Notes on Sticky Wages

Thomas J. Sargent

March 1974

Working Paper #28

Rsch. File #252.1

Thomas Sargent
March 1974

Notes on Sticky Wages

These notes describe a simple model that explains why money wages are "sticky," i.e., why wages don't adjust rapidly enough to assure that labor markets "clear" at every moment, so that layoffs never occur and the supply of labor always equals the demand.* The assumption that money wages are sticky in this sense is a key one in most macroeconomic models that purport to explain fluctuations in the unemployment rate. Here the sticky character of money wages is attributed to different attitudes of firms and workers toward risk bearing.

We consider a competitive firm that will be able to sell all that it wants of a perishable output in period t at the price $p(\theta)$. The price $p(\theta)$ depends on the state of the world θ that prevails at date t. For each date t we assume that there are two states of the world, indexed by $\theta = 1, 2$, and that $p(1) > p(2)$. We assume that the same two prices $p(1)$ and $p(2)$, contingent on states $\theta = 1$, and $\theta = 2$, respectively, hold for all t, a kind of stationarity assumption. The firm and its workers share a common view of the probabilities of states 1 and 2 emerging, denoted by $\pi(1)$ and $\pi(2)$, respectively. We assume that $\pi(1)$ and $\pi(2)$ are the same for each t, which together with our other assumptions imposes stationarity on the system. We assume that $0 < \pi(1) < 1$; of course $\pi(1) + \pi(2) = 1$.

The firm's output in state θ in period t is given by $f(n(\theta))$ where $n(\theta) \geq 0$ is the firm's employment, measured in number of men, in

---

*The model described in these notes is a much simplified version of the one developed by Costas Azariadis in "On the Incidence of Unemployment," (unpublished, 1973) although the setup here is not exactly identical with his.

state θ. The production function f satisfies $f' > 0$, $f'' < 0$; the marginal product of labor is positive but diminishing. We further assume that $\lim_{n\to 0} f'(n) = \infty$ and that $\lim_{n\to\infty} f'(n) = 0$.

The firm pays workers a money wage $w(\theta)$ in state θ in period t. The wage may be dependent on state θ, but is independent of time t. The latter specification is really no restriction, since our stationarity assumptions are sufficient to imply it as a consequence of optimal firm behavior. The firm's profits at time t in state θ are then given by

$$p(\theta)f(n(\theta)) - w(\theta)n(\theta).$$

The firm's objective is to maximize the expected discounted value of its stream of profits over the time interval t=1, ..., T:

$$V = \sum_{t=1}^{T}\sum_{\theta=1}^{2} \delta^t \pi(\theta)(p(\theta)f(n(\theta)) - w(\theta)n(\theta))$$

$$= D\sum_{\theta=1}^{2} \pi(\theta)(p(\theta)f(n(\theta)) - w(\theta)n(\theta))$$

where δ is the discount factor and $D = \sum_{t=1}^{T} \delta^t$. Since D is fixed, we may just as well assume that the firm attempts to maximize V/D, subject to the constraints imposed by the labor market. Positing that the firm maximizes expected profits means that the firm has a neutral attitude toward risk and is willing to accept fair bets in unlimited amounts.*

---

*That is, it is as if the firm were trying to maximize its expected utility of profits, but that its utility functionis linear in profits.

The representative workers possess a utility function that gives his happiness in period t as a function of the wages that he receives and his leisure. It is assumed that a worker either works full time in period t, in which case his level of leisure is $L=L_o$, or else he is unemployed, in which case he has leisure $L_1 > L_o$. There is no part-time or over-time work. The worker's happiness is given by the utility function

$$U = g(w(\theta), L) ; \qquad g_1 > 0, \; g_{11} < 0$$
$$g_2 \geq 0, \; g_{22} \leq 0$$

which is assumed to be concave and to possess continuous second partial derivatives. If the worker is employed, his utility can be written solely as a function of his wage.

$$U = U(w(\theta)) = g(w(\theta), L_o) ,$$

since L_o is a paremeter. Our assumptions on g imply that $U' > 0$, $U'' < 0$, so that the worker is assumed to be risk averse. We let r denote the pecuniary value the worker attaches to having leisure L_1 rather than L_o, i.e., it is the amount he would have to be paid to make him indifferent between working and not working. So r is defined by the equality

$$g(0, L_1) = g(r, L_o) .$$
$$= U(r) .$$

The worker's pecuniary income is thus $w(\theta)$ in state θ if he is employed in state θ, and r if he is unemployed. The worker maximizes expected utility, which given our definition of r, can be written as a function only of the distribution of his pecuniary income across states.

