MEASUREMENT ISSUES IN PRODUCTION SMOOTHING:

EVIDENCE FROM PHYSICAL-UNIT DATA

by

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Section 1. Introduction.

Early theoretical investigations of optimal inventory and production behavior established that if production costs are convex, then it is optimal for a firm to only partially adjust output in response to a change in its beginning-of-period inventory position. If such partial adjustment was prevalent among firms, then one would expect to observe sales much more variable than output. However, many studies (e.g. Blinder (1981), Blinder (1986), and West (1986)) have found that the variance of detrended production often exceeds that of shipments, casting doubt on the empirical validity of production smoothing models.

The theoretical results establishing the optimality of production smoothing behavior are based on the relationship between the physical unit flows of production and demand and the stock of final products held by the manufacturer. However, most studies of production-smoothing behavior have used value-based data derived from the Census Bureau's M-3 reports of the Monthly Survey of Manufacturers' Shipments, Orders, and Inventories. There is a long list of well known caveats concerning the use of these data to measure the physical-unit concepts. In this paper, we exploit data on physical-unit measures of production, inventories, and sales to measure more directly the relative variability of production and sales for a wide variety of industries. Our results indicate that production smoothing likely is more prevalent than studies using the M-3 based data would suggest.

Formal theoretical models also have emphasized that to completely characterize the production decisions of the firm, one must carefully account for both the costs of holding inventories and the (likely different) costs incurred if demand exceeds that amount of goods available for sale and a stockout occurs. However, such models are empirically difficult to implement, both because they often do not imply tractable decisions rules, and because in industries where firms do not record unfilled orders, there are no available measures of excess demand. Consequently, most empirical studies of production smoothing have not accounted for stockouts. We discuss the effects of excess demand on empirical tests of production smoothing, and in industries where we have unfilled orders data, we include these measures in our empirical investigation.

In section 2 below, we describe some of the well-known problems of M-3 based inventory data. In section 3, we examine the relative variability of production and shipments on a data set containing information on 48 physical-unit series. Section 4 discusses the standard quadratic-cost model tests of production smoothing and suggests some alternative variance bounds. In Section 5, we examine estimates of the quadratic cost function. Section 6 examines five of our series that contain information on unfilled orders. Section 7 concludes.

Section 2. The M-3 Based Inventory Data.

The primary source of the data used in most studies of production smoothing behavior is the Census Bureau's M-3 reports of the Monthly Survey of Manufacturers' Shipments, Orders, and Inventories. In these surveys, manufacturers give information about their shipments during a month as well as the book value of end-of-period levels of inventories and unfilled orders. Data are reported at the 2-digit SIC code level. Most researchers have used the seasonally adjusted, constant-dollar version of these inventory data that is produced by the Bureau of Economic Analysis. Because the BEA data are seasonally adjusted and the desire to smooth out seasonal fluctuations likely constitutes an important source of production smoothing behavior, many authors have used seasonal factors from the M-3 series to reintroduce seasonality into the BEA data. Production is calculated as the residual between the shipments data and the change in end-of-period (adjusted) inventories according to the familiar identity,

$$H_t = H_{t-1} + Q_t - S_t, \tag{1}$$

where H_t are the end-of-period inventory stocks, Q_t is the flow of production, and S_t is the flow of shipments during time t.

Inventory researchers have long recognized the shortcomings of these data. First, while the book-value of inventories are based on acquisition costs, shipment and order data are recorded at market value.

Hence, the inventory data must be adjusted to current market value to be comparable to the shipments series, or else the identity, (1), will not hold (see West (1983)). Second, accounting practices do not provide a uniform definition of book value; firms may use either FIFO or LIFO procedures as well as a fairly broad definition of acquisition costs in valuing their inventory stocks. Third, only about 50 percent of manufacturing firms who report inventories do so by stage of processing, so that further noise is introduced into the data when estimating allocations among stages of processing. Finally, if there is significant seasonality in inventory price deflators or other adjustments made to BEA constant-dollar series, the seasonality induced into implied production from the M-3 seasonals may be inconsistent with the movements in seasonally unadjusted shipments.

Although the BEA has made a careful effort in adjusting inventories for differences between recorded book and market value and deflating these series to constant-dollar levels, the procedures used in constructing these data likely add significant measurement error to any production series derived from equation (1), and consequently, obfuscate comparisons of the relative variability of production and sales. Indeed, Miron and Zeldes (1987) find that production is often less volatile than sales when industrial production indexes are used in place of implied production to measure output. However, there are a host of data construction issues associated with industrial production that make these indexes quite imperfect measures of production, also.

Finally, the theoretical arguments supporting production smoothing are all based on models of representative firm behavior. In most industries, the 2-digit SIC code level covers a large number of establishments. Within a 2-digit classification, the outputs of some firms will be used as inputs by other firms, so that shipment data may overstate actual product demand (see Lovell (1976)). Furthermore, it is possible that aggregation effects in industry data could offset production smoothing behavior at the firm level (see Paxton (1986) and Krane (1987)).

Section 3. The Relative Variability of Production and Shipments in Physical-Unit Data

The caveats concerning M-3 based data suggest that it might be informative to investigate production-smoothing questions using alternative data sources. In this section, we examine a number of series on production, inventories, and shipments collected by trade organizations and government agencies. These data have several advantages over the standard transformations of the M-3 data. First, at the reporting level, many of these series contain direct observations of production instead of relying on a series created from the inventory

identity.[1] Second, the data are recorded in terms of physical units, allowing us to avoid the complications associated with revaluing inventory stocks. Third, the data are not seasonally adjusted, so that we do not need to reintroduce seasonal variability using measures derived for different series. Finally, many series are available at levels of disaggregation below the 2-digit level, lessening the possibility of aggregation biasing the variability results.[2]

Table 1 describes our data set. The data are grouped into two categories. The first group consists of 29 industries where we have separate data on production, inventories, and shipments. The second group contains 19 industries for which we have data on production and inventories only. The physical measurement units and primary data source(s) are found in the second two columns. Within the first two groups, several series also include information on unfilled orders, and these are identified by the "U" superscript. All data are at a monthly frequency with the exception of the last 5 group 2 series, which are quarterly data.

