I have benefitted from the comments of John Boyd and Ed Prescott. They are not responsible for the contents of the paper, however.

The views expressed are those of the author and not necessarily those of the Federal Reserve System. The material contained is of a preliminary nature, is circulated to stimulate discussion, and is not to be quoted without permission of the author.
ABSTRACT

An overlapping generations model is developed that contains labor markets in which adverse selection problems arise. As a response to these problems, quantity rationing of labor occurs. In addition, the model is capable of generating (a) random employment and prices despite the absence of underlying uncertainty in equilibrium; (b) a statistical (nondegenerate) Phillips curve; (c) procyclical movements in productivity; (d) correlations between aggregate demand and unemployment (and output); (e) an absence of correlation between unemployment (employment) and real wages. In addition, the Phillips curve obtained typically has the "correct" slope. Finally, the model reconciles the theoretical importance and observed unimportance of intertemporal substitution effects, and explains why price level stability may be a poor policy objective.
The original work on the Phillips curve (Phillips (1958), Samuelson and Solow (1960)) derived correlations between the level of unemployment and the rate of change of money wage rates or money prices. However, little or none of the large theoretical literature on the Phillips curve proceeds in a setting where money and unemployment are present simultaneously in a general equilibrium model. As an example, perhaps the two most prominent explanations of unemployment have been developed in the search and the implicit contracting literatures respectively. These efforts at explaining unemployment have thus far not permitted the incorporation of money. On the other hand, perhaps the most prominent theoretical model of a Phillips curve with money—that of Lucas (1972)—does not permit labor to be unemployed. Thus it would seem that there is a gap to be bridged between theoretical and empirical Phillips curves. The object of this paper is to provide a model which permits the presence of both money and unemployment. This will be done in a way which generates a correlation between unemployment and (either wage or price level) inflation exactly of the nature observed by Phillips. In short, the model will provide an explanation of observed Phillips curves.

In addition, the model will provide a quite simple explanation of other observed features of the business cycle. In particular, the model is capable of generating the following phenomena: (a) the measured productivity of labor moves procyclically; (b) a correlation between "aggregate demand" and unemployment exists; (c) real wage movements are acyclic. It will be noted that together (b) and (c) imply that, under certain circumstances to be elaborated below, aggregate demand will appear to have more to do with the level of unemployment than do real wages. Moreover, this will be true even though in the model, aggregate demand policies may have no ability to influence the magnitude of output movements.
The economic setting which gives rise to these results is as follows. First, there is a sequence of overlapping generations. This feature permits a role for money in the model. Second, members of each young generation supply labor to firms. Their labor income, in turn, is required for savings and consumption. Third, each young generation consists of a heterogeneous group of agents who vary both in their preferences over alternative consumption-leisure streams, and in their marginal productivities when employed. Fourth, any agent's productive attributes are private information unless he can be induced to reveal them. The presence of this private information will create an "adverse selection" problem in labor markets. Fifth, in order to overcome this adverse selection problem, employers will offer workers wage-hours packages which use quantity rationing as a self-selection incentive. This feature generates the potential for unemployment in the model.

It will be seen that this structure can give rise to the following equilibrium outcomes. First, firms may use employment lotteries in equilibrium. This will provide a source of randomness in the economy despite the absence of shocks to preferences, endowments, or technology, and despite the fact that all uncertainty concerning agents' characteristics is resolved in equilibrium. Second, since some workers face randomized employment prospects (and, therefore, a random income stream), there will be shocks to the arguments of money demand functions, and hence price level fluctuations. This will provide a correlation between movements in unemployment and rates of change in money prices. Third, if the government attempts, for instance, to stabilize prices, this will generate a correlation between "aggregate demand" movements and the level of unemployment.

In short, then, this paper shows that the presence of adverse selection problems in labor markets is by itself sufficient to give rise to unem-
ployment. Such problems are also sufficient to generate prominent business cycle phenomenon of the type mentioned. This is why shocks to technology, preferences, or endowments, or shocks generated by government behavior are scrupulously excluded from the analysis. This is done merely in order to demonstrate that adverse selection problems alone can account for most of the qualitative features of the business cycle. When exogenously arising shocks to technology are also incorporated, Smith (1983) demonstrates that very simple versions of the model at hand can readily confront several quantitative features of the business cycle as well.

In addition, the model also permits a reconciliation of other theoretical and empirical results on economic behavior over the business cycle. In particular, intertemporal substitution effects have played an important role in recent theories of the business cycle. However, attempts to uncover such effects empirically have not been notably successful. In the economy studied below, intertemporal substitution effects are an important allocative mechanism. Specifically, they function so as to enhance the self-selection incentives created by employment lotteries. But, since self-selection is an equilibrium outcome, these effects will not be observable in any data the economy generates. Thus, even though intertemporal substitution effects play a role in determining levels of employment, current empirical attempts to find evidence of such effects will not be successful.

By way of a final comment prior to presenting the model, it is useful to relate this effort to some of the literature on quantity rationing in labor markets.¹ This literature proceeds in a setting where prices need not be market clearing, so that rationing schemes are equilibrating devices.

¹/E.g., Bennassy (1975), or Hildenbrand, Larocque, and Younes (1978).
Many of the models in this class also contain a commodity identified as money, although it is now recognized that this is inessential to the models. Thus, a literature does exist which, while often not explicitly concerned with the Phillips curve, does allow for unemployment and (one might argue) the presence of money. This paper differs from such literature in three main regards. First, the role for money is explicit. Second, the reason why quantity rationing arises is made explicit. This contrasts with the literature cited where there is no reason given why markets should not clear. Finally, in existing macroeconomic literature on quantity rationing, particular rationing schemes are imposed rather than derived. In the model of this paper quantity rationing schemes arise endogenously. This eliminates the arbitrariness of rationing schemes in the earlier literature.

The format of the paper is as follows. Despite the fact that the model is fairly simple, it has a large number of diverse features. Therefore, for expository purposes, Section I sets forth a static version of the model, and demonstrates that unemployment may be used as a self-selection device by firms. Some problems with the static model are pointed out. Section II presents an augmented version of the model which permits the introduction of money. A Nash equilibrium with money is then defined. Sections III and IV present the main results. These are as follows.

1) An overlapping generations model containing labor markets where adverse selection problems are a feature is capable of explaining observed Phillips curves.

2) The "shocks" which underly this Phillips curve are neither "real" nor "nominal," as these terms are typically used, but arise as an endogenous outcome in the model.
3) Intertemporal substitution effects play an important allocative role, but will be unobservable in economic data.

4) Certain government policies will imply a (perfect) correlation between government "stimuli" to "aggregate demand," and levels of output and unemployment.

5) This is true despite the fact that these "stimuli" may be purely passive, and unable to influence the magnitude of output movements.

6) The model explains observed procyclical movements in measured productivity.

7) "Problems" which arise in the static version of the model are resolved by the introduction of money and the focus on a dynamic setting.

In addition, it is demonstrated that some traditional views regarding roles for money are incorrect. Specifically, it is often suggested that the presence of money reduces the social costs of acquiring a given level of information about goods and services being exchanged. It is demonstrated as a by-product of the analysis that the social costs of fixed information acquisition can rise when money is introduced into an economy. Finally, these sections contain some additional results which serve to indicate the range of possible equilibrium outcomes for the set of economies at hand. Section V concludes.

At this point, a final comment on the mode of analysis is in order. Subsequent arguments are often illustrated by example. The reason for this is that a characterization of when certain outcomes will arise is impossible at any practical level given current knowledge of adverse selection settings.

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2/ See, e.g., Brunner and Meltzer (1971).
Section I contains some comments on this point, and some of the results of Section III may be viewed in part as further evidence on the impossibility of generalizing the analysis. In order to maintain a perspective on this lack of generality, it will be useful to keep in mind the relatively specific nature of other analyses of Phillips curves, such as Lucas' (1972), or of treatments of unemployment, such as search theoretic treatments.

I. A Static Model With Unemployment

The purpose of this section is to lay out a simple static model of a labor market in which unemployment arises in response to an adverse selection problem. This serves as a prelude to expanding the model so as to include money. As the tone of the paper is largely illustrative, the model presented is the simplest one possible which illustrates the points of interest.

A. The Model

We consider in this section a model in which there are three general classes of agents. The first we term entrepreneurs, whom throughout we equate with firms. The second and third classes consist of agents who will be firm employees, or workers. Thus, there are two types of workers, indexed by $i = 1, 2$. The difference between firms and workers is that workers are endowed with labor which they can sell, but are not endowed with access to a technology for converting labor into produced commodities. For entrepreneurs the situation is reversed. The differences between the two types of workers are twofold; type 1 and 2 agents differ in their preferences over consumption-leisure bundles, and in their marginal products in production.

There are only two goods in this economy, labor and a produced consumption good. In addition, there is an exogenously given set of events $E$, with a typical element of $E$ denoted by $e$. Then we indicate the consumption of
a type i agent in event e by $C_i(e)$, and the number of hours worked by a type i agent in e by $L_i(e)$. The preferences of a type i agent over consumption-labor pairs are denoted by $U_i: \mathbb{R}_+^2 \rightarrow \mathbb{R}$, and all agents have von Neumann-Morgenstern preferences. We will variously employ the following assumptions on the $U_i$:

(i) $U_i \in C^2$

(ii) $D_1 U_i(C,L) > 0$

(iii) $D_2 U_i(C,L) < 0$

(iv) $U_i$ concave,

where $D_j$ is the derivative of what follows with respect to its $j^{th}$ argument. Finally, all workers are endowed with a single unit of labor, and nothing else.