The firm employs $n(1)$ workers in state 1 and $n(2)$ workers in state 2 at each date. Its procedure is to offer jobs to a number $\max(n(1), n(2))$ workers, and then to lay off in a random fashion workers in the state for which $n(\theta) < \max(n(1), n(2))$. In our case, since $p(1) > p(2)$, there is a presumption that the firm will set $n(1) > n(2)$, so that $\max(n(1), n(2)) = n(1)$. Workers are aware of the firm's policy on this matter, and consequently realize that if state 2 occurs in a given period, only $n(2)/n(1)$ of the workers having jobs with the firm will actually be employed in that period. Since the firm lays off workers randomly, if state 2 occurs the worker believes that his chances of working are $n(2)/n(1)$ while his chances of being laid off are $\frac{n(1)-n(2)}{n(1)}$.

Laid off workers receive no wages from the firm.

The worker seeks to maximize his expected discounted utility, which is

$$u = \sum_{t=1}^{T} \delta^t (\pi(1)U(w(1))+\pi(2)\frac{n(2)}{n(1)}U(w(2))+\pi(2)\frac{n(1)-n(2)}{n(1)} U(r))$$

or

$$u = D[\pi(1)U(w(1))+\pi(2)\frac{n(2)}{n(1)} U(w(2))+\pi(2)(1-\frac{n(2)}{n(1)})U(r)]$$

where as before $D = \sum_{t=1}^{T} \delta^t$. Since D is a constant, we might as well assume that workers maximize

$$v \equiv \frac{u}{D} = \pi(1)U(w(1))+\pi(2)\frac{n(2)}{n(1)} U(w(2))+\pi(2)(1-\frac{n(2)}{n(1)})U(r)$$

The firm is assumed to be able to hire as large a labor force as it wants subject to the restraint that its jobs offer workers a level of expected utility v at least as great as $\bar{v}$, where $\bar{v}$ is a market-determined level of expected utility that workers can obtain by accepting jobs with other firms. The firm's problem is thus to maximize expected profits

(1) $$\pi(1)(p(1)f(n(1))-w(1)n(1))+\pi(2)(p(1)f(n(2))-w(2)n(2))$$

subject to the constraint

(2) $$\bar{v} = \pi(1)U(w(1))+\pi(2)\frac{n(2)}{n(1)} U(w(2))+\pi(2)(1- \frac{n(2)}{n(1)})U(r).$$

The firm chooses w(1), w(2), n(1), n(2) so as to maximize (1) subject to (2). The firm's problem is thus to choose a wage and layoff policy to maximize its profits subject to the constraints imposed by the labor market. Notice that according to (2), workers are willing to sacrifice some wages in state 2 for more security of employment, i.e., a higher $n(2)/n(1)$. Maximization of (1) subject to (2) is carried out through unconstrained maximization of

$$J(n(1),\ n(2),\ w(1),\ w(2),\ \lambda) = \pi(1)(p(1)f(n(1))-w(1)n(1)) + \pi(2)(p(2)f(n(2))-w(2)n(2))+\lambda[\bar{v}-\pi(1)U(w(1))-\pi(2)\frac{n(2)}{n(1)} U(w(2))-\pi(2)(1-\frac{n(2)}{n(1)})U(r)] \tag{3}$$

where λ is an undetermined Lagrange multiplier. The first-order conditions for a maximum of (3) are:

$$\frac{\partial J}{\partial n(1)} = \pi(1)p(1)f'(n(1))-\pi(1)w(1)+\lambda\pi(2)\frac{n(2)}{n(1)^2}(U(w(2))-U(r)) = 0 \tag{4}$$

$$\frac{\partial J}{\partial n(2)} = \pi(2)p(2)f'(n(2))-\pi(2)w(2)-\lambda\pi(2)\frac{1}{n(1)}(U(w(2))-U(r)) = 0 \tag{5}$$

$$\frac{\partial J}{\partial w(1)} = -\pi(1)n(1)-\lambda\pi(1)U'(w(1)) = 0 \tag{6}$$

$$\frac{\partial J}{\partial w(2)} = -\pi(2)n(2)-\lambda\pi(2)\frac{n(2)}{n(1)}U'(w(2)) = 0 \tag{7}$$

$$\frac{\partial J}{\partial \lambda} = \bar{v}-\pi(1)U(w(1))-\pi(2)\frac{n(2)}{n(1)}U(w(2))-\pi(2)(1-\frac{n(2)}{n(1)})U(r) = 0 \tag{8}$$

