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VINTAGE HUMAN CAPITAL, GROWTH, AND  
THE DIFFUSION OF NEW TECHNOLOGY

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ABSTRACT

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This paper develops a model of vintage human capital in which each technology requires vintage specific skills. We examine the properties of a stationary equilibrium for our economy. The stationary equilibrium is characterized by an endogenous distribution of skilled workers across vintages. The distribution is shown to be single peaked and, under general conditions, there is a lag between the time when a technology appears and the peak of it's usage, a phenomenon known as diffusion. An increase in the rate of exogenous technological change shifts the distribution of human capital to more recent vintages thereby increasing the diffusion rate.

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

## 1. Introduction

Why are new technologies often adopted slowly? Why do people often invest in new technologies even when apparently superior technologies are available? How are decisions to adopt new technologies affected by the prospect that even better technologies will arrive in the future? This paper develops a dynamic, general equilibrium model to deepen our understanding of the factors determining both the rate of adoption of new technologies and the rate of displacement of old ones.

The model has three distinctive features. First, capital is specific to a particular technology. Second, the marginal product of investment in a technology increases with the existing capital specific to that technology so that new and old capital are complementary inputs in production. Third, new and superior technologies arrive continually. Because capital is technology specific, old technologies will continue to be used even though better technologies are available. This feature implies that there is a distribution of capital in use by vintages (see Solow 1960, for an early analysis of a vintage capital model and Salter 1960, and Jugenfelt 1986, for analyses of diffusion in vintage capital models). Because old and new capital are complementary inputs, it can be desirable to create new capital specific to old technologies. Therefore the capital stock in a particular technology reaches its peak after some time following the technology's introduction. This phenomenon is known as diffusion. Because superior technologies arrive continually in the model, we are able to analyze the interplay between adoption rates and the rate of technological progress.

In our general equilibrium model, the diffusion rate depends on the current distribution of vintage capital, the relative superiority of the newest technology, and on the quality of technologies expected to arrive in the future.

The distribution of vintage capital in the future in turn depends upon the rate of investment in current technologies. Rather than starting at an arbitrary initial distribution, we examine the effect of changes in the quality of new technologies on the diffusion rate in a stationary equilibrium. We show that the diffusion rate in such an equilibrium increases with an increase in the relative superiority of new technologies over existing ones. We also show that the diffusion rate increases as the arrival rate of new technologies increases. In a classic paper, Rosenberg (1976) pointed to two opposing forces which determine how diffusion rates change with a change in the quality of new technologies. An increase in the relative superiority of currently available new technologies creates an incentive to switch to them. However, if substantially better technologies are expected to arrive in the future, firms have an incentive to wait for better technologies to arrive. In our model of ongoing technological change, the tradeoff between these incentives is decisively resolved in the direction of quicker adoption and more rapid diffusion.

Our model is consistent with several empirical observations which have now attained the stature of stylized facts. First, while the adoption rates of new technologies vary widely across industries and over time, it is undeniable that new technologies are often adopted on a large scale only after a prolonged period of time. As Mansfield (1968) points out "it took 20 years or more for all of the major firms (in several industries) to install centralized traffic control, car retarders, by-product coke ovens and continuous annealing." None of these inventions was patentable and so we must look to other sources to understand the apparently slow rate of adoption. Second, the empirical literature provides evidence that diffusion curves are generally S-shaped (see Jovanovic and Lach 1989, for a forceful expression of this view and also Griliches 1957, Davies

1979, Gort and Klepper 1982, and Jovanovic and Macdonald 1988). Third, Davies (1979) finds evidence that diffusion rates increase with an increase in the growth rate of the adopting industry.

We now turn to the relationship between our model and the existing theoretical literature on diffusion. Research in this area falls into three main categories. One focuses on uncertainty about the quality of new technologies (Jensen 1983, and Balcer and Lippman 1984). The second emphasizes strategic issues in technology adoption (Kamien and Schwartz 1972, Reinganum 1981, and Spence 1984). The third emphasizes spillover effects and learning by doing in generating slow diffusion (Jovanovic and Macdonald 1989, David 1969, David and Olsen 1987, and Jovanovic and Lach 1989). While uncertainty and strategic issues are undoubtedly important, in this paper we abstract away from these issues by considering a certainty model with competitive agents.

Our model differs from the existing literature in several substantive ways. First, in our model not only are old technologies used when apparently better technologies are available, people will invest in old technologies. The existing literature generally does not yield this implication. There is some evidence in Mansfield (1968, Chap. 8) suggesting that firms using older techniques grew in size, though obviously at a slower rate than firms using newer techniques. Second, in our general equilibrium model, both the cost of switching to a new technology and the cost of operating old technologies are endogenously determined rather than exogenously specified as in much of the literature. In particular, this endogeneity allows us to analyze how switching costs and operating costs are affected by the current distribution of technologies and by the anticipated arrival rate of new technologies. Third, much of the literature attempts to understand diffusion in environments where technological change is a

once-and-for-all event. For many industries (computers, for one example) it seems more appropriate to model technological change as a continual process. Such a model allows us to analyze industries where many different techniques simultaneously coexist and to develop a deeper understanding of how the rates of decay of older technologies interact with the growth of newer ones.

In our model, all the capital is technology-specific human capital which is acquired by using a particular technology. Learning by doing is, in this sense, an important feature of the model. However, there are no spillover effects across technologies. Therefore, the competitive equilibrium of our model is also Pareto optimal.

The plan of the paper is as follows. In Section 2 we set up the model, define a competitive equilibrium and prove existence and uniqueness of a stationary equilibrium. In Section 3 we characterize the stationary equilibrium. In Section 4 we show that competitive equilibria are Pareto-optimal. Section 5 concludes the paper.

## 2. The Model

We consider an infinite horizon overlapping generations model of agents who live for two periods. A new technology appears in every period. This technology is given by the production function  $\gamma^t f(N, Z)$  where  $t$  denotes the period in which the technology appeared,  $N$  is the input of unskilled workers,  $Z$  is the input of experienced workers, and  $\gamma > 1$ . A given set of technologies with  $t = \{0, -1, -2, \dots\}$  is available in period 0. We assume that the production function  $f$  has constant returns to scale and that  $f(\cdot, Z)$  is strictly increasing and strictly concave for each  $Z > 0$ . Let  $f(N, 0) = \omega_0 N$  where  $\omega_0 \geq 0$ . As will become apparent below, the form of this production function plays an important role in

determining the rate of diffusion. In particular, if the two types of labor are highly complementary, new technologies diffuse slowly. Note also that because  $\gamma > 1$ ,  $t$  in our economy output grows over time.

We will adopt standard notation for vintage capital models. The letter "t" will index time and the letter " $\tau$ " will index the vintage of the technology, with the following interpretation: A technology of vintage  $\tau$  in period  $t$  refers to the technology that appeared in period  $t - \tau$ . For example  $\tau = 2$  at date  $t$  denotes the technology that appeared in period  $t - 2$ . Notice also that the same technology in period  $t + 1$  will have vintage  $\tau = 3$ .

In every period, a constant population of agents is born who live for two periods. We normalize the population size to be 1. These agents have preferences defined over the two periods they live given by the utility function

$$u(c_1, c_2) = c_1 + \beta c_2$$

where  $c_1$  denotes consumption when young,  $c_2$  denotes consumption when old and  $0 < \beta < 1$ . In period 0, an old generation is alive which only cares about current consumption. We allow individuals of a given generation to borrow and lend among themselves. Thus, so long as aggregate consumption of a generation is positive in both periods of life, the market interest factor will be  $\beta$ .

Workers can choose to work in only one vintage. Experience is acquired by working in a firm using a particular technology as an unskilled worker for one period and is specific to that technology. In this sense, our model is one of learning by doing. The choices of young agents on which skills to learn induce a distribution of skilled workers across technologies in the following period. The distribution of old agents in period 0 is given.

Let  $\mu_t$  be the distribution of experienced old agents at date  $t$  across vintages  $\tau \in \{0, 1, 2, \dots\}$ . Thus  $\mu_t(\tau)$  indicates the number (more precisely mass) of old agents with experience in vintage  $\tau$ . These are the old people who when young worked in the technology that appeared in period  $t - \tau$ . Since there are no experienced workers in the "just born" vintage,  $\mu_t(0) = 0$  for all  $t$ . We will refer to  $\mu_t$  as the state of the economy.

