_k^{N_2})\tilde{k}_t + (q_{k'}^N - q_H^N q_{k'}^{N_2})\tilde{k}_{t+1} + (q_{k''}^N - q_H^N q_{k''}^{N_2})\tilde{k}_{t+2}$$

$$+ (q_w^N - q_H^N q_w^{N_2})\tilde{w}_{1t} + (q_{w'}^N - q_H^N q_{w'}^{N_2})\tilde{w}_{1t+1}$$

$$+ (q_n^N - q_H^N q_n^{N_2})\tilde{n}_t + (q_{n'}^N - q_H^N q_{n'}^{N_2})\tilde{n}_{t+1} = 0.$$

Solving for w_{1t} from the $Q^{L_1}(\cdot) = 0$ equation,

$$w_{1t} = -\frac{q_n^{L_1}}{q_w^{L_1}} n_t.$$

Substitute this into the $Q^k(\cdot) = 0$ equation and the modified Q^{N_2} equation:

$$(q_k^N - q_H^N q_k^{N_2})\tilde{k}_t + (q_{k'}^N - q_H^N q_{k'}^{N_2})\tilde{k}_{t+1} + (q_{k''}^N - q_H^N q_{k''}^{N_2})\tilde{k}_{t+2}$$

$$+ (q_n^N - q_H^N q_n^{N_2} - \frac{q_n^{L_1}}{q_w^{L_1}}[q_w^N - q_H^N q_w^{N_2})]\tilde{n}_t$$

$$+ (q_{n'}^N - q_H^N q_{n'}^{N_2} - \frac{q_n^{L_1}}{q_w^{L_1}}[q_{w'}^N - q_H^N q_{w'}^{N_2})]\tilde{n}_{t+1}.$$

$$q_k^k \tilde{k}_t + q_{k'}^k \tilde{k}_{t+1} + q_{k''}^k \tilde{k}_{t+2} + q_{k'''}^k \tilde{k}_{t+3} + (q_n^k - \frac{q_n^{L_1}}{q_w^{L_1}} q_w^k)\tilde{n}_t$$

$$+\left(q^k_{n'} - \frac{q_n^{L_1}}{q_w^{L_1}} q^k_{w'}\right)\tilde{n}_{t+1} + \left(q^k_{n''} - \frac{q_n^{L_1}}{q_w^{L_1}} q^k_{w''}\right)\tilde{n}_{t+2} = 0.$$

Now, lead the modified $Q^{N_2}(\cdot) = 0$ equation by one period and solve for $\tilde{n}_{t+2}$:

$$(q^n_k - q^N_H q_k^{N_2})\tilde{k}_{t+1} + (q^N_{k'} - q^N_H q_{k'}^{N_2})\tilde{k}_{t+2} + (q^N_{k''} - q^N_H q_{k''}^{N_2})\tilde{k}_{t+3}$$

$$+\left(q^N_n - q^N_H q_n^{N_2} - \frac{q_n^{L_1}}{q_w^{L_1}}[q^N_w - q^N_H q_w^{N_2}]\right)\tilde{n}_{t+1}$$

$$= -\left(q^N_{n'} - q^N_H q_{n'}^{N_2} - \frac{q_n^{L_1}}{q_w^{L_1}}[q^N_{w'} - q^N_H q_{w'}^{N_2}]\right)\tilde{n}_{t+2}.$$

Use this to substitute out for $\tilde{n}_{t+2}$ in $Q^k(\cdot) = 0$. Let,

$$a = -\left(q^N_{n'} - q^N_H q_{n'}^{N_2} - \frac{q_n^{L_1}}{q_w^{L_1}}[q^N_{w'} - q^N_H q_{w'}^{N_2}]\right)\tilde{n}_{t+2}$$

$$b = q^k_{n''} - \frac{q_n^{L_1}}{q_w^{L_1}} q^k_{w''}.$$

Then,

$$a q^k_k \tilde{k}_t + [b(q^N_k - q^N_H q_k^{N_2}) + a q^k_{k'}]\tilde{k}_{t+1} + [b(q^N_{k'} - q^N_H q_{k'}^{N_2}) + a q^k_{k''}]\tilde{k}_{t+2}$$

$$+ [b(q^N_{k''} - q^N_H q_{k''}^{N_2}) + a q^k_{k'''}]\tilde{k}_{t+3} + a\left(q^k_n - \frac{q_n^{L_1}}{q_w^{L_1}} q^k_w\right)\tilde{n}_t$$

$$+ \left[b\left(q^N_n - q^N_H q_n^{N_2} - \frac{q_n^{L_1}}{q_w^{L_1}}[q^N_w - q^N_H q_w^{N_2}]\right) + a\left(q^k_{n'} - \frac{q_n^{L_1}}{q_w^{L_1}} q^k_{w'}\right)\right]\tilde{n}_{t+1}.$$

After these substitutions, we are left with the following versions of $Q^N(\cdot) = 0$ and $Q^k(\cdot) = 0$:

$$\alpha_k \tilde{k}_t + \alpha_{k'} \tilde{k}_{t+1} + \alpha_{k''} \tilde{k}_{t+2} + \alpha_n \tilde{n}_t + \alpha_{n'} \tilde{n}_{t+1} = 0$$

$$\gamma_k \tilde{k}_t + \gamma_{k'} \tilde{k}_{t+1} + \gamma_{k''} \tilde{k}_{t+2} + \gamma_{k'''} \tilde{k}_{t+3} + \gamma_n \tilde{n}_t + \gamma_{n'} \tilde{n}_{t+1} = 0,$$

where

$$\gamma_k = a q^k_k$$

$$\gamma_{k'} = b(q^N_k - q^N_H q_k^{N_2}) + a q^k_{k'}$$

$$\gamma_{k''} = b(q^N_{k'} - q^N_H q_{k'}^{N_2}) + a q^k_{k''}$$

$$\gamma_{k'''} = b(q^N_{k''} - q^N_H q_{k''}^{N_2}) + a q^k_{k'''}$$

$$\gamma_n = a\left(q^k_n - q^N_H \frac{q_n^{L_1}}{q_w^{L_1}} q^k_w\right)$$

$$\gamma_{n'} = b\left(q_n^N - q_H^N q_n^{N_2} - \frac{q_n^{L_1}}{q_w^{L_1}}\left[q_w^N - q_H^N q_w^{N_2}\right]\right)$$

$$+ a\left(q_{n'}^k - \frac{q_n^{L_1}}{q_w^{L_1}} q_{w'}^k\right)$$

$$\alpha_k = q_k^N - q_H^N q_k^{N_2}$$

$$\alpha_{k'} = q_{k'}^N - q_H^N q_{k'}^{N_2}$$

$$\alpha_{k''} = q_{k''}^N - q_H^N q_{k''}^{N_2}$$

$$\alpha_n = q_n^N - q_H^N q_n^{N_2} - \frac{q_n^{L_1}}{q_w^{L_1}}\left[q_w^N - q_H^N q_w^{N_2}\right]$$

$$\alpha_{n'} = q_{n'}^N - q_H^N q_{n'}^{N_2} - \frac{q_n^{L_1}}{q_w^{L_1}}\left[q_{w'}^N - q_H^N q_{w'}^{N_2}\right].$$

Define:

$$X_t = \begin{bmatrix} \tilde{k}_t \\ \tilde{k}_{t+1} \\ \tilde{k}_{t+2} \\ \tilde{n}_t \end{bmatrix}.$$

Then, our two efficiency conditions can be written

$$AX_t + BX_{t+1} = 0,$$

$$A = \begin{bmatrix} \alpha_k & 0 & 0 & \alpha_n \\ \gamma_k & 0 & 0 & \gamma_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \alpha_{k'} & \alpha_{k''} & 0 & \alpha_{n'} \\ \gamma_{k'} & \gamma_{k''} & \gamma_{k'''} & \gamma_{n'} \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}.$$

It follows that

$$X_{t+j} = \Pi^j X_t,$$

where

$$\Pi = -B^{-1}A.$$

Write

$$\Pi = P \,\Lambda\, P^{-1}, \text{ so that}$$

$$P^{-1}X_{t+j} = \Lambda^j \, P^{-1}X_t,$$

or,

$$\begin{bmatrix} p_1X_{t+j} \\ p_2X_{t+j} \\ p_3X_{t+j} \\ p_4X_{t+j} \end{bmatrix} = \begin{bmatrix} \lambda_1^j p_1 X_t \\ \lambda_2^j p_2 X_t \\ \lambda_3^j p_3 X_t \\ \lambda_4^j p_4 X_t \end{bmatrix},$$

where

$$P^{-1} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix}$$

and p_i is 1×4, for i = 1, 2, 3, 4.