Our first exercise calculates the relative variances of production and sales for the data in the group 1 industries. Following West (1986), we assume that the firm's optimal production rule implies that, after adjusting for trend, production and shipments follow a covariance

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1. We do not, however, know if individual firms use the difference between shipments and the change in inventories to estimate production.
2. The series are compiled by the BEA. The data are found in current issues of the <u>Survey of Current Business</u>, as well as <u>Business Statistics</u>. We thank Ken Beckman at the BEA for providing us with time series of the data.

stationary process that can be approximated by a finite order vector autoregressive process. We use the VAR system in production and shipments,

$$S_t = a_0 + \sum_{i=1}^{3} a_i S_{t-i} + \sum_{i=1}^{3} b_i Q_{t-i} + a_{12} S_{t-12} + b_{12} Q_{t-12} + t_s time + u_{st} \quad (2)$$
$$Q_t = c_0 + \sum_{i=1}^{3} c_i S_{t-i} + \sum_{i=1}^{3} d_i Q_{t-i} + c_{12} S_{t-12} + d_{12} Q_{t-12} + t_q time + u_{qt}, \quad (2)$$

to calculate the (unconditional) variances of production and shipments, where "time" is a linear time trend, and the 12^{th} lag has been included to account for seasonal effects. The variances are calculated following Andersen (1971).[3] The resulting ratios of the variance of sales to the variance of production are tabulated in column A of table 2a.[4]

The results of the VAR between production and sales (column A) fail to indicate many cases of excess production volatility. In 18 out of 29 cases, the ratio of the sales variance to the production variance is greater than one--indicating some amount of production smoothing in most of the group 1 industries. The majority of ratios are grouped close to

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3. We are aware that this procedure need not produce an unbiased estimate of the population variance of production and shipments. For example, the time series properties of the series may imply that simple linear detrending does not induce stationarity. Alternatively, a longer order autoregressive process may be required to yield serially uncorrelated error terms. In addition, at the firm level, stockouts truncate inventories to zero if unfilled orders cannot be carried, so that the errors (2) may have a complicated distribution that is not well approximated by a normal density function.

4. We have not yet worked out the asymptotic distribution of the variance ratio. We plan to include standard errors of these statistics in the next version of the paper.

one, but several series, distilled spirits, iron and steel scrap, copper, zinc, kerosene, residual fuel oil, jet fuel, lubricants, asphalt, and liquefied gases all have ratios in excess of 2.0. The variance ratios are greatly below unity in only two out of 29 cases, still wines and bituminous coal, although beer and hardwood lumber have ratios below 0.8.[5] These results are consistent with those of Ghali (1987), who found that nonseasonaly adjusted detrended output was much more variable than shipments in only 1 out of the 7 physical product series that he examined.[6]

To be consistent with the calculations used with M-3 based data, we also calculated the variances of shipments and sales using a VAR between shipments and inventories,

$$S_t = a_0 + \sum_{i=1}^{3} a_i S_{t-i} + \sum_{i=1}^{3} b_i H_{t-i} + a_{12} S_{t-12} + b_{12} H_{t-12} + t_s time + u_{st} \quad (3)$$

$$H_t = c_0 + \sum_{i=1}^{3} c_i S_{t-i} + \sum_{i=1}^{3} d_i H_{t-i} + c_{12} S_{t-12} + d_{12} H_{t-12} + t_h time + u_{ht}, \quad (3)$$

5. It is interesting to note that the high variation in production of still wines is consistent with Blinder's (1986) explanation that production will be more variable than sales if cost factors are more volatile than demand. Here, the relative cost of producing wine outside of the autumn harvest season is extremely high, so production is bunched into September and October. In contrast, shipments from wineries are relatively constant over the year.

6. Ghali apparently calculates simple sample variances, and did not find the unconditional variance of a time-series process. If detrended output and shipments are autocorrelated, in finite samples, the sample variance is a biased estimator of the unconditional population variance.

and the inventory identity, (1). These results are found in column B, and indicate much more relative production volatility than the calculations based on equation (2). Here, implied production is less variable than shipments in only four industries, kerosene, lubricants, liquefied gases and Canadian newsprint. Furthermore, the ratios change dramatically for almost half of the series.

Finally, if we had information on only production and inventories (as in our group 2 data) then we would need to calculate the variances of production and sales from a VAR between production and inventories:

$$Q_t = a_0 + \sum_{i=1}^{3} a_i Q_{t-i} + \sum_{i=1}^{3} b_i H_{t-i} + a_{12} Q_{t-12} + b_{12} H_{t-12} + t_q \text{time} + u_{qt} \quad (4)$$

$$H_t = c_0 + \sum_{i=1}^{3} c_i Q_{t-i} + \sum_{i=1}^{3} d_i H_{t-i} + c_{12} Q_{t-12} + d_{12} H_{t-12} + t_h \text{time} + u_{ht} \quad (4)$$

The relative variances implied by this system are found in column C, and these results are consistent with production being smoother than sales for most industries. In most cases, the ratios based on equations (4) are the highest of our three cases, that is, more indicative of production smoothing. For distilled spirits and liquefied gases, these results indicate extreme production smoothing.