Equation (6) can be written

$$n(1) = -\lambda U'(w(1)) \ ,$$

while equation (7) can be written

$$n(1) = -\lambda U'(w(2)) \ .$$

Both of these equations can be satisfied only if

$$U'(w(2)) = U'(w(1)) \ . \tag{9}$$

But since $U'(w)$ is a monotone function of w--recall that we have assumed that $U'' < 0$--equation (9) can be satisfied only if

$$w(1) = w(2) = w \tag{10}$$

According to (10), the wage rate should be independent of the state that occurs. The firm should offer a fixed wage w and not adjust it, say, downward in state 2 simply because $p(1) > p(2)$.

To indicate the forces that lead to the wage being constant across states of nature, we study the worker's and the firm's indifference curves with respect to w(1) and w(2). Holding expected utility v and n(2)/n(1) both constant, w(1) and w(2) can vary so long as they satisfy

$$0 = dv = \pi(1)U'(w(1))dw(1)+\pi(2)\frac{n(2)}{n(1)}U'(w(2))dw(2).$$

Hence, in the w(1), w(2) plane, the slope of the worker's indifference curve is

$$\frac{dw(2)}{dw(1)} = -\frac{\pi(1)}{\pi(2)}\frac{n(1)}{n(2)}\frac{U'(w(1))}{U'(w(2))} < 0.$$

Differentiating the above equation with respect to w(1) gives

$$\frac{d^2w(2)}{dw(1)^2} = -\frac{\pi(1)}{\pi(2)}\frac{n(1)}{n(2)}\frac{U''(w(1))}{U'(w(2))} > 0 ,$$

which shows that the indifference curves are convex.

Holding both expected profits V/D and n(1) and n(2) constant, the firm is content with movements in w(1) and w(2) that satisfy

$$-\pi(1)n(1)dw(1)-\pi(2)n(2)dw(2) = d(\frac{V}{D}) = 0$$

or

$$\frac{dw(2)}{dw(1)} = -\frac{\pi(1)}{\pi(2)}\frac{n(1)}{n(2)} < 0 ,$$

which gives the slope of the firm's indifference curves in the w(1), w(2) plane, along which expected profits are constant. Differentiating the above equation with respect to w(1) gives

$$\frac{d^2w(2)}{dw(1)^2} = 0 ,$$

which shows that the firm's indifference curves are straight lines, a consequence of the firm's neutral attitude toward risk. Higher expected profits correspond to firm indifference curves closer to the origin, while higher expected utility for workers correspond to indifference curves further from the origin. (See figure (1)). The firm maximizes its expected profits subject to a fixed level of v by equating the slope of its iso-expected profit line to the slope of the worker's indifference curve:

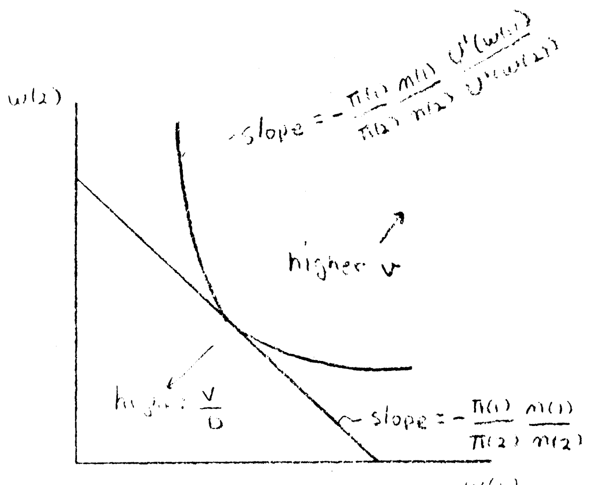

Figure 1

$$\frac{-\pi(1)}{\pi(2)}\,\frac{n(1)}{n(2)} = -\,\frac{\pi(1)}{\pi(2)}\,\frac{n(1)}{n(2)}\,\frac{U'(w(1))}{U'(w(2))} ,$$

or

$$U'(w(1)) = U'(w(2)) .$$

The above condition can only be satisfied where w(1) = w(2). Thus, given n(1) and n(2), the firm maximizes its expected profits subject to a constant level of v by eliminating any risk that the wage depend on the state of demand for the firm's product (i.e., on p(θ)).

