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Is Long-Run M1 Demand Stable?

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ABSTRACT

The money demand literature presents much conflicting evidence on this question. For example, Lucas (1988) reports unrestricted money demand regressions which seem to imply that long-run money demand elasticities are highly unstable across subsamples. At the same time, he also presents evidence from money demand regressions with the income elasticity restricted to unity which seem to suggest stability. We conduct a formal analysis which weighs these apparently conflicting facts to determine which hypothesis is more plausible; the hypothesis that money demand is stable, or the hypothesis that money demand is unstable. We find that the stability hypothesis is the more plausible one. Thus, according to our data set, the answer to the question in the title is "yes".

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1. Introduction

Until recently, the consensus in the money demand literature was that the answer to the question in the title is "no".[1] Lucas (1988) has criticized this consensus. He argues that 20th century data on real balances, income, and interest rates are consistent with a stable long-run demand for money with a unit income elasticity. To support this view, he points to the evidence summarized in Figure 1. That figure reports the scatter plot of (minus) the log of M1 velocity against a short-term interest rate for annual data covering 1900–1985, with the post-1957 observations being indicated by a different set of symbols than the earlier observations. The striking feature of Figure 1 is that although interest rates since 1957 have been substantially higher than in the earlier period, post-1957 velocity seems to be roughly what one would have predicted by extrapolating linearly from the pre-1958 scatter plot. Lucas conjectures that this stability in the empirical interest semi-elasticity of money demand that one gets when conditioning on a unit income elasticity is hard to reconcile with the view that M1 demand is unstable.

But if long-run M1 demand is in fact stable, then why has the literature concluded otherwise? It has done so because estimated money demand functions which leave the income elasticity unconstrained display significant subsample instabilities. For example, the estimated income elasticity and interest semi-elasticity are 1.0 and -0.08, for Lucas' entire sample while they are 0.2 and -0.01 using only the post-1957 data. Lucas hypothesizes that this instability reflects the contaminating effects of shifts in the high frequency interactions between the money demand disturbance and the explanatory variables in the money demand equation. The purpose of this paper is to evaluate this hypothesis quantitatively.

We formalize Lucas' hypothesis by building on recent developments in the theory of regression analysis of integrated and cointegrated variables. We identify the parameters of the long-run money demand equation with those of a cointegrating vector relating real balances, income, and the interest rate. The high frequency dynamics are identified with the parameters of a vector autoregression (VAR) involving the money demand disturbance, output, and the nominal interest rate. So, in these terms, Lucas' hypothesis is that the parameters of the cointegrating relation remained constant throughout the sample, but the VAR parameters governing the high-frequency dynamics shifted. There are two reasons to be skeptical of this hypothesis. First, the subsample instability in unrestricted money demand regressions is, after all, quantitatively quite large. And second, econometric theory tells us that breaks in the high frequency dynamics should not induce subsample instability in estimates of cointegrating relations, at least when each subsample is large. In view of these considerations, we were surprised to find substantial support for Lucas' hypothesis.

Our investigation proceeds in two steps. We first conduct classical tests of Lucas' null hypothesis. This requires making auxiliary assumptions about the nature of the high frequency dynamics. As it turns out, these assumptions have an important impact on the outcome of our analysis. The stable money demand hypothesis fails to be rejected under some specifications of the high frequency dynamics, but *is* rejected under others. This motivates the second step of our analysis, in which we take seriously the alternative that there has been a break of the magnitude documented above. We adopt a Bayesian perspective and evaluate the relative plausibility of a stable long-run money demand function, versus the alternative of a break. We do this under various specifications of the high frequency dynamics. We find that the specifications under which Lucas' hypothesis is rejected using classical methods are relatively implausible. In addition, our analysis establishes that the stability hypothesis is substantially more plausible than its alternative. Our classical and Bayesian analyses are conducted using bootstrap methods, as in Christiano and Ljungqvist (1988).

Our classical analysis is related to that of Stock and Watson (1993), who also test the null hypothesis of no shift in the money demand equation. However, our analysis differs from theirs in several key respects. First, as noted above, classical methods do deliver evidence against the null hypothesis. Second, Stock and Watson's test assumes there has been no shift at all in the dynamics of the data. In contrast, Figure 1 suggests that *something* changed in the post-1957 period, since the interest rate was so much higher. We formally document that the proposition of no change at all in the data dynamics is overwhelmingly rejected. Third, Stock and Watson specify that the interest rate is integrated of order 1. We reject this specification based on its implication, documented below, that the interest rate will go negative with high probability in samples that correspond to the length of our data set, which is 85 years.

But, why should we care about the income and interest elasticities of money demand and their stability? One set of motivations derives from the literature on empirically based dynamic, general equilibrium models of money.[2] Most of these models focus on the role of money in reducing various types of transactions costs associated with trading. As Lucas (1988,1993) in particular has emphasized, an important subset of these models imply a long-run money demand function which is stable in the sense that the elasticities are invariant to changes in monetary or other policies. Since our sample period covers the 20th century, it spans a wide variety of policy regimes, so a test of the stability of the long-run money demand function represents a potentially powerful specification test for this class of models.[3]

The numerical value of the long-run income and interest elasticities of money demand are also of interest to analysts of transactions cost models of money. For example, analysis of these models is greatly simplified by the assumption of a unitary income elasticity, so it is important to know whether this is consistent with the data. In addition, several analyses make use of the estimated long-run interest semi-elasticity of demand to calibrate certain preference parameters in monetary models (see, for example, Chari, Christiano, and Kehoe (1991), and Cooley and Hansen (1991), and Braun (1994b)).

Other reasons for being concerned about the value and stability of the money demand elasticities derive from direct policy considerations. One of these is that, assuming a fairly predictable trend in output growth and no trend in interest rates, the income elasticity determines the average rate of money growth that is needed to achieve a given inflation objective. Another reason is advanced in Chari, Christiano, and Kehoe (1991, 1993) and Braun (1994a). They describe a class of economic environments in which the magnitude of the income elasticity of money demand determines whether or not the Friedman rule of setting the nominal interest rate to zero is optimal. Also, the interest elasticity of money demand can be used to compute an estimate of the welfare cost of inflation. (See Lucas (1993) and the references he cites.) These considerations suggest that it is important to know the value of the money demand elasticities. At the same time, there would be little meaning or interest in knowing the values of these parameters if they were not stable across different policy environments.

Our work is related to that of Stock and Watson (1993), who also test the null hypothesis of no shift in the money demand equation. However, we differ in the specification of the maintained hypothesis in several key respects. First, Stock and Watson specify that the interest rate is integrated of order 1. We reject this specification based on its implication, documented below, that the interest rate will with high probability go negative in a sample of the length of Lucas' (85 years of data.) Second, under Stock and Watson's maintained hypothesis, there is no break in the high frequency dynamics. We document that this proposition is overwhelmingly rejected by the data.

