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# Two Illustrations of the Quantity Theory of Money Reloaded

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# Two Illustrations of the Quantity Theory of Money Reloaded

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## Abstract

In this paper, we review the relationship between inflation rates, nominal interest rates, and rates of growth of monetary aggregates for a large group of OECD countries. We conclude that the low-frequency behavior of these series maintains a close relationship, as predicted by standard quantity theory models. In an estimated model, we show those relationships to be relatively invariant to alternative frictions that can deliver very different high-frequency dynamics. We argue that these relationships are useful for policy design aimed at controlling inflation.

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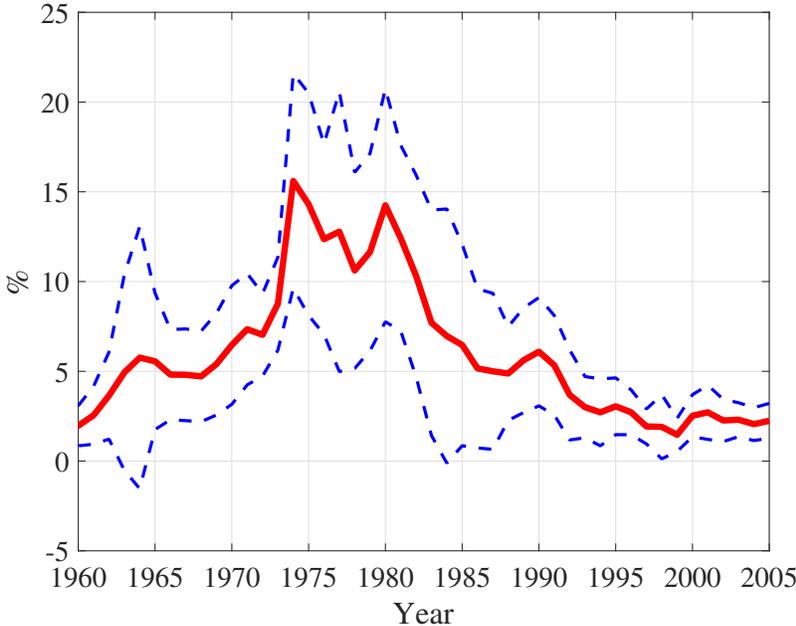
# 1 Introduction

The collapse of the Bretton Woods system in 1971 marked the beginning of an era of discretionary monetary policy characterized by a growing number of central banks that abandoned rigid rules pegging their currency to a precious metal or to a strong currency. This development came at a cost: the first years were characterized by rising and more volatile inflation.

In developed economies, good central banking progressively and successfully ended this high inflation period by the last decade of the 20th century. Figure 1 summarizes the rising inflation and its subsequent conquest. It depicts the average inflation rate for a set of OECD countries from 1960 to 2005, together with a one-standard deviation band.<sup>1</sup>

The figure shows the increase in inflation rates that started during the last years of the Bretton Woods system and got worse after its collapse. It also shows how inflation rates returned to low levels at the same time that the standard deviation went down to one of its smallest values in the period.

Figure 1: Average inflation for 13 OECD countries (1960–2005)



<sup>1</sup>We use the USA, Australia, Canada, Germany, Denmark, Italy, Japan, Netherlands, New Zealand, Portugal, South Korea, Spain, and the UK to compute means and standard deviations.

We argue that to understand the rise in inflation, its eventual conquest, and the essential role of central banks in the battle, it is sufficient to appeal to a simple and old theoretical tradition in monetary economics: the quantity theory of money. In its more traditional version, the theory has been presented and discussed by [Hume \(1741\)](#), [Mill \(1848\)](#), and [Fisher \(1911\)](#), among others. It has been further developed by [Friedman \(1959\)](#) and integrated into the modern dynamic general equilibrium theory by [Sidrauski \(1967\)](#) and [Lucas \(1982\)](#), among many others.

The empirical review we perform in this paper is organized around a simple model that belongs to that tradition. The model abstracts from a plethora of details that are relevant for monetary policy in general. In particular, the abstraction includes perfectly functioning markets populated by infinitely lived rational agents that possess perfect information regarding the economy in which they operate.

Day-to-day good central banking is a complicated task: it amounts to monitoring and assessing massive amounts of data, simulating alternative scenarios, studying the robustness of policies in each scenario, and deciding the right judgment in each policy decision. These decisions affect the actions of many different members of society, none of whom know exactly how the economy functions. Price setting in actual economies involves making forecasts of future events — including the actions taken by central banks themselves — and those price setting decisions affect the way markets function. It is very tempting, given the complicated nature of economic relationships, to disregard the lesson of very simple, almost naive theoretical constructions.