We investigated the source data references to see if data construction issues could give us a preferred VAR specification. The series with significant definitional issues are noted at the bottom of the table, and where these point to a particular VAR representation, that column has been marked with an asterisk (*). For example, the source

references indicated that for refined petroleum products, the shipment series are derived from information on production, stocks, imports, and exports. Consequently, the VAR between the variables that are closest to the original source data, production and inventories, likely gives us the best view of the relative variances of production and sales. It is interesting to note that none of the judgemental assessments favor the VAR system (3) between inventories and shipments. In about half the cases, the choices favor the system that produces the smoothest relative production reading among the three measures.

We also calculated the ratios of the variance of the change in production to the change in shipments implied by all three VAR systems. The results are found in table 2b. Although the results do not differ significantly from the computations based on detrended levels, they do indicate somewhat smoother relative production. Table 3 presents calculations of the relative variances of production and sales for the series in group 2 where we only have inventory and production data. Due to data reporting problems, we were forced to split our data sets in half for several series, and we've reported both sets of results. The variances are calculated from system (4), and show little evidence of excess production volatility, and many relative variance ratios are well over unity.

Finally, for the group 1 industries, we constructed the implied production series, $FQ_t = H_t - H_{t-1} + S_t$, and compared them to published production, Q_t, using the regression,

$$FQ_t = a + bQ_t + u_t, \tag{5}$$

along with tests of the joint hypothesis that a = 0 and b = 1 and separate tests for autocorrelation in the u_t. The results are found in table 4. The goodness of fit, as measured by the adjusted $\bar{R}^2$, is surprisingly low in a number of industries. As measured by a Q-statistic calculated over the first 12 errors, the u_t show significant autocorrelation at the 5 percent level in about half of the cases. Standard F-tests indicate that the null hypothesis of a = 0 and b = 1 is rejected at the 5 percent level in 10 cases, although because of the autocorrelation in the u_t, the sizes of these statistics are incorrect. In several industries, imports account for a significant portion of the inventories held by domestic producers, and where the data were available, we reran (5) with $FQ_t = H_t - H_{t-1} + S_t + M_t$, where M_t are imports. However, in most cases, this adjustment did not change the results much. The failure of imports to improve the fit of these equations may reflect the fact the import series generally are derived from Customs data, and there are notorious difficulties in the temporal allocation of these data.

Although there are a host of caveats concerning the use of such VAR systems to calculate the unconditional moments of production and shipments, this preliminary look at the data indicates that the excess volatility of production may not be as widespread as the M-3 based data would suggest. In addition, our results generally are consistent with Miron and Zeldes (1987). We also find that differences between reported

output and production implied by the inventory identity point to an important measurement problem, and that implied output measures generally yield higher relative production variation than readings based on more direct measures of output.

As demonstrated by West (1986), there is more to the evaluation of production smoothing models than the simple comparison of the variances of production and sales. Rather, these statistics must be evaluated in the context of estimates structural parameters of the underlying cost functions faced by the firm. We next turn to tests which rely on these estimates and examine some simple modifications to West's variance bound tests.

Section 4. Measuring Demand and its Implications for Variance Bounds Testing

One potential problem with the variance bounds test proposed by West (1986) centers around the differences between sales and demand. For firms that cannot backlog orders, unmet demand is lost forever, and sales will often fall short of demand. In these industries, (which have become known as "production-to-stock" industries following Belsley (1969)), sales are clearly a poor proxy for demand. In industries where firms are able to backlog orders, sales will differ from demand by the change in unfilled orders, so that these two series also may have quite different time-series properties. The balance of this section explores

a simple modification to the linear-quadratic framework that accounts for the differences between sales and demand.[7]

The derivation of the partial-adjustment decision rule for inventories begins with a quadratic-cost function of the type developed by Holt, Modigliani, Muth, and Simon (1960). Since the West paper is the point of departure, consider the cost-minimization problem that he analyzes,[8]

$$\min_{Q_t} E_0 \sum_{t=0}^{\infty} d^t \{a_0(\Delta Q_t)^2 + a_1(Q_t)^2 + a_2(H_t - a_3 S_{t+1})^2\}$$
$$\text{s.t. } Q_t = S_t + \Delta H_t \qquad (6)$$

where d is the one-period discount factor and E_0 is the mathematical expectations operator conditioned on all information known to the firm at time 0. All variables are measured in comparable units.

We first examine industries where firms cannot backlog orders. Even though we cannot observe actual demand in these industries, the exercise emphasizes the distiction between sales and demand. We consider two simple modifications of (6). First, rather than setting the target level of inventories proportional to sales, we let target inventories be proportional to demand, N. Second, we add a term to the cost function

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7. There is no pretention here that the quadratic cost function can be developed rigorously from first principles in the manner, for example, of Arrow, Karlin, and Scarf (1958). Rather, we work from this framework because it is a useful approximation to the cost function that yields tractable functional forms for empirical work.

