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Model of Financial Collapse

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by John Bryant*

The recurrent banking panics of the 19th century and the Great Depression of the 1930s are clear examples of the failure of our system of free enterprise. However, to this day, economics has failed to produce a satisfactory explanation for these events.^{1/} This paper presents a simple model of a financial collapse which reduces output and employment. The real disruption resulting from a financial collapse does not depend upon price rigidities, but rather is the direct consequence of reduced efficiency of transacting.

Although no satisfactory explanation for banking panics and depression has emerged, existing theory can give us guidance in searching for one. Indeed, it is argued in this paper that a possible explanation for these anomalous events is immediately at hand, an explanation which has simply been overlooked. The natural place to look for an explanation for failure of our free enterprise system is known failures in the economic model of competitive equilibrium. Twenty years ago Paul Samuelson introduced a failure of competitive equilibrium in his pure consumption-loans model [14]. He showed that with overlapping generations of finite-lived individuals in a model with no last period, the competitive equilibrium need not be Pareto optimal. Moreover, he introduced the concept of a negative net worth entity, the "social contrivance" of fiat (unbacked) money, the use of which makes everyone better off and can yield Pareto

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^{1/}See, for example, Lucas and Rapping [13].

optimality. We can, then, model recurrent banking panics and depression as recurrent and once-and-for-all collapse of a fiat money system, respectively.

To model the collapse of the banking system, one must determine the role of banking. We assume in this paper that the role of the banking system is to make transactions feasible or less costly, and the collapse of the banking system is a move to a less efficient means of transacting. Certainly this view is not unique to the author. Consider this quotation from the Federal Reserve System publication Banking and Monetary Statistics 1914-1941 [2, p. 281], "It is estimated that since 1865 depositors have sustained direct losses totalling some \$2 billion through bank failures. These losses alone have been of serious impact, but they represent only a part of the damage incident to insolvencies. . . . during periods of widespread suspensions, many areas have been left wholly without banking facilities. Business concerns, farmers, and other entrepreneurs have been unable to secure bank loans, and communities have suffered from the lack of a convenient means of payment." To model bank failure we should, then, model a loss in transactions efficiency. In the simple version of Samuelson's pure consumption-loans model of fiat money used in this paper there are two equilibria, a monetary and a nonmonetary equilibrium. In the monetary equilibrium transactions are costless, and in the nonmonetary equilibrium they are impossible. We treat the collapse of the banking system as if it were a move from the monetary to the nonmonetary solution. It is not the claim of this paper that we actually moved from a monetary to a nonmonetary equilibrium in the Depression, nor that we moved from free to impossible transactions. While we do not have a coherent model of financial institutions, is it not possible that their role is to provide an efficient means of transacting, and their collapse is a move to a less efficient means of transacting as in this model? In our simple model of fiat money there is global instability in output and unemployment. A fortiori

such pathological behavior can characterize a more sophisticated model of the financial system.

The model provides no explanation for which of the multiple equilibria will characterize the economy at a point in time, and therefore is devoid of dynamics. Moreover, the presented model is bare bones and thus does not have the richness to explain many phenomena, and it should be viewed as a polar case. First the model is presented, then some alternative ways to enrich the model will be sketched. However, the simplicity of the model is a virtue as it isolates the important elements and eliminates the extraneous. Indeed, as a paradigm the model may have use outside the context of this paper.

The Model

The model is a simple variation on the Samuelson [14] pure consumption-loans model. There are N identical individuals born each period and they live two periods. They have perfect foresight. In her first period an individual is endowed with L units of nontransferable leisure, while in the second she is endowed with nothing. There is a linear zero intercept technology available to the individual to transform leisure hours into a transferable but nonstorable consumption good. One hour of work yields w units of good where $w < 1$. The consumption good and leisure are perfect substitutes in the utility function of the individual where one hour of work equals one unit of goods. The individual maximizes her two-period utility using utility function $U(C_1, C_2)$. U is two-smooth, increasing in its arguments, concave, $U_1(0, C_2) = \infty = U_2(C_1, 0)$, C_1 and C_2 are strictly noninferior and strictly gross substitutes, and there exists an $S > 0$ such that $\lim_{\epsilon \rightarrow 0} \epsilon U_2(L-S, \epsilon S) > U_1(L-S, 0)$. There exists a quantity of NM dollars of fiat money which the young get from the old in exchange for goods.