As indicated above, production is carried out entirely by firms, with there being free entry among a finite number of entrepreneurs into the activity of goods production. Entrepreneurs care only about consumption, and are risk neutral. Technology is as follows. For each unit of type i labor hired by any firm, $\pi_i$ units of the consumption good can be produced. The $\pi_i$ are scalar constants obeying $\pi_1 > \pi_2$. In addition, the values of the $\pi_i$ are known by firms. However, the index of any worker is private information ex ante and, moreover, each firm observes only its total output ex post, i.e., the individual contribution of any worker to production is not directly observable by firms. Thus, an agent's index (and marginal product) is revealed to firms iff they can induce workers to sort themselves according to some observable attribute. In the static version of the model the only such attribute is the number of hours that any agent is willing to work under various circumstances. Thus, an agent's index (type) is identifiable by firms iff
for some \( e \in E \) in equilibrium and, therefore, different agents receive different wage rates if, and only if, \( L_1(e) \neq L_2(e) \) for some \( e \).

Let \( w_i(e) \) denote the (real) wage rate received by type \( i \) agents in event \( e \). If the equilibrium outcome has \( L_1(e) \neq L_2(e) \) for some \( e \), then agents' types are distinguishable, agents of type \( i \) receive wage rate \( w_i(e) \) in \( e \), and firm profits per type \( i \) agent hired are \([\mu_i - w_i(e)]L_i(e)\) in \( e \). (Again, profits are measured in terms of the numeraire consumption good.) If, on the other hand, \( L_1(e) = L_2(e) \) in equilibrium, agents' indices are indistinguishable, and there is only a single market wage rate for each event, \( w(e) \). Under such circumstances the firm views each agent's marginal product as a random variable drawn from the population distribution of marginal products. Let \( \theta \) denote the proportion of the population of consisting of type \( 1 \) agents. Then the mean population value for marginal products is \( \bar{\mu} = \theta \mu_1 + (1-\theta) \mu_2 \). In a case where types were unobservable in equilibrium, then, per capita expected profits in event \( e \) would be \([\bar{\mu} - w(e)]L(e)\) for each firm, where \( L(e) \) is the common value of the \( L_i(e) \). Each firm, as indicated above, is an expected profit maximizer, where expectations are taken with respect to the information revealed in equilibrium.

At this point, it may not be obvious why an exogenous set of events has been introduced given that preferences, endowments, and technology are not subject to randomness. The reason for its introduction is that firms may offer workers employment contracts which are different types of lotteries in order to induce sorting of worker types by contract accepted. The set \( E \) is important only in that its cardinality restricts the number of different lottery outcomes that can be specified in the contracts offered by firms.

Finally, it remains to make one additional assumption on preferences across workers of different types. This is
(v) \( \forall (C, L) \in [0, \pi_1] \times [0, 1], \)

\[
\begin{align*}
\frac{3C}{3L} \bigg|_{dU_1 = 0} & \neq \frac{3C}{3L} \bigg|_{dU_2 = 0}.
\end{align*}
\]

(v) implies the existence of some kind of correlation between productivities and preferences. In order to interpret this, one might think of the \( U_i( ) \) as indirect utility functions derived from a model of home production in which workers of different types have similar preferences over home-produced goods and other goods, but in which productivities at home and in the marketplace are correlated. One would think of the natural case as being a positive correlation, and in fact we focus below primarily on the case

\[
(\nu') \quad \frac{3C}{3L} \bigg|_{dU_1 = 0} > \frac{3C}{3L} \bigg|_{dU_2 = 0}
\]

for any value \((C, L)\). This also happens to be the case which leads to unemployment of labor. In any event, it is the attempt by firms to find self-selection incentives which exploit (v) that gives rise to the interesting features of the subsequent analysis.

B. Equilibrium

In this paper, a Nash equilibrium concept is imposed on firms which compete for the services of the workers described above. This competition occurs through the announcement of wage employment packages offered by each firm. The fact that there is free entry into production, or equivalently, that firms may compete for the same set of workers implies that in equilibrium

\[3/\text{I would like to thank Ron Michener for suggesting this interpretation.}\]
a number of "no surplus" conditions must be satisfied by these announcements. In order to elaborate on these, and to motivate the remainder of the equilibrium conditions, it will be useful to begin with a discussion of the set of wage-hours packages which firms might offer.

To begin, then, as firms choose the wage-hours packages which they offer, they must decide whether or not they wish to discriminate between workers of different types. If they wish to do so, then they must induce type 1 and 2 workers to select different sets of wage-hours combinations, i.e., they must induce self-selection among workers. This can typically be done in several ways, however. The simplest one is through exploitation of the correlation between preferences and productivity embodied in (v). Such exploitation will turn out to involve quantity rationing of labor, as we will see below. But to go beyond this simple version of quantity rationing, it might be the case that randomization of wages and employment levels can be used to induce self-selection (Smith (1982)). In this case firms will offer workers lotteries involving various wage-hours combinations.

It remains, then, to elaborate on the nature of these lotteries. As indicated previously, there is an exogenously-given set of states, $E$. The only function of this set is to determine the number of possible wage-employment packages that any worker may receive, which is $\#E$. In particular, firms may choose the wage received by a type $i$ worker in $e$, the number of hours worked in event $e$, and the probability that event $e$ will occur, $p(e)$, subject of course to

$$\sum_{e \in E} p(e) = 1.$$ 

In short, wages and hours might be determined by a coin flip, but we allow firms to choose the probability of the occurrence "heads." We assume also that lottery outcomes are independent across firms.
The choices of firms are subject to several restrictions, of course. The first of these is that the choices \( p(e) \), \( w^1(e) \), and \( L^1(e) \) must be consistent with self-selection, if this is the objective of the lottery. Second, the employees of any firm must not prefer to work for some other firm. Finally, it will be noted that lotteries are run by firms, i.e., firms determine the probabilities of their outcomes. Thus, we require that firms have no incentive to announce a probability \( p(e) \) for event \( e \), and then run a lottery with some other true probability for the occurrence of \( e \). This, in turn, requires that profits be equal across states of nature. In light of this restriction as well as the free entry assumption, then, firms are clearly restricted to choose \( w^1(e) = \pi_2 \circ e \) if \( L^1(e) \neq L^2(e) \) for some \( e \), and \( w^1(e) = \pi \circ I \circ e \) if \( L^1(e) = L^2(e) \circ e \).

These equilibrium conditions have several consequences. First, if \( L^1(e) \neq L^2(e) \), the lottery \( \{p(e), L^2(e)\}_{e \in E} \) must be maximal for type 2 agents over the set of such lotteries given that \( w^2(e) = \pi_2 \circ e \). If this were not the case, some other firm could offer an alternate lottery which type 2 workers prefer, and which earns nonnegative profits. Second, if \( L^1(e) \neq L^2(e) \), the lottery \( \{p(e), L^1(e)\}_{e \in E} \) must be maximal for type 1 agents over the set of lotteries consistent with self-selection, given that \( w^1(e) = \pi_1 \circ e \). Third, if \( L^1(e) = L^2(e) \), type 2 agents are in some sense "mimicking" type 1 agents. Then, for the same reason as above, the lottery \( \{p(e), L(e)\}_{e \in E} \) (where \( L(e) \) is the common value of \( L^1(e) \)) must be maximal for type 1 agents over the set of such lotteries given that \( w^1(e) = \pi \circ I \circ e \). Fourth, as we take firm behavior to be Nash, no firm can have an incentive in equilibrium to offer a different

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\(^1\)/Note that this restriction is a result of the assumed environment, i.e., of the assumption that the true probability of event \( e \) is not observable to workers.
lottery, given the actions of other firms. Finally, following Rothschild and Stiglitz (1976) and Wilson (1977), we also require that all lotteries are such that firms at least break even on each agent employed. Thus we have the following

**Definition.** An equilibrium is a mapping for each firm, \([w_1(e), w_2(e), L_1(e), L_2(e), p(e)] \): \(E \times R^4 \times [0,1]\), satisfying

(a) \[w_1(e) = \pi_1 \iff e \in E \text{ if } L_1(e) \neq L_2(e) \text{ for some } e, \]

(b) \[
\sum_{E} p(e) U_1 \left[ w_1(e)L_1(e), L_1(e) \right] > \\
\sum_{E} p(e) U_1 \left[ w_2(e)L_2(e), L_2(e) \right]
\]

(if \(L_1(e) \neq L_2(e) \) for some \(e\), this must be incentive compatible for type 1 agents, otherwise this holds trivially)

(c) \[
\sum_{E} p(e) U_2 \left[ w_2(e)L_2(e), L_2(e) \right] > \\
\sum_{E} p(e) U_2 \left[ w_1(e)L_1(e), L_1(e) \right]
\]

(self-selection for type 2 agents)

(d) \[p(e) \text{ and } L_1(e); e \in E \text{ maximize (given the values } L_2(e) \})
\[
\sum_{E} p(e)U_1 \left[ w_1(e)L_1(e), L_1(e) \right]
\]

subject to, (a), (c), and

\[
\sum_{E} p(e) = 1.
\]
(e) if \( L_2(e) \neq L_1(e) \) for some \( e \in E \), then \([C_2(e), L_2(e)]\) is maximal for type 2 agents in the set \([C_2(e), L_2(e)] : C_2(e) < w_2(e) L_2(e) \) \* \( e \in E \).