The paper is organized as follows. In Section 2 we report estimates of restricted and unrestricted money demand equations, based on the whole sample and based on subsamples. These results constitute the basic facts that motivate our analysis. Section 3 presents a formal description of our null and maintained hypotheses. In Sections 4 and 5 we present our classical and Bayesian analyses, respectively. Finally, Section 6 concludes.

2. Empirical Results for Money Demand

The money demand literature contains a great variety of analyses. In order to focus our analysis a sharply as possible, we concentrate on the functional form, data set, and econometric procedure used in Lucas (1988).

2.a. The Money Demand Function

Lucas (1988, 1993) displays a class of economic environments which imply a relationship between real balances, m_t, the rate of interest, r_t, and "permanent income," $y_{p,t}$:

$$(2.1) \quad \log m_t = \alpha_c + \alpha_y \log y_{p,t} + \alpha_r r_t + u_t$$

where $Eu_t = 0$. The disturbance, u_t, may reflect shocks to the transactions technology (that is, money demand shocks), and/or measurement error in log m_t, and log $y_{p,t}$. We follow Lucas by estimating the income elasticity of demand, α_y, and the interest semi-elasticity, α_r, by running an ordinary least squares regression using (2.1). One set of assumptions which guarantees that ordinary least squares estimates are consistent for α_y and α_r is that log $y_{p,t}$, log m_t, and r_t are integrated of order one and cointegrate. Roughly, this requires that u_t have finite variance and that log $y_{p,t}$ and r_t have a unit root, that is, Δ log $y_{p,t}$ and Δr_t have spectral densities which are nonzero at frequency zero.[4] Here, Δ is the first-difference operator. The intuition surrounding this result will be important for interpreting our empirical results, so we briefly review it. The fitted disturbance from the regression in (2.1) is

$$(2.2) \quad \hat{u}_t = (\alpha_y - \hat{\alpha}_y) \log y_{p,t} + (\alpha_r - \hat{\alpha}_r) r_t + (\alpha_c - \hat{\alpha}_c) + u_t$$

where a variable without a hat denotes its true value and a hat indicates an estimate. In a large sample, the variance of log $y_{p,t}$ and r_t are arbitrarily large under the unit root assumption, and the variance of $\hat{u}_t$ is infinite for any parameter estimates such that $\hat{\alpha}_y \neq \alpha_y$ or $\hat{\alpha}_r \neq \alpha_r$. Thus, the finite variance assumption on u_t implies that, in a large sample, the only way to get the variance of $\hat{u}_t$ to be finite is to set $\hat{\alpha}_y = \alpha_y$ and $\hat{\alpha}_r = \alpha_r$. Consistency of least squares follows from the fact that it chooses $\hat{\alpha}_y$, $\hat{\alpha}_r$ to minimize residual variance. Because in a large sample the variance of the dependent variable is infinite, while the residual variance is finite, it follows that, asymptotically, the R^2 of the least square regression is unity. In contrast to regression in the standard covariance stationary framework, interaction between u_t and the explanatory variables (as would be expected under our measurement error interpretation) does not affect the consistency of the least squares estimator. Basically, the low frequency considerations involved in achieving a finite variance in $\hat{u}_t$ completely swamp the impact on estimation of higher frequency interactions between u_t and the right hand variables. In a small sample, though, the role of the low frequency considerations may be reduced, in which case estimates of the parameters of long-run money demand are contaminated by high frequency dynamics, that is, simultaneity bias.[5] One possible signal that a sample may not be large enough to rule out simultaneity bias is a low estimated R^2.

A problem with the assumption that r_t is integrated, is that although this specification can capture the considerable persistence observed in r_t, it also has the counterfactual implication that r_t can be negative. Concern about this implication for r_t leads us to explore alternative representations, including ones in which r_t is stationary. As the intuition described above suggests, under these circumstances OLS is still a consistent estimator for α_y. At the same time, it is not consistent for α_r. However, we conjecture that as long as the variance of r_t is large relative to the covariation between r_t and u_t, then OLS will be approximately consistent.

2.b. Estimation Results

We use Lucas' annual data, covering the period 1901–1985. Real balances are measured by M1 divided by the deflator for net national product. Permanent income is measured as a geometrically weighted sum of current and past net national product with weight 1/3. Finally, r_t is measured as a short-run commercial paper rate. For further details about the data, see Lucas (1988, footnote 3).[6]

Empirical estimates of the money demand coefficients are reported in Table 1. That table provides results for the restricted case, $\alpha_y \equiv 1$, and for the unrestricted case. Also, results are provided for the whole sample, 1901–1985, and the pre-1958, and post-1957 subsamples. The 1958 date is taken from Lucas (1988). He broke the sample in 1958 because that was the end of the data sample used in Meltzer (1963), and Lucas' objective was to evaluate Meltzer's money demand equation in light of subsequent data. The fact that the 1958 break date was selected for reasons unrelated to the data themselves simplifies the sampling theory used in this paper considerably.[7]

For the evidence of instability in money demand, compare rows 1 and 3. These give the unrestricted regression results for the whole period and the post-1957 periods respectively. As noted above, the estimated income elasticity drops substantially, from around 1 to around 0.2. If we let δ_y denote the difference between the elasticity relevant for the whole sample and the one relevant for the post-1957 sample, then the estimate of this quantity, $\hat{\delta}_y$, implied by the results in Table 1, is 0.77. The interest semi-elasticity also drops substantially, from (in absolute value) 8 percent to 1 percent. (The interest rate data are measured in units of percent.) Letting $\hat{\delta}_r$ denote the estimate of this difference, Table 1 implies $\hat{\delta}_r = -0.07$.

Another indication of substantial instability is evidenced by the reduced performance of the unit income elasticity hypothesis as one goes from either the whole sample or the pre-1958 sample, to the post-1957 sample. To see this, consider the f-statistic reported in the sixth column of Table 1. That statistic measures the percent increase in the sum of squared residuals (SSR) induced by imposing the restriction $\alpha_y \equiv 1$. According to row 2, the restriction induces only a 1.6 percent increase in SSR over the whole sample, and a 5.9 percent increase when the pre-1958 sample alone is considered. By contrast, the restriction induces a 725 percent increase in SSR in the post-1957 sample. A way to get a sense of the magnitude of this increase is to note that the product of f and the degrees of freedom of the unrestricted regression is the usual f-statistic. This product is 181.25. Under simple textbook regression assumptions, this number would be a realization from a F-distribution with 1 numerator and 25 denominator degrees of freedom under the null hypothesis $\alpha_y = 0$. The tail area to the right of 181.25 under this distribution, is 0 to 6 digits after the decimal. We summarize the evidence of instability in the unrestricted regressions with the following three statistics:

(2.3) *Evidence of Instability*: $\hat{\delta}_y = 0.77, \quad \hat{\delta}_r = -0.07, \quad \hat{f} = 7.25.$

The evidence of instability just described may not reflect a change in the underlying money demand function, but instead may reflect the small sample simultaneity-type problems that we referred to above. This view seems consistent with the drastic reduction in the R^2 of the regression in the second part of the sample (compare rows 1 and 3). In addition, as noted in the introduction, Lucas (1988) draws attention to evidence in Figure 1 which supports a presumption that M1 demand is stable. We quantify that apparent stability using the restricted regression results in the even-numbered rows in Table 1. According to the results there, the estimated constant term in the regression covering the pre-1958 period, less the corresponding object covering the post-1957 period, is $\hat{\delta}_c = 0.209$. The analogous expression for the interest elasticity is $\hat{\delta}_s = -0.0231$ (the subscript s signifies "slope"). Thus,

(2.4) *Evidence of Stability*: $\hat{\delta}_c = 0.209, \quad \hat{\delta}_s = -0.0231.$

Lucas (1988) conjectures that it would be hard to reconcile this evidence of stability with the hypothesis that long-run M1 demand has been unstable.

3. Formalizing the Stable Money Demand Hypothesis

The null hypothesis considered in this paper is that the parameters of the money demand equation are constant throughout the sample. As usual, additional assumptions are required to actually implement a test of this hypothesis. These comprise the maintained hypothesis, which specifies such things as the functional form of the money demand equation, and the time series representation of the high frequency dynamics. Since there is no natural single specification of the maintained hypothesis, we are led to consider several. Our bootstrap procedure for testing the null hypothesis requires that we capture the null and maintained hypotheses in the form of a fully parameterized data generating mechanism (DGM). We explain how we do this below.

3.a. Modeling the High Frequency Dynamics

We adopt the following canonical representation:

$$(3.1)\quad A(L)Y_t = c + \epsilon_t$$

where,

$$(3.2)\quad Y_t = \begin{bmatrix} \Delta \log y_{p,t} \\ f(r_t, r_{t-1}) \\ u_t \end{bmatrix}.$$

Here, u_t is defined in (2.1), and $f(r_t,r_{t-1})$ is discussed below. In (3.1), ϵ_t is uncorrelated over time and

$$(3.3)\quad A(L) = I - A_1L - \cdots - A_pL^p$$

where L is the backshift operator. Also, c is a 3×1 vector of constants. Throughout the analysis, we set $p = 2$. Below, we report diagnostics on the adequacy of this specification.

When $f(r_t,r_{t-1}) = r_t - r_{t-1}$, (2.1) and (3.1) form a time series representation that has been used extensively for modeling cointegrated variables in other applications (see, for example, Hansen and Sargent (1991), Phillips (1991), Campbell (1987), and Campbell and Shiller (1987, 1989)). In addition, Hoffman and Rasche (1991), and Stock and Watson (1993) use it to model money demand data.

3.b. Three Versions of the Stable Money Demand Hypothesis

The null hypothesis of a stable money demand asserts that the parameters in the cointegrating relation between log $y_{p,t}$, r_t, and log m_t are constant. In order to test this null hypothesis, we have to take a stand on the nature of the high frequency dynamics, that is, the parameters in (3.1). We formulate three distinct maintained hypotheses about these parameters: The *No Break Maintained Hypothesis*, the *High Break, Innovation Maintained Hypothesis*, and the *High Break, VAR Maintained Hypothesis*.

Under the No Break Maintained Hypothesis (No Break), the parameters in (3.1) are constant throughout the sample. This maintained hypothesis, together with the specification $f(r_t, r_{t-1}) = r_t - r_{t-1}$, is the one underlying the test for stability of the long-run money demand function executed by Stock and Watson (1993).

Our method for testing the null hypothesis requires that we have a fully parameterized data generating mechanism which embodies the null and maintained hypotheses. We obtain parameter values for the money-demand equation from the first row of Table 1, and use the implied fitted disturbances to measure u_t. We then obtain values for the VAR parameters in (3.1) by applying least squares equation by equation to Y_t, using the period 1904–1985 as the estimation period, and 1901–1903 for initial conditions. This way of assigning values to the model parameters is statistically consistent under the No Break Maintained Hypothesis. We refer to this parameterized model, together with the distribution of the fitted values of the ϵ_t's, as the No Break Data Generating Mechanism (DGM). We analyze this DGM by simulating 5,000 artificial data sets from it, conditioning on the 1901–1903 values of log $y_{p,t}$, r_t, and log m_t. The ϵ_t's needed for the simulations are obtained by drawing, with replacement, from the empirical fitted values. Drawing the ϵ_t's in this way imposes the assumption that the ϵ_t's are independently and identically distributed over time. We investigate the empirical plausibility of this assumption in Subsection 3.d.

Lucas (1988) has little confidence in the No Break Maintained Hypothesis' assumption of stability in high frequency dynamics. Our other two maintained hypotheses allow for instability in the high frequency dynamics. The High Break, Innovation Maintained Hypothesis (HB/I), holds c and the parameters of A(L) constant throughout the sample, and draws the ϵ_t's from a different distribution in the pre- and post-1958 samples. Thus the High Break, Innovation DGM is just like the No Break DGM except that the first 54 ϵ_t's are drawn from the pre-1958 fitted ϵ_t's and the next 28 ϵ_t's are drawn from the corresponding post-1957 fitted values. Under the High Break, VAR Maintained Hypothesis (HB/VAR), the ϵ_t's are drawn from a different distribution in the post-1958 period and all other parameters in (3.1) are permitted to change as well, including the elements of c, which control the drift in the variables. We refer to the model estimated in this way as the High Break, VAR DGM.

Two of the three maintained hypotheses described above are capable of capturing arguments spelled out in Lucas (1988). His explanation of the apparently conflicting regression results in Table 1 is based in part on shifts in the trend in log m_t and r_t (in our context, a change in c), and in part on a change in the dynamic interaction between the variables (that is, a change A(L) and/or in the variance-covariance matrix of the ϵ_t's). To summarize his trend argument, it is useful to first difference (2.1) and then apply the expectations operator to both sides:

(3.5) $E\Delta \log m_t = \alpha_y E\Delta \log y_{p,t} + \alpha_r E\Delta r_t$

since $E\Delta u_t = 0$. According to (3.5), to be consistent with the basic trends in the data, the least squares estimates must satisfy a line formed by replacing the population means in (3.5) by their sample counterparts. In the pre-1958 data, r_t is roughly trendless. At the same time the trends in log $y_{p,t}$ and log m_t are roughly identical, so that matching trends implies $\hat{\alpha}_y$ close to unity. Trend considerations in this period place no restrictions on $\hat{\alpha}_r$. In the post-1957 data, there is a substantial trend in r_t, while the trend in log m_t falls and the trend in log $y_{p,t}$ remains roughly unchanged. As a result, being consistent with the trends in the post-1957 data implies a menu of possible values of $(\hat{\alpha}_y, \hat{\alpha}_r)$. It happens that the pre-1958 parameter estimates are an element in this menu, and this explains why the restricted post-1957 regression results (Table 1, row 4) are so similar to the unrestricted pre-1958 results (row 5). But, the unrestricted least squares algorithm applied to the post-1957 data obviously prefers a very different $(\hat{\alpha}_y, \hat{\alpha}_r)$ combination in this menu. This can be explained by appealing to a change in the dynamic interactions in the variables in the post-1957 subsample. According to Lucas, least squares is driven away from the pre-1958 point in part because the variability in r_t increases relative to the variability in log m_t. This results in a positive covariance between the money demand disturbance and the interest rate in the post-1957 subsample when the pre-1958 interest elasticity is imposed. This explains why the interest semi-elasticity is closer to zero in the post-1957 sample. This argument is similar to a standard simultaneity-bias argument.

The High Break, Innovation Maintained Hypothesis attributes all of the evidence of instability to a change in the variance-covariance matrix of the VAR disturbances. Drawing the post-1957 ϵ_t's from a different distribution allows the variance of r_t to change relative to that of log m_t. It also allows the correlation of r_t and u_t to change. By allowing the values of c and A(L) to also change in 1958, the High Break, VAR Maintained Hypothesis allows for a richer change in the dynamic interactions and has the potential to account for the trend shifts as reflecting a change in drift.

3.c. Interest Rate Implications of Our Time Series Representations

To complete our specification of the data generating mechanisms, we need to specify the interest rate transformation, f. When incorporated into one of the DGM's discussed above, f has implications for the dynamic behavior of the interest rate. In this section we consider several transformations and select two based on the plausibility of their implications for the mean and the sign of the interest rate.

A standard specification of f is $f(r_t,r_{t-1}) = r_t - r_{t-1}$ (see Stock and Watson (1993) and Hoffman and Rasche (1991)). We show that this transformation has the troublesome implication that r_t can go negative with high probability. In the real world, simple arbitrage considerations suggest that this is impossible. We rule out this transformation on the principle that models which predict the impossible with high probability are inadmissible (see Harvey 1990). We also consider the following alternative choices for f: $f(r_t,r_{t-1}) = \log(r_t) - \log(r_{t-1})$, $f(r_t,r_{t-1}) = \log(r_t)$, $f(r_t,r_{t1}) = r_t$, and $f(r_t,r_{t-1}) = g(r_t)$. The latter corresponds to $f(r_t,r_{t-1}) = r_t$, modified slightly to guarantee non-negativity of r_t:

$$(3.4)\quad g(r) = \left\{ \begin{array}{ll} r, & r \geq 1 \\ 1 + \log(r), & 0 < r < 1 \end{array} \right]. \tag*{}$$ [8]

Panels A and B in Table 3 reports the interest rate implications for two specifications of $f(r_t,r_{t-1})$ which allow r_t to go negative. The first row in each of Panels A and B reports the fraction of 5,000 realizations, each of length 85 periods, in which r_t was negative at least once. The second row reports the fraction of times that the sample average was negative. The last row reports the mean, across the 5,000 replications, of the sample average of r_t. With the exception of the last row, numbers in parentheses are Monte Carlo standard errors.[9] In the last row, numbers in parentheses are the mean, across replications, of the sample standard deviation of r_t.

The results in Table 3 show that the interest rate goes negative with very high probability under the two interest rate transformations considered in Panels A and B. For example, under the first difference specification the High Break, Innovation DGM produces a negative interest rate at least once in 53.3 percent of realizations of length 85 years. Even the 85 year average is negative 8.3 percent of the time. Thus, we are led to dismiss the level and first difference specifications of f.

Next consider panels C, D, and E in Table 3, which report results for transformations that restrict the interest rate to be nonnegative. In the U.S. data, the mean, and standard deviation of r_t are 4.41 and 2.91 percent, respectively. Two of the transformations are consistent with this. However, one is not. In particular, $f(r_t,r_{t-1}) = \log(r_t) - \log(r_{t-1})$, has the implication that the average, across replications, of the mean rate of interest ranges from 13 to 41 percent per year, depending on which representation we consider. The standard deviation across replications is 30 to 168 percent per year. These results lead us to focus on the transformations reported in Panels C and E in our analysis.[10]

3.d. Diagnostic Checks

As noted before, our model simulation procedure presumes that the ϵ_t's are i.i.d. Because this assumption plays such an important role in our analysis, it is particularly important that we test it. We apply standard diagnostic tests for residual autocorrelation and conditional heteroscedasticity. We report results for the representations based on the transformation $f(r_t,r_{t-1}) = g(r_t)$ in Table 5. Results for representations based on $f(r_t,r_{t-1}) = \log(r_t)$ are similar and so we do not report them. Panel A in Table 4 reports autocorrelation tests, while Panel B reports tests for first order conditional heteroscedasticity.

The first column which is labeled "No Break" reports results for (3.1) estimated over the whole sample, 1903–1985, assuming (2.1) is constant. The next two columns are relevant for the High Break, Innovation DGM. They analyze the pre-1958 and post-1957 subset of the column 1 residuals, respectively. The final two columns are relevant for the High Break, VAR DGM. They analyze the residuals of equations fit separately to the pre-1958 and post-1957 subsamples.

An examination of these statistics reveals some evidence of conditional heteroscedasticity in the output equation disturbance for the No Break DGM (see the 4.7 percent p-value in the first rows of Panel B). Also there is some evidence of residual autocorrelation in the disturbances of the High Break, Innovation DGM (see columns 2 and 3). However, there is little evidence of residual autocorrelation or heteroscedasticity in the High Break, VAR model. This latter result is important to us because, in the end it is the High Break, VAR DGM that we focus on most closely.

4. Testing the Null Hypothesis that Long-Run M1 Demand is Stable

Recall that in Section 2, we described three pieces of evidence characterizing the instability in empirical money demand equations. In this section we use these to test the null hypothesis that long-run M1 demand is stable. The main analysis is done using the specification, $f(r_t,r_{t-1}) = g(r_t)$. Our analysis is based on the three maintained hypotheses about the high frequency dynamics discussed in Section 3. We also examine the robustness of our results to using the $f(r_t,r_{t-1}) = \log(r_t)$ specification.