We now turn to analyze the determinants of n(1) and n(2), and so the probability of employment, n(2)/n(1). To start with an interesting extreme case, we begin by assuming that $r = 0$, so that workers derive no additional utility from their additional leisure when unemployed at time t. Under this assumption, it happens that $n(2) = n(1)$, so that employment is constant across states and the probability of employment is unity. To show this in a convenient way, note that if the firm employs $n(\theta)^o$ men in state θ, $n(2)^o < n(1)^o$, and pays a wage of w^o that is fixed across states of nature, its expected profits are

$$(\frac{V}{D})^o = \pi(1)p(1)f(n(1)^o)+\pi(2)p(2)f(n(2)^o)-n(1)^o w^o \\ (\pi(1)+\pi(2)\frac{n^o(2)}{n^o(1)}) \ .$$

Since workers are risk-averse, they prefer a certain wage of

$$\tilde{w} = w^o(\pi(1)+\pi(2)\frac{n^o(2)}{n^o(1)}) < w^o \ ,$$

to the uncertain wage package of w^o with probability $\pi(1)+\pi(2)\frac{n^o(2)}{n^o(1)}$ zero with probability $\pi(2)(1-n^o(2)/n^o(1))$. So if the firm employs $n^o(1)$ workers in each state, paying them the reduced wage $\tilde{w}$, workers are better off. The firm's expected wage bill is unchanged since the new expected wage bill is

$$n(1)^o\tilde{w} = n(1)^o w^o(\pi(1)+\pi(2)\frac{n^o(2)}{n^o(1)}) \ .$$

The firm's expected profits with $n(1) = n^o(1)$, $n(2) = n^o(1)$ and the certain money wage $\tilde{w}$ are given by

$$(\frac{V}{D})' = \pi(1)p(1)f(n^o(1))+\pi(2)p(2)f(n^o(1))-n^o(1)\tilde{w} \ .$$

Subtracting $(V/D)^o$ from $(V/D)'$ gives

$$(\frac{V}{D})'-(\frac{V}{D})^o = \pi(2)p(2)(f(n^o(1))-f(n^o(2))) ,$$

which is positive so long as $n^o(1) > n^o(2)$. Thus, the firm's expected profits are greater and workers are happier with $n(2) = n^o(1)$, so that in a sense additional workers in state 2 are free to the firm so long as $n(2) < n(1)$. Since workers have a positive marginal product, it pays the firm to set $n(1) = n(2)$.

So with $r = 0$, employment, output, and the wage rate are each constant across states, this in spite of the fact that the price of the firm's output varies across states. Evidently, then, the firm's supply curve is vertical, while its demand for workers is independent of the real wage in terms of the firm's own good. Notice that the equilibrium in which $w(1) = w(2)$ and $n(1) = n(2)$ is one in which the worker bears no risk since he receives a certain wage of $w(1)$. The firm bears all the risk. This is a consequence of the firm's risk neutrality and the worker's risk-aversion. The firm is willing to accept whatever fair bets are offered to it, while the worker attempts to avoid any fair bets. There is thus incentive for the firm and individuals to trade risks so that the firm accepts whatever risks must be borne. The character of our results stems directly from the asymetrical attitudes toward risk that we have attributed to the firm, on the one hand, and to the worker, on the other hand.

We now examine the case in which $r > 0$. We begin by taking the total differential of the firm's expected profits,

$$d(\frac{V}{D}) = -(\pi(1)n(1)+\pi(2)n(2))dw+\pi(1)[p(1)f'(n(1))-w]dn(1)$$

$$+\pi(2)[p(2)f'(n(2))-w]dn(2).$$

Assume that $n(1) = n(2)$ initially and hold $n(1)$ fixed (i.e., set $dn(1) = 0$). Then we have

$$d(\frac{V}{D}) = -n(1)dw+\pi(2)[p(2)f'(n(2))-w]dn(2) \; .$$

The firm is willing to bear variations in w and n(2) so long as expected profits remain unchanged, ($d(\frac{V}{D}) = 0$), i.e., so long as the variations of w and n(2) satisfy

$$(11) \qquad dw = \pi(2)(p(2)f'(n(2))-w)\frac{dn(2)}{n(1)} \; .$$

Since $p(2)f'(n(2))-w < 0$,* the firm is willing to increase w if it can decrease n(2). Notice that

$$(12) \qquad d(\frac{n(2)}{n(1)}) = \frac{dn(2)}{n(1)} - \frac{n(2)}{n(1)}\;\frac{dn(1)}{n(1)} \; ,$$

so that if $dn(1) = 0$, as we are assuming, then $d(n(2)/n(1)) = dn(2)/n(1)$.

