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AMBIGUITIES AND IMPLICIT ASSUMPTIONS IN RECENT ANALYSES OF THE INTRODUCTION OF FUTURES MARKETS AND BUFFER STOCKS

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Economists have long sought insights into the difficult problem of determining the effects of commodity futures markets and buffer stock policies on the welfare of commodity producers, inventory holders, and consumers. Recently Kawai (1983) and Turnovsky (1983) proposed a method for computing these effects within a class of infinite horizon, rational expectations models with stochastic production and demand. This method has been applied by Turnovsky and Campbell (1985), Campbell and Turnovsky (1985), Courchane and Nickerson (1985), and, with generalization for asymmetric information, Stein (1986).

Although the Kawai-Turnovsky method improves on earlier efforts in this area, it is still not adequate to answer the questions it is being used to address. Its inadequacies involve both basic ambiguities in questions about the effects of introducing a futures market or buffer stock and the manner in which those ambiguities are implicitly resolved by Turnovsky and Kawai.

At a general equilibrium level, the presence or absence of an economic institution is an endogenous, not an exogenous, property of an economy. Strictly speaking, it is ambiguous to ask about the effects of adding a futures market or buffer stock. We can only ask about the overall effects of some of the many possible changes in the structure of the economy that might, among other things, cause these institutions to arise.

Even if this general equilibrium ambiguity can be resolved or skirted, we face the additional partial equilibrium ambiguity that the effects of a new economic institution can depend critically on what economic institutions are exogenously assumed to be already in place. We can't ask about the effects of a futures market, for example, without specifying what other risk-shifting mechanisms are already in use.

None of those applying the Kawai-Turnovsky method to practical problems explicitly resolve these ambiguities. Besides discussing these ambiguities in general terms, this note also provides an interpretation of how Kawai and Turnovsky implicitly resolve them. For the partial equilibrium ambiguity this is done by embedding a Kawai-Turnovsky economy in an intertemporal model with a preexisting and minimally restricted credit market. Such a framework--perhaps the simplest intertemporal choice model for rigorously interpreting the Kawai-Turnovsky method--shows that in the Kawai-Turnovsky economy, credit markets are implicitly assumed to be very tightly restricted. In other words, compared to a solution of the more conventional intertemporal equilibrium problem that allows for consumption smoothing through borrowing and lending, the Kawai-Turnovsky method can be interpreted as giving a solution for an economy in which futures markets are free (or buffer stocks are operating) but credit markets are tightly restricted.

This interpretation places serious limitations on the usefulness of empirical results obtained by this method. Because futures markets and buffer stocks are rarely if ever introduced into economies where producers cannot use at least some credit to at least partially smooth consumption, use of the Kawai-Turnovsky method to realistically estimate the qualitative or quantitative effects of introducing a futures market or a buffer stock cannot be recommended.

A General Equilibrium Ambiguity

From a pure general equilibrium view, markets arise as agents seek to maximize subject to constraints. Agents' preferences and endowments (including their endowments of capital, information, and production and communication technology) constitute the exogenous structure of the economy. Based on that structure and the presumption that all feasible and mutually advantageous trades will be executed, an economic equilibrium may be determined. If so, the presence or absence of a commodity or asset market is determined by whether a nonzero volume of the commodity or asset is traded in at least some time period in equilibrium. For a previously inactive market to become active, something in the structure of the economy must change.

In terms of various criteria, including producer and consumer surpluses and price volatility, the difference between an economy with and an economy without a futures market can depend on exactly what structural characteristics differentiate them. Futures markets might arise in one economy because its agents hold very different attitudes toward risk than the agents in another economy. This would make welfare comparisons very problematic and would mean that even comparisons of price volatility would incorporate the effects of the preference differences that gave rise to the market differences. Alternatively, two economies might have identical agents but differing communication technologies. In this case, a question about the "introduction" of a futures market might be interpretable as shorthand for a more proper question about the effects of the communication technology. Even then the answer might be different than if the underlying structural change had been to the variability of output.

This basic ambiguity in questions about the effects of introducing futures markets may not always be a problem. For example, when policymakers decide whether to subsidize, penalize, or prohibit futures trading on the basis of factors essentially exogenous to the commodity markets affected, it may be reasonable for economists to compare two economies whose structures are identical except for the exogenous presence or absence of these government policies. This seems to be the type of question the Kawai-Turnovsky method has been designed and used to answer. Similarly, use of the method to analyze buffer stocks presumes that they are introduced without change in the underlying preferences and endowments of the model's producers, speculators, and consumers.