8. Actually, West considers the analogous profit-maximization problem, but the inclusion of output prices has no implications for this paper.

to reflect the opportunity cost of lost sales.[9] These changes yield the cost function:

$$\min_{Q_t} E_0 \sum_{t=0}^{\infty} d^t (a_0(\Delta Q_t)^2 + a_1(Q_t)^2 + a_2(H_t - a_3 N_{t+1})^2 + a_4(N_t - S_t)^2)$$

$$\text{s.t. } Q_t = S_t + \Delta H_t \tag{7}$$

An interesting contrast can be derived from comparing the variance bounds tests developed from (6) and (7) according to the methodology developed by West (1986). Suppose C^*, Q^*, H^*, and S^* are the costs, production, inventories, and sales that result from the minimum-cost solution to the programming problem specified by (7). That is,

$$C^* = E_0 \sum_{t=0}^{\infty} d^t (a_0(\Delta Q_t^*)^2 + a_1(Q_t^*)^2 + a_2(H_t^* - a_3 N_{t+1})^2 + a_4(N_t - S_t^*)^2) \tag{8}$$

This cost function suggests two separate variance-bounds tests. The first is derived by comparing C^* with the costs that would occur if the firm had maintained the same sales path S^*, but had done so without the benefit of inventory holdings. The second variance bound compares C^* with the costs associated with the plan where the firm adjusts

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9. In principle we should also add the constraint that $S_t \leq N_t$, but then there would not be a neat and closed-form solution to the programming problem (see Karlin, p.158). Rather, the quadratic-cost approximation replaces this constraint with a penalty proportional to the square of the excess of sales over orders. Obviously this is not perfect.

production to equal demand in every period--a plan that also dispenses with the need for inventories.

In the first case, production moves in lock-step with sales, (with $Q^a=S^*$ and $H=0$), and results in expected costs,

$$C^a = E_0 \sum_{t=0}^{\infty} d^t \{a_0(\Delta S_t^*)^2 + a_1(S_t^*)^2 + a_2(-a_3 N_{t+1})^2 + a_4(N_t - S_t^*)^2\} \quad (9)$$

Since C^* is the optimal plan, it follows that $C^a > C^*$, so that subtracting (9) from (8) yields:

$$0<C^a-C^* = E_0 \sum_{t=0}^{\infty} d^t \{a_0(\Delta S_t^{*2}-\Delta Q_t^{*2}) + a_1(S_t^{*2}-Q_t^{*2}) + a_2(H^{*2}-2a_3 H_t^* N_{t+1})\} \quad (10)$$

If all variables have been converted to stationarity by computing them as the difference from their trends[10], it follows that $E(x^2) = var(x)$, and (10) implies the following variance bound:

$$0 < a_0(var(\Delta S)-var(\Delta Q)) + a_1(var(S)-var(Q)) - a_2 var(H) - a_2 a_3 cov(H,N_{+1}) \quad (11)$$

In contrast, using the the cost function (6) and the alternative strategy that $Q^A = S^*$ and $H^A = 0$, West derives the variance bound,

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10. It is an assumption that these series can be converted to stationarity by extracting a time trend. Obviously, the best method of inducing stationarity depends on the nature of the underlying economic process.

$$0 < a_0(\text{var}(\Delta S)-\text{var}(\Delta Q)) + a_1(\text{var}(S)-\text{var}(Q)) - a_2\text{var}(H)-a_2a_3\text{cov}(H,S_{+1}) \qquad (12)$$

West finds that (12) is rejected for most of the industries that he examines--a result that apparently rejects the production-smoothing model. There is only one minor difference between (11) and (12)--the replacement of $\text{cov}(H,N_{+1})$ with $\text{cov}(H,S_{+1})$--and even if demand could be accurately measured, it is likely that any dataset that would reject (12) would also reject (11). Yet there remains the technical point that the term (N-S) is missing from the Euler equations that provide the rejections of model (6). If the Euler equation really should be based on the cost function (7), the coefficients estimated from the standard instrumental variable techniques will be biased because this missing term likely is highly correlated with the instrument set (particularly lagged stocks and shipments).

The second variance bound that arises from (7) compares C^* with the cost that would have resulted from a path where Q = S = N. From this path, we can derive the bound,

$$0 < a_0[\text{var}(\Delta N)-\text{var}(\Delta Q)] + a_1[\text{var}(N)-\text{var}(Q)] - a_2\text{var}(H) -a_2a_3\text{cov}(H,N_{+1}) - a_4[\text{var}(N) - 2\text{cov}(N,S) + \text{var}(S)] \qquad (13)$$

The existence of this alternative condition raises a major procedural problem: suppose that production was smoother than demand so that the variance bound (13) is met, but more volatile than sales so

that condition (12) is violated. Would that be judged a sufficient reason to discard the linear-quadratic framework? At first it may seem frivolous that the resolution of the variance-bounds paradox may lie in the distinction between demand and sales; because demand is not available to the econometrician for many industries, this resolution may have little empirical content.[11] Yet for a large class of industries--those that report unfilled orders--data exits for both orders and sales. By examining these industries very carefully, we possibly can shed some light on the industries that do not report the distinction between sales and demand.

These "production-to-order" industries have been studied using cost functions proposed by Childs (1967). This cost function, which has separate roles for both inventories and unfilled orders, is:

$$C^* = \min E_0 \sum_{t=0}^{\infty} d^t (a_0 (\Delta Q_t)^2 + a_2 (H_t - a_3 N_{t+1})^2 + a_4 (U_t - a_5 Q_t)^2) \quad (14)$$

In (14), inventories have a desired stock that is proportional to new orders while unfilled orders have a target level that varies with production. The justification for the last term is that a longer backlog of unfilled orders makes it easier to schedule production. As in the cost function (7) above, two variance bounds tests can be developed from Childs' cost function; one compares the optimal path with

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11. Even though demand cannot be observed, there may exist proxies that, in principle, allow these models to be estimated. See Krane (1985) for a discussion of these issues and an (unsuccessful) attempt to estimate such a model using refined gasoline data.

the scenario that production exactly follows S^* while the second compares the optimal path with the case where production exactly meets demand.