Where $r > 0$, is it still true that contracts will be written so that $n(2) = n(1)$? The worker's expected pecuniary income is

$$E(\tilde{w}) = \pi(1)w+\pi(2)\frac{n(2)}{n(1)}\,w+\pi(2)(1-\frac{n(2)}{n(1)})r \; ,$$

where now $r > 0$. Taking the total differential of the above equation and setting $n(2)/n(1) = 1$, we have

---

*The marginal conditions (6) and (7) imply $\lambda = -n(1)/U'(w)$. Substituting this into marginal condition (5) gives

$$p(2)f'(n(2))-w = -(\frac{U(w)-U(r)}{U'(w)}) < 0 \; ,$$

since $U(w)-U(r) > 0$ and $U'(w) > 0$.

(13) $$dE(\tilde{w}) = dw+\pi(2)[w-r]d(\frac{n(2)}{n(1)}) .$$

Now if n(2) = n(1) initially, firms are just willing to raise wages and decrease n(2) so long as dw and dn(2) obey (11), which with use of (12) can be written as

(14) $$dw = \pi(2)(p(2)f'(n(2))-w)d(\frac{n(2)}{n(1)}) .$$

The effects on the expected income of the worker of such a variation in an initial w and n(2), starting from a position where n(1) = n(2), are found by substituting (14) into (13):

(15) $$dE(\tilde{w}) = \pi(2)[p(2)f'(n(2))-r]d(\frac{n(2)}{n(1)}) .$$

Now if $p(2)f'(n(2))-r < 0$, the worker's expected pecuniary income will increase as a result of a <u>lowering</u> of n(2)/n(1), since then sign $dE(\tilde{w}) = -\text{ sign } d(\frac{n(2)}{n(1)})$. This means that if $p(2)f'(n(2))-r < 0$ and $\frac{n(2)}{n(1)} = 1$, the firm, in effect, is in a position to offer the worker a favorable bet. By bearing a little uncertainty i.e., accepting a decrease in n(2)/n(1) below unity, the worker can increase his expected pecuniary income. As we have seen above, a risk-averse individual who behaves as our worker does will always take at least a small part of a favorable bet (this is Arrow's proposition, which we have encountered in several guises already). This means that the worker will be anxious to get, and the firm willing to offer, a contract in which $n(2)/n(1) < 1$. Thus, if $p(2)f'(n(2))-r< 0$, n(2)/n(1) cannot equal unity; some workers will be unemployed in state 2.

The following considerations provide a heuristic way of understanding what is going on here. Suppose we begin from a situation where $n(2)/n(1) = 1$. Since $p(2)f'(n(2))-w < 0$, given w, the firm would have higher expected profits if $n(2)$ were smaller. By lowering $n(2)$ by $dn(2)$, the firm's expected profits would increase by $(p(2)f'(n(2))-w)dn(2)$. So the firm would actually be willing to pay unemployment compensation in an amount up to $-[p(2)f'(n(2))-w]$ per man in order to have $dn(2)$ fewer people working in state 2; i.e., the firm would be willing to pay this much in order to have some people not work. In order not to work in state 2, a worker would want the firm to pay him at least $w-r$, the excess of his pecuniary income when he is working over that when he isn't. Then the firm is willing to pay the worker not to work more than he requires to be induced not to work if

$$(w-p(2)f'(n(2))-(w-r) > 0$$

$$r-p(2)f'(n(2)) > 0 \ , \tag{16}$$

which is our condition for $n(2)/n(1)$ to be less than unity. In our setup, the firm is constrained from actually paying unemployment compensation, but part of the "surplus" indicated if condition (16) is met is distributed to workers in the form of the firm's offering workers what amounts to a favorable bet for them. In our setup, workers can share in the "surplus" only by bearing some risk. That is, we have ruled out the possibility that the firm directly offers to insure workers against unemployment, offering to pay them some amount when they are unemployed. Since workers are risk averse there seems to be an incentive for such an institution to emerge.