(f) there does not exist an alternate mapping \([\hat{v}_1(e), \hat{w}_2(e), \hat{L}_1(e), \hat{L}_2(e), \hat{p}(e)] : E \times R_+^+ \times [0,1] \) which, given the actions of other firms,

i) attracts any workers, and

ii) earns nonnegative expected profits given the workers it attracts.

We have used several facts to simplify the definition. First, (b) never holds with equality in equilibrium (as shown in Smith (1982)). Therefore, (second) type 2 agents will always face a trivial lottery if self-selection obtains, so the values \( p(e) \) are determined strictly by condition (d).

Finally, it will be noted that randomization of employment will occur only if \( L_1(e) \neq L_2(e) \) for some \( e \), and (c) holds with strict equality. If these conditions do not hold, concavity will dictate nonrandom outcomes.

Prior to proceeding, several comments about this definition are in order. First, it will be noted that the consumption of each agent may be contingent on at most \#E events in the conditions defining an equilibrium. This implicitly rules out the trading of "unemployment insurance" claims between agents employed by different firms. The reason for such a restriction is that it substantially simplifies the analysis. It is readily verified that none of the results of the paper, as well as none of the results referred to in Smith (1982) are dependent on this assumption; i.e., the restriction is not binding in the demonstrations of the propositions which follow. However, for more general economies the restriction will bind. We do not comment further
on this, but it should be noted that other models of unemployment and the Phillips curve, as well as other adverse selection models, also rule out trading of this type.\footnote{E.g., Lucas (1972) rules out trading between "islands." Also, the implicit contracting literature clearly requires an absence of markets in "employment" insurance. In the adverse selection insurance literature, it is a standard assumption that individuals deal only with a single insurance company.} Thus, this is a common restriction.

Second, in adverse selection settings of the type under discussion the appropriate equilibrium concept is itself a matter of controversy. The equilibrium notion employed here is a straightforward variant of the Nash concept of Rothschild and Stiglitz (1976). However, two problems with this definition are that an equilibrium need not exist, and even if equilibria do exist, they may be suboptimal. Thus, some comments on the choice of equilibrium concept are in order.

One point of note is that we could easily have adopted the notion of equilibrium introduced by Wilson (1977), for which an equilibrium here would always exist. This would necessitate only minor and obvious changes in the wording of what follows, since when a Nash equilibrium exists, it coincides with the Wilson equilibrium. This approach has not been taken since the phenomena of macroeconomic interest arise here precisely in the case when a Nash equilibrium does exist. Thus, employing the Wilson equilibrium concept would not expand the scope of the discussion.

With regard to the second problem, game forms which induce optimal allocations have yet to be devised for the settings under discussion. However, it is readily checked that optimal allocations (supported through the use of money) have all of the qualitative features of the Nash equilibria discussed below (when these exist), so that again the discussion could be focused
on Pareto optimal allocations with only minor and obvious changes in the wording of the arguments which follow.

Finally, it will be useful as a matter of terminology to define two additional terms. The first, a nonstochastic equilibrium, is defined for two reasons. In particular, it will often be useful to consider the simplest possible circumstance, which is one where firms are assumed not to employ lotteries. Moreover, this situation will serve as a useful reference point in what follows. Therefore, we provide the following

**Definition.** A nonstochastic equilibrium is an equilibrium satisfying the additional restriction

\[(g) \quad L^e_i = L^e_{i*}, \quad \nu^e_i = \nu^e_{i*}, \quad \nu^e_i, \quad \nu^e_{i*} \in \mathbb{E}, \quad i = 1, 2,\]

and with (f) amended to

\[(f') \quad \text{there is no alternate mapping} \quad (\hat{\nu}_1(e), \hat{\nu}_2(e), \hat{L}_1(e), \hat{L}_2(e), \hat{p}(e)) : \mathbb{E} \to \mathbb{R}_+ \times [0, 1] \text{ which}
\]

\[\text{i) attracts any workers,}
\]

\[\text{ii) earns nonnegative expected profits given the workers it attracts and}
\]

\[\text{iii) satisfies} \quad \hat{L}_i(e) = \hat{L}_i(e*), \quad \hat{\nu}_i(e) = \hat{\nu}_i(e*) \forall e, \quad e* \in \mathbb{E}, \quad i = 1, 2.\]

Finally, we wish to define what is meant by unemployment. The usage of this term is quite conventional. Let \(\hat{L}_i(e)\) be the utility maximizing value of labor supply for type i agents in event e given the prevailing equilibrium (real) wage rate. Thus \(\hat{L}_i\) is the "notional labor supply" of type i agents, which is independent of e by condition (a). Then
Definition. An unemployment equilibrium is an equilibrium in which $L_i(e) < \tilde{L}_i$ for some $i$ for some $e$.

C. Results

The following propositions are demonstrated in Smith (1982), and are merely repeated here.

(1) There exist economies where a nonstochastic equilibrium fails to exist, but which do have an equilibrium with nondegenerate employment lotteries.

(2) There exist economies with a nonstochastic equilibrium, and a Pareto superior (stochastic) equilibrium.

(3) Any equilibrium has $L_1(e) \neq L_2(e)$ for some $e$ (i.e., types are revealed in any equilibrium).

(4) There exist economies where unemployment of labor is an equilibrium outcome.

Notice that together (1) and (2) imply that nondegenerate employment lotteries are sometimes an equilibrium outcome. This is the case despite the following implication of (3): in any equilibrium all ex ante uncertainty is resolved. In particular, there are no shocks to preferences, technology, or endowments. In addition, uncertainty about agents' types is resolved prior to the occurrence of any economic activity. Therefore, there is no objective uncertainty in equilibrium. Nevertheless, employment and output may be stochastic.

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6/ This is a straightforward extension of a result in Rothschild-Stiglitz (1976).
Unfortunately, useful conditions characterizing when employment lotteries occur do not appear possible to obtain in this setting. In order to illustrate this, Smith (1982) presents an example in which two economies are identical except that the marginal utility of consumption differs across type 2 agents in the two economies. Moreover, preferences in the two economies can be made arbitrarily close in the sense of Kannai (1970). Nevertheless, one economy displays adverse selection problems, unemployment, and employment lotteries in equilibrium. The other behaves as if there were full (public) information. Therefore, small perturbations of economic data can drastically effect the nature of an equilibrium, preventing useful characterizations of when certain outcomes arise. For this reason, the remainder of this paper is illustrative, as it is difficult to comment on the generality of certain results to be obtained.

It remains in this section to indicate the nature of an equilibrium with unemployment. It suffices for this purpose to focus on a nonstochastic equilibrium. An economy with a nonstochastic unemployment equilibrium is depicted in Figure 1. In this figure, the loci labelled $C = \pi_1 L$ are the zero profit loci for firms employing type 1 agents, and the loci labelled $U_i$ are indifference curves for type i agents. If the economy depicted were one with full public information regarding workers' types, the equilibrium values of the $L_i$ for this economy would be $L_1^*$ and $L_2^*$, which are the levels of employment where an indifference curve for a type i agent is tangent to the $C = \pi_1 L$ locus. Now consider the same economy with private information. As indicated above, in a nonstochastic equilibrium $L_1 \neq L_2$. Therefore, agents' types are observable, so

$$L_2^* = \arg\max \{U_2(\pi_2 L, L)\} = L_2^*$$
Figure 1
An Unemployment Equilibrium
as before. \( L_1 \) is determined in equilibrium by the condition that \((C_1, L_1)\) must be maximal for type 1 agents among the set of \((C, L)\) pairs resulting in non-negative firm profits, and which are incentive compatible. Thus, the equilibrium value of \( L_1 \) is the solution to the problem

\[
\max U_1(\pi_1 L, L) \text{ subject to } U_2(\pi_1 L_1, L_1) < U_2(\pi_2 L^*, L^*).
\]

Now, clearly the incentive compatibility constraint is binding for the economy at hand, as \((\pi_1 L^*, L^*)\) is preferred to point A by type 2 agents. Therefore, \( L_1 \) is determined by

\[
U_2(\pi_1 L_1, L_1) = U_2(\pi_2 L^*, L^*),
\]

the solution to which is point B in Figure 1.\(^{I/}\)

It will be noted that in this equilibrium \( L_1 < L^* \), so that there is unemployment of labor. This quantity constraint arises to resolve adverse selection problems in the economy. Thus, it is the case that the economy under consideration provides a rationale for rationing; i.e., quantity rationing is not imposed \textit{a priori} as in much of the macroeconomic literature on quantity constrained equilibria.

Finally, it will also be noted that the set of unemployed agents in the economy are workers with high marginal products. This will be the case in any such static economy with unemployment. While this is actually not at variance with U.S. experience (Smith (1982)), it does contrast with normal perceptions of which agents in an economy experience unemployment. Also it

\(^{I/}\)We can guarantee that this is, in fact, an equilibrium (i.e., rule out existence problems) by appropriate choice of \( \theta \).
should probably be viewed as a shortcoming of the model that only one type of worker experiences unemployment in this setting. However, as we shall demonstrate in the next two sections, these aspects of the model can be rectified as we move to consideration of a dynamic model with money.

II. The Model With Money

A. Description

In this section we expand the model of Section I to permit the introduction of money. This will allow an analysis of the relationship between rates of increase in money prices (or money wage rates) and the rate of unemployment in the model. As the economic setting of this section is a fairly straightforward extension of the static one, the description of the economy can be fairly brief.