We now describe the evolution of the state of the economy. Let  $N_1(t, \tau)$ ,  $t = 0, 1, \dots$ , denote the number of young workers who enter vintage  $\tau$  at date  $t$ . In period  $t + 1$  these workers will be skilled in vintage  $\tau + 1$ . Therefore,

$$(1) \quad \mu_{t+1}(\tau+1) = N_1(t, \tau) \quad \tau = 0, 1, \dots, \text{ all } t.$$

Only experienced old workers in a particular vintage can supply the skilled labor input in that vintage. We allow experienced workers to move freely and supply unskilled labor at any vintage. Let  $N_2(t, \tau)$  denote the number of old workers who work as unskilled workers in vintage  $\tau$  and let  $Z(t, \tau)$  denote the skilled labor input to vintage  $\tau$  at date  $t$ . Then we have

$$(2) \quad 0 \leq Z(t, \tau) \leq \mu_t(\tau) \quad \tau = 0, 1, \dots, \text{ all } t.$$

We now turn to the decision problems of the workers. Let  $w(t, \tau)$  denote the wage of unskilled workers in vintage  $\tau$  in period  $t$  and let  $v(t, \tau)$  denote the wage paid to skilled workers at vintage  $\tau$  in period  $t$ . First consider the decision problem of old workers. Since old workers can work at any vintage as unskilled workers, they must be paid at least as much as unskilled workers in any vintage. That is, if  $Z(t, \tau) > 0$  then  $v(t, \tau) \geq w(t, s)$  for all  $s$ . Furthermore, if the last inequality is strict, skilled workers in vintage  $\tau$  will only supply skilled labor, or  $Z(t, \tau) = \mu_\tau$ . Clearly old workers who work as unskilled workers will

choose the vintage offering the highest wage rate; that is, if  $N_2(t, \tau) > 0$  then  $w(t, \tau) \geq w(t, s)$  for all  $s$ .

Now consider the decision problem of young workers. If a young worker decides to enter vintage  $\tau$ , his or her earnings in the following period will be (at least)  $v(t+1, \tau+1)$  since in the following period he or she will be skilled in vintage  $\tau + 1$ . Since young agents maximize discounted earnings, we have the following inequality<sup>1</sup>,

$$(3) \quad w(t, \tau) + \beta \max\{v(t+1, \tau+1), w(t+1, 0), w(t+1, 1), \dots\} \\ \geq w(t, s) + \beta \max\{v(t+1, s+1), w(t+1, 0), w(t+1, 1), \dots\}$$

for all  $\tau$ ,  $s$ , and  $t$  such that  $N_1(t, \tau) > 0$ .

In each vintage profit maximizing firms operate the production process. Because of our assumption of constant returns to scale, the distribution of property rights of firms is irrelevant. In each vintage we have

$$(4) \quad \max_{N < Z} \{\gamma^{t-\tau} f(N, Z) - w(t, \tau)N - v(t, \tau)Z\} = 0 \text{ for all } \tau \geq 1, \text{ for all } t.$$

In the "just-born" vintage we have the profit maximization condition,

$$(5) \quad \max_N \{\gamma^t f(N, 0) - w(t, 0)N\} = 0.$$

Recall that  $f(N, 0) = \omega_0 N$ . Thus, (5) implies that if  $N_1(t, 0) > 0$  for some  $t$ , then  $w(t, 0) = \gamma^t \omega_0$ .

A *competitive equilibrium* for this economy is a collection of wage functions  $w(t, \tau)$  and  $v(t, \tau)$ ; employment functions  $N_1(t, \tau)$ ,  $N_2(t, \tau)$ , and  $Z(t, \tau)$ ; and a sequence of distribution functions  $(\mu_t)$ , such that:



- (i) Young workers are indifferent among vintages: The wage functions satisfy (3). Old workers maximize their income:  $Z(t, \tau) > 0$  implies  $v(t, \tau) \geq w(t, s)$  for all  $s$ ,  $v(t, \tau) > w(t, s)$  for all  $s$  implies  $Z(t, \tau) = \mu_\tau$  and  $N_2(t, \tau) > 0$  implies  $w(t, \tau) \geq w(t, s)$  for all  $s$ .
- (ii) Profit maximization: The employment functions solve (4) and (5).
- (iii) Resource constraints:  $\sum_{\tau=0}^{\infty} N_1(t, \tau) = 1$  and  $\sum_{\tau=0}^{\infty} (N_2(t, \tau) + Z(t, \tau)) = 1$  for all  $t$  and the employment and distribution functions satisfy (1) and (2).

In the rest of the paper we will concentrate our attention on the *stationary equilibrium*, which is a competitive equilibrium with the additional conditions:

- (iv)  $\mu_t = \mu$ ,  $N_2(t, \tau) = N_{2\tau}$ ,  $Z(t, \tau) = Z_\tau$ ,  $w(t, \tau) = \gamma^t w_\tau$ , and  $v(t, \tau) = \gamma^t v_\tau$  for all  $t$ , where  $w_\tau$  and  $v_\tau$  denote the wage rates at period 0.

To interpret the conditions imposed by a stationary equilibrium it is convenient to think of the technologies as lying on the real line. Then in a stationary equilibrium the distribution of agents across technologies advances to the right at a constant rate. However, relative to the newest technology the distribution is stationary. So, at the beginning of every period the economy is identical to what it was in the previous period, except that all the technologies are more productive by a factor of  $\gamma$ . We therefore require that the allocations at each vintage be the same over time in a stationary equilibrium and that wages rise at the rate  $\gamma$ .<sup>2</sup>

The key condition in an equilibrium is the present value condition in (3). This condition is somewhat unwieldy but we can use our constant returns to scale assumption to simplify it considerably. To use this assumption, let us define

the input of unskilled labor relative to skilled labor by  $n$  and the profit function  $\pi_\tau(w)$  by

$$(6) \quad \pi_\tau(w) = \max_n \{ \gamma^{-\tau} f(n, 1) - wn \}.$$

From the constant returns to scale assumption, it is clear that if  $Z_\tau > 0$  for some  $\tau$  then  $v_\tau = \pi_\tau(w_\tau)$ . Of course, if  $Z_\tau < \mu_\tau$  then  $v_\tau$  equals the maximum wage. Therefore,

$$(7) \quad v_\tau = \max \{ \pi_\tau(w_\tau), \max_\tau w_\tau \} \text{ for all } \tau \text{ such that } \mu_\tau > 0.$$

Recalling that wages rise at rate  $\gamma$  in a stationary equilibrium, the present value conditions (3) can be simplified to read,

$$(8) \quad w_{\tau-1} + \beta\gamma \max \{ \pi_\tau(w_\tau), \max_\tau w_\tau \} \leq k$$

with equality for all  $\tau$  such that  $\mu_\tau > 0$  where  $k$  is the present value of income for a young worker at date zero.

In the appendix we establish the following

*Proposition 1 (Unskilled workers' wages increase with vintage).*

In a stationary equilibrium,

- (i) The support of the distribution is finite: there is some number  $T$  such that  $\mu_\tau > 0$  for  $\tau \leq T$  and  $\mu_\tau = 0$  for  $\tau > T$ .
- (ii)  $Z_\tau = \mu_\tau$  for  $\tau \leq T - 2$  and  $Z_{T-1} > 0$ .
- (iii)  $w_\tau \geq w_{\tau-1}$  and  $v_{\tau+1} \leq v_\tau$  for  $\tau = 1, \dots, T - 1$ .

Proposition 1 says that, in a stationary equilibrium, only a finite number of vintages will be used. Thus, all technologies are eventually discarded. It

also says that young workers only enter vintages where old workers supply skilled labor and that unskilled workers wages increase with the age of a technology while skilled workers wages decline with the age of a technology. Thus, young workers who enter new technologies invest when young and reap the benefits of their human capital when they are old.

Proposition 1 together with the present value condition (8) suggests a simple recursive procedure for computing the wage sequence in a stationary equilibrium. To develop this procedure, first note that using Proposition 1 in (8) we have

$$(9) \quad w_{\tau-1} + \beta\gamma\pi_{\tau}(w_{\tau}) = k \quad \text{for } \tau = 1, \dots, T-1$$

Next, using the result in Proposition 1 that young workers enter the just-born vintage and (5) we have

$$(10) \quad w_0 = \omega_0.$$

Thus, if the present value of earnings,  $k$ , and the number of vintages,  $T$ , are known we can use (9) and (10) to compute the wage sequence for unskilled workers. We turn now to determining  $k$  and  $T$ . Suppose first that  $z_T = 0$  so that all old workers skilled in vintage  $T$  work as unskilled workers. In a stationary equilibrium, any young worker who enters vintage  $T-1$  will then work as an unskilled worker when old as well. Such a worker will therefore choose the vintage paying the highest wages to unskilled workers in both periods of his life. Thus, young workers entering vintage  $T-1$  receive a wage  $w_{T-1}$  when young and  $\gamma w_{T-1}$  when old. We then have the equation for determining  $k$  given by,

$$(11) \quad w_{T-1} + \beta\gamma w_{T-1} = k.$$

To determine the number of vintages,  $T$ , note that if  $z_T = 0$ , the wage of workers entering vintage  $T - 1$ ,  $w_{T-1}$ , must be the highest wage paid to unskilled workers at any vintage. In particular,  $w_{T-1} \geq w_T$ . Furthermore, since workers skilled in vintage  $T$  also receive a wage of  $w_{T-1}$ , we have that  $\pi_T(w_T) \leq w_{T-1}$ . Now  $\pi_T(\cdot)$  is a decreasing function of the wage. Thus, we have that

$$(12) \quad \pi_T(w_{T-1}) \leq w_{T-1}.$$

Next, from Proposition 1 we have that workers skilled in vintage  $T - 1$  choose to work as skilled workers. Therefore,

$$(13) \quad \pi_{T-1}(w_{T-1}) \geq w_{T-1}.$$

Inequalities (12) and (13) can now be used together with equations (9) through (11) to construct the wage sequence. The algorithm is to guess at a value of  $T$  and then use (9) through (11) to construct a wage sequence. We then verify whether the constructed wage sequence satisfies (12). If it does not, we guess a larger value of  $T$ . If the wage sequence fails to satisfy (13) we guess a smaller value of  $T$ . Note that because  $\pi_r(\cdot)$  is decreasing in  $r$ , if the wage sequence fails to satisfy (12) it will satisfy (13).