We consider the representative consumer of generation t where the subscript t is dropped for simplicity. Let P be the current rate of exchange of goods for dollars and P' be the next period value of that variable. $1/P$ is the price level as usually interpreted. The individual must choose hours of work, W , and dollars of money holding, m , to maximize her utility given P, P' .

Her problem is

$$\max_{W,m} U(C_1, C_2)$$

subject to

$$C_1 = L - W + wW - Pm$$

$$C_2 = P'm$$

$$Pm \leq wW$$

$$W \leq L.$$

If $P' = 0$, $W = 0$ and $C_2 = 0$. As $w < 1$, the individual will never produce for consumption, but only for sales, $wW = Pm$. As $U_1(0, C_2) = \infty$, $W < L$ always. As $U_2(L, 0) = \infty$, $P' > 0$ implies $W > 0$.

For $P, P' > 0$ the problem can be written

$$\max_W U[L-W, \frac{P'}{P} w W].$$

The first-order condition is

$$-U_1[L-W, \frac{P'}{P} w W] + \frac{P'}{P} w U_2[L-W, \frac{P'}{P} w W] = 0.$$

This can be written as $W = f(\frac{P'}{P})$. f is continuous and single valued by strict noninferiority of C_1, C_2 , is strictly increasing by the strict gross substitutes assumption, and is bounded below by the assumption that $\lim_{\epsilon \rightarrow 0} \epsilon U_2(L-S, \epsilon S) > U_1(L-S, 0)$. The domain of f is $(0, \infty)$ and its range is within $[S, L]$.

The current old get no benefit from dollar holding so they trade all their NM dollars to the young for goods, NwW goods. Our equilibrium condition is that

$$(I) \quad \frac{NwW}{P} = NM.$$

Substituting in the optimal decision rule for W and rearranging yields

$$(II) \quad Nwf\left(\frac{P'}{P}\right) = PNM.$$

We are now ready for our central proposition.

Theorem I:

- A. There is a unique monetary equilibrium characterized by a constant price level and $W > 0$.
- B. There is a nonmonetary equilibrium (an equilibrium with $P = 0$ in all periods) characterized by $W = 0$.
- C. The monetary equilibrium is Pareto superior to the nonmonetary equilibrium.^{2/}

Proof:

- A. From (II) there is a unique constant positive price equilibrium at price $\bar{P} = \frac{W}{M}f(1) = \frac{W}{M}W$. Consider any other positive equilibrium price sequence $\{P_t\}$ with $P_t \neq \bar{P}$ in some period t . Suppose $P_t > \bar{P}$. Then from (II) and the monotonicity of f , $P_{t+1}/P_t > 1$. Indeed, by using (II) iteratively we see that $\{P_{t+k}\}$ must be growing at an increasing percentage rate. But this is not feasible as $\{P_t\}$ is bounded above uniformly by $\frac{W}{M}L$ from (II) and the upper bound of f . Suppose $0 < P_t < \bar{P}$. Then $\{P_{t+k}\}$ must be falling at an

^{2/}This proof follows proofs in [11] and [15].

increasing percentage rate. This implies $\lim_{k \rightarrow \infty} P_{t+k} = 0$, but this is impossible from (II) and the lower bound on f . Suppose that for some equilibrium price sequence $\{P_t\}$ not zero in every period, for some k_0 , $P_{k_0+1} = 0$. Then $W_{k_0} = 0$, which from the equilibrium condition (I) implies $P_{k_0} = 0$. Let the first nonzero element occur at time $k_1 + 1$ where $k_1 > k_0$. Then $W_{k_1} > 0$, which implies $P_{k_1} > 0$ from the equilibrium condition (I), contradiction.