4.a. Results for the g(r) Specification

Consider Figure 2, which reports three scatter plots, each containing 5,000 realizations of $(\hat{\delta}_r,\hat{\delta}_y)$. These realizations are based on calculations done on artificial data generated by our three DGM's. The vertical and horizontal lines in each scatter plot indicate the empirical magnitudes of $\hat{\delta}_r$ and $\hat{\delta}_y$ (see Section 2). The frequency of points falling in each quadrant is reported, and the number in parentheses is the associated Monte Carlo standard error.[11] Consider the scatter plot with the heading, "No Break." There, the null hypothesis of no break in the interest elasticity is rejected at the 3.7 percent significance level (see the sum of the frequencies in the bottom two quadrants). The null hypothesis of no break in the income elasticity is rejected at the 5.5 percent significance level. The other two scatter plots, which accommodate a change in high frequency dynamics, fail to reject the null hypothesis that money demand is stable.

The results in Table 5 provide additional insight to the findings in Figure 2. The first column of Panels A and B in Table 5 report estimated money demand elasticities taken from Table 1. The next three columns report the mean and standard deviation across 5,000 simulated data sets of OLS estimates of money demand elasticities. The three columns correspond to the three DGM's incorporating the g(r) specification discussed in the previous section.[12] Note that significant post-1957 subsample instability only occurs in simulated data that allow for a break in the high frequency dynamics.

Panel C pertains to our third piece of evidence of instability: The value of the f-statistic, $\hat{f}$ (see (2.3)). The statistics reported in Panel C overwhelmingly reject the null hypothesis (at the 1.3 percent significance level) under the No Break Maintained Hypothesis. However, the null hypothesis is not rejected when the maintained hypothesis allows for a break in the high frequency dynamics.

4.b. Evidence of Robustness

To investigate the robustness of our conclusions to the choice of f, we redid the calculations in 4.a. above, using DGM's that incorporate $f(r_t, r_{t-1}) = \log(r_t)$. We redid the scatter plots in Figure 2, and found essentially the same results: The null hypothesis is rejected under the No Break Maintained Hypothesis[13], but the null hypothesis is not rejected under the other two maintained hypotheses. However, when we test the null hypothesis using the f statistic, we reject it for all three maintained hypotheses. The p-values are: 0.0 (0.0), 0.002 (0.001), 0.004 (0.001), for the No Break, High Break, Innovations and High Break, VAR Maintained Hypotheses. Numbers in parentheses are Monte Carlo standard errors. Thus, the finding of the previous subsection is not robust to this change in the maintained hypothesis.

5. Which is More Plausible, Stability or Instability of M1 Demand?

In the previous section we reported several maintained hypotheses under which the null hypothesis fails to be rejected. We found it surprising that it is possible to reconcile the observed instability in estimated money demand equations with the null hypothesis. Still, this result seems fragile: It is overturned by an apparently small change in the maintained hypothesis. In particular, the null hypothesis is rejected when we replace the g(r) specification of f with the log(r) specification. This drives us to pose the question: "Which perspective is more plausible, the one suggested by the results based on log(r), according to which M1 demand is unstable, or the one suggested by g(r), according to which M1 demand is stable?"

We investigate this question by pursuing the strategy of Christiano and Ljungqvist (1988). Namely, we identify data generating mechanisms which formalize each perspective, and select the most plausible one based on its ability to account for the relevant facts. In this study, these are the evidence of stability and instability reported in Section 2. We find that by this criterion, the perspective suggested by the g(r) specification is the most plausible.

We already have two DGM's which capture the perspective that money demand is stable, namely, the High Break, Innovation and High Break, VAR DGM's which incorporate the g(r) specification. In contrast, the log(r) results drive us to think about breaks in money demand. In the context of the present study, it is natural to posit a break in 1958. To capture the instability perspective, we estimate three new DGM's using the log(r) specification. The cointegrating vectors used in these DGM's are the pre-1958 and post-1957 estimates reported in Table 1. The three DGM's are differentiated according to how they handle the high frequency dynamics. In particular, we adopt the three specifications analyzed in the previous section. We call these DGM's the Low Break DGM, the Low Break, Innovation DGM and the Low Break, VAR DGM. For the sake of symmetry, we also estimate these DGM's for the g(r) specification. Counting the three DGM's based on the log(r) specification studied before, gives us a total of 12 DGM's to use in evaluating the question that motivates this section.

Our results are reported in Table 6. The entries in this table are the frequency of times that various events occur in 5,000 artificial data sets simulated from each of our 12 DGM's. The numbers in parentheses are Monte Carlo standard errors. We identify seven different events that summarize the evidence of stability and instability discussed in Section 2. The events, I and S, are boxes constructed about $(\hat{\delta}_y,\hat{\delta}_r)$ and $(\hat{\delta}_c,\hat{\delta}_s)$ respectively. The event, F, is an interval constructed about $\hat{f}$. For details about these intervals, see the notes to Table 6. We also consider four additional events defined by the various combinations of I, S, and F. Results pertaining to the g(r) and to the log(r) specifications appear in Panels A and B, respectively. We use the entries in the table to evaluate the relative likelihood of alternative DGM's.

Consider first the results pertaining to the event $I \cap S \cap F$ in Panel A. Consistent with our previous results, the High Break, VAR DGM is more likely than the No Break DGM. Interestingly, the Low Break, VAR DGM is about one-third as likely as the High Break, VAR DGM. That is to say, according to the g(r) specification, the hypothesis of a stable money demand is substantially more plausible than the alternative, when we jointly take into account the evidence of stability and instability. It is particularly notable that the stable money demand hypothesis is more likely than the unstable version, even when we focus exclusively on the evidence of instability (that is, the events $I \cap F$, I and F). So the g(r) specification strongly favors the stable money demand hypothesis.

The results for the log(r) specification in Panel B are quite different. Relative to the $I \cap S \cap F$ event, the stable money demand hypothesis is roughly as plausible as the alternative that there was a substantial break in 1958.

Comparing across Panels A and B, we find that the best DGM which incorporates the g(r) specification is over five times more likely than the best DGM which incorporates the log(r) specification. We conclude from the results in this table that the null hypothesis of a stable money demand equation is much more plausible than the alternative.

We investigate the robustness of this finding by comparing the likelihood of the best g(r) DGM with that of 54 other DGM's. To begin, we considered the 24 DGM's obtained by constructing the entries in Table 6 for the other specifications of $f(r_t, r_{t-1})$ described in Section 3. Our best g(r) DGM is four times more likely than the best of the $\log(r_t)-\log(r_{t-1})$, $g(r_t)-g(r_{t-1})$, and $r_t - r_{t-1}$ DGM's. The results for the r_t specification are virtually identical to those reported in Panel A of Table 6 for $g(r_t)$. For example, the frequency of the event $I \cap S \cap F$ is 0.037 with a Monte Carlo standard error of (0.003) and 0.004 (.001) for the High Break, VAR and Low Break, VAR DGM's, respectively.[14]

Next, we considered the 30 DGM's formed by constructing the entries in Table 6 for all five specifications of f, using 3-lagged VAR's rather than 2-lagged VAR's (that is, we set $p = 3$). Our best 2-lagged DGM based on the g(r) specification dominates all these models too. Of these 30 DGM's the 15 which capture the unstable money demand hypothesis are of particular interest. Of these, the most plausible DGM is the log(r) Low Break, VAR DGM. For it, the frequency of the event, $I \cap S \cap F$, is 0.0304, with Monte Carlo standard error, (0.0024). This frequency is less than that of our best $p = 2$, g(r) model, even when Monte Carlo sampling uncertainty is taken into account. Presumably, the latter model dominates even more when account is taken of the fact that the Low Break, VAR DGM with $p = 3$ has 21 more parameters.[15] The results of these calculations are discussed in detail in Braun and Christiano (1993).