We have derived this algorithm for the case when  $z_T = 0$ . A slight modification is required to accommodate the possibility that  $z_T > 0$ . In this case, it could be that  $w_{T-1}(1+\beta\gamma) < k$ , so that the algorithm described above apparently cannot be used. Suppose, provisionally, that the present value of earnings,  $k$ , is known. Let  $S$  denote the smallest vintage such that  $\pi_s(k/(1+\beta\gamma)) \leq k/(1+\beta\gamma)$ . For  $r = T, T + 1, \dots, S - 1$  we define  $w_r$  so that  $w_{r-1} + \beta\gamma\pi_r(w_r) = k$  and let  $w_{S-1} = k/(1+\beta\gamma)$ . Clearly, this constructed wage sequence satisfies all the conditions of a stationary equilibrium. We have established that, in a

stationary equilibrium, there exists a vintage  $S$  and a wage sequence  $w_\tau$ ,  $\tau = 0, \dots, S - 1$  satisfying

$$(14) \quad w_\tau + \beta\gamma\pi_{\tau+1}(w_{\tau+1}) = k \quad \text{for } \tau = 0, \dots, S - 2$$

$$(15) \quad w_0 = \omega_0, \quad w_{S-1} = \frac{k}{(1+\beta\gamma)}$$

$$(16) \quad \pi_{S-1} \left( \frac{k}{1+\beta\gamma} \right) \geq \frac{k}{(1+\beta\gamma)}$$

$$(17) \quad \pi_S \left( \frac{k}{1+\beta\gamma} \right) \leq \frac{k}{(1+\beta\gamma)}.$$

Note that (16) together with (14) implies that  $w_{\tau+1} \geq w_\tau$ ,  $\tau = 0, \dots, S - 2$ , and that  $\pi_\tau(w_\tau) > k/(1+\beta\gamma)$  for  $\tau = 1, \dots, S - 2$  since  $\pi$  is strictly decreasing in the vintage.

This construction shows that the algorithm is the same both when  $z_T = 0$  and when  $z_T > 0$ . We turn now to the problem of uncovering the number of active vintages  $T$  from the wage sequence satisfying (14) through (17) and, more generally, constructing the employment allocations.

Suppose we find some sequence of wages,  $w_0, w_1, \dots, w_{S-1}$  which satisfies (14)-(17). Let  $n_\tau(w_\tau)$  denote the solution to the problem

$$\pi_\tau(w_\tau) = \max_n \gamma^{-\tau} f(n, 1) - w_\tau n.$$

Then, given the sequence of wages, the employment sequence is constructed as follows. Suppose first that  $n_{S-1}(w_{S-1}) > 0$ . Then, let all young workers who enter vintage  $S - 1$  work as unskilled workers in the following period in vintage  $S-1$ . Therefore, we have  $N_{1S-1} = N_{2S-1}$ . In all preceding vintages, it follows from the fact that  $\mu_{\tau+1} = N_{1\tau}$  and  $N_{1\tau} = n_\tau \mu_\tau$  that  $\mu_{\tau+1} = \mu_1 \sum_{s=1}^T n_s$ . We now show that  $\mu_1$

can be chosen to satisfy labor market clearing. The market clearing condition can be written as

$$(18) \quad \sum_{\tau=1}^{S-2} \mu_{\tau} n_{\tau} + \mu_{S-1} \frac{n_{S-1}}{2} = 1 - \mu_1.$$

From the recursive construction of  $\mu$ , the left side of (18) is a continuous increasing function of  $\mu_1$  which equals zero if  $\mu_1$  equals zero and the right side is a continuous strictly decreasing function ranging from zero to one. Therefore, there is a unique value of  $\mu_1$  which solves (18). From this value of  $\mu_1$ , the remaining allocations can be constructed.

If  $n_{S-1}(w_{S-1}) = 0$ , let  $T - 1$  denote the largest vintage such that  $n_{T-1}(w_{T-1}) > 0$ . Replace the second term on the left side of (18) by  $(\mu_{T-1} n_{T-1})/2$ . The same argument for the existence for a distribution then applies.

The problem of proving the existence of a stationary equilibrium then reduces to the problem of verifying that a wage sequence can be found which satisfies (14) through (17). We show that such a wage sequence can be found in the following proposition which is proved in the appendix.

*Proposition 2 (Existence of a wage sequence).*

There exists a unique number  $S$  and a unique sequence of wages,  $w_0, w_1, \dots, w_{S-1}$  satisfying (14)-(17).

If  $n_{S-1}(w_{S-1}) > 0$ , the number of active vintages,  $T$  equals  $S$ . If  $n_{S-1}(w_{S-1}) = 0$ , the number of active vintages is determined by the largest vintage  $T$  such that  $n_{T-1}(w_{T-1}) > 0$ . We have established the following theorem.<sup>3</sup>

*Theorem (Equilibrium existence).*

A stationary equilibrium exists for the vintage capital model.

From Proposition 2 the wage sequence is unique. The employment sequence is also unique except for two knife-edge cases. One case occurs when workers skilled in vintage  $T$  are indifferent between working in vintage  $T$  as skilled workers and working in vintage  $T - 1$  as unskilled workers. Then there is an indeterminacy in allocating old workers skilled in vintage between vintage  $T - 1$  and  $T$ . The other case occurs when  $w_{T-1} = v_{T-1}$ ,  $N_{2T-1} > 0$ , and  $Z_{T-1} < \mu_{T-1}$ . Then there is an indeterminacy in allocating old workers skilled in vintage  $T - 1$  between skilled and unskilled tasks. It is clear that both cases are knife-edge in nature. The equilibrium is therefore unique in general.<sup>4</sup>

### 3. Properties of the Equilibrium

The stationary distribution provides a picture of the rise and fall of a particular technology in our model. Say a new technology arrives in period 0. In period 1, this technology is of vintage 1 and therefore has exactly the same capital stock as the vintage 1 technology in period 0. In period 2, this technology has the same capital stock as the vintage 2 technology in period 0, and so on. Therefore, the rate of diffusion of new technology is closely related to the properties of the stationary distribution. We establish that the distribution is single peaked and log concave in the following proposition.

*Proposition 3 (Single peakedness and log concavity).*

There is a vintage  $R$  such that for all  $\tau \leq R$ ,  $\mu_\tau \geq \mu_{\tau-1}$  and for  $\tau > R$ ,  $\mu_\tau \leq \mu_{\tau-1}$ . Furthermore,  $\log \mu_\tau$  is concave in  $\tau$ .

*Proof.* Since  $w_r$  is increasing in  $r$ , it follows that  $n_r$  is decreasing in  $r$ . Recall that  $\mu_r = \mu_{r-1}n_{r-1}$ . Therefore, if  $n_{r-1} > 1$ ,  $\mu_r > \mu_{r-1}$ , and if  $n_{r-1} < 1$ ,  $\mu_r < \mu_{r-1}$ , and all subsequent values of  $\mu$  are also decreasing. Since the number of vintages in use is finite, the result follows.

Log concavity follows because the increment between  $\log \mu_{r+1}$  and  $\log \mu_r$  is given by  $\log n_r$  which is decreasing in  $r$ .  $\square$

Proposition 3 illustrates the sense in which our model is one of diffusion. If the peak of the diffusion curve is at vintages greater than 1 the distribution of skills (or capital) rises and then falls over time. This will occur if relative employment at vintage 1 is greater than unity. We examine this possibility below. Furthermore, the diffusion curve is log concave and the growth rate of capital decreases monotonically as some of the empirical literature suggests.

We now discuss the factors determining the shape of the distribution function. In particular, we examine the conditions under which the peak of the distribution occurs at vintages greater than 1. The factors in determining whether relative employment in vintage 1 is greater than unity are the production function,  $f(\cdot, \cdot)$  the rate of growth of the economy,  $\gamma$ , and the discount factor,  $\beta$ . We first consider the role of the production function. A large number of young workers will enter vintage 1 and later vintages if the demand for their services is high, that is, if their marginal product is high in these vintages. These young workers will be willing to work as skilled workers when old and continue to attract large numbers of young workers into their vintages if the marginal product of the young workers they hire increases with the number of skilled workers. That is, if skilled and unskilled labor are complementary inputs in production then slow adoption of new technologies is likely to occur.



To illustrate this role of complementarities in generating diffusion we conducted some simulations.