The variance bound for the first alternative ($Q^A = S^*$, and $H^A = 0$) is,

$$0 < a_0[\mathrm{var}(\Delta S) - \mathrm{var}(\Delta Q)] + a_4 a_5^2[\mathrm{var}(S) - \mathrm{var}(Q)] - a_2\mathrm{var}(H)$$
$$+2a_2 a_3 \mathrm{cov}(H, N_{+1}) + 2a_4 a_5[\mathrm{cov}(Q,U) - \mathrm{cov}(U,S)] \qquad (15)$$

The first four components of this variance-bound are almost identical to the terms in the variance bound for the "production-to-stock" industries (11). However, (15) contains an additional term, involving the covariance of unfilled orders with production and shipments. Thus, the variance bound may not be violated when production is more volatile than shipments if: 1) inventories vary strongly with orders, or 2) if unfilled orders vary more strongly with production than with shipments.

In the second alternative stragtegy, the firm dispenses with both inventories and unfilled orders, as $Q^A = N$ and $H^A = U^A = 0$. The variance bound is:

$$0 < a_0[\mathrm{var}(\Delta N) - \mathrm{var}(\Delta Q)] + a_4 a_5^2[\mathrm{var}(N) - \mathrm{var}(Q)] - a_2\mathrm{var}(H)$$
$$+2a_2 a_3 \mathrm{cov}(H, N_{+1}) + a_4[\mathrm{var}(U) - 2a_5\mathrm{cov}(Q,U)] \qquad (16)$$

This condition, which is similar to (13) above, is most sensitive to the relative variance of production and new orders.

If condition (16) were satisfied, then firms would be successfully using their inventories and unfilled orders to smooth production relative to demand. Satisfaction of (15) would indicate that firms are using their inventories to smooth production relative to sales. In principle, both (15) and (16) should hold, but it is possible that (15) could hold while (16) fails.

Section 5. Euler Equation Estimation to Evaluate the Variance Bounds for Production and Shipments

Evaluation of the variance bounds proposed in the last section requires explicit estimates of the cost parameters $\{a_i\}$. We attempted to determine the values of these structural parameters using the standard method of estimating the first-order conditions of the cost functions (6) and (14). These Euler equations are shown at the top of tables 5 and 6. Instrumental variable estimation proceeded according to Hayashi and Sims (1983)--a method that corrects for the moving-average error encountered when estimating a rational expectations system. The results (not shown) were disappointing. Although all of the $\{a_i\}$ ought to be positive for the cost functions to be well defined, the estimates of individual parameters were positive only about half the time; as shown in table 6, only 86 of the 156 parameter estimates (4 parameters in 39 industries) were of the correct sign. In only 2 of the 39

industries did all four $\{a_i\}$ estimates turn out to be positive.[12] Furthermore, the frequency of positive estimates did not vary with any of the data problems discussed in section 3. As shown in the three right-hand columns, the success rate did not depend on whether measured production was used or whether a production estimate was inferred from inventories and shipments. In addition, although we have not yet calculated Hansen-type J-statistics, the general fits of the equations were poor, and in many cases we experienced convergence difficulties when iterating over the covariance matrix.

For the 5 industries that report unfilled orders, estimation of the cost parameters of the variance bound (15) was equally unsuccessful (table 6). Only 9 of the 20 parameters estimated had the correct sign, and none of the industries produced positive estimates for all of its parameters.

Given this unfortunate lack of success in Euler-equation estimation, it seems pointless to proceed in evaluating the variance bounds given by equations (12), (13), (15), or (16). Instead, we examine the industries where firms carry unfilled orders more closely to see if, at least on the surface, these data are consistent with the production smoothing paradigm.

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12. Ten series in group 2 were dropped because of some sampling problems. They will be included in future revisions.

Section 6: Industries that Report Unfilled Orders

As discussed in section 4, a major measurement issue is the difference between sales and demand. While we assume that sales are a poor measure of demand in all of the industries we study, five of the industries report a measure of new orders. Because new orders are a more suitable measure of demand, it is a useful exercise to examine the relative variances of production, shipments and new orders for these industries--despite the lack of success in estimating the Euler equations.

We estimated the variances in a fashion similar to that used in section 3--except we now add a variable to the VAR system to account for the difference between sales and demand. In principle, this difference is a relevant factor in every other VAR system estimated above. In three of the series, douglas firs, southern pine, and western pine, we have information on production, shipments, inventories, unfilled orders and new orders. In cotton and synthetics & silk, we have information on the level of unfilled orders, production, and inventories. Consequently, we estimate the relevant variances for the lumber industries from both a VAR between production, shipments, and new orders as well as a VAR using production, inventories and the level of unfilled orders. Only the latter VAR may be used for the fabric industries.

The results are shown on table 5. The relative variability of production and shipments are quite similar to those found using the 2-variable VAR's, with the variance of production fairly close to that of

shipments in the lumber industries and production slightly smoother than shipments in the fabric categories. In general, production is smoother relative to demand than it is relative to sales.[13] This finding substantiates the recent work of West (1987) indicating that orders are more volatile than production in the M3 data. In addition, shipments are smoother than demand for four out of the five industries. This result is consistent with the implications of the Childs model, as well as with Kahn's (1987) proposition that firms will use order backlogs to smooth shipments relative to demand.

Section 7: Conclusion

We have addressed the problem of the excess volatility of production relative to sales by examining two separate measurement issues in inventory models. First, we find that the excess volatility results appear to be sensitive to how production is measured. For 29 industries, we have obtained physical-unit measures of both production and shipments--data that circumvents many of the pitfalls of inferring production from the book values of sales and inventories. For the majority of these industries, production is at least as smooth as sales when measured directly from the production and shipment data. More interestingly, this result does not hold when we infer production from sales and inventories. Indeed, we find that the inventory identity

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13. The only exception to this is for western pine, and then only for var(N)/var(Q). However, var(ΔN)/var(ΔQ) does indicate production smoothing.

often does not hold in our data, suggesting that the identity probably does not hold in the M-3 data, either. Therefore, the volatility results derived from the Census data could be misleading.