Now, three elements of X_t are undetermined: $\tilde{k}_{t+1}, \tilde{k}_{t+2}, \tilde{n}_t$. If $|\lambda_i| > 1$, i = 1, 2, 3, and $|\lambda_4| < 1$, then there is a unique way of selecting these so that $\{\tilde{k}_t\}$ and $\{\tilde{n}_t\}$ converge to zero (i.e., steady–state). This is defined by the condition:

$$p_1 x_t = 0$$
$$p_2 x_t = 0$$
$$p_3 x_t = 0.$$

For the parameter values used in the illustration in Section D,

$$\lambda_1 = 1.6 \times 10^8$$
$$\lambda_2 = -2.21$$
$$\lambda_3 = 1.0387$$
$$\lambda_4 = .9723.$$

Thus, the uniqueness condition is satisfied.

F. The Linearized Decision Rules.

This section describes the computation of the linearized decision rules under five specifications of Ω_t^0, Ω_t^1. The first, which we call the "limited information" specification, is the following:

$$\Omega_t^0 = \{k_t, \theta_{t-1}, x_{t-1}\}$$

$$\Omega_t^1 = \{k_t, \theta_{t-1}, \theta_t, x_{t-1}\}.$$

In the "full information" specification,

$$\Omega_t^0 = \Omega_t^1 = \{k_t, \theta_t, x_t\}.$$

In the "intermediate information" specification,

$$\Omega_t^0 = \Omega_t^1 = \{k_t, \theta_t, x_{t-1}\} = \tilde{\Omega}_t.$$

In the "Fuerst specification," we set

$$\Omega_t^0 = \{k_t, \theta_{t-1}, x_{t-1}\}$$

$$\Omega_t^1 = \{k_t, \theta_{t-1}, \theta_t, x_{t-1}, x_t\}.$$

Finally, we also work with the "sluggish saving" specification. This is like the Fuerst specification, with the single exception that $\tilde{k}_{t+1}$ is determined prior to observing x_t.

The basic structure of the decision rules is as follows:

$$\tilde{k}_{t+1} = k^0\tilde{k}_t + k^1\tilde{\theta}_{t-1} + k^2\tilde{\theta}_t + k^3\tilde{x}_{t-1} + k^4\tilde{x}_t$$

$$\tilde{H}_{1t} = H^0\tilde{k}_t + H^1\tilde{\theta}_{t-1} + H^2\tilde{\theta}_t + H^3\tilde{x}_{t-1} + H^4\tilde{x}_t$$

$$\tilde{w}_{1t} = w^0\tilde{k}_t + w^1\tilde{\theta}_{t-1} + w^2\tilde{\theta}_t + w^3\tilde{x}_{t-1} + w^4\tilde{x}_t$$

$$\tilde{n}_t = n^0\tilde{k}_t + n^1\tilde{\theta}_{t-1} + n^2\tilde{\theta}_t + n^3\tilde{x}_{t-1} + n^4\tilde{x}_t$$

All specifications have the same values for coefficients with superscript 0. The specifications differ in terms of the other coefficients. For example, in the full information specification, all coefficients with the superscript 1 and 3 are zero.

We can obtain more model specifications than were described simply by "mixing" them. In particular, our "basic sluggish saving" and "limited information employment" models both have the values of coefficients with superscrips 1 and 2 from the full information model. The basic sluggish sluggish saving model takes the coefficients for parameters with superscripts 3 and 4 from the sluggish saving model and the limited information employment model takes these coefficients from the limited information model.

We apply the undetermined coefficients method described in section C to each version of our model economy.

1. The Limited Information Economy.

It is convenient to write $k(\cdot)$, $H_1(\cdot)$, $w_1(\cdot)$ and $n(\cdot)$ out in detail. In particular,

$$\tilde{k}_{t+1} = k^0\tilde{k}_t + k^1\tilde{\theta}_{t-1} + k^2\tilde{\theta}_t + k^3\tilde{x}_{t-1}, \tag{3}$$

where k^i, i = 0, 1, 2, 3 denote the undetermined coefficients and a tilde over a variable denotes deviation from steady state, i.e., $\tilde{k}_t \equiv k_t - k$. It is useful to solve (3) forward recursively, and to make use of our assumption,

$$(4) \qquad E[\tilde{\theta}_t | \Omega_t^0] = \rho_\theta \tilde{\theta}_{t-1}, \ E[\tilde{x}_t | \Omega_t^1] = E[\tilde{x}_t | \Omega_t^0] = \rho_x \tilde{x}_{t-1}.$$

Then,

$$(5) \qquad E[\tilde{k}_{t+2} | \Omega_t^1] = (k^0)^2 \tilde{k}_t + k^0 k^1 \tilde{\theta}_{t-1} + [k^0 k^2 + k^1 + k^2 \rho_\theta] \tilde{\theta}_t + [k^0 k^3 + k^3 \rho_x] \tilde{x}_{t-1}.$$

$$(6) \qquad E[\tilde{k}_{t+3} | \Omega_t^1] = (k^0)^3 \tilde{k}_t + (k^0)^2 k^1 \tilde{\theta}_{t-1} + \left\{ k^0 [k^0 k^2 + k^1 + k^2 \rho_\theta] + k^1 \rho_\theta + k^2 \rho_\theta^2 \right\} \tilde{\theta}_t$$
$$+ \left\{ (k^0)^2 k^3 + \rho_x [k^0 k^3 + k^3 \rho_x] \right\} \tilde{x}_{t-1}.$$

Similarly,

$$(7) \qquad \tilde{H}_{1t} = H^0 \tilde{k}_t + H^1 \tilde{\theta}_{t-1} + H^2 \tilde{\theta}_t + H^3 \tilde{x}_{t-1}$$

$$(8) \qquad E[\tilde{H}_{1t+1} | \Omega_t^1] = H^0 k^0 \tilde{k}_t + H^0 k^1 \tilde{\theta}_{t-1} + [H^0 k^2 + H^1 + H^2 \rho_\theta] \tilde{\theta}_t +$$
$$[H^0 k^3 + \rho_x H^3] \tilde{x}_{t-1}.$$

$$(9) \qquad E[\tilde{H}_{1t+2} | \Omega_t^1] = H^0 (k^0)^2 \tilde{k}_t + H^0 k^1 k^0 \tilde{\theta}_{t-1}$$
$$+ \left\{ H^0 [k^0 k^2 + k^1 + k^2 \rho_\theta] + H^1 \rho_\theta + H^2 \rho_\theta^2 \right\} \tilde{\theta}_t$$
$$+ \left\{ H^0 k^0 k^3 + \rho_x [H^0 k^3 + H^3 \rho_x] \right\} \tilde{x}_{t-1}.$$

(10) $$\tilde{w}_{1t} = w^0\tilde{k}_t + w^1\tilde{\theta}_{t-1} + w^2\tilde{\theta}_t + w^3\tilde{x}_{t-1}.$$

(11) $$E[\tilde{w}_{1t+1}|\Omega_t^1] = w^0k^0\tilde{k}_t + w^0k^1\tilde{\theta}_{t-1} + [w^0k^2+w^1+w^2\rho_\theta]\tilde{\theta}_t$$
$$+ [w^0k^3+\rho_x w^3]\tilde{x}_{t-1}.$$

(12) $$E[\tilde{w}_{1t+2}|\Omega_t^1] = w^0(k^0)^2\tilde{k}_t + w^0k^0k^1\tilde{\theta}_{t-1}$$
$$+ \left\{w^0[k^0k^2+k^1+k^2\rho_\theta] + w^1\rho_\theta + w^2\rho_\theta^2\right\}\tilde{\theta}_t$$
$$+ \left\{w^0k^0k^3+\rho_x[w^0k^3+w^3\rho_x]\right\}\tilde{x}_{t-1}.$$

(13) $$\tilde{n}_t = n^0\tilde{k}_t + n^1\tilde{\theta}_{t-1} + n^3\tilde{x}_{t-1}.$$

(14) $$E\{\tilde{n}_{t+1}|\Omega_t^1] = n^0k^0\tilde{k}_t + n^0k^1\tilde{\theta}_{t-1} + (n^0k^2+n^1)\tilde{\theta}_t + (n^0k^3+\rho_x n^3)\tilde{x}_{t-1}.$$

(15) $$E[\tilde{n}_{t+2}|\Omega_t^1] = n^0(k^0)^2\tilde{k}_t + n^0k^0k^1\tilde{\theta}_{t-1}$$
$$+ \left\{n^0[k^0k^2+k^1+k^2\rho_\theta] + n^1\rho_\theta\right\}\tilde{\theta}_t$$
$$+ \left\{n^0k^0k^3+\rho_x[n^0k^3+n^3\rho_x]\right\}\tilde{x}_{t-1}.$$

We will also need the conditional expectations of the above objects based on Ω_t^0, i.e., $E[\tilde{n}_{t+2}|\Omega_t^0]$. These can be obtained in the obvious way, using the law of iterated mathematical expectations: $E[\tilde{n}_{t+2}|\Omega_t^0] = E\{\ E[n_{t+2}|\Omega_t^1]\ |\Omega_t^0\}$, so that

$$E[\tilde{n}_{t+2}|\Omega_t^0] = n^0(k^0)^2\tilde{k}_t + n^0k^0k^1\tilde{\theta}_{t-1}$$

$$+\left\{n^0[k^0k^2+k^1+k^2\rho_\theta]+n^1\rho_\theta\right\}\rho_\theta\tilde{\theta}_{t-1}$$

$$+\left\{n^0k^0k^3+\rho_x[n^0k^3+n^3\rho_x]\right\}\tilde{x}_{t-1}$$

In effect, we just replaced $\tilde{\theta}_t$ in the formula for $E[\tilde{n}_{t+2}|\Omega_t^1]$ by $\rho_\theta\tilde{\theta}_{t-1}$.