To show that this indeed can be the case, suppose that firms now consider paying workers an amount per worker of $w(3)$ for not working in state 2. The firm's expected profits are then

$$\frac{V}{D} = \pi(1)(p(1)f(n(1))-w(1)n(1))+\pi(2)(p(2)f(n(2))-w(2)n(2))$$
$$-\pi(2)w(3)(n(1)-n(2)) \ ,$$

where n(1)-n(2) is the number of men in state 2 receiving unemployment compensation from the firm. The worker's expected utility is

$$v = \pi(1)U(w(1))+\pi(2)\frac{n(2)}{n(1)} U(w(2))+\pi(2)(1-\frac{n(2)}{n(1)})g(w(3)), L_1)$$

The firm's problem can then be formulated as the unconstrained maximization of

$$J = \pi(1)(p(1)f(n(1))-w(1)n(1))+\pi(2)(p(2)f(n(2))-w(2)n(2))$$
$$-\pi(2)w(3)(n(1)-n(2))$$
$$+\lambda[\bar{v}-\pi(1)U(w(1))-\pi(2)\frac{n(2)}{n(1)} U(w(2))-\pi(2)(1-\frac{n(2)}{n(1)})g(w(3)), L_1)]$$

where λ is again a Lagrange multiplier. The marginal conditions for w(1) and w(2) are identical with (6) and (7), which can be written as

(6') $$n(1) = -\lambda U'(w(1))$$

(7') $$n(1) = -\lambda U'(w(2)) \ .$$

Equating to zero the partial derivative of J with respect w(3) gives

$$-\pi(2)(n(1)-n(2))-\lambda\pi(2)(\frac{n(1)-n(2)}{n(1)})\frac{\partial g(w(3), L_1)}{\partial w(3)} = 0$$

or

(17) $$n(1) = -\lambda\frac{\partial g(w(3), L_1)}{\partial w(3)}$$

Together, equations (6'), (7') and (17) imply

$$(18) \qquad U'(w(1)) = U'(w(2)) = \frac{\partial g(w(3),\ L_1)}{\partial w(3)} \quad .$$

Given the monotone nature of $U'(\ )$, the above equality implies that $w(1) = w(2)$.

Consider now the particular utility function

$$(19) \qquad U = g(w,\ L) = h(w+BL) \ ; \qquad B > 0;\ h' > 0,\ h'' < 0.$$

This utility function is characterized by straight-line indifference curves between wages and leisure, so that wages and leisure are perfect substitutes. Given this utility function, $U(w)$ is given by

$$U(w) = h(w+BL_o) \ ;$$

r is defined by

$$h(r+BL_o) = h(BL_1)$$

or

$$r+BL_o = BL_1$$

$$r = B(L_1-L_o) \quad .$$

Notice, that

$$\begin{aligned} g(w(3),\ L_1) &= h(w(3)+BL_1) \\ &= h(w(3)+r+BL_o) \\ &= g(w(3)+r,\ L_o) \\ &= U(w(3)+r) \ . \end{aligned}$$

This implies that

$$\frac{\partial g(w(3),\ L_1)}{\partial w(3)} = U'(w(3)+r) \quad .$$

Given the above equality, equation (18) becomes

$$U'(w(1)) = U'(w(2)) = U'(w(3)+r)$$

which together with the monotone nature of $U'(\ )$ implies

$$w(1) = w(2) = w(3)+r = w.$$

Then if at $n(2) = n(1)$, $r > p(2)f'(n(2))$, condition (16), the firm will set $n(2) < n(1)$ but pay unemployment compensation at the rate $w(3) = w-r$. In this fashion, for the particular utility function (19), the labor contract is fashioned so that workers bear no risks, trading them all to the firm.

For utility functions not of the form (19), it will not in general be true that $w(3)+r = w(1) = w(2)$. Still, for many utility functions it will be true that (19) can be satisfied with $w(3) > 0$, so that the firm will opt to set $n(1) > n(2)$ and pay unemployment compensation if condition (16) is met. Evidently, this institutional arrangement is Pareto superior to the one posited at the beginning of these notes, which had the effect of preventing the firm from offering workers unemployment insurance, thereby ruling out a certain "market."