A Partial Equilibrium Ambiguity and Its Implicit Resolution

Even in a partial equilibrium framework with the existence of markets or buffer stocks taken to be exogenous, the Kawai-Turnovsky method can give dubious measurements of their impacts. This is because Kawai and Turnovsky make (at least implicitly) rigid and generally not very realistic assumptions about what other markets already exist when the futures market or buffer stock is introduced. In particular, Kawai and Turnovsky achieve their solution by assuming that credit markets operate only in a very limited sense that rules out consumption smoothing via borrowing and lending. This strips the wealth accumulation dynamics from the standard intertemporal consumption-investment problem and gives rise to a much simpler sequence of essentially static input decisions (in the face of a still dynamic forecasting problem). Unfortunately it strips much of the empirical interest and relevance from the problem as well.

These comments will be developed by contrasting producers' and speculators' decisions in a Kawai-Turnovsky economy to their decisions in a more general economy that allows intertemporal consumption smoothing via borrowing and lending. The Kawai-Turnovsky economy turns out to be a special case of the general economy where, in each period, producers and speculators are constrained to consume exactly their current profits and to borrow exactly the value of their current purchases of inputs. With borrowing and lending so limited, futures markets or buffer stocks may take on some of the consumption-smoothing role normally associated with credit markets. This additional function for futures markets or buffer stocks in the Kawai-Turnovsky model means that the model may exaggerate or distort the effects of introducing these institutions into economies with less restricted credit markets. This possibility is a specific reason for not using the Kawai-Turnovsky method to answer empirical questions about the effects of futures markets or buffer stocks in such economies. It is also an illustration of the general point that the effects of new economic institutions depend on which economic institutions already exist.

<u>A General Model With Flexible Borrowing</u>. At time t, representative producers have accumulated wealth equal to w_t^p. They choose $\bar{y}_{t+1}$, their mean output of corn at time t + 1. This entails purchasing $(\frac{\delta}{2})(\bar{y}_{t+1}+e_t^p)^2$ dollars worth of production inputs, where $\{e_t^p\}_{t=0}^{\infty}$ is a sequence of uncorrelated normal variates with mean zero and variance σ_{ep}^2. Producers observe e_t^p before choosing $\bar{y}_{t+1}$. They also choose current consumption (c_t^p) and savings (s_t^p) and enter into z_t futures contracts, each contract requiring delivery of a unit of corn at t + 1 for price p_t^f.

At time t + 1, producers receive a risk-free return of (1+r) on their savings and realize corn output of $\bar{y}_{t+1}+v_{t+1}$, where $\{v_t\}_{t=0}^{\infty}$ is a sequence of uncorrelated normal variates with mean zero and variance σ_v^2. The producers sell $(\bar{y}_{t+1}+v_{t+1}-z_t)$ units of corn at the spot price p_{t+1} and deliver z_t units to settle their futures contracts. All this changes their wealth to

$$(1) \qquad w_{t+1}^p = [w_t^p-c_t^p-(\tfrac{\delta}{2})(\bar{y}_{t+1}+e_t^p)^2](1+r) + p_{t+1}(\bar{y}_{t+1}+v_{t+1}-z_t) + p_t^f z_t$$

$$= [s_t^p+(\tfrac{\delta}{2})(\bar{y}_{t+1}+e_t^p)^2](1+r) + \pi_{t+1}^p,$$

where

$$\pi_{t+1}^p = p_{t+1}(\bar{y}_{t+1}+v_{t+1}-z_t) + p_t^f z_t - (\tfrac{\delta}{2})(1+r)(\bar{y}_{t+1}+e_t^p)^2$$

represents profits from producing corn. Having realized w^p_{t+1}, the producers again choose current (for t + 1) consumption, savings, futures position, and planned (for t + 2) output.