In order to accommodate a role for money, then, the economy of Section I is embedded here in an overlapping generations model. The nature of this model is as follows. Time is discrete, and indexed by \( t = 0, 1, \ldots \).

At \( t = 0 \), there is an initial old generation, which is endowed with the entire stock of fiat money, \( M \), and nothing else. For the present, we take this stock to be fixed across time and states of nature (this will be relaxed below). At \( t = 0 \), there is also an initial young generation consisting of three types of agents: entrepreneurs, and workers of types 1 and 2. Entrepreneurs are again endowed only with access to technology. The endowments of type \( i \) workers are as follows. When young, all workers are endowed with a single unit of labor, and nothing else. When old, all workers are retired (they have no endowment of labor), and type \( i \) workers have endowment \( n_i \) of the single consumption good, which is also produced as previously. As is clear from the context, all agents are two period lived, and at \( t = 1 \), there is a new young generation,
etc. In order to keep complications to a minimum, all generations are identical in size and composition (except for possibly the initial old), and all generations have access to the same technology (described in Section I). Ex ante availability of information, and the transmission of information are the same as before, with one possible exception to be discussed below. Finally, we assume that there are \( N \) entrepreneurs at each date (\( N \) finite), and hence at most \( N \) firms.

Each young agent, it will be noted, cares only about consumption (of the single good) when young, consumption when old, and hours worked when young. As previously, consumption and employment levels may be determined by lotteries run by firms, despite the absence of underlying uncertainty in the model. We retain our assumption that trading of claims in employment insurance is ruled out (this is inessential for all of the results in Section III below), and also rule out insurance markets for old age consumption. (This is inessential as well, and serves merely to economize on notation.) Nevertheless, for reasons to be elaborated, consumption in both periods may depend for any agent on the outcome of the lotteries of all firms. Recall that there are at most \( N \) firms. Then, as before, each firm has at most \( \#E \) possible outcomes for its lottery. Let \( E = \bigtimes_{j=1}^{N} E_j \), so that \( \#E = N(\#E_j) \). Let a typical element of \( E \) be denoted \( e \). Then we denote the hours worked by a type \( i \) agent in event \( e \) by \( L_i(e) \) (this is determined by the lottery of his employer only), the youth consumption of a type \( i \) agent in event \( e \) by \( C_i^1(e) \), and the old age consumption of a type \( i \) agent by \( C_{i1}^2(e, e') \) if the sequence of events \( e, e' \) is experienced. Throughout we use \( e' \) (or \( e' \)) to denote "next period's event." We focus only on (stochastic) steady states, so we omit dates on these variables.

In order to complete our description of trading in this economy, we describe the instruments available to any agent for transferring goods between
periods. There are two of these. First, agents may borrow and lend. We let $x_i$ denote the borrowing of a type $i$ agent (which will depend typically both on whether or not agents are distinguishable, and on the event $g$), which takes place at the gross interest rate $R(g)$ in $g$. Second, agents may hold money. We let $M_i$ denote the nominal balances of a type $i$ agent (which will also depend on the identifiability of agents, and on $g$). We retain our choice of the consumption good as numeraire, and let $S(g)$ be the goods value of money in $g$. ($S(g)$ is the inverse price level in $g$.)

Type $i$ workers have preferences over nonnegative triples $(C_1, C_2, L)$ denoted by $U_i: \mathbb{R}^3_+ \times R$. Given these preferences, it remains only to describe the manner in which type $i$ agents choose their portfolios. To this end, notice that agents accept a particular wage-employment lottery prior to the occurrence of production. Thus agents discover their period one income only subsequent to accepting a given set of wage-hours opportunities. They then select their portfolios after the realization of the outcome of the complete set of lotteries. Thus their income depends on how, and whether, they have been identified, and (possibly) on the outcome of a lottery run by their employer. Therefore, consider an arbitrary type $i$ agent who has received income $y$ and level of employment $L$ as a result of the state of information and the realization $e$ at his firm. The current value of money is $S(g)$, the current interest rate $R(g)$, and the agent knows the objective probabilities of future events, and hence of future values $S(g')$. Therefore, his choice of portfolio is made so as to solve the (competitive) problem

$$\max E U_i \left[ C_{1i}(g), C_{2i}(e, g'), L \right]$$

subject to
\[ C_{1i}(e) < y + x_1(e) - S(e)M_1(e) \]
\[ C_{2i}(e,e') < n_1 + S(e')M_1(e) - R(e)x_1(e) \]

by choice of \( x_1(e) \) and \( M_1(e) \); \( M_1(e) > 0 \). This problem gives rise to the following demand correspondences for loans and money:

\[
M_i(e) = \Phi_i[S(e),\{S(e')\}_{e' \in E},R(e),y,L] \\
x_i(e) = \Psi_i[S(e),\{S(e')\}_{e' \in E},R(e),y,L]; i = 1, 2.
\]

We assume throughout that agents' portfolios are not observable by firms, so that these correspondences govern the portfolio choice of workers under any circumstances.\(^8\)

Finally, it will be convenient to make the following assumptions.

(vi) \( D_1 U_1(C_1,C_2,L) > 0; i, j = 1, 2 \)

(vii) \( D_3 U_2(C_1,C_2,L) < 0 \)

(viii) \( U_2 \) strictly concave, \( U_1 \) concave.

It will also prove convenient to restrict the set of lottery outcomes which can be employed by firms. To this end, we assume that a feature of the environment in which firms operate is as follows:

(ix) if \( L_1(e) \neq L_1(e^*) \) for some \( i \), for some \( e, e^* \in E \), then

either \( L_1(e) = 1 \), or \( L_1(e) = 0 \) for this \( i \).

\(^8\)/This is not essential in any results. It does seem the most natural assumption, however, in that agents have not selected their portfolios at the time they begin their association with a firm. It should also be noted that the timing of the events described in the text is not essential to the results which follow.
Assumption (ix) says that firms may offer only two types of lotteries: a degenerate lottery, or a lottery where workers are either employed "full time," or not at all.

Some comments on assumption (ix) are in order. First, the restriction on outcomes of a nondegenerate lottery is imposed only because we wish to solve for the equilibrium of some sample economies. This would not be a tractable problem without (a version of) (ix). Second, (ix) closely resembles some assumptions made in the implicit contracting literature regarding the indivisibility of labor. It should be thought of as somewhat analogous to these assumptions, but with the difference that here it is an inessential convenience. Third, the fact that firms are allowed to offer any level of employment in a degenerate lottery demonstrates that equilibrium employment lotteries are not a result of some "indivisibility" assumption. Finally, in light of (ix), it is obviously not a further restriction to also assume that \#E = 2. With these comments in mind, we are now prepared to define an equilibrium for this economy.

B. Equilibrium

The definition of a (Nash) equilibrium for this monetary economy closely parallels that for the static version of the same economy. The changes which are required are as follows. First, there are additional conditions requiring the clearing of (competitive) loan and money markets. Second, the labor market behavior of firms is amended to take account of the optimizing portfolio choices of each agent.

For the purpose of defining this equilibrium, it will be useful to provide some additional notation. Let n index operating firms, n=1, ..., N'; N' < N. Then let elements of the vector e take the form e = (e_1, ..., e_N'), where e_n is the event realization for firm n. Denote the N' - 1 vector e with
its $n^{th}$ element removed by $e_{-n} = (e_1, \ldots, e_{n-1}, e_{n+1}, \ldots, e_N)$. (It will be recalled that lottery outcomes are independent across firms.) Also, let

$$E_{-n} = E_1 \times \cdots \times E_{n-1} \times E_{n+1} \times \cdots \times E_N,$$

where $E_n$ is the set of events for firm $n$. Finally, define

$$V_1[w(e)L(e),L(e);g] = \sum_{E} p(g') u_1[w(e)L(e) +$$

$$\psi_i[-,w(e)L(e),L(e)] - S(g)\psi_i[-,w(e)L(e),L(e)], e_i +$$

$$S(g')\phi_i[-] - R(g)\phi_i[-],L(e)],$$

which is simply the expected utility of a type $i$ agent given employment $L(e)$ and income $w(e)L(e)$, in current period state $g$.