Recall (see step (i) of the undetermined coefficients method in section C) that we define the functions $Q^N(\,)$, $Q^{L_1}(\,)$, $Q^{N_2}(\,)$, $Q^K(\,)$ as the first order Taylor series expansions of $q^N(\,)$, $q^{L_1}(\,)$, $q^{N_2}(\,)$, $q^K(\,)$ about the nonstochastic steady state values of their arguments. (For a definition of these latter functions, see section C above.) The Taylor series expansions are written in detail as follows. We suppress an explicit statement of all the arguments of the functions in order to conserve on space.

(16) $$Q^N(\,)=q_k^N\tilde{k}_t+q_{k'}^N\tilde{k}_{t+1}+q_{k''}^N\tilde{k}_{t+2}+q_H^N\tilde{H}_{1t}+q_{H'}^N\tilde{H}_{1t+1}$$

$$+q_w^N\tilde{w}_{1t}+q_{w'}^N\tilde{w}_{1t+1}+q_n^N\tilde{n}_t+q_{n'}^N\tilde{n}_{t+1}$$

$$+q_x^N\tilde{x}_t+q_{x'}^N\tilde{x}_{t+1}+q_\theta^N\tilde{\theta}_t+q_{\theta'}^N\tilde{\theta}_{t+1}.$$

(17) $$Q^{L_1}(\,)=q_k^{L_1}\tilde{k}_t+q_{k'}^{L_1}\tilde{k}_{t+1}+q_H^{L_1}\tilde{H}_{1t}+q_w^{L_1}\tilde{w}_{1t}+q_n^{L_1}\tilde{n}_t+q_x^{L_1}\tilde{x}_t+q_\theta^{L_1}\tilde{\theta}_t.$$

(18) $$Q^{N_2}(\,)=q_k^{N_2}\tilde{k}_t+q_{k'}^{N_2}\tilde{k}_{t+1}+q_{k''}^{N_2}\tilde{k}_{t+2}+q_H^{N_2}\tilde{H}_{1t}+q_{H'}^{N_2}\tilde{H}_{1t+1}$$

$$+q_w^{N_2}\tilde{w}_{1t}+q_{w'}^{N_2}\tilde{w}_{1t+1}+q_n^{N_2}\tilde{n}_t+q_{n'}^{N_2}\tilde{n}_{t+1}$$

$$+q_x^{N_2}\tilde{x}_t+q_{x'}^{N_2}\tilde{x}_{t+1}+q_\theta^{N_2}\tilde{\theta}_t+q_{\theta'}^{N_2}\tilde{\theta}_{t+1}.$$

(19) $$Q^k(\,)=q_k^k\tilde{k}_t+q_{k'}^k\tilde{k}_{t+1}+q_{k''}^k\tilde{k}_{t+2}+q_{k'''}^k\tilde{k}_{t+3}$$

$$+ q_H^k \tilde{H}_{1t} + q_{H'}^k \tilde{H}_{1t+1} + q_{H''}^k \tilde{H}_{1t+2}$$

$$+ q_w^k \tilde{w}_{1t} + q_{w'}^k \tilde{w}_{1t+1} + q_{w''}^k \tilde{w}_{1t+2}$$

$$+ q_n^k \tilde{n}_t + q_{n'}^k \tilde{n}_{t+1} + q_{n''}^k \tilde{n}_{t+2}$$

$$+ q_x^k \tilde{x}_t + q_{x'}^k \tilde{x}_{t+1} + q_{x''}^k \tilde{x}_{t+2}$$

$$+ q_\theta^k \tilde{\theta}_t + q_{\theta'}^k \tilde{\theta}_{t+1} + q_{\theta''}^k \tilde{\theta}_{t+2}.$$

Since the coefficients in (16) – (19) are derivatives of the first order conditions, evaluated at nonstochastic steady state, they are functions of the model's parameters. We computed them by numerical differentiation methods, and they are treated as known from here on.

We now move to steps (ii) and (iii) of the undetermined coefficients procedure. In particular, write

(20) $$E[Q_t^N | \Omega_t^0] = Q_k^N \tilde{k}_t + Q_\theta^N \tilde{\theta}_{t-1} + Q_x^N \tilde{x}_{t-1}$$

(21) $$E[Q_t^{L_1} | \Omega_t^1] = Q_k^{L_1} \tilde{k}_t + Q_\theta^{L_1} \tilde{\theta}_{t-1} + Q_{\theta'}^{L_1} \tilde{\theta}_t + Q_x^{L_1} \tilde{x}_{t-1}$$

(23) $$E[Q_t^{N_2} | \Omega_t^1] = Q_k^{N_2} \tilde{k}_t + Q_\theta^{N_2} \tilde{\theta}_{t-1} + Q_{\theta'}^{N_2} \tilde{\theta}_t + Q_x^{N_2} \tilde{x}_{t-1}$$

(24) $$E[Q_t^k | \Omega_t^1] = Q_k^k \tilde{k}_t + Q_\theta^k \tilde{\theta}_{t-1} + Q_{\theta'}^k \tilde{\theta}_t + Q_x^k \tilde{x}_{t-1}.$$

Recall from the discussion of step (ii) that Q_t^N denotes the function $Q^N(\ )$, after the period t decisions have been substituted out using the linear decision rules. The objects $Q_t^{L_1}$, $Q_t^{N_2}$, Q_t^k are defined analogously. Thus, the coefficients to the right of the equalities in (20)–(24) are functions of the Taylor series expansion coefficients and the undetermined coefficients in the linear decision rules, (3), (7), (10), (13). The requirement that the conditional expectations, (20) – (24), be zero implies:

$$Q_k^N = Q_\theta^N = Q_x^N = 0$$

$$Q_k^{L_1} = Q_\theta^{L_1} = Q_{\theta'}^{L_1} = Q_x^{L_1} = 0$$

(25)

$$Q_k^{N_2} = Q_\theta^{N_2} = Q_{\theta'}^{N_2} = Q_x^{N_2} = 0$$

$$Q_k^k = Q_\theta^k = Q_{\theta'}^k = Q_x^k = 0,$$

Conditions (25) represents 15 equations to be solved for the 15 decision rule parameters. We proceed now to solve these equations. We exploit the fact that, conditional on k^0, they are linear in the remaining undetermined coefficients. Also, these equations are block recursive. This enables us to solve first for k^0, H^0, n^0 and w^0. Then we solve for k^3, H^3, w^3 and n^3. The remaining 7 parameters are then solved in a final block.

Substituting (3)–(15) into (16)–(19):

$$Q_k^N = q_k^N + q_{k'}^N k^0 + q_{k''}^N (k^0)^2 + q_H^N H^0 + q_{H'}^N H^0 k^0 + q_w^N w^0 + q_{w'}^N w^0 k^0 + q_n^N n^0 + q_{n'}^N n^0 k^0. \tag{25}$$

$$Q_k^{L_1} = q_k^{L_1} + q_{k'}^{L_1} k^0 + q_H^{L_1} H^0 + q_w^{L_1} w^0 + q_n^{L_1} n^0. \tag{26}$$

$$Q_k^{N_2} = q_k^{N_2} + q_{k'}^{N_2} k^0 + q_{k''}^{N_2} (k^0)^2 + q_H^{N_2} H^0 + q_{H'}^{N_2} H^0 k^0 + q_w^{N_2} w^0 + q_{w'}^{N_2} w^0 k^0 + q_n^{N_2} n^0 + q_{n'}^{N_2} n^0 k^0. \tag{27}$$

$$(28) \qquad Q_k^k = q_k^k + q_{k'}^k k^0 + q_{k''}^k (k^0)^2 + q_{k'''}^k (k^0)^3$$
$$+ q_H^k H^0 + q_{H'}^k H^0 k^0 + q_{H''}^k H^0 (k^0)^2$$
$$+ q_w^k w^0 + q_{w'}^k w^0 k^0 + q_{w''}^k w^0 (k^0)^2$$
$$+ q_n^k n^0 + q_{n'}^k n^0 k^0 + q_{n''}^k n^0 (k^0)^2.$$

Note that the only unknowns in (25) – (28) are k^0, H^0, n^0, and w^0. We use these equations to determine these four unknowns. Conditional on a value for k^0, the three equations $Q_k^N = Q_k^{L_1} = Q_k^{N_2} = 0$ determine H^0, n^0, w^0 as the solution to a system of linear equations. In particular, let

$$(29) \qquad z = \begin{bmatrix} q_k^N + q_{k'}^N k^0 + q_{k''}^N (k^0)^2 \\ q_k^{L_1} + q_{k'}^{L_1} k^0 \\ q_k^{N_2} + q_{k'}^{N_2} k^0 + q_{k''}^{N_2} (k^0)^2 \end{bmatrix}$$

$$(30) \qquad A = \begin{bmatrix} q_H^N + q_{H'}^N k^0 & q_w^N + q_{w'}^N k^0 & q_n^N + q_{n'}^N k^0 \\ q_H^{L_1} & q_w^{L_1} & q_n^{L_1} \\ q_H^{N_2} + q_{H'}^{N_2} k^0 & q_w^{L_1} + q_{w'}^{L_1} k^0 & q_n^N + q_{n'}^N k^0 \end{bmatrix}$$

Then, if $\gamma = [H^0\ w^0\ n^0]'$,

$$(31) \qquad A\gamma + z = 0,$$

which is easy to solve for γ. After substituting H^0, w^0, and n^0 expressed as functions of k^0 into (28), we have a function, $Q_k^k(k^0)$. The value of k^0 we seek satisfies the property, $Q_k^k(k^0) = 0$.