The second issue involves the measurement of demand. Sales often will not equal demand because of stockouts or sales out of backlogged orders, and we suggest different variance bounds to account for these differences. Although we rarely can observe excess demand, industries that report unfilled orders do show a measure of the discrepancy between sales and demand--opening a window on how the world might look if all industries reported demand measures. However, our attempts to estimate Euler equations were unsuccessful regardless of whether we used actual production data or the series implied from the inventory identity. This prevented us from calculating the alternative variance bounds tests for these industries. In general, it appears that some part of the model used to test production-smoothing behavior is misspecified. The model is a joint hypothesis standing on at least three principle pillars: 1) a linear-quadratic model of the costs facing an optimizing firm, 2) rational expectations, and 3) constant coefficients. At least one of these pillars is shaky.

Table 1

Group 1: Industries with data on production, inventories, and shipments.

Industry name	Units	Primary data source
Denatured alcohol	millions of wine gallons	Bureau of Mines
Beer	millions of barrels	Treasury Department
Distilled spirits	millions of tax gallons [1]	Treasury Department
Effervescent wines	millions of wine gallons	Treasury Department
Still wines	millions of wine gallons	Treasury Department
Lumber (total)	millions of board feet	National Forest Product Assoc.
Lumber (hardwood)	millions of board feet	National Forest Prodcut Assoc.
Lumber (softwood)	millions of board feet	National Forest Product Assoc.
Lumber (douglas fir)[U]	millions of board feet	National Forest Product Assoc.
Lumber (south. pine)[U]	millions of board feet	National Forest Product Assoc.
Lumber (west. pine)[U]	millions of board feet	National Forest Product Assoc.
Iron and steel scrap	thousands of short tons	Census and Interior
Pig iron	thousands of short tons	Amer. Iron & Steel Inst.& Census and Interior
Refinery copper	thousands of metric tons	Bureau of Mines
Slab zinc	thousands of metric tons	Bureau of Mines and Amer. Bureau of Metal Statistics
Bituminous coal	thousands of short tons	Department of Energy
Gasoline	millions of barrels	Department of Energy
Kerosene	millions of barrels	Department of Energy
Distillate fuel oil	millions of barrels	Department of Energy
Residual fuel oil	millions of barrels	Department of Energy
Jet fuel	millions of barrels	Department of Energy
Lubricants	millions of barrels	Department of Energy
Asphalt	millions of bareels	Department of Energy
Liquefied gases	millions of barrels	Department of Energy
Newsprint (Canada)	thousands of metric tons	Newsprint Assoc. of Canada
Newsprint (US)	thousands of metric tons	American Paper Institute
Synthetic rubber	thousands of metric tons	Census Bureau, Rubber Manufacturers Assoc.
Pneumatic casing	thousands	Census Bureau, Rubber Manufacturers Assoc.
Glass containers	thousand gross	Census Bureau

U: Series with data on new and unfilled orders.

1. Shipments are recorded in millions of wine gallons.

Table 1 (cont.)

Group 2: Data on production and inventories only.

Industry name	Units	Primary data source
Sulfur	thousands of metric tons	Bureau of Mines
Superphosphates	thousands of short tons	Census Bureau
Ethyl alcohol	millions of tax gallons	Treasury Department
Whisky	millions of tax gallons	Treasury Department
Butter	millions of pounds	Agriculture Department
Cheese (total)	millions of pounnds	Agriculture Department
Cheese (American)	millions of pounds	Agriculture Department
Milk (cond. & evap.)	millions of pounds	Agriculture Department
Milk (whole dry)	millions of pounds	Agriculture Department
Milk (nonfat dry)	millions of pounds	Agriculture Department
Petroleum coke	thousands of short tons	Department of Energy
Crude petroleum	millions of barrels	Department of Energy
Fabric (cotton) U	millions of linear yards	Census Bureau
Fabric (manmade&silk) U	millions of linear yards	Census Bureau
Fibers (acetate yarn)	millions of pounds	Textile Economics Bureau, Inc.
Fibers (rayon)	millions of pounds	Textile Economics Bureau, Inc.
Fibers (noncel. yarn)	millions of pounds	Textile Economics Bureau, Inc.
Fibers (noncel.staple)	millions of pounds	Textile Economics Bureau, Inc.
Fibers (text. glass)	millions of pounds	Textile Economics Bureau, Inc.

U: Series with data on unfilled orders.

Table 2a

Relative variance of shipments and production for group 1 industries.

	Var[S]/Var[Q]		
	Direct	Implied	Implied
Industry	(A)	(B)	(C)
Denatured alcohol	.895	.607	1.525*
Beer (1)	.761	.483	1.783
Distilled spirits (2)	2.156	.040	50.438
Effervescent wines (1)	1.218	n.c.	5.112*
Still wines (1)	.009*	.004	.215
Lumber (total) (3)	.963*	.974	1.005
Lumber (hardwood) (3)	.718*	.725	.888
Lumber (softwood) (3)	1.031	.985	1.016
Lumber (douglas fir)	.990*	.984	.984
Lumber (southern pine) (3)	1.074*	.985	1.199
Lumber (western pine) (3)	.956	.950	.930
Iron and steel scrap (4)	2.960	.941	1.050
Pig iron	.936	.988	1.029
Refinery copper (5)	3.704	.504	9.610*
Slab zinc (6)	2.632*	.838	1.281
Bituminous coal (3)	.188	.278	.347*
Gasoline (7)	1.024	.789	1.125*
Kerosene (7)	2.623	1.685	2.018*
Distillate fuel oil (	1.793	.849	1.843*
Residual fuel oil (7)	5.045	.709	2.806*
Jet fuel (7)	1.493	.515	.974*
Lubricants (7)	1.418	1.178	1.618*
Asphalt (7)	2.223	n.c.	1.770*
Liquefied gases (7)	5.017	1.914	20.651*
Newsprint (Canada) (8)	1.091	1.233	1.216*
Newsprint (US) (8)	.936	.923	.922
Synthetic rubber	.954	.413	2.025
Pneumatic casing (9)	1.250	n.c.	1.427
Glass containers	1.106	.751	1.210

(A) Based on VAR between production and shipments.
(B) Based on VAR between shipments and inventories.
(C) Based on VAR between production and inventories.