With this additional notation, we are now prepared to present the following

Definition. A (Nash) equilibrium with money is a mapping for each firm

$[\nu_1(e),\nu_2(e),L_1(e),L_2(e),p(e)]: E \times [0,1]$, and a mapping $[S(g),R(g)]: E \times \mathbb{R}^+$ satisfying

(i) either $L_1(e) = 1$ or $L_1(e) = 0 \forall e \in E$, or $L_1(e) = L_1(e^*)$

$\forall e, e^* \in E$

(ii) $\nu_1(e) = \pi_1 \forall e \in E$ if $L_1(e) \neq L_2(e)$ for some $e$,

$\nu_1(e) = \pi \forall e \in E$, $i = 1, 2$ if $L_1(e) = L_2(e) \forall e$

(k) $\sum_{E} p(g) V_1[w_1(g)L_1(g),L_1(g);g] >$

$\sum_{E} p(g) V_1[w_2(g)L_2(g),L_2(g);g]$
\[ \sum_{E} p(e) \mathbb{V}_{2}[w_2(e)L_2(e),L_2(e);e] > \]
\[ \sum_{E} p(e) \mathbb{V}_{2}[w_1(e)L_1(e),L_1(e);e] \]

(conditions (k) and (l) are the self-selection conditions; they hold trivially if \( L_1(e) = L_2(e) \), \( e \in E \))

(m) \( p(e) \) and \( L_1(e); e \in E \) maximize (taking the values \( L_2(e) \) and the choices of other firms as given)

\[ p \sum_{E-n} p(e-n) \mathbb{V}_{1}[w_1(1)L_1(1),L_1(1);(e-n,1)] + \]
\[ (1-p) \sum_{E-n} p(e-n) \mathbb{V}_{1}[w_1(2)L_1(2),L_1(2);(e-n,2)], 2/ \]

where \( p \equiv p(1) \), subject to (i), (j) and (l)

(n) if \( L_1(e) \neq L_2(e) \) for some \( e \) then,

\[ L_2(e) = \text{argmax} \{ \mathbb{V}_{2}[w_2(e)L_2(e),L_2(e);e] \}, e \in E. \]

(o) there does not exist for any firm an alternate mapping \( [\hat{w}_1(e),\hat{w}_2(e),\hat{L}_1(e),\hat{L}_2(e),\hat{p}(e)] \) satisfying (i) which, given the actions of other firms,

i) attracts any workers, and

ii) earns nonnegative expected profits given the workers it attracts

(p) \( \sum_{E} \psi_1(-) = 0 \), \( e \in E \)

(q) \( \sum_{E} \phi_1(-) = M \), \( e \in E \),

\[ 2/(e^n,e) \equiv e \] as a notational convenience.
where the summations in (p) and (q) are summations over members of each young
generation.\(^{10}\)

At this point, it will be useful to present a preliminary result
which will aid in interpreting some of the propositions to be presented in
Section III. This is

**Proposition 1.** If

\[
\frac{\partial y}{\partial L} \bigg|_{Dv_1(y,L;e)=0} \neq \frac{\partial y}{\partial L} \bigg|_{Dv_2(y,L;e)=0}
\]

\((y,L,e)\) such that \(y = \pi L\), then each worker's type is revealed in equilib­
rium.

Proof. Suppose the contrary. Then there will be a common value of \(L(e)\) for
all workers \(v\) (at each firm, since all firms have equivalent "best" strate­
gies with regard to inducing revelation of type). Therefore, by condition (m)
and concavity of \(U_i\), \(L\) is nonstochastic, and by condition (j), \(w_i = \pi \cdot v_i\).
Also, in order for such an arrangement to be an equilibrium, no firm can have
an incentive to change its actions, given the actions of other firms.

**Case 1.** Suppose that

\[
\frac{\partial y}{\partial L} \bigg|_{Dv_2=0} > \frac{\partial y}{\partial L} \bigg|_{Dv_1=0}
\]

at the values \(y = \pi L\) and \(L\) dictated by (m). Then suppose that some firm
alters its (nonstochastic) wage-hours package by offering income of \(y + \varepsilon\) and
hours \(L + \delta; \varepsilon, \delta > 0\), with \(\varepsilon\) and \(\delta\) satisfying

\(^{10}\)It should be noted that conditions (m) and (n) are consistent
with (i). This is true since (j) implies constant wages, and in light of
this, (m), (n), and concavity of the \(U_i\) will imply constant values of employ­
ment in the absence of lotteries.
Then it is readily verified that for $\varepsilon, \delta$ sufficiently small, type 1 agents prefer the new wage-hours package while type 2 agents prefer the initial package. Therefore, the deviant firm attracts only type 1 agents. Thus, for $\varepsilon, \delta$ again sufficiently small, the deviant firm must earn a positive profit. This contradicts the supposition that the initial arrangement could be an equilibrium. Hence, this case is impossible.

Case 2. This case is symmetrical to Case 1, but with \( (2) \) replaced by

\[
\left. \frac{\partial y}{\partial L} \right|_{d\nu_2=0} > \frac{\varepsilon}{\delta} > \left. \frac{\partial y}{\partial L} \right|_{d\nu_1=0}
\]

A contradiction can be derived in the same way, but with $\varepsilon, \delta < 0$, and with $\varepsilon, \delta$ satisfying

\[
\left. \frac{\partial y}{\partial L} \right|_{d\nu_2=0} < \frac{\varepsilon}{\delta} < \left. \frac{\partial y}{\partial L} \right|_{d\nu_1=0}
\]

which is possible by \( (4) \). This contradiction establishes the proposition.

Proposition 1 states that all uncertainty regarding workers' marginal products must be resolved in any equilibrium (so long as the fairly weak condition (1) is satisfied). Therefore, since there are no shocks to preferences, endowments, or technology, this class of economies displays no underlying uncertainty in equilibrium. Nevertheless, some economies in this class will give rise to endogenous business cycles.
Finally, it will be useful as a reference to define a nonstochastic equilibrium; i.e., an equilibrium in which firms are restricted to offer only degenerate lotteries. Thus, we provide the following

**Definition.** A nonstochastic (monetary) equilibrium is an equilibrium with (i) replaced by

\[(i') \quad L^*_i(e) = L^*_i(e^*) = e, \quad e^* \in E; \quad i = 1, 2,\]

and (o) replaced by

\[(o') \quad \text{there does not exist for any firm a degenerate lottery which, given the actions of other firms,}
\]
\[
\text{i) attracts any workers, and}
\]
\[
\text{ii) earns nonnegative expected profits given the workers it attracts.}
\]

An unemployment equilibrium is defined as previously.

**III. The Phillips Curve**

This section establishes the potential existence of a (stochastic) Phillips curve, and some of its properties for the economy of Section II. In demonstrating the presence and features of this Phillips curve little effort is made at attaining maximum generality. The reason for this is as follows. As indicated in Section I, there is little of generality to be said regarding when an equilibrium will exist, and if one exists, when employment lotteries will be an equilibrium outcome. As the model of Section I is a special case of the Section II economy, this comment applies here as well. In short, we have nothing useful to say about when stochastic Phillips curves will arise. However, we do show, for a certain class of economies, that if they arise they
will "normally" have an appropriate slope (relative to actual Phillips curves). Thus, while it is difficult to know in advance when these Phillips curves will be observed in equilibrium, our model requires very little structure to obtain an "appropriate" slope for them when do occur. This is not the case for other models of the Phillips curve.

Put briefly, the results of this section are as follows.

(1) An adverse selection economy with money can give rise to a statistical Phillips curve and a business cycle. This is true despite the absence of underlying uncertainty and the constancy of underlying parameter values.

(2) If there is only one operating firm in equilibrium, normality of second period consumption and a nonnegative derivative of savings with respect to real interest rates are sufficient for the Phillips curve to be "normally" sloped (if it exists).

(3) It is possible, though, for the Phillips curve to be incorrectly sloped.

(4) In the presence of money, type 2 agents may be unemployed. This is in contrast to the static setting, where only high productivity workers were unemployed.

In addition to these results, we digress to consider two other issues. First, we show that an intertemporal substitution-like effect plays an important allocative role in an equilibrium with a Phillips curve. In spite of this, however, an econometrician would be unable to discern the presence of such an effect in any data. Second, we digress even further to consider the social costs of obtaining certain information with and without money. We show that, in contrast to general assertions, the presence of money may increase the social costs of obtaining a given amount of information.
A. Existence of a Phillips Curve

The main result of this paper is, of course, that this economy with no friction other than an adverse selection problem can give rise to a statistical Phillips curve and a business cycle which are endogenous to the model, that is, which are not produced by exogenous shocks to the economy. This is Proposition 2. There exist economies in the class presented in Section II which display a negative correlation between the level of unemployment and the rate of inflation (in either wages or prices), both of which are stochastic.

The "proof" of this proposition consists of demonstrating the existence of such an economy. This is done in Example 1. The economy consists of \( N > 2 \) entrepreneurs, and a single worker of each type at each date. The fact that each generation has only two workers implies at most two firms. As type 2 agents will not face any randomness in equilibrium, we can without loss of generality focus on the case where there is a single firm in equilibrium, which, however, must deter the entry of other entrepreneurs into the activity of production. Technology, endowments, and preferences are as follows: \( \pi_1 = 2, \pi_2 = 1, \eta_1 = 0; i = 1, 2, \)

\[
U_2(C_1,C_2,L) = \frac{5}{4}C_1 + 2C_2 - \frac{1}{2}(L + 1)^2
\]

\[
U_1(C_1,C_2,L) = \begin{cases} 
10C_2 - \frac{(1 - \frac{1}{10})L}{10}; & C_2 > 2L \\
(79.6)C_2 - (139.3)L; & C_2 < 2L 
\end{cases}
\]

(The "kink" in the preferences of type 1 agents guarantees existence of both a nonstochastic and a stochastic equilibrium. It plays no other role in the analysis.)

For this first example, it may be useful to explicitly solve for both a nonstochastic and a stochastic equilibrium. (Here we focus only on the
case with valued fiat money.) To consider the nonstochastic case first, we begin as follows. It is easily verified that \( L_2 = 1 \). If self-selection constraints did not bind, \( L_1 = 1 \) would also hold. But, since \( w_1 > w_2 \), such an arrangement is not incentive compatible. Therefore, \( L_1 \) is determined by the self-selection constraint and feasibility considerations. Using the fact that \( \hat{w}_i = \pi_i \) if self-selection obtains, and the fact that the value of money is constant across time and events, this may be written

\[
2\hat{w}_i L_1 - \left( \frac{1}{2} \right) (L_1 + 1)^2 = 2w_2 L_2 - \left( \frac{1}{2} \right) (L_2 + 1)^2.
\]

In obtaining this expression, we have used the fact that, in a nonstochastic steady state, all workers save their entire first period earnings.