We studied the function $Q_k^k(\cdot)$ for the model parameter values used in the illustrative, steady–state calculations. We plotted it over the range $k^0 \in [-1,2]$ and noted that it appears quadratic and concave. Also, the function has exactly two zeros; $k^0 = 0.971335$ and $k^0 = 1.039765$. The product of β^* and the higher zero is 1.029. We rule it out on account of its large magnitude.

There are 11 remaining coefficients to be determined, and 11 conditions to do it with (e.g., all the expressions in (25), except those with a k subscript). It turns out that k^3, H^3, w^3, and n^3 can be determined from $Q_x^N = Q_x^{L_1} = Q_x^{N_2} = Q_x^k = 0$. Let

$$z = [k^3\ H^3\ w^3\ n^3]'.$$

Then,

$$(32) \qquad Q_x^N = q_{k'}^N k^3 + q_{k''}^N (k^0 k^3 + k^3 \rho_x) + q_H^N H^3 + q_{H'}^N (H^0 k^3 + H^3 \rho_x)$$

$$+ q_w^N w^3 + q_{w'}^N [w^0 k^3 + \rho_x w^3] + q_n^N n^3 + q_{n'}^N (n^0 k^3 + \rho_x n^3)$$

$$+ q_x^N \rho_x + q_{x'}^N \rho_x^2$$

$$= B_1 z + q_x^N \rho_x + q_{x'}^N \rho_x^2.$$

Let

$$B_{1,1} = q_{k'}^N + q_{k''}^N (k^0 + \rho_x) + q_{H'}^N H^0 + q_{w'}^N w^0 + q_{n'}^N n^0.$$

Then,

$$B_1 = [B_{1,1},\ q_H^N + q_{H'}^N \rho_x,\ q_w^N + q_{w'}^N \rho_x,\ q_n^N + q_{n'}^N \rho_x].$$

$$(33) \qquad Q_x^{L_1} = q_{k'}^{L_1} k^3 + q_H^{L_1} H^3 + q_w^{L_1} w^3 + q_n^{L_1} n^3 + q_x^{L_1} \rho_x$$

$$= B_2 z + q_x^{L_1} \rho_x.$$

$$B_2 = [q_{k'}^{L_1}, q_{H'}^{L_1}, q_{w'}^{L_1}, q_{n}^{L_1}].$$

(34) $$Q_x^{N_2} = q_{k'}^{N_2} k^3 + q_{k''}^{N_2}[k^0 k^3 + k^3 \rho_x] + q_H^{N_2} H^3 + q_{H'}^{N_2}[H^0 k^3 + \rho_x H^3]$$

$$+ q_w^{N_2} w^3 + q_{w'}^{N_2}[w^0 k^3 + \rho_x w^3] + q_n^{N_2} n^3 + q_{n'}^{N_2}[n^0 k^3 + \rho_x n^3]$$

$$+ (q_x^{N_2} + \rho_x q_{x'}^{N_2})\rho_x$$

$$= B_3 z + (q_x^{N_2} + \rho_x q_{x'}^{N_2})\rho_x.$$

$$B_{3,1} = q_{k'}^{N_2} + q_{k''}^{N_2}(k^0 + \rho_x) + q_{H'}^{N_2} H^0 + q_{w'}^{N_2} w^0 + q_{n'}^{N_2} n^0.$$

$$B_3 = [B_{3,1}, q_H^{N_2} + \rho_x q_{H'}^{N_2}, q_w^{N_2} + q_{w'}^{N_2} \rho_x, q_{n'}^{N_2} + q_{n'}^{N_2} \rho_x].$$

(35) $$Q_x^k = q_{k'}^k k^3 + q_{k''}^k [k^0 k^3 + k^3 \rho_x] + q_{k'''}^k \left\{(k^0)^2 k^3 + \rho_x [k^0 k^3 + k^3 \rho_x]\right\}$$

$$+ q_H^k H^3 + q_{H'}^k (H^0 k^3 + \rho_x H^3) + q_{H''}^k \left\{H^0 k^0 k^3 + \rho_x [H^0 k^3 + H^3 \rho_x]\right\}$$

$$+ q_w^k w^3 + q_{w'}^k [w^0 k^3 + \rho_x w^3] + q_{w''}^k \left\{w^0 k^0 k^3 + \rho_x [w^0 k^3 + w^3 \rho_x]\right\}$$

$$+ q_n^k n^3 + q_{n'}^k [n^0 k^3 + \rho_x n^3] + q_{n''}^k \left\{n^0 k^0 k^3 + \rho_x [n^0 k^3 + n^3 \rho_x]\right\}$$

$$+ q_x^k \rho_x + q_{x'}^k \rho_x^2 + q_{x''}^k \rho_x^3$$

$$= B_4 z + q_x^k \rho_x + q_{x'}^k \rho_x^2 + q_{x''}^k \rho_x^3.$$

$$B_{4,1} = q_{k'}^k + q_{k''}^k (k^0 + \rho_x) + q_{k'''}^k \left[(k^0)^2 + \rho_x (k^0 + \rho_x)\right]$$

$$+ q_{H'}^k H^0 + q_{H''}^k [H^0 k^0 + \rho_x H^0] + q_{w'}^k w^0 + q_{w''}^k [w^0 k^0 + \rho_x w^0]$$

$$+ q^k_{n'} n^0 + q^k_{n''}[n^0 k^0 + \rho_x n^0].$$

$$B_{4,2} = q^k_H + q^k_{H'}\rho_x + q^k_{H''}\rho_x^2.$$

$$B_{4,3} = q^k_w + q^k_{w'}\rho_x + q^k_{w''}\rho_x^2.$$

$$B_{4,4} = q^k_n + q^k_{n'}\rho_x + q^k_{n''}\rho_x^2.$$

Let

(36) $$Z = \begin{bmatrix} q^N_x \rho_x + q^N_{x'}\rho_x^2 \\ q_x^{L_1}\rho_x \\ (q_x^{N_2} + \rho_x q_{x'}^{N_2})\rho_x \\ q^k_x\rho_x + q^k_{x'}\rho_x^2 + q^k_{x''}\rho_x^3 \end{bmatrix}$$

(37) $$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix}$$

Then,

(38) $$Bz + Z = 0,$$

which can easily be solved for z.

This leaves 7 coefficients, Q, where values need to be determined, where

(39) $$Q = [k^1\ k^2\ H^1\ H^2\ w^1\ w^2\ n^1]'.$$

We nail these coefficients using the 7 conditions, $Q_\theta^N = Q_\theta^{L_1} = Q_\theta^{N_2} = Q_\theta^k = 0$ and $Q_{\theta'}^{L_1} = Q_{\theta'}^{N_2} = Q_{\theta'}^k = 0$. Now,