If the producers' utility of consumption is time separable, then the producers' decision problem has a recursive structure that permits it to be written as a dynamic program with state variable w^p_t and choice variables s^p_t, $\bar{y}^p_{t+1}$, and z_t. Letting $U^p(c^p_t)$ denote the producers' utility of current consumption, the producers' objective is to

$$\text{(2)} \qquad \max_{s^p_t, \bar{y}_{t+1}, z_t} \{U^p(c^p_t) + \beta E_t V^p(w^p_{t+1})\}$$

$$= \max_{s^p_t, \bar{y}_{t+1}, z_t} \{U^p[w^p_t - s^p_t - (\tfrac{\delta}{2})(\bar{y}_{t+1} + e^p_t)^2]$$

$$+ \beta E_t V^p[s^p_t(1+r) + p_{t+1}(\bar{y}_{t+1} + v_{t+1} - z_t) + p^f_t z_t]\},$$

subject to

$$\text{(3)} \qquad \lim_{k\to\infty} E_t \beta^k w^p_{t+k} = 0,$$

which rules out unlimited borrowing as a way of financing unlimited consumption. Here β is a discount factor and E_t denotes expectation conditional upon all information available at t. $V^p(w^p_{t+1})$ is the value function which gives the maximized value of

$$\text{(4)} \qquad E_{t+1} \sum_{j=0}^{\infty} \beta^j U^p(c^p_{t+1+j}),$$

subject to a given w^p_{t+1}, the law of motion (1), and the borrowing constraint (3).

Speculators (in corn inventories) face a similar decision problem. At time t they have accumulated wealth of w^i_t. They purchase an inventory i_t to carry over to the next period, thereby also incurring current storage costs of $(\gamma/2)(i^p_t+e^i_t)^2$, where $\{e^i_t\}^\infty_{t=0}$ is a sequence of uncorrelated normal variates with mean zero and variance $\sigma_{ei}{}^2$. The speculators see e^i_t before choosing i_t. They also choose current consumption (c^i_t) and savings (s^i_t) and enter into x_t futures contracts. At time t + 1, they receive the risk-free (1+r) rate of return on savings, sell (i_t-x_t) at the spot price, and deliver x_t units against their futures contracts. This changes their wealth to

$$
\begin{aligned}
(5) \qquad w^i_{t+1} &= [w^i_t-c^i_t-(\gamma/2)(i_t+e^i_t)^2-p_t i_t](1+r) + p_{t+1}(i_t-x_t) + p^f_t x_t \\
&= [s^i_t+(\gamma/2)(i^2_t+e^i_t)^2+p_t i_t](1+r) + \pi^i_{t+1},
\end{aligned}
$$

where

$$
\pi^i_{t+1} = p_{t+1}(i_t-x_t) + p^f_t x_t - (1+r)[(\gamma/2)(i^2_t+e^i_t)^2+p_t i_t]
$$

represents the profits from inventory holding. Assuming that speculators also have time separable utility of consumption, their decision problem can also be written as a dynamic program, with state variable w^i_t and decision variables s^i_t, i_t, and x_t.

Consumers' behavior is represented by the demand function $D_t = A - ap_t + u_t$, where D_t is aggregate consumption of corn and $\{u_t\}^\infty_{t=0}$ is a sequence of uncorrelated normal variates with mean zero and variance σ_u^2. The random variables u, v, e^p, and e^i are also uncorrelated at all dates. This closes the futures trading version of the model. The no-futures version is obtained by setting $z_t \equiv x_t \equiv 0$ for all t.

Except possibly for the case of linear utility, which is of limited interest for analyzing the effects of futures trading, this dynamic programming problem does not permit an analytical solution such as Kawai and Turnovsky achieved for their problem. This means that analysis of the futures versus no-futures versions of this model would in general require much more computing power and expense than for its special case, the Kawai-Turnovsky model. 1/

The Kawai-Turnovsky Model. Though rather intractable for empirical work, the above choice problem with flexible borrowing and lending can be used to interpret the Kawai-Turnovsky model. The latter can be rigorously obtained by making more specific assumptions about utility functions and random terms and by imposing strict borrowing constraints on producers and speculators. These constraints simplify the solution of the model considerably, but by nearly shutting down credit markets may also seriously distort measures of the effects of futures trading or buffer stocks.