There is a single solution to this problem, given that \( L_2 = 1 \) and \( L_1 < 1 \). This is \( L_1 = .172 \). It is, therefore, the case that the level of real balances in equilibrium obeys \( SM = 1.172 \), since all workers save their earnings. Finally, \( R(e) = 1 \forall e \).

Having solved for the nonstochastic equilibrium, we now see whether employment lotteries can emerge in equilibrium. To this end, we begin by noting that if there are employment lotteries, agents must accept contracts prior to the realization of the current period state. Therefore, the equilibrium value of \( L_2 \) solves

\[
\max E V_2[w_2(e) L_2(e), L_2(e), e],
\]

where concavity of \( V_2 \) implies that \( L_2 \) will be nonstochastic. For this example,

\[
V_2(w_2 L_2, L_2, e) = 2 E\left[ \frac{ES(e')}{S(e)} \right] w_2 L_2 - \left( \frac{1}{2} \right) (L_2 + 1)^2,
\]
where we have assumed that type 2 agents hold money in both states. (This assumption will be borne out in equilibrium.) It is now easy to check that \( L_2 = 1 \) in equilibrium. If self-selection constraints did not bind, it is again the case that \( L_1 = 1 \) would hold. As this is clearly not incentive compatible, some self-selection device must be employed by firms. If this is a nondegenerate employment lottery, the value \( p \) must solve (since \( L_1(1) = 1, L_1(2) = 0 \))

\[
2p\left[\frac{E[S(e')]}{S(1)}\right]w_1 - 2p - \frac{1}{2}(1-p) = 2E\left[\frac{E[S(e')]}{S(e)}\right]L_2 - \left(\frac{1}{2}\right)(L_2+1)^2.
\]

This expression states that the expected utility obtained by mimicking a type 1 agent must not exceed that from behaving as a type 2 agent. The expected payoff from mimicking, on the left-hand side of (6), is the expected period two consumption derivable from a unit of labor receiving wage rate \( w_1 \), which is held in the form of money earning as a real return the expected rate of deflation, minus the expected disutility of labor. As a mimicking agent works only in \( e = 1 \), this is the only state in which money could be held.

To complete the set of equilibrium conditions, we note that if all agents save all period one earnings, then

\[
S(1)M = w_1 + w_2L_2
\]

\[
S(2)M = w_2L_2.
\]

Using the zero profit conditions, (6)-(8) can be solved for \( p = .671, S(1)M = 3, S(2)M = 1 \), and \( ES(e)M = 2.342 \). We now verify that this is an equilibrium outcome. To do so, we need only show that no degenerate lottery dominates it. The best degenerate lottery was computed above. It generates (expected)
payoffs $U_2 = 0$, and $U_1 = 3.423$. The lottery generates expected payoffs $EU_2 = 0.589$, and $EU_1 = 10.381$. Hence, the lottery outcome is not dominated, so the equilibrium for this economy does display random employment.\footnote{11}

We now demonstrate that this economy has a Phillips curve with "normal" slope. To see this, we note that $e = 1$ is the high employment state. If the current period state is $e = 1$, the expected rate of deflation (plus one) is

$$\frac{ES(e')}{S(1)} = \frac{pS(1)+(1-p)s(2)}{S(1)} < 1.$$ 

Therefore, on average prices will rise when unemployment is low, and will fall when unemployment is high. This is, of course, simply the Phillips curve correlation. Finally, we note that there is an identical relationship here between money wage movements and unemployment for the following reason. The zero profit condition implies constant real wages. Therefore, price level movements must always be matched by proportional movements in the nominal wage, and we have derived a Phillips curve for both wage and price level inflation.\footnote{12}

We have, then, derived a Phillips curve and an endogenous business cycle. Moreover, the Phillips curve in this model has the property that its presence reduces the social costs associated with sorting workers. To see

\footnote{11}{It is readily verified that this economy satisfies the conditions of Proposition 1. Hence there is no equilibrium with $L_1(e) = L_2(e)$.}

\footnote{12}{It will be noted that (ix) is inessential to the argument. In particular, we have shown that some lottery outcome dominates the nonstochastic equilibrium. If firms were allowed to choose $L_1(1)$ and $L_2(2)$, this would clearly continue to be true. Furthermore, wlog let $L_1(1) > L_2(2)$. Then, since $S(e)M = \pi_1 L_1(e) + \pi_2 L_2$, high employment states will continue to be high expected inflation states as well. Thus (ix) merely simplifies computation; it does not affect the qualitative nature of the results.}
this in the context of example 1, it is useful to consider equation (6). The interpretation of the first term on the left-hand side of (6) is that it represents labor income, and the return to saving this income. Since a type 2 agent mimicking a type 1 agent will work only in $e = 1$, and since the Phillips curve has the "usual" slope, this means that he will work only in states where the intertemporal terms of trade are adverse. This constitutes an additional deterrent, over and above the risk associated with lotteries, to the mimicking of type 1 agents. Put otherwise, firms use lotteries as a technique to induce self-selection of labor. The use of these lotteries gives rise to a Phillips curve. This Phillips curve functions not only as a by-product of employment lotteries, however, but as an unintentional aid in sorting workers.

Notice that this adverse terms of trade effect operates much like the intertemporal substitution mechanism of Lucas (1972). In particular, the fact that working is less attractive in certain states due to the intertemporal terms of trade serves to deter type 2 agents from pretending to be type 1 agents. However, despite its important allocative role, an econometrician observing this economy would never discover an intertemporal substitution effect. This is because type 2 agents are deterred from mimicking type 1 agents, and because employment lotteries are specified prior to the state realization. Thus, actual employment cannot be based on the ex post intertemporal terms of trade.

The presence of this "intertemporal substitution effect" is suggestive that price stabilization policies would be undesirable here. We take up this issue below. The remaining feature of this example to be examined here is yet another aspect of observed business cycles which is explained by the model. This is the procyclical nature of movements in average productivity in the labor force. Notice that when output and employment are high in this
setting, a relatively large number of type 1 workers are employed. Employment of type 2 workers does not vary over the cycle. Hence average productivity must move procyclically here, since at peaks high productivity workers must constitute a larger portion of the labor force than they do in troughs.

The issue of productivity of the labor force leads into the next proposition. Recall that in the static version of the model, all unemployment was among high productivity workers. In the dynamic version, this feature is rectified. In fact, we now demonstrate that it is possible for high productivity workers to often be on their notional labor supply curves, and for low productivity workers to be unemployed. This is

Proposition 3. In an economy with valued fiat money type 2 workers may be unemployed.

This is again established by example.

Example 2. Let \( \pi_2 = (3/4) \), \( \pi_1 = 2 \),

\[
U_2(C_1,C_2,L) = C_1 + (0.99)C_2 - (1/2)(L+1)^2,
\]

and

\[
U_1(C_1,C_2,L) = \begin{cases} 
5C_2 - \left(\frac{3}{2}\right)L; & C_2 > 2L \\
38C_2 - (6.5)L; & C_2 < 2L 
\end{cases}
\]

with \( \eta_i = 0; i = 1, 2 \). Again, let each generation consist of a single agent of each type, so that we may (wlog) focus on the case of a single firm in equilibrium. Then it is straightforward to verify that the equilibrium outcome has stochastic employment with the following equilibrium values: (probability \( e = 1 \)) \( p = .5827 \), \( L_2 = .7633 \), \( S(1)M = 2 \), and \( S(2)M = 1.14 \). It will be noted that the Phillips curve has a "normal" slope here, since in \( e = 1 \),
Also, it is readily established that type 2 agents are unemployed in \( e = 2 \); i.e., \( L_2(2) < \tilde{L}_2(2) \). To see this, notice that in \( e = 2 \), type 2 agents save their entire earnings. Then

\[
V_2(wL, L; 2) = (.99)\left[\frac{ES(e')}{S(2)}\right] wL - \left(\frac{1}{2}\right)(L+1)^2.
\]

Therefore, the value \( \tilde{L}_2(2) \) maximizes this expression, subject to \( L_2 < 1 \), at the values \( w_2 = \pi_2 \), and \( ES(e')/S(2) = 1.435 \). Then \( \tilde{L}_2(2) = 1 \), but \( L_2(2) = .7633 \approx e \). Thus, type 2 agents can be off their notional labor supply curves, and be "unemployed" (ex post). This establishes the proposition.

It is the case, then, that moving to a setting with money can rectify certain shortcomings of the static model. It will also be noted that examples 1 and 2 display a Phillips curve with the "standard" slope. We now turn to the question of whether this is a "typical" outcome for the class of economies at hand.