(40) $$Q_\theta^N = q_{k'}^N k^1 + q_{k''}^N k^0 k^1 + q_H^N H^1 + q_{H'}^N H^0 k^1$$

$$+ q_w^N w^1 + q_{w'}^N w^0 k^1 + q_n^N n^1 + q_{n'}^N n^0 k^1$$

$$+ q_\theta^N \rho_\theta + q_{\theta'}^N \rho_\theta^2$$

$$+ q_{k'}^N k^2 \rho_\theta + q_{k''}^N [k^0 k^2 + k^1 + k^2 \rho_\theta] \rho_\theta$$

$$+ q_H^N H^2 \rho_\theta + q_{H'}^N [H^0 k^2 + H^1 + H^2 \rho_\theta] \rho_\theta$$

$$+ q_w^N w^2 \rho_\theta + q_{w'}^N [w^0 k^2 + w^1 + w^2 \rho_\theta] \rho_\theta$$

$$+ q_{n'}^N (n^0 k^2 + n^1) \rho_\theta$$

$$= A_1 Q + q_\theta^N \rho_\theta + q_{\theta'}^N \rho_\theta^2$$

where $A_1 = [A_{1,1}, A_{1,2}, \ldots, A_{1,7}]$ and,

$$A_{1,1} = q_{k'}^N + q_{k''}^N k^0 + q_{H'}^N H^0 + q_{w'}^N w^0 + q_{n'}^N n^0 + q_{k''}^N \rho_\theta.$$

$$A_{1,2} = q_{k'}^N \rho_\theta + q_{k''}^N (k^0 + \rho_\theta) \rho_\theta + q_{H'}^N H^0 \rho_\theta + q_{w'}^N w^0 \rho_\theta + q_{n'}^N n^0 \rho_\theta.$$

$$A_{1,3} = q_H^N + q_{H'}^N \rho_\theta \qquad A_{1,6} = q_w^N \rho_\theta + q_{w'}^N \rho_\theta^2$$

$$A_{1,4} = q_H^N \rho_\theta + q_{H'}^N \rho_\theta^2 \qquad A_{1,7} = q_n^N + q_{n'}^N \rho_\theta$$

$$A_{1,5} = q_w^N + q_{w'}^N \rho_\theta.$$

(41) $$Q_\theta^{L_1} = q_k^{L_1} k^1 + q_H^{L_1} H^1 + q_w^{L_1} w^1 + q_n^{L_1} n^1$$

$$= A_2 Q$$

where

$$A_2 = [q_{k'}^{L_1} \;\; 0 \;\; q_H^{L_1} \;\; 0 \;\; q_w^{L_1} \;\; 0 \;\; q_n^{L_1}].$$

(42) $$Q_\theta^{N_2} = q_{k'}^{N_2} k^1 + q_{k''}^{N_2} k^0 k^1 + q_H^{N_2} H^1 + q_{H'}^{N_2} H^0 k^1 + q_w^{N_2} w^1$$

$$+ q_{w'}^{N_2} w^0 k^1 + q_n^{N_2} n^1 + q_{n'}^{N_2} n^0 k^1$$

$$= A_3 Q.$$

$$A_3 = [q_{k'}^{N_2} + q_{k''}^{N_2} k^0 + q_{H'}^{N_2} H^0 + q_{w'}^{N_2} w^0 + q_{n'}^{N_2} n^0, 0, q_H^{N_2}, 0, q_w^{N_2}, 0, q_n^{N_2}].$$

(43) $$Q_\theta^k = q_{k'}^k k^1 + q_{k''}^k k^0 k^1 + q_{k'''}^k (k^0)^2 k^1 + q_H^k H^1 + q_{H'}^k H^0 k^1$$

$$+ q_{H''}^k H^0 k^1 k^0 + q_w^k w^1 + q_{w'}^k w^0 k^1 + q_{w''}^k w^0 k^0 k^1 + q_n^k n^1$$

$$+ q_{n'}^k n^0 k^1 + q_{n''}^k n^0 k^0 k^1$$

$$= A_4 Q,$$

where,

$$A_{4,1} = q_{k'}^k + q_{k''}^k k^0 + q_{k'''}^k (k^0)^2 + q_{H'}^k H^0 + q_{H''}^k H^0 k^0$$

$$+ q_{w'}^k w^0 + q_{w''}^k w^0 k^0 + q_{n'}^k n^0 + q_{n''}^k n^0 k^0.$$

$$A_4 = [A_{4,1} \;\; 0 \;\; q_H^k \;\; 0 \;\; q_w^k \;\; 0 \;\; q_n^k].$$

(44) $$Q_{\theta'}^{L_1} = q_{k'}^{L_1} k^2 + q_H^{L_1} H^2 + q_w^{L_1} w^2 + q_\theta^{L_1}$$

$$= A_5 Q + q_\theta^{L_1}.$$

$$A_5 = [0 \;\; q_{k'}^{L_1} \;\; 0 \;\; q_{H}^{L_1} \;\; 0 \;\; q_{w}^{L_1} \;\; 0].$$

(45) $$Q_{\theta'}^{N_2} = q_{k'}^{N_2} k^2 + q_{k''}^{N_2}[k^0 k^2 + k^1 + k^2 \rho_\theta]$$

$$+ q_{H}^{N_2} H^2 + q_{H'}^{N_2}[H^0 k^2 + H^1 + H^2 \rho_\theta]$$

$$+ q_{w}^{N_2} w^2 + q_{w'}^{N_2}[w^0 k^2 + w^1 + w^2 \rho_\theta]$$

$$+ q_{n'}^{N_2}[n^0 k^2 + n^1] + q_{\theta}^{N_2} + q_{\theta'}^{N_2} \rho_\theta$$

$$= A_6 Q + q_{\theta}^{N_2} + q_{\theta'}^{N_2} \rho_\theta.$$

$$A_{6,1} = q_{k''}^{N_2},$$

$$A_{6,2} = q_{k'}^{N_2} + q_{k''}^{N_2}(k^0 + \rho_\theta) + q_{H'}^{N_2} H^0 + q_{w'}^{N_2} w^0 + q_{n'}^{N_2} n^0.$$

$$A_{6,3} = q_{H'}^{N_2}.$$

$$A_{6,4} = q_{H}^{N_2} + q_{H'}^{N_2} \rho_\theta.$$

$$A_{6,5} = q_{w'}^{N_2}.$$

$$A_{6,6} = q_{w}^{N_2} + q_{w'}^{N_2} \rho_\theta.$$

$$A_{6,7} = q_{n'}^{N_2}.$$

$$A_6 = [A_{6,1}, A_{6,2}, A_{6,3}, A_{6,4}, A_{6,5}, A_{6,6}, A_{6,7}].$$

$$(46)\qquad Q^k_{\theta'} = q^k_{k'}k^2 + q^k_{k''}[k^0k^2+k^1+ k^2\rho_\theta]$$

$$+ q^k_{k'''}\Big\{k^0[k^0k^2+k^1+k^2\rho_\theta] + k^1\rho_\theta + k^2\rho^2_\theta\Big\}$$

$$+ q^k_H H^2 + q^k_{H'}[H^0k^2+H^1+H^2\rho_\theta]$$

$$+ q^k_{H''}\Big\{H^0[k^0k^2+k^1+k^2\rho_\theta] + H^1\rho_\theta + H^2\rho^2_\theta\Big\}$$

$$+ q^k_w w^2 + q^k_{w'}[w^0k^2+w^1+w^2\rho_\theta]$$

$$+ q^k_{w''}\Big\{w^0[k^0k^2+k^1+k^2\rho_\theta] + w^1\rho_\theta + w^2\rho^2_\theta\Big\}$$

$$+ q^k_{n'}[n^0k^2+n^1] + q^k_{n''}\Big\{n^0[k^0k^2+k^1+k^2\rho_\theta] + n^1\rho_\theta\Big\}$$

$$+ q^k_\theta + q^k_{\theta'}\rho_\theta + q^k_{\theta'}\rho^2_\theta$$

$$= A_7Q + q^k_\theta + q^k_{\theta'}\rho_\theta + q^k_{\theta''}\rho^2_\theta.$$

$$A_{7,1} = q^k_{k''} + q^k_{k'''}\{k^0+\rho_\theta\} + q^k_{H''}H^0 + q^k_{w''}w^0 + q^k_{n''}n^0.$$

$$A_{7,2} = q^k_{k'} + q^k_{k''}(k^0+\rho_\theta) + q^k_{k'''}\Big[k^0(k^0+\rho_\theta) + \rho^2_\theta\Big]$$

$$+ q^k_{H'}H^0 + q^k_{H''}H^0(k^0+\rho_\theta) + q^k_{w'}w^0 + q^k_{w''}[w^0(k^0+\rho_\theta)]$$

$$+ q^k_{n'}n^0 + q^k_{n''}n^0(k^0+\rho_\theta).$$

$$A_{7,3} = q^k_{H'} + q^k_{H''}\rho_\theta.$$

$$A_{7,4} = q^{k}_{H} + q^{k}_{H'}\rho_\theta + q^{k}_{H''}\rho^2_\theta.$$

$$A_{7,5} = q^{k}_{w'} + q^{k}_{w''}\rho_\theta.$$

$$A_{7,6} = q^{k}_{w} + q^{k}_{w'}\rho_\theta + q^{k}_{w''}\rho^2_\theta.$$

$$A_{7,7} = q^{k}_{n'} + q^{k}_{n''}\rho_\theta.$$

$$A_7 = [A_{7,1,}, A_{7,2}, A_{7,3}, A_{7,4}, A_{7,5}\ A_{,7,6}, A_{7,7}].$$

Let

(47) $$F = [q^{N}_{\theta}\rho_\theta + q^{N}_{\theta'}\rho^2_\theta,\ 0,\ 0,\ 0,\ q_\theta^{L_1},\ q_\theta^{N_2} + q_{\theta'}^{N_2}\rho_\theta,\ q^{k}_{\theta} + q^{k}_{\theta'}\rho_\theta + q^{k}_{\theta''}\rho^2_\theta]'$$

(48) $$A = \begin{bmatrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_7 \end{bmatrix}.$$

Then,

(49) $$AQ + F = 0,$$

which is a system of equations that can be solved easily for Q.