The Kawai-Turnovsky model of producer behavior is essentially identical to the model presented above except that producers are constrained to set $s_t^p = (-\delta/2)(\bar{y}_{t+1}+e_t^p)^2$. That is, they are assumed to borrow exactly the cost of their production inputs. As shown by equation (1), this also means that each period they consume exactly the profits, π_t^p, that they realize in that period. This collapses the producers' objective at t from (2) to

$$\max_{\bar{y}_{t+1}, z_t} \quad \beta E_t U^p(\pi_{t+1}^p). \tag{6}$$

The model no longer contains a state variable, so there are no dynamic feedback effects by which individual producers' current decisions constrain their future decisions. Rather than having to solve a dynamic program to determine current actions, producers in the Kawai-Turnovsky model face an essentially static problem, albeit one with a production lag and a still-dynamic price forecasting component (since current aggregate inventories affect the distribution of all future prices). Kawai and Turnovsky similarly simplify the speculators' problem by imposing $s_t^i = -[(\gamma/2)(i_t^2+e_t^i)^2+p_t i_t]$. These simplifications, combined with specific assumptions about the variances of random terms, linear mean-variance forms for producers' and speculators' utility functions, and rational expectations methods for handling the expectational difference equations that arise in the first-order conditions, allow Kawai and Turnovsky to analytically solve their model's competitive equilibrium. 2/

Comparison of the Models. Neither the general model nor the Kawai-Turnovsky model is uniformly superior to the other for all purposes. The general model can capture some effects that are assumed away in the Kawai-Turnovsky model, while the latter is more tractable. The interesting insights into futures markets that have been obtained by Kawai, Turnovsky, and others using their method show that this tractability is valuable.

However, the Kawai-Turnovsky model could give very misleading answers to practical questions because of its implicit and generally highly unrealistic assumptions about borrowing and lending. In particular, the incremental effects on producers, speculators, and consumers of opening a futures market (or buffer stock) will depend on whether those agents already have access to relatively unrestricted credit markets. With flexible credit, they can already reduce risks by borrowing and lending to smooth consumption. The additional risk-shifting opportunities provided by a futures market (or buffer stock) may appear much different when the futures market (buffer stock) is opened to agents who cannot borrow or lend (except to borrow exactly their current production expenses). Since as a historical matter it seems reasonable to say that credit markets developed before futures markets (buffer stocks) in most actual economies, the assumptions underlying the Kawai-Turnovsky model appear counterfactual. [3]

Summary

The welfare effects of opening any market or institution depend on what structural changes allow the new market or institution to open and on which markets are already open. At a minimum, therefore, those who use the Kawai-Turnovsky method to evaluate the effects of futures markets or buffer stocks should explicitly state which markets they assume to be open in the no-futures or no-buffer-stock economy. A rigorous embedding of their model into the framework of a more general intertemporal consumption-investment model implies that they have assumed virtually nonexistent credit markets. This probably makes the Kawai-Turnovsky method unreliable for analyzing the effects of introducing futures markets or buffer stocks in economies with active credit markets.

Footnotes

1/ One result obtained by Kawai and Turnovsky and earlier analysts of similar models [Danthine (1978), Holthausen (1979)]--that with futures trading the producers' production decision can be made using the futures price as a certainty equivalent--does generalize to the multiperiod dynamic programming problem, as shown by examination of the first-order conditions for the producers' problem.

2/ At this point Kawai and Turnovsky diverge temporarily in justifying the linear mean-variance objective they both use. Kawai gets rid of output uncertainty by setting $\sigma_v^2 = 0$, which allows him to assume that profits are normally distributed in equilibrium. He then posits constant absolute risk aversion utility functions and rigorously derives the linear mean-variance form. Turnovsky adopts a more realistic specification of production uncertainty by allowing $\sigma_v^2 > 0$. However, even after setting $\sigma_{ep}^2 = \sigma_{ei}^2 = 0$, his profit functions involve quadratic random terms and are thus not normally distributed. Turnovsky justifies the linear mean-variance objective as an acceptable approximation to the true objective. [See Turnovsky and Campbell (1985; fn. 9)].

3/ A possible alternative interpretation of the Kawai-Turnovsky model is that producers and speculators can borrow and lend but the Kawai-Turnovsky mean-variance objective function in profits is simply a crude local approximation to the value function for current wealth. This interpretation is not rigorous. In particular, it would imply that the risk-aversion parameters in the Kawai-Turnovsky objective functions are state dependent and therefore time varying [as the location of the local approximation to $V(w_t)$ shifts]. This in turn would render invalid the constant-coefficient difference equation techniques used by Kawai and Turnovsky to solve their models.

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