B. Slope of the Phillips Curve

In this section we argue that, under relatively weak conditions, single (operating) firm versions of the model generate inverse correlations between unemployment and inflation (if a Phillips curve exists in equilibrium). This is significant because of the difficulty in other models of producing Phillips curves with "correct" slopes. In particular, the slope of Phillips curves in search models depends on which parties engage in search, and the slope of the Phillips curve in Lucas (1972) depends on the nature of monetary injections. Here we give sufficient conditions much weaker than these for the Phillips curve to have a normal slope with a single firm.
In order to do this, we make two assumptions. The first is that \( \psi_i(-) = 0; i = 1, 2, \ldots, t \). This merely simplifies notation and reduces the number of cases to be considered. The case where some agents borrow may be treated analogously. The second is that (the number of operating firms) \( N' = 1 \). There are two reasons for making this assumption. One is that if \( N' > 1 \), the nature of the Phillips curve correlation can take many forms. A one-firm economy requires \( \#E = 2 \), so that the nature of this correlation is quite clear. Second, with \( N' > 1 \), a more sophisticated technology is required to prove a certain uniqueness result below. The one-firm case is relatively simple (but restrictive). Therefore, we confine ourselves to it.

With these comments in mind, we state

Proposition 4. The following conditions are sufficient for an inverse correlation between unemployment and inflation if a nondegenerate Phillips curve exists.

(7) \( N' = 1 \)

(8) \[ D_j \phi_i \left[ \frac{S(1)}{S(e)} \frac{S(2)}{S(e)} \nu_1 L_1(e); p \right] > 0; i = 1, 2, j = 1, 2, \]

(where \( p \) is a parameter of these demand functions, which has previously been suppressed notationally.)

(9) \[ D_3 \phi_1 \left[ \frac{S(1)}{S(e)} \frac{S(2)}{S(e)} \nu_1 L_1(e); p \right] > 0; \]

(10) \[ U_i(C_1, C_2, L) = W_i(C_1, C_2) + T_1(L); i = 1, 2. \]

(8) is simply that savings do not fall as real returns rise, (9) is that second period consumption is a normal good, and (10) is separability of utility functions in consumption streams and leisure.
Proof. To begin, note that since \( N' = 1 \), only one lottery can be used in any period. The focus on steady states implies that this lottery will be used in all periods. With this in mind, the proof proceeds by establishing (a) that there exists for any \( p \) a unique stochastic steady state mapping \( s(e) \) with \( s(e) > 0 \), and (b) that this has \( s(1) > s(2) \).

To prove the first fact, note that money market equilibrium requires

\[
S(1)M = \theta \phi_1[1, \frac{s(2)}{s(1)}, \pi_1, p] + (1-\theta) \phi_2[1, \frac{s(2)}{s(1)}, \pi_2, L_2, p]
\]

\[
S(2)M = \theta \phi_1[1, \frac{s(1)}{s(2)}, 1, 0, p] + (1-\theta) \phi_2[1, \frac{s(1)}{s(2)}, 1, \pi_2, L_2, p].
\]

Without loss of generality, let \( M = 1 \). Now notice that for any \( p \), (11) and (12) are two equations in \( s(1) \) and \( s(2) \). It is readily verified that (using (8))

\[
\frac{\partial s(2)}{\partial s(1)} \bigg|_{(11)} > \frac{s(2)}{s(1)}
\]

and that (again using (8))

\[
0 < \frac{\partial s(2)}{\partial s(1)} \bigg|_{(12)} < \frac{s(2)}{s(1)}
\]

at any point such that \( s(1), s(2) > 0 \).

Now consider Figure 2. As should be clear, \( s(1) = s(2) = 0 \) is one equilibrium pair of prices for this economy. By the assumption that a non-trivial Phillips curve exists, there must be at least one other with \( s(e) > 0 \); \( e = 1, 2 \). But by (13) and (14) there is only one other, as at any interior intersection of (11) and (12), (11) must intersect (12) from below, and (11) and (12) are continuous (by assumption).
We now show that $\nu p \in (0,1)$, (11) and (12) imply $S(1) > S(2)$. To see this, consider the intersections of (11) and (12), respectively, with the locus $S(2) = S(1)$ in Figure 2. The intersection of (11) with this locus occurs at

$$\tilde{S}(1) = \theta \Phi_1(1,1,\pi_1) + (1-\theta) \Phi_2(1,1,\pi_2L_2)$$

and (12) intersects it at

$$\tilde{S}(2) = \theta \Phi_1(1,1,0) + (1-\theta) \Phi_2(1,1,\pi_2L_2).$$

By (10), $\tilde{S}(1) > \tilde{S}(2)$. Therefore, since (11) and (12) are both positively sloped, and intersect only once, an equilibrium must be as shown in Figure 2, with the intersection of (11) and (12) below the 45° line. This is true for any $p \in (0,1)$, and hence must be true for the equilibrium $p$ if the Phillips curve is (stochastically) nondegenerate.

Thus the normal case in the single-firm economy is for the Phillips curve to have its observed slope. The intuition behind this result is straightforward. In particular, in high employment states income is high. If consumption in both periods is a normal good for type 1 agents, then in high employment states they will save more (borrow less). Barring "abnormal" slopes of excess money demand schedules, this will cause prices to be "below average" in high employment states, so that in such states the "average" outcome must be inflation. Note that this argument is independent of the number of firms, suggesting that the single firm case is not special. It is substantially simpler than other cases, however, so we do not attempt to generalize Proposition 4 here.

It is possible, however, for the Phillips curve to have an "incorrect" slope in these models. To show this, we relax the assumption that $U_1$ is concave. Then we present
Example 3. Each generation has a single worker of each type (so wlog, $N' = 1$). $\pi_1 = 1.25$, $\pi_2 = 1$, $n_1 = .1$, $n_2 = 0$, $\pi_2 = 0$.

\[
U_2(C_1, C_2, L) = C_1 + 2C_2 - (.5)(L+1)^2
\]

\[
U_1(C_1, C_2, L) = C_1L + (.5)C_2 - (1.04)L.
\]

It is tedious but straightforward to demonstrate that this economy has no nonstochastic equilibrium (i.e., this fails to exist). It does have a stochastic equilibrium, however. This can be solved for in the following manner. First, note that at $w_2 = \pi_2 = 1$, the maximal value for $L_2$ is $L_2 = 1$. Therefore, the equilibrium lottery must satisfy

\[
E V_2[\pi_1 L_1(e), L_1(e), e] = p 2\pi_1 \frac{ES(e')}{S(1)} - 2p (1/2)(1-p) - 2 = E V_2[\pi_2 L_2, L_2, e].
\]

In addition, we may derive asset market equilibrium conditions as follows. Inspection of $U_1$ indicates that

\[
\psi_1(-) = \begin{cases} 
\frac{n_1}{R(e)} & \text{if } L_1 > (1/2) \\
\Psi & \text{if } L_1 < (1/2)
\end{cases}
\]

and that

\[
\phi_1(-) = \begin{cases} 
\nu_1 L_1 & \text{if } L_1 < (1/2) \\
0 & \text{if } L_1 > (1/2)
\end{cases}
\]

Inspection of $U_2$ indicates that loans and money must bear equal expected returns, or that $R(e) = \frac{ES(e')}{S(e)} w e$. Also,
\[ \Phi_2(-) + \psi_2(-) = \pi_2 L_2 \forall e \text{ such that } \frac{ES(e')}{S(e)} > (1/2). \]

Using these observations and \( L_2 = 1 \), \( S(1) \) and \( S(2) \) satisfy

\[ S(1)M = 1 - \left( \frac{1}{10} \right) \frac{S(1)}{ES(e')} \]

\[ S(2)M = 1, \]

where \( ES(e') = pS(1) + (1-p)S(2) = pS(1) + (1-p)\left( \frac{1}{M} \right) \). While the equilibrium conditions are difficult to solve for \( p \) explicitly, \( p \in (0.4, 0.5) \), an equilibrium with a nondegenerate lottery can be shown to exist, and it is obvious that \( S(1) < ES(e) \). This latter fact implies that in \( e = 1 \) there is deflation, so that the Phillips curve for this economy slopes the "wrong" way. Clearly this example has some very special features, though, and does not contradict the notion that a "normal" Phillips curve displays an inverse correlation between unemployment and inflation.

C. A Remark

In all of the discussion to date, we have focused on versions of the economy in which only a single firm operates in equilibrium. This has been for simplicity of exposition. However, some remarks are in order regarding conditions which are necessary for a stochastically nondegenerate Phillips' curve to emerge in equilibrium. First, in the general discussion, we have not elaborated on "how many" workers there are in the economy. This is because the number of workers (as opposed to the proportion of type 1 and 2 agents in the population) plays no important role in the analysis. In particular, workers play a completely passive role in labor markets. Firm actions determine the important features of equilibrium. Thus, there can be finitely many workers, a countably infinite number, or a measure space of workers with no effect on the arguments of the paper.
The number of potential firms, on the other hand, is not irrelevant. If there are infinitely many firms conducting independent lotteries, there will be no aggregate uncertainty in any equilibrium and, hence, there will not be an observed Phillips' curve for the economy as a whole. Thus, it is important that the number of agents with access to the production technology (N) be finite. This finiteness condition on the number of firms is the only essential assumption on population in the analysis, however.

D. Money, and the Cost of Acquiring Information

As a digression, we now consider the social cost of acquiring a given amount of information in economies with and without valued fiat money. Specifically, we provide a counterexample to the following assertion: the (social) cost of acquiring a certain amount of information about goods and services (when agents are asymmetrically informed) falls with the introduction of valued fiat money.

Example 4. This is the same as Example 1, except that we consider the case in which M = 0, and we contrast the case of a full (public) information economy with that of the adverse selection economy.

Case 1: Public Information, M = 0. It is readily verified that the maximal (and labor market clearing) value of $L_2$ is $L_2 = 1/4$ (if $M = 0$). The maximal (and market clearing) value of $L_1$ is $L_1 = 0$ since there is no medium for carrying earnings over into the second period.