The decision rules were computed using the parameter values studied in the steady–state section. The linearized first order conditions were obtained by numerical differentiation, i.e., letting f(x) be the function of interest, then $f'(x) = [f(x+eps)-f(x)]/eps$, where x is a steady–state value. We set $eps = x\Delta$ when $|x| > 0$ and $eps = \Delta$ when $|x| = 0$, for two values of Δ. Table 1 reports our results.

2. The Full Information Economy.

We now turn to the case in which

$$\Omega_t^0 = \Omega_t^1 = \{k_t, \theta_t, x_t\} \equiv \Omega_t.$$

The discussion in this section closely parallels that in the previous subsection. First, we use the same linearized first order conditions that we used there. Second, we must re–specify the decision rules, to reflect the changed information set and modify the equations.

The decision rules are:

$$\tilde{k}_{t+1} = k^0\tilde{k}_t + k^2\tilde{\theta}_t + k^4\tilde{x}_t$$

$$E[\tilde{k}_{t+2}|\Omega_t] = (k^0)^2\tilde{k}_t + k^2(k^0 + \rho_\theta)\tilde{\theta}_t + k^4(k^0 + \rho_x)\tilde{x}_t$$

$$E[\tilde{k}_{t+3}|\Omega_t] = (k^0)^3\tilde{k}_t + k^2[k^0(k^0 + \rho_\theta) + \rho_\theta^2]\tilde{\theta}_t$$

$$+ k^4[k^0(k^0 + \rho_x) + \rho_x^2]\tilde{x}_t$$

$$\tilde{H}_{1t} = H^0\tilde{k}_t + H^2\tilde{\theta}_t + H^4\tilde{x}_t$$

$$E[\tilde{H}_{1t+1}|\Omega_t] = H^0k^0\tilde{k}_t + [H^0k^2 + H^2\rho_\theta]\tilde{\theta}_t + [H^0k^4 + H^4\rho_x]x_t$$

$$E[\tilde{H}_{1t+2}|\Omega_t] = H^0(k^0)^2\tilde{k}_t + [H^0k^2(k^0 + \rho_\theta) + H^2\rho_\theta^2]\tilde{\theta}_t$$
$$+ [H^0k^4(k^0 + \rho_x) + H^4\rho_x^2]\tilde{x}_t$$

$$\tilde{w}_{1t} = w^0\tilde{k}_t + w^2\tilde{\theta}_t + w^4\tilde{x}_t$$

$$E[\tilde{w}_{1t+1}|\Omega_t] = w^0k^0\tilde{k}_t + (w^0k^2 + w^2\rho_\theta)\tilde{\theta}_t + (w^0k^4 + w^4\rho_x)\tilde{x}_t$$

$$E[\tilde{w}_{1t+2}|\Omega_t] = w^0(k^0)^2\tilde{k}_t + [w^0k^2(k^0+\rho_\theta) + w^2\rho_\theta^2]\tilde{\theta}_t + [w^0k^4(k^0+\rho_x) + w^4\rho_x^2]\tilde{x}_t$$

$$\tilde{n}_{1t} = n^0\tilde{k}_t + n^2\tilde{\theta}_t + n^4\tilde{x}_t$$

$$E[\tilde{n}_{1t+1}|\Omega_t] = n^0k^0\tilde{k}_t + (n^0k^2 + n^2\rho_\theta)\tilde{\theta}_t + (n^0k^4 + n^4\rho_x)\tilde{x}_t$$

$$E[\tilde{n}_{1t+2}|\Omega_t] = n^0(k^0)^2\tilde{k}_t + [n^0k^2(k^0+\rho_\theta) + n^2\rho_\theta^2]\tilde{\theta}_t + [n^0k^4(k^0+\rho_x) + n^4\rho_x^2]\tilde{x}_t.$$

Substituting these into the linearized first order conditions,

$$E[Q_t^N|\Omega_t] = Q_k^N\tilde{k}_t + Q_\theta^N\tilde{\theta}_t + Q_x^N\tilde{x}_t$$

$$E[Q_t^{L_1}|\Omega_t] = Q_k^{L_1}\tilde{k}_t + Q_\theta^{L_1}\tilde{\theta}_t + Q_x^{L_1}\tilde{x}_t$$

$$E[Q_t^{N_2}|\Omega_t] = Q_k^{N_2}\tilde{k}_t + Q_\theta^{N_2}\tilde{\theta}_t + Q_x^{N_2}\tilde{x}_t$$

$$E[Q_t^k|\Omega_t] = Q_k^k\tilde{k}_t + Q_\theta^k\tilde{\theta}_t + Q_x^k\tilde{x}_t.$$

The coefficients to the right of the above 4 equalities are functions of the 12 undetermined decision rule parameters. As before, efficiency requires that all 12 coefficients be equated to zero and this (in addition to a transversality condition) is enough to determine the 12 decision rule parameters. It is easy to verify that the formulas for Q_k^N, $Q_k^{L_1}$, $Q_k^{N_2}$, Q_k^k coincide exactly with (25)–(28). Thus, the conditions $Q_k^N = Q_k^{L_1} = Q_k^{N_2} = Q_k^k = 0$ determine k^0, H^0, n^0, w^0 in precisely the way that was described there.

Conditional on values for k^0, H^0, w^0, and n^0, values for the parameters k^2, H^2,

w^2, n^2 can be obtained from the conditions $Q_\theta^N = Q_\theta^{L_1} = Q_\theta^{N_2} = Q_\theta^k = 0$. In particular, if $B_\theta = [Q_\theta^N, Q_\theta^{L_1}, Q_\theta^{N_2}, Q_\theta^k]'$ then

$$B_\theta = A_\theta z_\theta + F_\theta,$$

where $z_\theta = [k^2\ H^2\ w^2\ n^2]'$ and

$$F_\theta = \begin{bmatrix} q_\theta^N + q_{\theta'}^N \rho_\theta \\ q_\theta^{L_1} \\ q_\theta^{N_2} + q_{\theta'}^{N_2} \rho_\theta \\ q_\theta^k + q_{\theta'}^k \rho_\theta + q_{\theta''}^k \rho_\theta^2 \end{bmatrix}.$$

Also, A_θ is constructed as follows:

$$A_{1,1} = q_{k'}^N + q_{k''}^N (k^0 + \rho_\theta) + q_{H'}^N H^0 + q_{w'}^N w^0 + q_{n'}^N n^0$$

$$A_{1,2} = q_H^N + q_{H'}^N \rho_\theta, \quad A_{1,3} = q_w^N + q_{w'}^N \rho_\theta$$

$$A_{1,4} = q_n^N + q_{n'}^N \rho_\theta$$

$$A_1 = [A_{1,1}, A_{1,2}, A_{1,3}, A_{1,4}]$$

$$A_2 = [q_{k'}^{L_1}, q_H^{L_1}, q_w^{L_1}, q_n^{L_1}].$$

The 1×4 vector A_3 is constructed in the same way as A_1, except that the superscript N is replaced by N_2 everywhere. Now consider the 1×3 vector A_4

$$A_{4,1} = q^k_{k'} + q^k_{k''}(k^0 + \rho_\theta) + q^k_{k'''}[k^0(k^0 + \rho_\theta) + \rho^2_\theta]$$
$$+ q^k_{H'}H^0 + q^k_{H''}H^0(k^0 + \rho_\theta) + q^k_{w'}w^0 + q^k_{w''}w^0(k^0 + \rho_\theta)$$
$$+ q^k_{n'}n^0 + q^k_{n''}n^0(k^0 + \rho_\theta)$$

$$A_{4,2} = q^k_H + q^k_{H'}\rho_\theta + q^k_{H''}\rho^2_\theta$$

$$A_{4,3} = q^k_w + q^k_{w'}\rho_\theta + q^k_{w''}\rho^2_\theta$$

$$A_{4,4} = q^k_n + q^k_{n'}\rho_\theta + q^k_{n''}\rho^2_\theta$$

$$A_4 = [A_{4,1}, A_{4,2}, A_{4,3}, A_{4,4}].$$

Finally, the 4 × 4 matrix A

$$A_\theta = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}.$$

Thus, B_θ is a simple linear function of k^2, H^2, w^2, n^2. The condition $B_\theta = 0$ implies

$$z_\theta = -A_\theta^{-1}F_\theta.$$

Finding $z_x = [k^4, H^4, w^4, n^4]'$ requires a similar set of calculations. That is,

$$z_x = -A_x^{-1}F_x,$$

where A_x and F_x are obtained simply by replacing θ subscripts by x subscripts.