We considered constant elasticity of substitution production functions of the form  $[a_2 N^\rho + a_1 Z^\rho]^{1/\rho}$  with the elasticity given by  $1/(\rho-1)$ . We normalize  $a_2$  to be 1. As  $\rho$  decreases from one to zero, the inputs become more and more complementary. In Figure 1 we plot the capital stock  $\mu_t$  at each vintage against the vintages for a number of different values of  $\rho$ . Notice that the rate of diffusion slows as the inputs become more and more complementary. Again, the intuition is that the marginal gain to investing in an old technology is high when new investment is a complementary input to the existing capital stock. Few workers then join the newest technology, even though it is better. Notice that the distribution is not symmetric about the peak. Therefore, the adoption rate of new technologies can be quite different from the decay rate of old technologies. Figure 2 reproduces Figure 1 for  $\rho = 0.1$  up to the peak of the diffusion curve. The adoption curve has a classic S-shape as much of the empirical literature suggests it should.

We now examine the effect of a change in the rate of technological change on the stationary distribution. We consider two economies with growth rates given by  $\gamma'$  and  $\gamma$  with  $\gamma' > \gamma$ . Our main result is that the distribution associated with the higher growth rate, say  $\mu'$ , will be dominated in the sense of stochastic dominance by the original distribution. In other words, when the growth rate increases the distribution of skilled workers shifts to more recent vintages. This result also implies that the rate of diffusion of new technologies is higher if the economy grows more rapidly.

*Proposition 4 (Relative employment decreases with an increase in the growth rate).*

Consider two economies with  $\gamma' > \gamma$  and associated stationary distributions  $\mu'$  and  $\mu$  respectively. Let  $w(\tau, \mu')$  and  $w(\tau, \mu)$  denote the wage rates in the two economies and  $n(\tau, \mu')$ ,  $n(\tau, \mu)$  denote the relative labor input decisions in the two economies. Then fewer vintages are used in the high growth economy, the wages satisfy

$$w(\tau, \mu') \geq \left( \frac{\gamma}{\gamma'} \right)^\tau w(\tau, \mu) \quad \text{for all } \tau \text{ with } \mu'_\tau > 0,$$

and  $n(\tau, \mu') \leq n(\tau, \mu)$  for all  $\tau$  such that  $n(\tau, \mu) > 0$ .

*Proof.* See Appendix.

We use this result to prove:

*Proposition 5 (Stochastic dominance).*

Consider two economies with  $\gamma' > \gamma$ . Let  $\mu'$ ,  $\mu$  denote the respective stationary distributions. Then  $\mu$  stochastically dominates  $\mu'$ , i.e.,

$$\sum_{\tau=1}^S \mu_\tau \leq \sum_{\tau=1}^S \mu'_\tau \quad \text{for all } S.$$

*Proof.* We first establish that  $\mu_1 \leq \mu'_1$ . Suppose by way of contradiction that  $\mu_1 > \mu'_1$ . Then, from Proposition 4 it follows that  $n(1, \mu_1)\mu_1 \geq n(1, \mu'_1)\mu'_1$  or by definition of  $n$  that  $\mu_2 \geq \mu'_2$ . Repeating the use of Proposition (4) it follows that  $\mu_\tau \geq \mu'_\tau$  for  $\tau \geq 2$ . Thus  $\sum_{\tau=1}^\infty \mu_\tau > \sum_{\tau=1}^\infty \mu'_\tau$ . Of course, both sides of this inequality sum to 1 and we have a contradiction. Let  $T$  be the smallest number

such that  $\mu_T' < \mu_T$ . The Proposition follows immediately for  $S < T$ . Consider now the Proposition for  $S \geq T$ . Using Proposition 4, it follows that  $\mu_\tau' \leq \mu_\tau$  for all  $\tau \geq T$ . Thus, for any  $S \geq T$ , we have  $\sum_{\tau=S}^{\infty} \mu_\tau' \leq \sum_{\tau=S}^{\infty} \mu_\tau$ . But since  $\sum_{\tau=1}^{\infty} \mu_\tau' = 1 - \sum_{\tau=1}^{\infty} \mu_\tau$ , we have that  $1 - \sum_{\tau=1}^{S-1} \mu_\tau' \leq 1 - \sum_{\tau=1}^{S-1} \mu_\tau$  which establishes the result.  $\square$

Proposition 5 shows that with an increase in the growth rate of the economy, diffusion becomes more rapid. The distribution of capital shifts towards more recent vintages in the first order stochastic dominance sense. This result is consistent with the evidence presented in Davies (1979). The intuition for the result is straightforward except for some complications induced by the general equilibrium structure of the model. With an increase in the growth rate, the relative inferiority of older technologies measured by  $\gamma^{-\tau}$  increases. Therefore, fewer young workers join older technologies. The distribution of capital then shifts towards more recent vintages. However, one should not ignore general equilibrium effects. Young workers' decisions to join relatively new vintages offering lower wages drives down the wages of unskilled workers thereby tending to make continued operation of older vintages profitable. Proposition 4 bounds the decline in unskilled worker wages and limits this general equilibrium effect.

Our next result shows that the earnings profile becomes flatter as the growth rate of the economy increases. From Proposition 4

$$w(\tau, \mu') \geq \left[ \frac{\gamma}{\gamma'} \right]^\tau w(\tau, \mu).$$

Using this result, we have

$$(19) \quad \pi'_\tau(w') = \max_n \gamma'^{-\tau} f(n) - w' n \leq \left[ \frac{\gamma}{\gamma'} \right]^\tau \max \left[ \gamma^\tau f(n) - \left( \frac{\gamma'}{\gamma} \right)^\tau w' n \right] \leq \left[ \frac{\gamma}{\gamma'} \right]^\tau w_\tau(w).$$

Since  $\gamma' > \gamma$ , it follows that

$$(20) \quad \frac{\gamma' \pi'_{\tau+1}(w'_{\tau+1})}{w'_\tau} \leq \frac{\gamma \pi_{\tau+1}(w_{\tau+1})}{w_\tau}.$$

The numerator of (19) is the wage of young workers after they have become skilled. The denominator is the wages of young unskilled workers. Therefore, the earnings profile becomes flatter with a higher growth rate. Because preferences are linear, the wage profile does not pin down the intertemporal allocation of consumption of workers. We examine how consumption profiles of a generation change with the growth rate of the economy. The aggregate (or per capita) consumption of young workers is defined by

$$(21) \quad c_y = \sum_{\tau=0}^{T-1} w_\tau \mu_{\tau+1}$$

and the consumption of old workers is similarly defined. The consumption profile of a generation is given by

$$(22) \quad \frac{c_0}{c_y} = \frac{\sum_{\tau=1}^T \gamma^V \mu_\tau}{\sum_{\tau=0}^{T-1} w_\tau \mu_{\tau+1}}.$$

In Figure 3, we plot this consumption profile against the growth parameter  $\gamma$  for three values of the substitution parameter  $\rho$ . The discontinuities in the figure occur when the number of vintages used changes. As can be seen from the figure, the consumption profiles are not monotone. There are two effects working in opposite directions. The wage profile at any given vintage becomes flatter

when the growth rate increases thus making the consumption profile flatter. However, the distribution of workers shifts towards more recent vintages where wage profiles are steeper thus making the consumption profile steeper. When an increase in the growth rate leaves the number of vintages unchanged, the consumption profile becomes flatter. The effect of a flattening wage profile can be clearly seen in such a case. The effect of a shift in the distribution can be seen most clearly when the number of vintages used changes. The consumption profile becomes steeper. Interestingly, the figure shows that consumption profiles are generally steep when complementarities are high even though more workers join older technologies with relatively flat wage profiles. The reason is that with higher complementarities the wage profiles at each vintage become steeper.

We can also use the model to study human capital accumulation. One measure of investment in human capital is foregone earnings. That is, the investment of a worker joining vintage  $\tau$  is given by  $(w_{T-1} - w_\tau)$ . Total investment<sup>5</sup> in the economy is then  $\sum_{\tau=0}^{T-1} (w_{T-1} - w_\tau) \mu_{\tau+1}$ . Since the capital stock depreciates completely after one period of use, the investment is also the amount of capital in the economy. In Figure 4, we plot the ratio of investment to output against the growth rate parameter for three values of the substitution parameter. The investment-output ratio and the consumption profile are obviously closely linked. Again, the discontinuities in Figure 4 occur when the number of vintages changes. The effect of the flattening wage profile is to reduce measured investment at each vintage while the shift of workers to more recent vintages where the wage profile is steeper increases investment. As can be seen from the figure, the latter effect eventually dominates, raising the investment-output or capital-

output ratio. As in Figure 3, stronger complementarities tend to raise the amount of investment in the economy.

The flattening of the wage profile is also associated with a change in the cost of switching from the oldest technology to the newest one. We measure these switching costs by  $w_{T-1} - \omega_0$ . The sense in which  $(w_{T-1} - \omega_0)$  is a switching cost can be made clearer if we assume that a single, competitive firm operates all the technologies. Then  $(w_{T-1} - \omega_0)$  measures the cost to this firm of switching an unskilled worker from the oldest to the newest technology. Note from (20) that  $\gamma' \pi_1'(w_1') \leq \gamma \pi_1(w_1)$ . Therefore,  $\omega_0 + \beta \gamma' \pi_1'(w_1') \leq \omega_0 + \beta \gamma \pi_1(w_1)$ . Using (8) we have that  $w_{T-1}'(1 + \beta \gamma') \leq w_{T-1}(1 + \beta \gamma)$ . Therefore  $w_{T-1}$  falls with an increase in the growth rate  $\gamma$ . That is, switching costs decline with an increase in the growth rate. This decline in switching costs stimulates young workers to move towards more recent vintages. In Figure 5, we plot the logarithms of switching costs against the growth rate and the substitution parameter. Note that the switching costs decline with an increase in the growth rate and rise with an increase in the complementarities of the inputs.