* Preferred representation due to data construction issues.
n.c.--Not computable, variance calculations did not converge.

NOTES:
(1) Shipments represent taxable withdrawals. In wine, there are significant nontaxable withdrawals (e.g. wine used as an intermediate inputs).
(2) Depending on the reporting State, shipments represent shipments at either the wholesale and producers level.
(3) BEA comments indicate survey stock coverage is not on par with production and shipment coverage.
(4) Shipments represent consumption. Inventory and consumption data include receipts of scrap from nonproduction sources, which are of similar magnitude to production.
(5) Shipments represent consumption by mills and smelters. Stocks include holdings by Commodity Exchange.
(6) Shipments represent consumption by fabricators. Stocks exclude stocks held by fabricators.
(7) Shipments represent domestic demand, which is calcualted as production plus inventory accumulation plus imports less exports.
(8) Shipment data include tonnage invoiced but not shipped.
(9) Tires on consignment included in both stocks and shipments.

Table 2b

Relative variance of the change in shipments and production for group 1 industries.

Industry	Var[ΔS]/Var[ΔQ] Direct (A)	Implied (B)	Implied (C)
Denatured alcohol	.836	.330	2.513*
Beer (1)	.804	.302	3.027
Distilled spirits (2)	4.333	.032	137.61
Effervescent wines (1)	2.262	n.c.	9.963*
Still wines (1)	.008*	.004	.363
Lumber (total) (3)	1.044*	.999	1.133
Lumber (hardwood) (3)	1.006*	.689	2.131
Lumber (softwood) (3)	1.095	1.013	1.141
Lumber (douglas fir)	1.143*	1.111	1.106
Lumber (southern pine) (3)	1.297*	.754	2.380
Lumber (western pine) (3)	1.001	.944	.940
Iron and steel scrap (4)	3.209	.678	1.782
Pig iron	1.016	.861	1.380
Refinery copper (5)	3.369	.512	11.072*
Slab zinc (6)	4.469*	.932	3.054
Bituminous coal (3)	.246	.345	.458
Gasoline (7)	1.561	.985	2.026*
Kerosene (7)	4.095	2.622	3.277*
Distillate fuel oil (7)	2.403	1.264	2.366*
Residual fuel oil (7)	5.170	.643	2.657*
Jet fuel (7)	1.182	.503	.937*
Lubricants (7)	1.967	1.286	1.998*
Asphalt (7)	2.475	n.c.	2.195*
Liquefied gases (7)	4.113	1.683	5.234*
Newsprint (Canada) (8)	1.200	1.258	1.288*
Newsprint (US) (8)	.742	.740	.750
Synthetic rubber	1.923	.203	10.207
Pneumatic casing (9)	1.749	n.c.	2.297
Glass containers	1.071	.612	1.469

(A) Based on VAR between production and shipments.
(B) Based on VAR between shipments and inventories.
(C) Based on VAR between production and inventories.

* Preferred representation due to data construction issues.
n.c.--Not computable, variance calculations did not converge.

NOTES: References for notes 1-6 are found on Table 2a.

Table 3

Relative variances of production and implied shipments for group 2 industries.

Industry	Var[S]/Var[Q]		Var[ΔS]/Var[ΔQ]	
Sulfur	n.c.		n.c.	
Superphosphates	1.128		1.977	
Ethyl alcohol	3.277		4.885	
Whisky	18.20		27.303	
Petroleum coke	1.529		1.463	
Crude petroleum	2.123		2.073	
Fabric (cotton)	1.207		1.205	
Fabric (manmade & silk)	1.145		1.188	
Variables with split data:	A	B	A	B
Butter	1.722	1.133	2.111	2.516
Cheese (total)	1.292	n.c.	1.875	n.c.
Cheese (American)	1.667	.892	2.021	2.364
Milk (cond. & evap.)	2.767	5.150	3.734	6.236
Milk (whole dry)	.703	1.135	.738	1.365
Milk (nonfat dry)	.639	.845	.500	1.076
Fibers (acetate yarn)	.994	.973	1.231	.863
Fibers (rayon)	.866	n.c.	1.076	n.c.
Fibers (noncellul. yarn)	1.184	.926	1.396	.879
FIbers (noncellul. staple)	1.041	.853	.918	.781
Fibers (textile glass)	3.708	.794	4.138	1.266

Based on VAR between production and inventories.

A: first part of sample.

B: second part of sample.

n.c.--Not computable, variance calculations did not converge.

Table 4

Regressions of constructed on actual production for group 1 industries.