- B. We have seen that for $\{P_t\} = \underline{0}$, $W = 0$ satisfies the equilibrium condition and the individual maximization problem.
- C. As $W = 0$ is feasible for the individual in the monetary equilibrium, the monetary equilibrium is revealed preferred to the nonmonetary equilibrium for the current young and future generations. As the current old consume only the real value of money holdings, the monetary equilibrium is superior for them as well. Note the role of the unusual assumption on utility which yields the lower bound on f . Without this assumption there could be multiple monetary equilibrium price sequences characterized by different inflation rates--all Pareto superior to the nonmonetary equilibrium, however.

We take "depression" to be a completely surprise shift from the monetary to the nonmonetary equilibrium.

Refinements

In this model people work only for future consumption, all they can consume in the future is the fruits of current labor, and fiat money is the only means of transacting for the fruits of one's labor. In the nonmonetary equilibrium transactions are impossible, yielding zero output, employment, and second-period consumption. This should be viewed as a polar case of what we do see in reality. We do transact for the fruits of our labor, and the banking system provides an efficient means of transacting. The model can easily be

modified to include endowments in the second period of existence and other means of transacting.^{3/}

As with the "new-new" labor economics, people not working at all does depend upon the utility of "leisure."^{4/} For example, in our simple model if $C_1 = wW - P_m$, then $W = L$ if $w > 0$. However, as unemployment does not imply starvation, utility of "leisure" does not seem unreasonable. That unemployment means that there are better opportunities available outside the economy than in it is, nonetheless, cold comfort!

In the simple model "firms" are factored into the individuals' problem. This can be changed to have separate firm entities.

In the nonmonetary equilibrium nobody works at all, but they are not searching for work and, therefore, are not involuntarily unemployed. This is easily fixed in a model with multiple means of transacting and two-period endowments by having individuals drawn from a pool and randomly assigned to one of two production technologies, more profitable and less profitable. Individuals have to pay a small "search cost" to belong to the pool. With the monetary equilibrium both technologies are used, but in the nonmonetary equilibrium only the more profitable is used (if the model is rigged correctly). In this way not everyone is idle in a "depression," but those who are are involuntarily unemployed. And, of course, productivity is higher in a "depression." If there are separate firms, a similar result can be achieved by having individuals come in two kinds, skilled and unskilled, but having the only means of discriminating being to employ them a short time.

The model has collapse of the banking system, the change from monetary to nonmonetary equilibrium, a complete surprise. Instead, one could assume that

^{3/} See [15].

^{4/} See [1] and [3].

there are subjective probabilities of moving from one to the other. One could further suppose that the probabilities are not independent and assuming learning.

In short, there are an innumerable number of ways to enrich the model to make it more "realistic."

It should be noted that, more generally, the model demonstrates that losses in transactions efficiency are not to be taken lightly. For example, the model can be modified to generate a positively sloped Phillips curve; it can be used to explain the coexistence of inflation and unemployment. The model also can be grafted to the nonneutral models of monetary policy of Bryant and Wallace, [7] and [8], to yield unemployment effects.

Conclusions

The problem in a depression may not be inadequate demand per se, but the collapse of the banking system. This implies that demand management may not be the way to avoid global instability, rather careful regulation (or deregulation!) of banking may be the answer. It also raises the possibility that depression and cyclical downturn are very different phenomena rather than essentially similar events of different magnitude.

We have produced a simple general equilibrium model with multiple equilibria, one of them being a low output, unemployment equilibrium. Moreover, if so simple a model can generate multiple equilibria, a fortiori more complex models of the banking system can exhibit instability. This is not to deny that developing such complex models is a valuable pursuit, quite the contrary. Hopefully, it will be possible to develop models in which the banking system, and more generally the means of transactions, are endogenous. For such models to support our simple model it is necessary that the banking system, either because

of some incompleteness or because of ill-advised government regulation, be a noncompetitive solution, just as fiat money is a noncompetitive solution in the pure consumption-loans model.

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