We now examine the effect of a change in the discount factor on the stationary distribution. One interpretation of an increase in the discount factor is a decrease in the length of the time interval. This decrease can be interpreted as increasing the arrival rate of new technologies. But shrinking the length of the time interval also shrinks the training time of young workers. So we are changing two variables at once. We show in Propositions 6 and 7 below that an increase in the discount factor results in a more rapid rate of diffusion.

*Proposition 6.* Consider two economies with  $\beta' > \beta$  with associated stationary distributions  $\mu'$  and  $\mu$  respectively. Let  $w(\tau, \mu')$  and  $w(\tau, \mu)$  denote the wage rates in vintage  $\tau$  and  $n(\tau, \mu')$ ,  $n(\tau, \mu)$  denote the input decisions in vintage  $\tau$  in the two economies. Then  $w(\tau, \mu') \geq w(\tau, \mu)$  and  $n(\tau, \mu') \leq n(\tau, \mu)$  all  $\tau$ .

*Proof.* See Appendix.  $\square$

We then have

*Proposition 7.* Let  $\mu'$  and  $\mu$  denote the steady state distributions for two economies characterized by discount factors  $\beta'$  and  $\beta$  respectively. Then  $\mu$  stochastically dominates  $\mu'$ .

*Proof.* Parallels Proposition 5 exactly and is omitted.  $\square$

#### 4. Optimality of the Competitive Equilibrium

In this section, we establish that if the growth rate of the economy is not too large, the competitive equilibrium is Pareto optimal. In fact, the competitive equilibrium maximizes the discounted value of output. We will need to assume  $\beta\gamma < 1$ . This is a standard condition in models of economic growth. We need to ensure that the discounted consumption stream is bounded to ensure that our social welfare function is well-defined. Let  $c_t = (c_{1t}, c_{2t})$  denote the consumption when young and old of a representative agent born at time  $t$ . We have assumed that  $U(c_{1t}, c_{2t}) = c_{1t} + \beta c_{2t}$ . The initial old care only about consumption in the first period denoted by  $c_{2,-1}$ . Let  $c = (c_{2,-1}, (c_t)_{t=0}^{\infty})$ . We will assume that a planner has preferences given by

$$(23) \quad W(c) = c_{2,-1} + \sum_{t=0}^{\infty} \beta^t (c_{1t}, c_{2t}).$$

Let  $y_t = c_{2t-1} + c_{1t}$ . Thus  $y_t$  denotes the total output at time  $t$ . The planner is assumed to have available a total labor endowment of two units in each period. Let  $N_1(t, \tau)$  denote labor of young workers allocated to vintage  $\tau$  at time  $t$  and  $N_2(t, \tau)$  denote labor of old workers to unskilled tasks. Let  $Z(t, \tau)$  denote skilled labor allocated to vintage  $\tau$  at time  $t$ . In keeping with our assumptions,  $Z(t, \tau) \leq N_1(t-1, \tau-1)$ . The problem faced by the planner is then

$$(24) \quad \max \sum_{t=0}^{\infty} \beta^t y_t$$

subject to

$$(25) \quad 0 \leq y_t \leq \sum_{\tau=0}^{\infty} \gamma^{t-\tau} f(N_1(t, \tau) + N_2(t, \tau), Z(t, \tau))$$

$$(26) \quad \sum_{\tau=0}^{\infty} N_1(t, \tau) \leq 1 \quad \text{all } t$$

$$(27) \quad Z(t+1, \tau+1) \leq N_1(t, \tau) \quad \text{all } t, \tau \geq 0$$

$$(28) \quad \sum_{\tau=0}^{\infty} (Z(t+1, \tau) + N_2(t+1, \tau)) \leq \sum_{\tau=0}^{\infty} N_1(t, \tau)$$

given  $N_1(-1, \tau)$ .

Let  $\lambda_t$  denote the Lagrange multiplier on constraint (26). The necessary first order conditions are then given after some simplification by

$$(29) \quad \beta^t \gamma^{t-\tau} f_1(N(t, \tau), Z(t, \tau)) + \beta^{t+1} \gamma^{t+1-(\tau+1)} f_2(N(t+1, \tau+1), Z(t+1, \tau+1)) - \lambda_t$$



for all vintages with  $N(t, \tau) > 0$ ,  $Z(t+1, \tau+1) > 0$ . Let  $\hat{\lambda}_t = (\beta\gamma)^{-\tau} \lambda_t$ . Then equation (29) can be rewritten to read

$$(30) \quad \gamma^{-\tau} f_1(N(t, \tau), Z(t, \tau)) + \beta \gamma^{-\tau} f_2(N(t+1, \tau+1), Z(t+1, \tau+1)) = \hat{\lambda}_t.$$

Let  $w(t, \tau) = \gamma^{t-\tau} f_1(N(t, \tau), Z(t, \tau))$  and  $v(t, \tau) = \gamma^{t-\tau} f_2(N(t, \tau), Z(t, \tau))$ . Equation (30) is then obviously identical to equation (3). Similar arguments apply to vintages where  $Z$  or  $N$  equals zero. The fact that  $f(\cdot, \cdot)$  is homogenous of degree one implies that  $f_1(\cdot, \cdot)$  and  $f_2(\cdot, \cdot)$  are homogenous of degree zero. Let  $n(t, \tau) = N(t, \tau)/Z(t-1, \tau-1)$ . Euler's Theorem implies that

$$f_2(n(t, \tau), 1) = f(n(t, \tau), 1) - n(t, \tau) f_1(n(t, \tau), 1).$$

Hence,

$$v(t, \tau) = f(n(t, \tau), 1) - w(t, \tau) n(t, \tau).$$

Therefore, profit maximization follows. We need to verify that the competitive equilibrium satisfies the transversality condition that  $(\beta\gamma)^t \gamma^{-\tau} f_2(N(t, \tau), Z(t, \tau)) Z(t, \tau)$  goes to zero. But this obviously follows because  $f_2(\cdot, \cdot) Z(\cdot, \cdot) \leq f(1, 1)$  which is bounded. Therefore, the competitive equilibrium solves the same problem as does the planner. Note that consumers are indifferent in the competitive equilibrium about the timing of their consumptions. Hence, the competitive equilibrium is Pareto optimal. Clearly, any solution to the programming problem described above can also be supported as a competitive equilibrium.

### 5. Concluding Remarks

We have presented a model of investment in technology specific human capital. Such specificities lead to a lag between the time that a new technology becomes available and the peak of its usage. In other words, technologies diffuse slowly. We show that an increase in the rate of change of technology arrival implies an increase in the rate of diffusion. We also show that the wage profiles over time are flatter in older technologies than in newer ones. In that sense, people who learn newer technologies invest in technology-specific skills and our model is one of human capital accumulation. The equilibrium we describe is Pareto optimal.

An obvious extension of our model would be to allow for uncertainty in the rate of technological innovation. We conjecture that in such a case, a technological innovation which is substantially better than average will attract a large number of young workers and lead to larger than average investment in the newest technology. Since this capital is specific to the technology, in subsequent periods relatively few young workers will be attracted to even newer technologies. These technologies will then be adopted and diffused at a slower rate than average. Thus, this extension of the model can account for bursts in technological advance followed by a slowdown.

The assumption of exogenous technical change obviously does not do justice to the reality of the process of innovation which requires the use of resources. It would be interesting to examine a model where technological innovation as well as adoption are jointly and endogenously determined. One possible modification of our model would be to let the productivity of the newest vintage relative to the previous one,  $\gamma$ , be determined by the number of workers who enter the newest

industry. In such a case workers in the newest vintages can be thought of as engaging in innovative activity.

An alternative possibility is to think of spillover effects as implying that if workers are skilled in relatively new technologies then the productivity of the newest technology is higher. In such a case, the rate of growth of the economy and the rate of adoption of new technologies are both determined endogenously. The resulting equilibrium is not necessarily Pareto optimal and the model can be used to study the effect of policy interventions on the growth rate of the economy.

We conjecture that an exogenous improvement in the technology of innovation will lead, as in this paper, to an increase in the rate of diffusion. The earning profiles will also likely get flatter with such an improvement.

## Footnotes

<sup>1</sup>We assume for now that the interest rate is  $1/\beta - 1$  and show that this is true in equilibrium.

<sup>2</sup>In an earlier version of the paper we showed that if the distribution is stationary the equilibrium allocations are stationary and wages rise at rate  $q$ . Proofs with this weaker condition are available upon request.

<sup>3</sup>It is also clear from the proof of Proposition 2 that the maximum wage rate is strictly positive. Therefore, no worker is at a corner in his consumption and the equilibrium interest rate is  $1/\beta - 1$ .