Case 2: No Information, M = 0. $L_2 = 1/4$ and $L_1 = 0$ are still equilibrium values, as type 2 agents have no incentive to mimic type 1 agents under these circumstances. Thus self-selection constraints do not bind, $L_1 \neq L_2$ so there is complete information, ex post, and the social cost of obtaining this information is zero.

Which appears, e.g., in Brunner and Meltzer (1971).
Case 3: Full Information, M > 0. In steady state $S_{t+1} = S_t$. Then it is readily verified that $L_2 = L_1 = 1$ in equilibrium, that $SM = 3$, and that $U_1 = 39.9, U_2 = 0$.

Case 4: No Information, M > 0. We computed the equilibrium for this case above. Clearly self-selection constraints bind. Hence, there is a social cost to obtaining information when $S(e)M > 0$. This is the desired result. Also, note that in this equilibrium, $S(1)M = 3$ and $S(2)M = 1$, i.e., relative to the full information case demand for money has fallen. Thus the common assertion that limited information augments money demand is also false here.

IV. Policy and Other Considerations

In this section we take up several issues. The first concerns the scope for (price level) stabilization policy. In the context of Example 1 we show that such stabilization is feasible, and that the optimal stabilization policy (in a certain class) is Pareto improving. This is somewhat paradoxical in light of our previous comments regarding the Phillips curve. In fact, it can be shown that income transfers associated with the stabilization policy are responsible for the welfare gain, and that price stabilization is undesirable.

Second, we show that this adverse selection economy is capable of generating a (perfect) correlation between the level of government expenditures (money supply) and output, even of generating an observed multiplier. This is true despite the fact that the level of expenditures is completely incapable of influencing the level of output.
A. Price Stabilization

Consider the economy of Example 1, except relax the assumption that the money stock must be constant across dates and events. In addition, a government is introduced which uses monetary tax-transfer mechanisms to attempt to stabilize the price level. Clearly, if a lottery continues to be used under a stabilization policy, the demand for real balances will be three in \( e = 1 \), and one in \( e = 2 \). Let \( M(e) \) denote the money stock in current period event \( e \). Obviously, if the price level is to be completely stabilized, \( S(e) = S \cdot e \). Therefore, \( M(e) \) must satisfy \( S(1) = 3 \), \( S(2) = 1 \).

Now suppose that to accomplish its objectives, the government may use tax-transfers levied on the current period old in the model. As is apparent, if the current state is \( e = 2 \) and last period's state was \( e = 1 \), the government must levy a tax (due in money) of two units of the good. If the ordering of the states is reversed, the government must inject (in money) two units of real balances. Otherwise it does nothing. Then, let \( \lambda \) be the tax payable by old type 1 agents if last period's state was \( e = 1 \) and the state is now \( e = 2 \), and let \( \beta \) be the transfer they receive if the ordering of states is reversed. (Type 2 agents pay \( 2 - \lambda \), and receive \( 2 - \beta \) in the corresponding sequences of event realizations.) Then, ignoring the initial old (who will turn out to be better off), the optimal stabilization policy in this class is the solution to

\[
\max (19.9)p + 10p(1-p)(\beta - \lambda)
\]

subject to

\[
2p(1-p)[(2-\beta)-(2-\lambda)] = \bar{U}_2
\]

\[
(1.25)p - .25 = \bar{U}_2
\]
(17) \( \lambda, \beta > 0; \lambda, \beta < 2, \)

by choice of \( \lambda, \beta, \bar{U}_2 \). The maximand is the expected utility of type 1 agents, as \( p(1-p) \) is the probability of realizing either sequence of states. (15) states that type 2 agents must receive expected utility \( \bar{U}_2 \), which is a choice variable for the government (although obviously there are only two independent choice variables). (17) is a feasibility requirement (nonnegative consumption), and (16) is the self-selection constraint.

Collapsing (15) and (16) and substituting into the maximand we may rewrite this optimal taxation problem as

\[
\max_{\{U_2\}} 3.98 + (10.92)\bar{U}_2 \text{ subject to (17).}
\]

This clearly requires setting \( \lambda = 2 \) and \( \beta = 0 \). The equilibrium value of \( p \) is then \( p = 0.769 \), and under this regime \( EU_2 = \bar{U}_2 = 0.711, EU_1 = 11.747 \); i.e., all young agents are better off under this regime. Also note that \( SM(1) = 3, SM(2) = 1 \), and that \( e = 1 \) is more probable than previously, so that the initial old are better off under this regime as well. Thus, this price stabilization policy is Pareto improving.

In light of our arguments regarding the role of the Phillips curve in inducing self-selection of workers this may seem paradoxical. However, it will be noted that there is both a price stabilization and a tax-transfer component to this policy. We now show that price stabilization is undesirable, i.e., that a greater welfare gain is attainable through tax-transfers alone.

To see this, it is sufficient to compare the price stabilization regime with a regime where the government transfers 2 units from old type 1 agents to old type 2 agents with probability \( p(1-p) \) (if last period's state was \( e = 1 \), and the current state is \( e = 2 \)). Under this arrangement prices are
not stabilized and M is constant, so S(e) is as for Example 1. However, with the self-selection condition adjusted to take account of the transfer scheme the equilibrium value of p is now p = .792. Under this arrangement EU₂ = 1.098, EU₁ = 12.466, and the initial old are better off for the following reason. The level of real balances in each state is unaffected, but e = 1 (the favorable state) is even more likely than under price stabilization. Thus, all agents would be better off if the government only conducted a tax-transfer scheme and did not attempt to stabilize prices.

This example illustrates a number of important points. One is that noise in prices may be desirable, despite the fact that it is not related to any "real" shocks to the economy, i.e., to changes in preferences, endowments, technology, population, etc. The second is that this adverse selection setting is capable of explaining observed correlations between the level of government expenditures (money supply) and output. In particular, consider the government price stabilization policy above. When state e = 2 is followed by e = 1, the government injects money (increases expenditures). This is, obviously, accompanied each time it occurs by a rise in output. Conversely, when e = 1 is followed by e = 2, output and government expenditures fall together. Moreover, while in the example the output changes equal the expenditure changes, if type 1 agents consumed a portion of their period one earnings an empirical "multiplier effect" would be observed. Thus, the model is capable of explaining observed correlations other than just the Phillips curve.

It is useful to reflect briefly on what one observes in this price stabilization regime. Periods of low output which are followed by high government expenditure are also followed by periods of high output. If government expenditure does not rise, we will not observe higher output. Moreover, periods of low output are followed by periods of high output without any rise in the price level.
One might ask how a Keynesian would interpret these observations. It seems that such an interpretation would be that in low output periods there is "slack" in the economy. Therefore, expansionary fiscal (monetary) policy can be used to raise output without fear of inflation. In fact, of course, the level of output is randomly determined, and is not influenced by these government expenditures. Inflation or deflation do not occur because these expenditures passively stabilize prices. Thus, the adverse selection economy explains most observations frequently cited as supportive of "Keynesian" models, i.e., the existence of a Phillips curve, and the correlation between "aggregate demand" and output. It does not support their analyses or policy implications, however.

Finally, though, it will be noted that Pareto improvements were obtained in the example above by making transfers from the "rich" to the "poor." Other Pareto improvements are obtainable through "distorting" tax schemes (which operate much like the presence of a Phillips curve). Thus, the adverse selection setting does not typically favor laissez-faire arrangements or abstention from government intervention. It merely suggests that policies based on observed "multiplier" like correlations are unlikely to be successful, and that price stabilization may be a poor policy objective.

V. Conclusions

The adverse selection economies examined are capable of explaining why quantity rationing arises in labor markets. They can also explain observed Phillips curve correlations, and correlations between "aggregate demand" and output. Moreover, they are consistent with the procyclical nature of labor productivity, and the weak relation between observed real wages and employment. In short, these economies are capable of capturing the primary features of the business cycle, and of doing so using endogenously arising
disturbances. They thus avoid, and may help to resolve the as yet inconclusive debate regarding sources of variation in actual business cycles.

Moreover, these results are obtained in the context of a model which is in the spirit of very traditional macroeconomic analysis. In particular, it will be noted that we have permitted employers to determine almost unilaterally the level of employment in the economy. Thus the Rothschild-Stiglitz (1976) equilibrium concept adopted is not at variance with standard Keynesian equilibrium notions. In fact, it is consistent with a number of "Keynesian observations." As an example, many Keynesian economists argue that macro models should be constructed consistent with the observation that relative wages play an important role in the behavior of labor markets. This is a feature of our model; one which is more apparent in the setting with stochastic marginal products examined by Smith (1983). However, despite these Keynesian aspects of the model, and despite the fact that many "Keynesian features" of the economy can be explained by it, the model is highly unsupportive of existing Keynesian analyses and policy prescriptions.

It is, of course, the case that the model here cannot claim to capture all features of observed business cycles. In particular, in economies with large (but finite) numbers of firms, the model of this paper can give rise to a nondegenerate Phillips curve. However, it cannot give rise to relatively frequent disturbances which affect the entire economy. Thus, the argument of the paper should not be taken to imply that either "nominal" or "real" disturbances are unimportant. In fact, the adverse selection setting can be usefully coupled with economies where real and nominal disturbances are a feature. However, the paper does demonstrate that economies with no labor market frictions other than an adverse selection problem can give rise to virtually all of the qualitative features of observed business cycles.
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