We conclude that the stable money demand hypothesis is more plausible than the alternative.

6. Conclusion

In a recent paper, Lucas (1988) presented an example of the kind of evidence that has led many in the literature to conclude that 20th century data cannot be characterized by a single, stable long-run money demand function. He showed that the long-run empirical money demand elasticities based on 85 years of data differ greatly from the corresponding quantities estimated using only the post-1957 data. He sketched an argument that is capable, in principle, of reconciling this evidence of instability with the existence of a stable long-run money demand equation that emerges from Figure 1. To our surprise, we found a simple linear data generating mechanism which (i) verifies Lucas' argument and (ii) passes stringent tests for empirical plausibility.

Footnotes

[1]See Laidler (1977), Judd and Scadding (1982), and Goldfeld and Sichel (1990) for a review of this literature.

[2]See, for example, Braun (1994a), Chari, Christiano, and Kehoe (1991), Kydland (1989), den Haan (1991), Christiano (1991), Christiano and Eichenbaum (1992a,b), Cooley and Hansen (1989, 1991), Cho and Cooley (1990), King (1990), Hodrick, Kocherlakota, and Lucas (1991), Marshall (1992), and Sims (1989).

[3]Kareken and Wallace (1980 pp. 2-3) argue that transaction cost models fail this specification test by pointing to the alleged instability in empirical money demand equations.

[4]There is a large literature on the subject of cointegration and integration, and on the consistency result described in the text. See Engle and Granger (1987), Phillips (1991), Stock (1987), and West (1988), and the references they cite.

[5]Note, although consistency of ordinary least squares is unaffected by simultaneity bias considerations, inference generally is not. In particular, the asymptotic sampling distribution of the ordinary least squares estimator is influenced by the degree of interaction between right-hand variables and the disturbance term, Phillips (1991).

[6]Our data reflect a slight correction made by Lucas after publishing his paper.

[7]For a discussion of the complications that arise in contexts where the break date is selected after looking at the data, see Christiano (1992).

[8]We thank Julio Rotemberg for suggesting the $g(\cdot)$ transformation to us.

[9]To compute the Monte Carlo standard errors, we note that the first four rows in Table 3 are the average, across artificial data sets, of an indicator function, $I_r(A)$, which is one if a specified event A is true on the r^{th} data set, and 0 otherwise. Denote the average of this function by $I'(A)$. Then, because $I_r(A)$ is an i.i.d. random variable, the standard deviation of $I'(A)$ is just the standard

deviation of $I_r(A)$ (which we estimate by the standard deviation of $I_r(A)$ across our 5,000 data sets), divided by the square root of the number of replications, 5,000. We refer to this as the Monte Carlo standard error of $I'(A)$.

[10]In results not reported in Table 3, we also investigated and rejected the $f(r_t,r_{t-1}) = g(r_t) - g(r_{t-1})$ transformation. We found that this transformation has the implication that interest rates can often be very close to zero for extended periods of time. To see why this happens, it is useful to note that the estimated VAR parameters for all three representations corresponding to this transformation are virtually identical to the representations corresponding to the $f(r_t,r_{t-1}) = r_t - r_{t-1}$, transformation. This is because $g(r_t) = r_t$ for virtually every observation in the U.S. data. Therefore, to understand the interest implications of representations based on $f(r_t,r_{t-1}) = g(r_t) - g(r_{t-1})$, one can look at the results in Panel A of Table 3. They indicate that these representations imply a substantial amount of variation in $g(r_t)$, and, in particular, imply that this object can go negative for long periods of time. But, this maps into r_t being roughly zero for long periods of time.

[11]These plots leave out six observations for the No Break DGM and eight observations for the HB/VAR DGM. Including these points would make it difficult to discern the patterns in the plots.

[12]Notice that the standard errors are quite large in the post-57 subsample for the No Break and the High Break, Innovation models. These large standard errors are due to the influence of a small number of realizations. On rare occasions these two models produce realizations of the interest rate that are very close to zero for the entire post-1957 subsample. This in turn results in estimates of the interest semi-elasticity that are very large. For the No Break model there are eight realizations where the estimated interest semi-elasticity is greater than two in the short sample and for the High Break, Innovation model there are 12 such events. Leaving out the eight realizations for the No Break model results in a mean and standard error for the post-1957 interest semi-elasticity of -0.027

(0.053). Leaving out the 12 realizations for the High Break, Innovation DGM results in an interest elasticity mean and standard error of -0.005 (0.074).

[13]The p-value testing the null hypothesis that there is no instability in the income elasticity is 1.9 percent. For testing that there is no instability in the interest semi-elasticity, it is 12 percent.

[14]These results are available on request from the authors.

[15]An extra lag in the VAR with a break in 1958 introduces 18 extra parameters, and the break in money demand introduces three extra parameters. We did not formally investigate the quantitative importance of the degrees of freedom issue.

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Table 1: Ordinary Least Squares Regression Results

Dependent Variable: $\log m_t$

Line	Years	Coefficient on: constant	$\log y_{p,t}$	r_t	f[1]	R^2
1	1901-1985	1.830	.975	−.0847		.9631
2	1901-1985	1.722	1.0	−.0874	.0163	
3	1958-1985	5.683	.205	−.0148		.3281
4	1958-1985	1.548	1.0	−.0683	7.251	
5	1901-1957	1.371	1.08	−.0788		.9629
6	1901-1957	1.757	1.0	−.0914	.0593	

[1] f = (RSS-USS)/USS, where RSS is the sum of squared residuals of the money demand regression restricted to have a unit income elasticity and USS is the sum of squared of residuals of the unrestricted regression.