<sup>4</sup>An interesting question is whether the economy converges to a stationary equilibrium from an arbitrary initial distribution. This question is difficult to answer analytically for multiple capital goods models like this one. For the parametric examples described below the dynamical system implied by the equilibrium conditions is locally stable. Indeed, we found that for  $\beta\gamma$  sufficiently close to unity all the examples we considered were locally stable.

<sup>5</sup>Our measure of investment is the standard one in multiple capital goods model and is formally identical to the way investment is measured in the National Income and Product Accounts. Investment in different sectors is measured at market prices and therefore measures the decrease in consumption at the margin due to the investment activity.

## Appendix

*Proof of Proposition 1.* We prove this proposition by proving a series of claims.

*Claim 1 (No holes in the stationary distribution).*

If  $Z_r = 0$  for some  $r$ , then  $\mu_{r+1} = 0$ .

*Proof.* Suppose  $Z_T = 0$  and  $\mu_{T+1} > 0$ . Then  $w_T = \gamma^{-T}\omega_0$  for all  $t$  from (5). Furthermore,  $w_{T-1} \geq \gamma^{-T+1}\omega_0$  and  $w_{T+1} \geq \gamma^{-T-1}\omega_0$  since firms could make infinite profits otherwise. Now, if  $\omega_0 = 0$ ,  $\pi_T(w_T)$  is infinite implying that the present value of income is infinite which is inconsistent with the fact that output is bounded. If  $\omega_0 > 0$ ,  $w_T < w_{T-1}$ . From (8) this implies that  $\pi_{T+1}(w_{T+1}) > \pi_T(w_T)$ . But  $w_{T+1} > \gamma w_T$  which implies that  $\pi_{T+1}(w_{T+1}) \leq \pi_T(w_T)$ . We have a contradiction.

*Claim 2 (The support of  $\mu$  is finite).*

There exists  $T$  such that  $\mu_r = 0$  for all  $r > T$ .

*Proof.* The argument is by contradiction. Using Claim 1 repeatedly and noting that  $0 \leq Z_r \leq \mu_r$ , it follows that  $\mu_r > 0$  for all  $r$  and that  $Z_r > 0$  for all  $r$ . Therefore, from (8) we have that

$$(A1) \quad w_r + \beta\gamma\pi_{r+1}(w_{r+1}) = k \quad \text{for all } r.$$

Suppose first that for some  $r$ ,  $w_r > w_{r+1}$ . Then, clearly, from (A1)  $\pi_S(w_S) < \pi_{S+1}(w_{S+1})$  for all  $S \geq r$  and  $w_S > w_{S+1}$ . We next show that there is some  $T$  such that  $n_r(w_r) \geq 1$  for all  $r \geq T$ . Define  $T$  as the smallest number which satisfies  $\gamma^{-T}f(1,1) \leq w_T$ . Since  $w_r > 0$  such a number exists. Since  $\pi_T(w_T) \geq w_T$  and  $\pi_T(w_T) \leq \gamma^{-T}f(n_T(w_T), 1)$ , clearly  $n_T(w_T) \geq 1$ . It also follows that  $n_r(w_r) \geq 1$  for all  $r \geq T$ . Since wages are strictly decreasing,  $N_{2r} = 0$  and  $Z_r = \mu_r$ . Therefore,  $\mu_{r+1} =$

$n_\tau(w_\tau)\mu_\tau$  and  $\mu_\tau$  is an increasing sequence. But  $\sum_\tau \mu_\tau = 1$ . We have a contradiction. Consider next the case when, for all  $\tau$ ,  $w_\tau \leq w_{\tau+1}$ . But  $\pi_\tau$  is a decreasing function of the vintage and so for  $\tau$  sufficiently large,  $\pi_\tau(w_\tau) < w_1$  implying that  $Z_\tau = 0$ . This contradiction establishes the desired result.  $\square$

*Claim 3 (Unskilled workers' wages are nondecreasing).*

Let  $T$  denote the last vintage. In a stationary equilibrium,  $w_\tau \geq w_{\tau-1}$  and  $v_{\tau+1} \leq v_\tau$  for  $\tau = 1, \dots, T-1$ . Furthermore,  $Z_\tau = \mu_\tau$ ,  $\tau \leq T-2$ , and  $Z_{T-1} > 0$ .

*Proof.* Let  $T$  be the last vintage. We show that the wage sequence is nondecreasing up to vintage  $T-1$ . Suppose, by way of contradiction, that  $w_{T-1} < w_{T-2}$ . Then using equation (8), we have

$$(A2) \quad \pi_T(w_T) > \pi_{T-1}(w_T).$$

Therefore,  $w_T < w_{T-1}$ , which implies that  $N_{2T} = 0$ . Since  $N_{1T} = 0$  inequality (A2) cannot hold and we have a contradiction. Therefore,  $w_{T-1} \geq w_{T-2}$ . This implies that  $Z_{T-1} > 0$  and  $\pi_{T-1}(w_{T-1}) < \pi_{T-2}(w_{T-2})$ . From the present value conditions,  $w_{T-2} > w_{T-3}$ . By induction, the wages of unskilled workers are strictly increasing for  $\tau = 0, \dots, T-2$ . Furthermore, since the wages of skilled workers are strictly decreasing for  $\tau = 1, \dots, T-1$ ,  $Z_\tau = \mu_\tau$ .  $\square$

*Proposition 2 (Existence of a wage sequence).*

There exists a unique number  $S$  and a unique sequence of wages satisfying (14)–(17).

*Proof.* The proof is by construction. To construct the wage sequence, fix a number  $T$ . Define a sequence of wages as functions of  $w_{T-1}$  recursively as follows. Let the wage at vintage  $T-2$  be given by

$$(A3) \quad w_{T-2}(w_{T-1}) = w_{T-1}(1+\beta\gamma) - \beta\gamma\pi_{T-1}(w_{T-1}).$$

Clearly,  $w_{T-2}(\cdot)$  is a strictly increasing function, and for  $w_{T-1}$  sufficiently large,  $w_{T-2} \geq 0$ . Define the rest of the wage sequence recursively as follows:

$$(A4) \quad w_r(w_{T-1}) = w_{T-1}(1+\beta\gamma) - \beta\gamma\pi_{r+1}(w_{r+1}(w_{T-1})).$$

Clearly,  $w_r(w_{T-1})$  is a continuous strictly increasing function, which is positive and therefore well-defined for sufficiently large  $w_{T-1}$ . In particular,  $w_0(w_{T-1})$  is a continuous strictly increasing function. Therefore, there is a unique value of  $w_{T-1}$  for each  $T$  such that  $w_0(w_{T-1}) = \omega_0$ . Denote the wage sequence at this value of  $w_{T-1}$  by  $(w_0(T), w_1(T), \dots, w_{T-1}(T))$  and the associated present value by  $k(T)$ . If  $\pi_1(\omega_0) \leq \omega_0$ , the number of vintages is 1. So suppose  $\pi_1(\omega_0) > \omega_0$  and let  $T = 2$ . Then (A3) reads  $\omega_0 + \beta\gamma\pi_1(w_1(2)) = w_1(2)(1+\beta\gamma)$ . If  $\pi_1(w_1(2)) \leq w_1(2)$ , then  $\omega_0 \geq w_1(2) \geq \pi_1(w_1(2))$  which contradicts the assumption that  $\pi_1(\omega_0) > \omega_0$ . So  $\pi_1(w_1(2)) > w_1(2)$  and the constructed sequence at  $T = 2$  satisfies (14)-(16). We now use induction to argue that if a sequence satisfies (14)-(16) but does not satisfy (17) at  $T$ , the sequence at  $T + 1$  also satisfies (14)-(16). We have constructed wage sequences satisfying (14) and (15) for each  $T$ . Suppose  $\pi_T(w_T(T+1)) < w_T(T+1)$ . Since  $\pi_T(w_{T-1}(T)) > w_{T-1}(T)$ , it follows that  $w_{T-1}(T) < w_T(T+1)$ . Therefore  $k(T) < k(T+1)$ . Furthermore, because  $\pi_T(w_T(T+1)) < w_T(T+1)$ , using (A3) we have that  $w_{T-1}(T) < w_{T-1}(T+1)$ . Using the result that  $k(T) < k(T+1)$ , we can use (A4) recursively to show that  $w_0(T) < w_0(T+1)$  which clearly contradicts the fact that  $w_0(T) = \omega_0 = w_0(T+1)$ . We have established that  $\pi_T(w_T(T+1)) \geq w_T(T+1)$ .

We now show that  $w_T(T+1) \geq w_{T-1}(T)$  if the sequence at  $T$  satisfies (14)-(16). Suppose that  $w_T(T+1) < w_{T-1}(T)$ . Then  $k(T+1) < k(T)$  and  $w_{T-1}(T+1) < w_{T-1}(T)$ . Again, using (A4) it is easy to show that  $w_{T-2}(T+1) < w_{T-2}(T)$  and recursively that

$w_0(T+1) < w_0(T)$ , establishing a contradiction. Since  $w_T(T+1) \geq w_{T-1}(T)$  and  $\pi_{T+1}(\cdot) < \pi_T(\cdot)$ , it is clear that, for  $T$  sufficiently large, (17) is satisfied.