3. Intermediate Information

This time, let

$$\Omega_t^0 = \Omega_t^1 = \{k_t, \theta_t, x_{t-1}\} = \tilde{\Omega}_t$$

and

$$\tilde{k}_{t+1} = k^0 \tilde{k}_t + k^2 \tilde{\theta}_t + k^3 \tilde{x}_{t-1}$$

$$\tilde{H}_{1t} = H^0 \tilde{k}_t + H^2 \tilde{\theta}_t + H^3 \tilde{x}_{t-1}$$

$$\tilde{w}_{1t} = w^0 \tilde{k}_t + w^2 \tilde{\theta}_t + w^3 \tilde{x}_{t-1}$$

$$\tilde{n}_t = n^0 \tilde{k}_t + n^2 \tilde{\theta}_t + n^3 \tilde{x}_{t-1}.$$

To compute k^0, H^0, w^0, n^0, k^2, H^2, w^2, n^2, use the procedure used for the full information economy. Let Q_x^N, $Q_x^{L_1}$, $Q_x^{N_2}$, Q_x^k denote the coefficient on $\tilde{x}_{t-1}$ in $E[Q_t^N | \tilde{\Omega}_t]$, $E[Q_t^{L_1} | \tilde{\Omega}_t]$, $E[Q_t^{N_2} | \tilde{\Omega}_t]$, and $E[Q_t^k | \tilde{\Omega}_t]$. Working out the algebra, we get

$$\begin{aligned} Q_x^n = {} & q_{k'}^N k^3 + q_{k''}^N k^3(k^0 + \rho_x) + q_H^N H^3 + q_{H'}^N [H^0 k^3 + H^3 \rho_x] \\ & + q_w^N w^3 + q_{w'}^N [w^0 k^3 + w^3 \rho_x] + q_n^N n^3 + q_{n'}^N [n^0 k^3 + n^3 \rho_x] \\ & + q_x^N \rho_x + q_x^N \rho_x^2 \end{aligned}$$

$$Q_x^{L_1} = q_{k'}^{L_1} k^3 + q_H^{L_1} H^3 + q_w^{L_1} w^3 + q_n^{L_1} n^3 + q_x^{L_1} \rho_x$$

$$\begin{aligned} Q_x^{N_2} = {} & q_{k'}^{N_2} k^3 + q_{k'}^{N_2} k^3(k^0 + \rho_x) + q_H^{N_2} H^3 + q_{H'}^{N_2} [H^0 k^3 + H^3 \rho_x] \\ & + q_w^{N_2} w^3 + q_{w'}^{N_2} [w^0 k^3 + w^3 \rho_x] + q_n^{N_2} n^3 + q_{n'}^{N_2} [n^0 k^3 + n^3 \rho_x] \\ & + q_x^{N_2} \rho_x + q_{x'}^{N_2} \rho_x^2 \end{aligned}$$

$$\begin{aligned} Q_x^k = {} & q_{k'}^k k^3 + q_{k''}^k k^3(k^0 + \rho_x) + q_{k'''}^k k^3[k^0(k^0 + \rho_x) + \rho_x^2] \\ & + q_H^k H^3 + q_{H'}^k (H^0 k^3 + H^3 \rho_x) + q_{H''}^k [H^0 k^3(k^0 + \rho_x) + H^3 \rho_x^2] \\ & + q_w^k w^3 + q_{w'}^k (w^0 k^3 + w^3 \rho_x) + q_{w''}^k [w^0 k^3(k^0 + \rho_x) + w^3 \rho_x^2] \\ & + q_n^k n^3 + q_{n'}^k (n^0 k^3 + n^3 \rho_x) + q_{n''}^k [n^0 k^3(k^0 + \rho_x) + n^3 \rho_x^2] \\ & + q_x^k \rho_x + q_{x'}^k \rho_x^2 + q_{x''}^k \rho_x^3. \end{aligned}$$

Let

$$A_{1,1} = q_{k'}^N + q_{q''}^N (k^0 + \rho_x) + q_{H'}^N H^0 + q_{w'}^N w^0 + q_{n'}^N n^0$$

$$A_{1,2} = q_H^N + q_{H'}^N \rho_x$$

$$A_{1,3} = q_w^N + q_{w'}^N \rho_x$$

$$A_{1,4} = q_n^N + q_{n'}^N \rho_x$$

$$A_1 = [A_{1,1}, A_{1,2}, A_{1,3}, A_{1,4}]$$

$$A_2 = [q_{k'}^{L_1}\ q_H^{L_1}\ q_w^{L_1}\ q_n^{L_1}]$$

$$A_{3,1} = q_{k'}^{N_2} + q_{k'}^{N_2}(k^0 + \rho_x) + q_{H'}^{N_2} H^0 + q_{w'}^{N_2} w^0 + q_{n'}^{N_2} n^0$$

$$A_{3,2} = q_H^{N_2} + q_{H'}^{N_2} \rho_x$$

$$A_{3,3} = q_w^{N_2} + q_{w'}^{N_2} \rho_x$$

$$A_{3,4} = q_n^{N_2} + q_{n'}^{N_2} \rho_x$$

$$A_3 = [A_{3,1}, A_{3,2}, A_{3,3}, A_{3,4}].$$

$$\begin{aligned} A_{4,1} = {} & q^k_{k'} + q^k_{k''}(k^0 + \rho_x) + q^k_{k'''}[k^0(k^0 + \rho_x) + \rho_x^2] \\ & + [q^k_{H'} + q^k_{H''}(k^0 + \rho_x)]H^0 + [q^k_{w'} + q^k_{w''}(k^0 + \rho_x)]w^0 \\ & + [q^k_{n'} + q^k_{n''}(k^0 + \rho_x)]n^0 \end{aligned}$$

$$A_{4,2} = q^k_H + q^k_{H'}\rho_x + q^k_{H''}\rho_x^2$$

$$A_{4,3} = q^k_w + q^k_{w'}\rho_x + q^k_{w''}\rho_x^2$$

$$A_{4,4} = q^k_n + q^k_{n'}\rho_x + q^k_{n''}\rho_x^2$$

$$A_4 = [A_{4,1}, A_{4,2}, A_{4,3}, A_{4,4}].$$

Let

$$\tilde{A}_x = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}.$$

Let

$$\tilde{F}_x = \begin{bmatrix} q^N_x + q^N_x \rho_x \\ q_x^{L_1} \\ q_x^{N_2} + q_{x'}^{N_2}\rho_x \\ q^k_x + q^k_{x'}\rho_x + q^k_{x''}\rho_x^2 \end{bmatrix}\rho_x.$$

$$\tilde{z}_x = \begin{bmatrix} k^3 \\ H^3 \\ w^3 \\ n^3 \end{bmatrix}, \quad B_x = \begin{bmatrix} Q_x^N \\ Q_x^{L_1} \\ Q_x^{N_2} \\ Q_x^k \end{bmatrix}.$$

Then,

$$\tilde{A}_x \tilde{z}_x + \tilde{F}_x = B_x,$$

or, $B_x = 0$ implies

$$\tilde{z}_x = -\tilde{A}_x^{-1} \tilde{F}_x.$$

But, $\tilde{F}_x = F_x \rho_x$ and $\tilde{A}_x = A_x$, so that

$$\begin{aligned} \tilde{z}_x &= -A_x^{-1} F_x \rho_x \\ &= z_x \rho_x. \end{aligned}$$

4. <u>Fuerst Specification.</u>

Now,

$$\Omega_t^0 = \{k_t, \theta_{t-1}, x_{t-1}\}$$

$$\Omega_t^1 = \{k_t, \theta_{t-1}, \theta_t, x_{t-1}, x_t\}.$$

Then,

$$\tilde{k}_{t+1} = k^0 \tilde{k}_t + k^1 \tilde{\theta}_{t-1} + k^2 \tilde{\theta}_t + k^3 \tilde{x}_{t-1} + k^4 \tilde{x}_t$$

$$\tilde{H}_{1t} = H^0\tilde{k}_t + H^1\tilde{\theta}_{t-1} + H^2\tilde{\theta}_t + H^3\tilde{x}_{t-1} + H^4\tilde{x}_t$$

$$\tilde{w}_{1t} = w^0\tilde{k}_t + w^1\tilde{\theta}_{t-1} + w^2\tilde{\theta}_t + w^3\tilde{x}_{t-1} + w^4\tilde{x}_t$$

$$\tilde{n}_t = n^0\tilde{k}_t + n^1\tilde{\theta}_{t-1} + n^3\tilde{x}_{t-1}.$$

We have 18 decision rule parameters to be determined. We do this by substituting the decision rules into the efficiency conditions and coefficients in the following expressions to zero:

$$E[Q_t^N | \Omega_t^0] = Q_k^N\tilde{k}_t + Q_\theta^N\tilde{\theta}_{t-1} + Q_x^N\tilde{x}_{t-1}$$

$$E[Q_t^{L_1} | \Omega_t^1] = Q_k^{L_1}\tilde{k}_t + Q_\theta^{L_1}\tilde{\theta}_{t-1} + Q_{\theta'}^{L_1}\tilde{\theta}_t + Q_x^{L_1}\tilde{x}_{t-1} + Q_{x'}^{L_1}\tilde{x}_t$$

$$E[Q_t^{N_2} | \Omega_t^1] = Q_k^{N_2}\tilde{k}_t + Q_\theta^{N_2}\tilde{\theta}_{t-1} + Q_{\theta'}^{N_2}\tilde{\theta}_t + Q_x^{N_2}\tilde{x}_{t-1} + Q_{x'}^{N_2}\tilde{x}_t$$

$$E[Q_t^k | \Omega_t^1] = Q_k^k\tilde{k}_t + Q_\theta^k\tilde{\theta}_{t-1} + Q_{\theta'}^k\tilde{\theta}_t + Q_x^k\tilde{x}_{t-1} + Q_{x'}^k\tilde{x}_t.$$

The formulas for $Q_k^N = Q_k^{L_1} = Q_k^{N_2} = Q_k^k = 0$ are precisely what they were in the limited information economy (see equations (25)–(28)). Therefore, just like there, these formulas can be used to solve for k^0, H^0, n^0, and w^0.