Industry	Q(12)	F-stat.	$\bar{R}^2$
Denatured alcohol	16.0	21.0	.79
Beer	25.1	8.78	.80
Distilled spirits	21.3	5.93	.0
adjusted for imports	20.7	6.36	.0
Effervescent wines	20.3	8.09	.92
adjusted for imports	13.4	13.4	.10
Still wines	44.6*	6.21	.01
adjusted for imports	42.4*	8.4	.90
Lumber (total)	10.0	5.55	.99
adjusted for imports	128.4*	42.9*	.67
Lumber (hardwood)	2.2	5.46	.93
Lumber (softwood)	17.6	5.45	.99
Lumber (douglas fir)	n.c.	n.c.	1.0
Lumber (southern pine)	31.3*	.072	.92
Lumber (western pine)	n.c.	n.c.	1.0
Iron and steel scrap	1191*	73.97*	.80
Pig iron	8.78	5.55	.99
Refinery copper	7.95	5.73	.13
adjusted for imports	7.76	8.70	.06
Slab zinc	113*	20.54*	.71
Bituminous coal	42.2*	32.47*	.89
Gasoline	36.9*	5.47	.83
Kerosene	20.0	18.34	.97
Distillate fuel oil	62.6*	31.58*	.88
adjusted for imports	54.2*	24.2*	.96
Residual fuel oil	49.6*	20.55*	.0
adjusted for imports	24.7*	26.8*	.0
Jet fuel	5.8	21.72*	.11
Lubricants	25.9*	19.92*	.85
Asphalt	476*	9.96	.99
Liquefied gases	52.6*	7.46	.55
Newsprint (Canada)	n.c.	n.c.	1.0
Newsprint (US)	n.c.	n.c.	1.0
Synthetic rubber	41.5*	71.07*	.50
Pneumatic casing	182.7*	5.76	.65
Glass containers	21.1	5.67	.83

* Significant at the .05 percent level.

n.c.--Not computable.

Table 5

Summary of Euler-Equation Estimation

West (1986) Cost Function:

$$\min E_0 \sum_{t=0}^{\infty} d^t \{a_0(\Delta Q_t)^2 + a_1(Q_t)^2 + a_2(H_t - a_3 S_{t+1})^2\} \qquad (6)$$

Euler Equation:

$$q_{t+1} = a_0(dq_{t+2} + q) + a_2 H_t - a_2 a_3 S_{t+1} + \text{(time trend, seasonal dummies)}$$

$$\text{where } q_t \equiv Q_t - dQ_{t-1} \quad \text{and } d = \text{discount factor } (.995)$$

	All	Industries Reporting Q, H, S, Measured Production	Industries Reporting Q, H, S, $Q \equiv S + \Delta H$	Industries Reporting Q, H only
Number of Correct Signs/ Number of Coefficients	86/156	66/116	63/116	20/40
Number of Industries with all Correct Signs/ Number of Industries	2/39	2/29	3/29	0/10
Instruments		S_{-1}, S_{-2}, S_{-3} Q_{-1}, Q_{-2}, Q_{-3}, H_{-4}	S_{-1}, S_{-2}, S_{-3} H_{-1}, H_{-2}, H_{-3},	S_{-1}, S_{-2}, S_{-3} Q_{-1}, Q_{-2}, Q_{-3},

Table 6

Summary of Euler Equation Estimation For Firms Carrying Unfilled Orders

Childs (1967) Cost Function:

$$\min E_0 \sum_{t=0}^{\infty} d^t \{a_0 (\Delta Q_t)^2 + a_2 (H_t - a_3 N_{t+1})^2 + a_4 (U_t - a_5 Q_t)^2\} \quad (14)$$

Euler Equation:

$$0 = a_0 (q_{t+1} - 2q_{t+1} + q_t) + a_2 (H - a_3 N) + a_4 a_5 u_{t+1} - a_4 a_5^2 q_{t+1}$$

where $q \equiv Q_t - dQ_{t-1}$ and $u \equiv U_t - dU_{t-1}$

	All	Industries Reporting Q, H, S, U	Industries Reporting Q, H, U only
Number of Correct Signs/ Number of Coefficients	9/20	6/12	3/8
Number of Industries with all Correct Signs/ Number of Industries	0/5	0/3	0/2
Instruments		N_{-1}, N_{-2}, N_{-3} Q_{-1}, Q_{-2}, Q_{-3}, U_{-1}, U_{-2}, U_{-3}	N_{-1}, N_{-2}, N_{-3} Q_{-1}, Q_{-2}, Q_{-3}, U_{-1}, U_{-2}, U_{-3}

1. These three are softwood lumber industries: douglas fir, southern pine, and western pine.
2 These two industries are both woven fabric: 1) cotton, and 2) synthetics and silk.

Table 7a

Variances of production, shipments, and demand for industries with unfilled orders.

Industry	Var[S]/ Var[Q]	Var[N]/ Var[Q]	Var[N]/ Var[S]
Based on VAR between Q, S, and N:			
Lumber (douglas fir)	.991	1.026	1.036
Lumber (southern pine)	1.074	1.164	1.084
Lumber (western pine)	.975	.929	.953
Based on VAR between Q, U, and H:			
Lumber (douglas fir)	.990	1.074	1.085
Lumber (southern pine)	1.217	1.269	1.043
Lumber (western pine)	.964	.965	1.001
Fabric (cotton)	1.214	1.356	1.116
Fabric (manmade & silk)	1.146	2.312	2.018

Table 7b

Variances of changes in production, shipments, and demand for industries with unfilled orders.

Industry	Var[ΔS]/ Var[ΔQ]	Var[ΔN]/ Var[ΔQ]	Var[ΔN]/ Var[ΔS]
Based on VAR between Q, S, and N:			
Lumber (douglas fir)	1.161	1.797	1.548
Lumber (southern pine)	1.271	2.556	2.010
Lumber (western pine)	1.015	1.299	1.280
Based on VAR between Q, U, and H:			
Lumber (douglas fir)	1.124	3.805	3.387
Lumber (southern pine)	2.471	6.255	2.532
Lumber (western pine)	1.001	6.175	6.172
Fabric (cotton)	1.210	2.201	1.819
Fabric (manmade & silk)	1.186	3.036	2.559

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