We now establish the uniqueness of the wage sequence. Let  $w_r(T)$  satisfy (9)–(12) and let  $w_r(T^*)$  satisfy the same conditions for a larger number,  $T^*$ . Since  $\pi_{T^*-1}(w_{T^*-1}(T^*)) \geq w_{T^*-1}(T^*)$  and  $\pi_T(w_{T-1}(T)) \leq w_{T-1}(T)$  it follows that  $w_{T^*-1}(T^*) < w_{T-1}(T)$ . Therefore,  $k(T^*) < k(T)$ . It is also straightforward to show that  $w_{T-1}(T^*) < w_{T-1}(T)$ . Recursively using (A4) it is easy to establish a contradiction.  $\square$

*Proposition 4.* If  $\gamma' \geq \gamma$  then  $n(\tau, \mu') \leq n(\tau, \mu)$  where  $\mu'$  and  $\mu$  are the invariant distributions corresponding to  $\gamma'$  and  $\gamma$ , respectively.

*Proof.* We will show that

$$w(\tau, \mu') \geq \left[ \frac{\gamma}{\gamma'} \right]^\tau w(\tau, \mu) \quad \text{for } \tau \leq T'.$$

Since  $n'_\tau[(\gamma/\gamma')^\tau w] = n_\tau(w)$ , this suffices to prove the result. We will assume throughout that  $Z_{T'} = Z_T = 0$ . The proof goes through with straightforward modification if  $Z_{T'}$  or  $Z_T$  is positive.

We start by assuming that  $T' \leq T$ . Suppose by way of contradiction that  $w'_1 < (\gamma/\gamma')w_1$ . Then, by definition of  $\pi$ ,  $\pi'_1(w'_1) > (\gamma/\gamma')\pi_1(w_1)$  which, in turn, implies, if  $T' \geq 2$  that

$$k' = \omega_0 + \beta\gamma'\pi'_1(w'_1) > \omega_0 + \beta\gamma\pi_1(w_1) = k.$$

But then we also have that

$$k' = w'_1 + \beta\gamma'v'_2 > w_1 + \beta\gamma v_2 = k.$$



Since  $w'_1 < w_1$ , we have that  $v'_2 > v_2$  which, in turn, implies if  $T' \geq 3$  that  $\gamma' v'_2 = \gamma' (\pi'_2(w'_2)) > \gamma \pi_2(w_2) = \gamma v_2$ . From the definition of  $\pi$  we have that  $w'_2 < (\gamma/\gamma')^2 w_2$ . Repeating this argument we have that

$$(A7) \quad w'_{T'-1} < (\gamma/\gamma')^{T'-1} w_{T-1}.$$

But we also have

$$k' = w'_{T'-1} + \beta \gamma' w'_{T'-1} > k \geq w_{T-1} + \beta \gamma w_{T-1}.$$

We have a contradiction, and  $w'_1 \geq (\gamma/\gamma') w_1$ .

We now complete the argument with an inductive step. Suppose there is some  $\tau$  such that  $w'_{\tau-1} \geq (\gamma/\gamma')^{\tau-1} w_{\tau-1}$  but  $w'_\tau < (\gamma/\gamma')^\tau w_\tau$ . Then,  $\pi'_\tau(w'_\tau) > (\gamma/\gamma')^\tau w_\tau$  and

$$(A9) \quad k' = w'_{\tau-1} + \beta \gamma v_\tau > \left(\frac{\gamma}{\gamma'}\right)^{\tau-1} [w_{\tau-1} + \beta \gamma v_\tau] = \left(\frac{\gamma}{\gamma'}\right)^{\tau-1} k.$$

Also note that  $v'_{\tau+1} = (k' - w'_\tau)/\beta \gamma' > (\gamma/\gamma')^\tau ((k - w_\tau)/\beta \gamma) = (\gamma/\gamma')^\tau v_{\tau+1}$ . Therefore,  $w'_{\tau+1} < (\gamma/\gamma')^{\tau+1} w_{\tau+1}$ . Repeating this argument, we get that  $w'_{T'-1} < (\gamma/\gamma')^{T'-1} w_{T-1}$  and using (A8) and (A9) we get a contradiction. Therefore, if  $T' \leq T$ , then  $w'_\tau \geq (\gamma/\gamma')^\tau w_\tau$  for all  $\tau \leq T'$ .

We now show that  $T' \leq T$ . Suppose  $T' > T$ . Using the same argument as above it can be shown that  $w'_T \geq (\gamma/\gamma')^T w_T$ . But then  $\pi'_T(w'_T) \leq (\gamma/\gamma')^T \pi_T(w_T)$  so that  $\pi_T(w_T) > w_T$ . This obviously contradicts a necessary condition of a stationary equilibrium.

*Proposition 6.* Consider two economies with discount factors  $\beta'$  and  $\beta$ , with  $\beta' > \beta$ . Then  $w'_\tau \geq w_\tau$  all  $\tau$  and  $n'_\tau \leq n_\tau$  all  $\tau$ .

*Proof.* Suppose  $T' \leq T$  and  $w'_r \leq w_1$ . We will show that this leads to a contradiction. From (8) we have that

$$(A10) \quad \omega_0 + \beta\gamma\pi_1(w_1) = w_1 + \beta\gamma\pi_2(w_2) = k$$

$$(A11) \quad \omega_0 + \beta'\gamma\pi_1(w'_1) = w'_1 + \beta'\gamma\pi_2(w'_2) = k'.$$

Subtracting the first equation from the second and noting that  $w'_1 \leq w_1$  we have after some rearranging that

$$(A12) \quad \beta'[\pi_1(w'_1) - \pi_2(w'_2)] \leq \beta[\pi_1(w_1) - \pi_2(w_2)].$$

Recall that  $\beta' > \beta$  and  $\pi(\cdot)$  is decreasing function. Hence  $w'_2 < w_2$ . By induction it is easy to show that  $w'_r < w_r$  all  $r$ . In particular,  $w'_{T'-1} < w_{T'-1}$ . It follows that  $k' < k$ . But

$$(A13) \quad w'_{T'-1}(1+\beta'\gamma) = k' < k = w_{T'-1}(1+\beta\gamma).$$

Subtracting the left side of (A13) from (A10) and the right side of (A13) from (A11) we can easily establish a contradiction. Therefore  $w'_1 \geq w_1$ .

Repeating the same argument, it follows that  $w'_r \geq w_r$  for all  $r$ . The argument that  $T' \leq T$  follows exactly the same lines as in the proof of Proposition 4.  $\square$