Next, consider the 7 coefficients $Q = [k^1, k^2, H^1, H^2, w^1, w^2, n^1]'$. These can be obtained using the given values of k^0, H^0, n^0, w^0, and the 7 conditions, $Q_\theta^N = Q_\theta^{L_1} = Q_\theta^{N_2} = Q_\theta^k = Q_{\theta'}^{L_1} = Q_{\theta'}^{N_2} = Q_{\theta'}^k = 0$. Now,

$$\begin{bmatrix} Q_{\theta}^{N} \\ Q_{\theta}^{L_1} \\ Q_{\theta}^{N_2} \\ Q_{\theta}^{k} \\ Q_{\theta'}^{L_1} \\ Q_{\theta'}^{N_2} \\ Q_{\theta'}^{k} \end{bmatrix} = AQ + F,$$

where F and A are defined in (47)–(48). Thus,

$$Q = -A^{-1}F.$$

Finally, consider the 7 coefficients $Z = [k^3, k^4, H^3, H^4, w^3, w^4, n^3]$. These can be obtained using the given values of k^0, H^0, n^0, w^0, and the 7 conditions

$$Q_x^N = Q_x^{L_1} = Q_x^{N_2} = Q_x^k = Q_{x'}^{L_1} = Q_{x'}^{N_2} = Q_{x'}^k = 0.$$

Then,

$$\begin{bmatrix} Q_x^{n} \\ Q_x^{L_1} \\ Q_x^{N_2} \\ Q_x^{k} \\ Q_{x'}^{L_1} \end{bmatrix} = \tilde{A}Z + \tilde{F}.$$

$$\begin{bmatrix} Q_{x'}^{N_2} \\ Q_{x'}^{k} \end{bmatrix}$$

Here, $\tilde{A}$ is the same as A in (48), except that ρ_x replaces ρ_θ everywhere. Also, $\tilde{F}$ is the same as F in (47), except that the subscript x replaces the subscript θ everywhere.

5. Sluggish Saving.

Now, suppose H_{1t} and H_{2t} are contingent on x_t, θ_t, but I_t is not contingent on x_t. Thus,

$$\tilde{k}_{t+1} = k^0\tilde{k}_t + k^1\tilde{\theta}_{t-1} + k^2\tilde{\theta}_t + k^3\tilde{x}_{t-1}$$

$$\tilde{H}_{1t} = H^0\tilde{k}_t + H^1\tilde{\theta}_{t-1} + H^2\tilde{\theta}_t + H^3\tilde{x}_{t-1} + H^4\tilde{x}_t$$

$$\tilde{w}_{1t} = w^0\tilde{k}_t + w^1\tilde{\theta}_{t-1} + w^2\tilde{\theta}_t + w^3\tilde{x}_{t-1} + w^4\tilde{x}_t$$

$$\tilde{n}_t = n^0\tilde{k}_t + n^1\tilde{\theta}_{t-1} + n^3\tilde{x}_{t-1}.$$

These 17 coefficients can be determined by setting the 17 coefficients in the following expressions to zero:

$$E[Q_t^N | \tilde{k}_t, \tilde{\theta}_{t-1}, \tilde{x}_{t-1}] = Q_k^N\tilde{k}_t + Q_\theta^N\tilde{\theta}_{t-1} + Q_x^N\tilde{x}_{t-1}$$

$$E[Q_t^{L_1} | \tilde{k}_t, \tilde{\theta}_{t-1}, \tilde{\theta}_t, \tilde{x}_{t-1}, \tilde{x}_t] = Q_k^{L_1}\tilde{k}_t + Q_\theta^{L_1}\tilde{\theta}_{t-1} + Q_{\theta'}^{L_1}\tilde{\theta}_t + Q_x^{L_1}\tilde{x}_{t-1} + Q_{x'}^{L_1}\tilde{x}_t$$

$$E[Q_t^{N_2} | \tilde{k}_t, \tilde{\theta}_{t-1}, \tilde{\theta}_t, \tilde{x}_{t-1}, \tilde{x}_t] = Q_k^{N_2}\tilde{k}_t + Q_\theta^{N_2}\tilde{\theta}_{t-1}$$

$$+ Q_{\theta'}^{N_2}\tilde{\theta}_t + Q_x^{N_2}\tilde{x}_{t-1} + Q_{x'}^{N_2}\tilde{x}_t$$

$$E[Q_t^k|\tilde{k}_t,\tilde{\theta}_{t-1},\tilde{\theta}_t,\tilde{x}_{t-1}] = Q_k^k\tilde{k}_t + Q_\theta^k\tilde{\theta}_{t-1} + Q_{\theta'}^k\tilde{\theta}_t + \tilde{Q}_x^k\tilde{x}_{t-1}.$$

As before, $Q_k^N = Q_k^{L_1} = Q_k^{N_2} = Q_k^k = 0$ are used to determine k^0, H^0, n^0, and w^0. Similarly, as in Section 4, $Q_\theta^N = Q_\theta^{L_1} = Q_\theta^{N_2} = Q_\theta^k = Q_{\theta'}^{L_1} = Q_{\theta'}^{N_2} = Q_{\theta'}^k = 0$ are used to determine k^1, k^2, H^1, H^2, w^1, w^2, n^1. Now, use $Q_x^N = Q_x^{L_1} = Q_x^{N_2} = \tilde{Q}_x^k = Q_{x'}^{L_1} = Q_{x'}^{N_2} = 0$ to determine k^3, H^3, H^4, w^3, w^4, n^3. To do this, simply modify the equations used in Section 4 to get $z = [k^3,k^4,H^3,H^4,w^3,w^4,n^3]'$. Define

$$\tau = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \rho_x \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Let $\tilde{\tilde{A}}$ be the 6×6 matrix given by $\tau\tilde{A}$ with the second column removed. Here, $\tilde{A}$ is as defined in Section 4. Also, let $\tilde{z}$ denote the 6×1 vector given by z with the second element removed. Then,

$$\begin{bmatrix} Q_x^N \\ Q_x^{L_1} \\ Q_x^{N_2} \\ \tilde{Q}_x^k \\ Q_{x'}^{L_1} \\ Q_{x'}^{N_2} \end{bmatrix} = \tilde{\tilde{A}}\tilde{z} + \tau\tilde{F},$$

where $\tilde{F}$ is as defined in Section 4. Then,

$$\tilde{z} = -\tilde{\tilde{A}}^{-1}{}_{T}\tilde{F}.$$

Table 1

Model Parameters

$\delta = .02$, $\beta = 1.03^{-.25}$, $\mu = .004$, r = .01, $\gamma = 2/3$, $\alpha = .36$,
$\theta = 0$, $\psi = -2$, x = .012, $\rho_\theta = .95$, $\rho_x = .81$, $\rho = 10/9$.

Decision Rule Parameters, $\Delta = .1e{-5}$ ($\Delta = .1e{-7}$)

$$k_{t+1} = (1-k^0)k + k^0 k_t + k^1(\theta_{t-1}-\theta) + k^2(\theta_t-\theta) + k^3(x_{t-1}-x)$$
$$H_{1t} = H + H^0(k_t-k) + H^1(\theta_{t-1}-\theta) + H^2(\theta_t-\theta) + H^3(x_{t-1}-x)$$
$$w_{1t} = w_1 + w^0(k_t-k) + w^1(\theta_{t-1}-\theta) + w^2(\theta_t-\theta) + w^3(x_{t-1}-x)$$
$$n_t = n + n^0(k_t-k) + n^1(\theta_{t-1}-\theta) + n^3(x_{t-1}-x)$$

	k_{t+1}		H_{1t}		w_{1t}		n_t
k	5.8766	H	.1476	w_1	2.8479	n	.8374
k^0	.9717 (.9713)	H^0	–.002655 (–.002744)	w^0	–.02085 (–.02155)	n^0	–.02149 (–.02222)
k^1	.1172 (.1164)	H^1	.04960 (.04926)	w^1	.3895 (.3868)	n^1	.4015 (.3988)
k^2	.2146 (.2139)	H^2	0 (0)	w^2	0 (0)	n^2	— (—)
k^3	.0826 (.0824)	H^3	–.04389 (–.04393)	w^3	1.9348 (1.9345)	n^3	–.4855 (–.4857)