Table 2: Sample Averages

Line	Years	average annual changes in: $\log m_t$	$\log y_{p,t}$	r_t
1	1901-1985	.0242	.0310	.0427
2	1901-1957	.0336	.0313	−.0100
3	1958-1985	.0052	.0304	.1500

Table 3: Interest Rate Implications of Alternative Specifications of $f(r_t, r_{t-1})$[1]

	No Break	HB/I	HB/VAR
Panel A: $f(r_t, r_{t-1}) = r_t - r_{t-1}$			
freq($r_t < 0$ at least once)	.323	.533	.498
	(.007)	(.007)	(.007)
freq($\bar{r}_t < 0$)	.027	.083	.062
	(.161)	(.277)	(.242)
$\bar{r}$	7.001	4.828	5.862
	(4.100)	(3.381)	(3.585)
Panel B: $f(r_t, r_{t-1}) = r_t$			
freq($r_t < 0$ at least once)	.291	.524	.590
	(.006)	(.007)	(.007)
freq($\bar{r}_t < 0$)	.034	.103	.042
	(.001)	(.002)	(.001)
$\bar{r}$	6.401	4.712	4.542
	(2.660)	(2.702)	(1.174)
Panel C: $f(r_t, r_{t-1}) = \log(r_t)$			
$\bar{r}$	5.788	5.085	4.550
	(3.174)	(2.795)	(1.587)
Panel D: $f(r_t, r_{t-1}) = \log(r_t) - \log(r_{t-1})$			
$\bar{r}$	18.799	13.470	41.098
	(46.533)	(29.902)	(168.309)
Panel E: $f(r_t, r_{t-1}) = g(r_t)$[2]			
$\bar{r}$	6.448	4.912	4.591
	(2.555)	(2.403)	(1.114)

[1] freq (A) denotes the fraction of replications, out of 5,000, in which the event A is satisfied. r_t is the nominal rate of interest, in percent terms. $\bar{r}_t$ denotes the sample average of the interest rate. Numbers in parentheses for the "freq" rows are Monte Carlo standard errors. Numbers in parentheses in the $\bar{r}$ rows are the average, across 5,000 replications, of $\bar{r}_t$. The associated numbers in parentheses are standard deviations.

[2] $g(r) = r$ for $r > 1$, $g(r) = 1 + \log r$ for $0 < r \leq 1$.

Table 4: Analysis of VAR Residuals

For Level, $g(r_t)$ Models[1]

	No Break	HB/I		HB/VAR	
	1903-1985	1903-1957	1958-1985	1903-1957	1958-1985
Panel A: Autocorrelation Analysis[2]					
ϵ_y					
1st order	−.009	−.047	.253	−.034	.017
	(.933)	(.728)	(.181)	(.797)	(.927)
3rd order	.109	.094	.011	.097	.198
	(.316)	(.484)	(.955)	(.467)	(.295)
Q stat	32.828	21.039	34.730	21.831	13.409
	(.203)	(.457)	(.002)	(.409)	(.495)
ϵ_r					
1st order	.051	.119	−.044	−.017	−.089
	(.642)	(.374)	(.815)	(.897)	(.639)
3rd order	.199	.390	.008	.169	.083
	(.068)	(.003)	(.965)	(.207)	(.662)
Q stat	18.739	37.332	8.429	18.993	8.761
	(.879)	(.015)	(.866)	(.586)	(.846)
ϵ_u					
1st order	.037	.130	−.040	.010	−.055
	(.738)	(.332)	(.832)	(.943)	(.771)
3rd order	.092	.255	−.032	.130	.022
	(.397)	(.056)	(.867)	(.332)	(.908)
Q stat	15.748	32.730	7.450	32.503	6.058
	(.958)	(.049)	(.916)	(.052)	(.965)
Panel B: Test for First Order Conditional Heteroscedasticity[3]					
ϵ_y	3.931	1.018	.580	.639	.066
	(.047)	(.313)	(.446)	(.424)	(.797)
ϵ_r	.503	1.010	.001	.307	.453
	(.478)	(.315)	(.977)	(.580)	(.501)
ϵ_u	.043	.012	.305	.005	1.217
	(.835)	(.915)	(.581)	(.944)	(.270)

[1] ϵ_y, ϵ_r, ϵ_u are the first, second and third elements of ϵ_t in (3.2). The VAR's indicated in column headings are the ones we had to estimate to construct our six Level, log r_t data generating mechanisms.

[2] We report 1st and 3rd order autocorrelations and the corresponding p-values that would be appropriate if these were autocorrelations of independent random variables. Also reported are Q-statistics and their associated p-values.

[3] We report the product of the number of observations and the R-squared of the regression of the squared fitted VAR disturbance on one lag of itself. In parentheses is the p-value of this statistic assuming it is a realization from a chi-square distribution with one degree of freedom. (See Engle (1982) for a discussion of this statistic.)

Table 5: Regression Results in Data and $f(r_t, r_{t-1}) = g(r_t)$ Specification

	MODELS[3]			
Variable[1]	U.S. Data[2]	No Break	HB/I	HB/VAR
Panel A: Whole Sample Unrestricted OLS (1901–85)				
α_y	.975	.816	.901	.936
		(.185)	(.135)	(.063)
α_r	−.085	−.051	−.064	−.072
		(.018)	(.017)	(.014)
Panel B: Post-1957 Unrestricted OLS (1958–85)				
α_y	.205	.766	.304	.302
		(.468)	(.456)	(.247)
α_r	−.015	−.016	.049	−.011
		(.386)	(1.945)	(.008)
Panel C: Post-1957 f Statistic				
f	7.251	1.049	4.811	9.861
		(1.734)	(5.628)	(9.545)
freq(f >7.25)		.013	.210	.493
		(.002)	(.006)	(.007)

[1]See Equation (2.1) and Table 1, note 2 for an explanation of the variables.

[2]Numbers taken from Table 1.

[3]Numbers are the average, over 5,000 artificial datasets generated by the indicated model, of the indicated variable. Numbers in parentheses represent standard deviations across datasets. The final three columns impose the restriction that the money demand disturbance is independent of y and r.

Table 6: Evaluation of the $g(r_t)$ and $\log(r_t)$ Specifications[1]