Figure 1

DIFFUSION CURVES

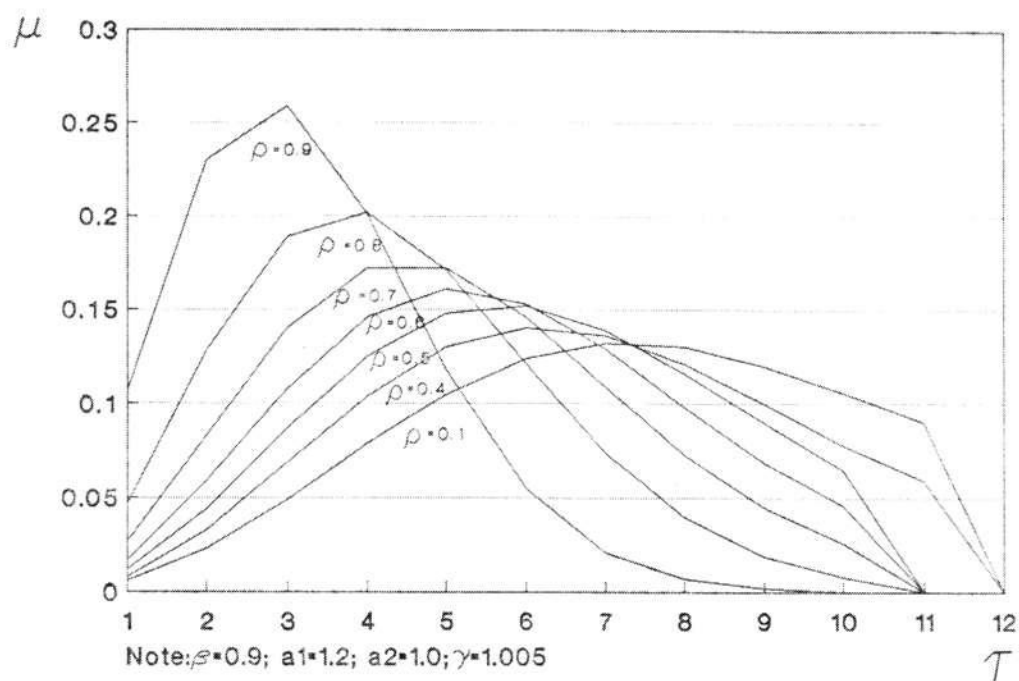


Figure 2

DIFFUSION CURVES UPTO ADOPTION PEAK

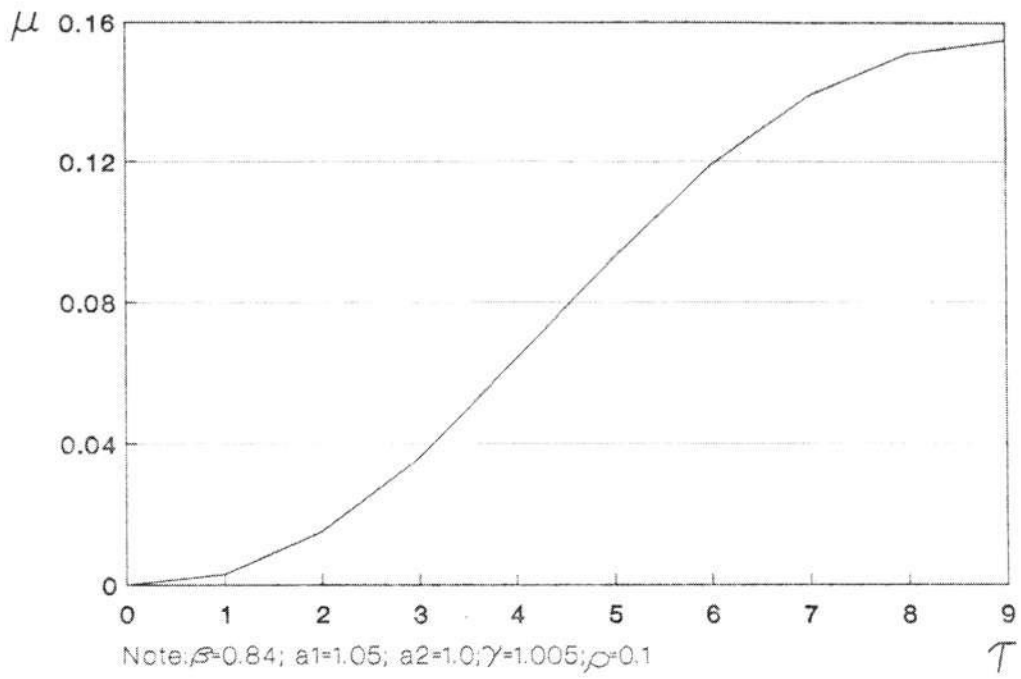
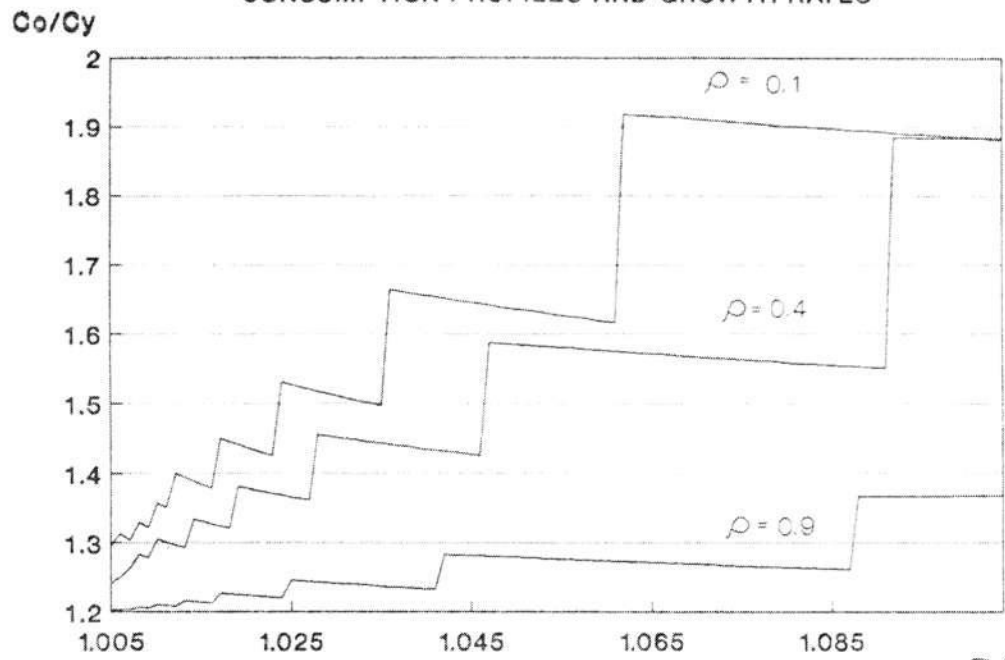


Figure 3

CONSUMPTION PROFILES AND GROWTH RATES



Note:  $\beta=0.9$ ,  $a_1=1.2$ ,  $a_2=1.0$

Figure 4

INVESTMENT-OUTPUT RATIO AND GROWTH RATES

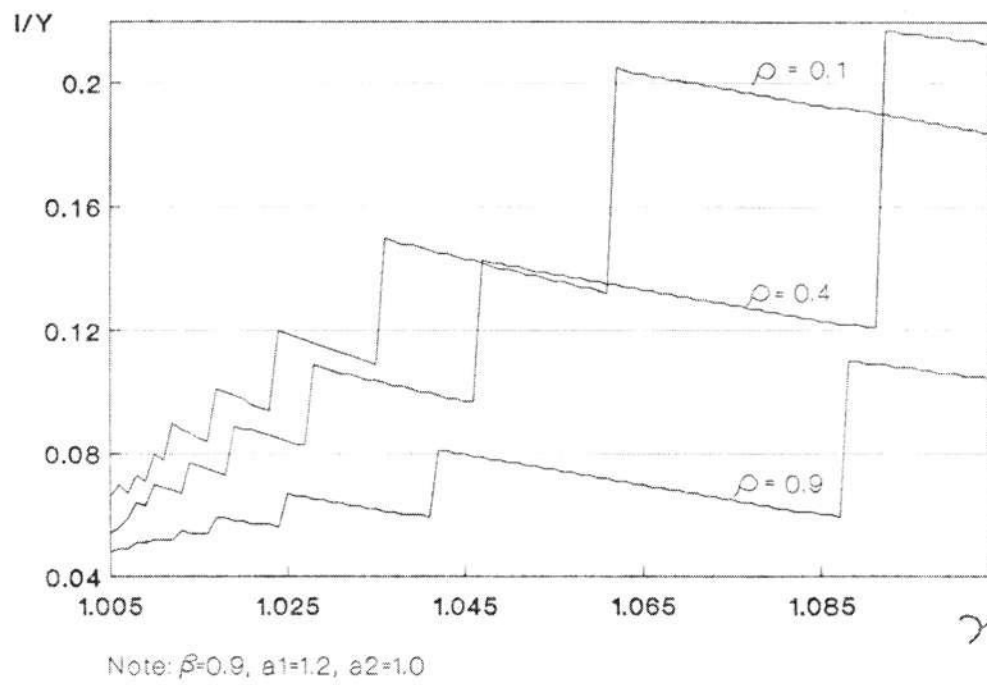
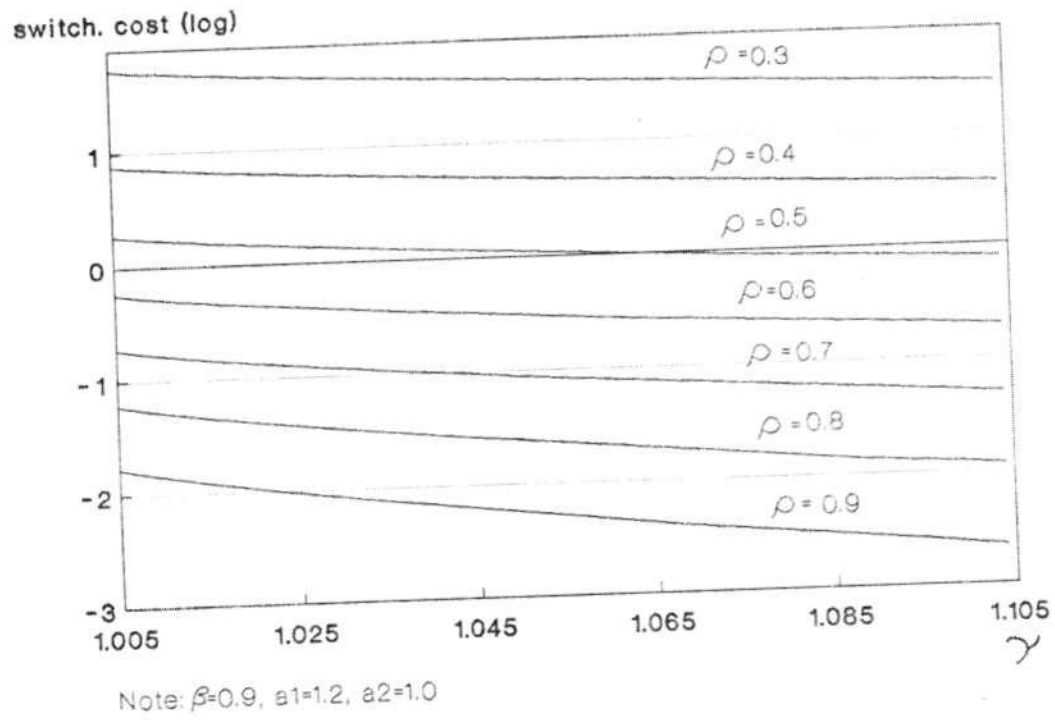


Figure 5

SWITCHING COSTS AND GROWTH RATES



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