	No Break	HB/I	HB/VAR	Low Break	LB/I	LB/VAR
Panel A: $f(r_t,r_{t-1}) = g(r_t)$ Specification						
freq(I)[2]	.010	.120	.285	.131	.125	.135
	(.001)	(.005)	(.006)	(.005)	(.005)	(.005)
freq(S)[3]	.172	.196	.309	.049	.117	.114
	(.005)	(.006)	(.007)	(.003)	(.005)	(.004)
freq(F)[4]	.043	.272	.342	.292	.226	.153
	(.003)	(.006)	(.007)	(.006)	(.006)	(.005)
freq(I ∩ F)	.003	.048	.110	.039	.028	.019
	(.001)	(.003)	(.004)	(.003)	(.002)	(.002)
freq(I ∩ S)	.001	.027	.103	.010	.017	.023
	(.000)	(.002)	(.004)	(.001)	(.002)	(.002)
freq(S ∩ F)	.006	.057	.107	.016	.030	.015
	(.001)	(.003)	(.004)	(.002)	(.002)	(.002)
freq(I ∩ S ∩ F)	.000	.010	.039	.004	.004	.003
	(.000)	(.001)	(.003)	(.001)	(.001)	(.001)
Panel B: $f(r_t,r_{t-1}) = \log(r_t)$ Specification						
freq(I)	.004	.050	.138	.131	.148	.195
	(.001)	(.003)	(.005)	(.005)	(.005)	(.006)
freq(S)	.186	.202	.354	.049	.107	.122
	(.006)	(.006)	(.007)	(.003)	(.004)	(.005)
freq(F)	.002	.013	.057	.302	.295	.153
	(.001)	(.002)	(.003)	(.006)	(.006)	(.005)
freq(I ∩ F)	.000	.001	.013	.045	.045	.035
	(.000)	(.000)	(.002)	(.003)	(.003)	(.003)
freq(I ∩ S)	.000	.010	.060	.008	.019	.038
	(.000)	(.001)	(.003)	(.001)	(.002)	(.003)
freq(S ∩ F)	.000	.002	.023	.014	.032	.021
	(.000)	(.001)	(.002)	(.002)	(.002)	(.002)
freq(I ∩ S ∩ F)	.000	.000	.006	.003	.004	.007
	(.000)	(.000)	(.001)	(.001)	(.001)	(.001)

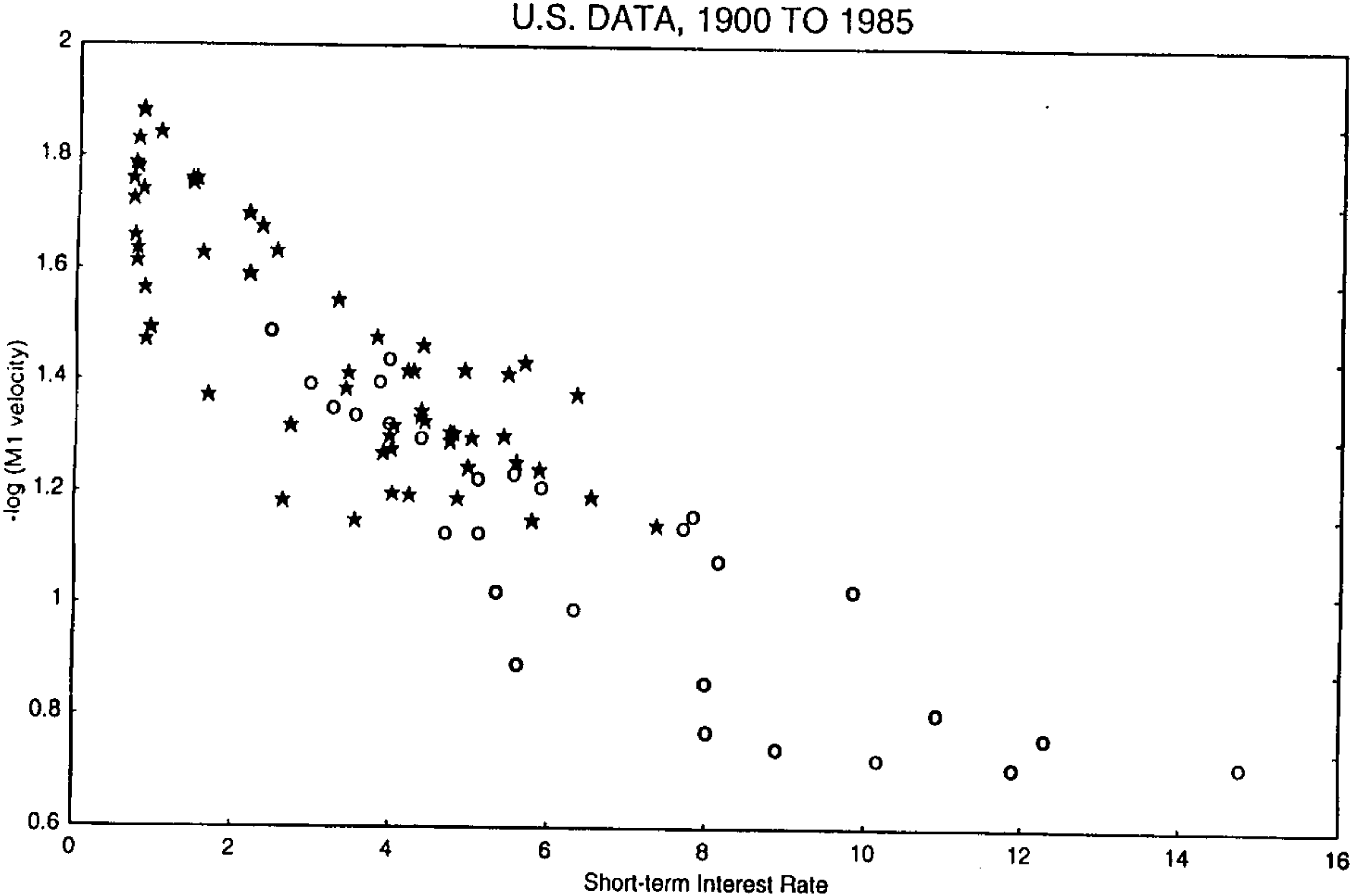
[1]Table contains frequency, out of 5,000 artificial datasets generated by the indicated model, that the indicated event, or intersection of events, is satisfied.

[2]I = event $\{\hat{\delta}_{y,r} \in \hat{\delta}_y \pm 0.25, \hat{\delta}_{r,r} \in \hat{\delta}_r \pm 0.01\}$, where $\hat{\delta}_y$, $\hat{\delta}_r$ are defined in equation (2.3), and $\hat{\delta}_{y,r}$, $\hat{\delta}_{r,r}$ are the simulated values of $\hat{\delta}_y$, $\hat{\delta}_r$ in the r−th artificial dataset, r = 1 to 5,000.

[3]S = event $\{\hat{\delta}_{c,r} \in \hat{\delta}_c \pm 0.25, \hat{\delta}_{s,r} \in \hat{\delta}_s \pm 0.01\}$, where $\hat{\delta}_c$, $\hat{\delta}_s$ are defined in equation (2.4), and $\hat{\delta}_{c,r}$, $\hat{\delta}_{s,r}$ are the simulated values of $\hat{\delta}_c$, $\hat{\delta}_s$ in the r−th artificial dataset, r = 1 to 5,000.

[4]F = event $\{\hat{f}_r \in \hat{f} \pm 3\}$, where $\hat{f}$ is defined in equation (2.3) and $\hat{f}_r$ is the simulated value of $\hat{f}$ in the r−th artificial dataset, r = 1 to 5,000.

Figure 1



★ - denotes observations from 1900 to 1957

o - denotes observations from 1958 to 1985

Figure 2
ACCOUNTING FOR THE EMPIRICAL EVIDENCE OF INSTABILITY