The purpose of this paper is to make a case for not falling into that temptation. The immediate effect of a monetary policy change may very much depend on details of the environment, and relatively minor changes can sometimes substantially affect the conclusions. But to understand medium-term inflation, we argue that the simple, utterly unrealistic abstraction suffices.

The notion that sustained periods of very high inflation are associated with prolonged

periods of both high money growth and high nominal interest rates (or financial repression) is not disputed. These notions are evident in the data analyzed in the by now classic paper of [Sargent \(1982\)](#) on the end of four big inflations following the First World War.<sup>2</sup> The purpose of our paper is to argue that the same forces are behind the data in [Figure 1](#).

We follow a tradition of separating the data into a short-run (or high-frequency) component and a long-run (or low-frequency) component, which was pioneered by [Lucas \(1980\)](#) — which explains the title of this paper — and used by [Benati \(2009\)](#) and [Sargent and Surico \(2011\)](#), among others. The separation involves the use of a statistical filter. The filter we use differs from the previous studies and the theoretical implications are somewhat different, as we will make precise in Section 2, where we present the model. In Section 3, we discuss and rationalize the decisions we made regarding the filtering technique and present the evidence for a relatively large set of countries. It is in this section that we forcefully argue that the simple model does an extremely good job of explaining the medium-term behavior of the data.

As is well known, the simple model we use notably fails at explaining the short-run behavior of the data. It is because of this failure that the monetary literature has developed models accounting for more realistic features, like frictions in the setting of prices. In Section 4, we therefore estimate one such model, but we allow for changes in the medium-term inflation target. Our estimates show that those slow-moving changes in policy are the drivers of the low frequency of the data we focus on. We also simulate the model for different degrees of price frictions and filter the simulated data as we did for the true data. We use this exercise to argue that the behavior of medium-run inflation is almost invariant to the degree of price frictions and essentially the same as in the simple model of Section 2. Thus, the degree of frictions seem to have little role, if any, in explaining the low-frequency movements evident in [Figure 1](#). We conclude with a discussion of the policy implications of the evidence discussed in the paper.

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<sup>2</sup>See also the evidence on Latin American hyperinflation in the '80s and '90s collected in [Kehoe and Nicolini \(forthcoming\)](#), for example.

## 2 The Model

We study a labor-only representative agent economy with uncertainty, in which making transactions is costly.<sup>3</sup> The preferences of the representative agent are

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t), \quad (1)$$

where  $x_t$  is consumption at date  $t$  and  $U$  is differentiable, increasing, and concave. The goods production technology is given by

$$y_t = x_t = z_t l_t,$$

where  $l_t$  is time devoted to the production of the final consumption good and  $z_t$  is an exogenous stochastic process. Each period, the representative agent is endowed with a unit of time with  $l_t$  used to produce goods and  $1 - l_t$  used to carry out transactions.

We assume that households choose the number  $n$  of “trips to the bank,” in the manner of the classic Baumol-Tobin model. Thus, purchases over a period are then subject to a cash-in-advance constraint

$$P_t x_t \leq M_t n_t, \quad (2)$$

where  $M_t$  is money and  $n_t$  is the velocity of money.

We assume that the cost of going to the bank is linear in the number of trips, as in the Baumol-Tobin case, according to

$$\theta n_t \nu_t,$$

where  $\theta$  is a positive parameter and  $\nu_t$ , is an exogenous stochastic process. The variable  $\nu_t$ , introduces unobserved randomness into the model and is meant to capture changes in the technology to adjust portfolios available to households. We assume that the logs of  $\nu_{t+1}$  and  $\nu_t$  are jointly stationary, so their difference is also stationary.

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<sup>3</sup>The model is a special case of the one developed in detail in [Benati et al. \(2020\)](#).

Total time available for production is therefore

$$l_t = 1 - \theta n_t \nu_t,$$

so consumption must satisfy

$$x_t = z_t(1 - \theta n_t \nu_t). \quad (3)$$

The real wage is equal to  $z_t$ , and the nominal wage is  $z_t P_t$ .

At the beginning of each period, an agent begins with nominal wealth  $\Psi_t$ , which can be allocated to money  $M_t$  or to interest bearing bonds  $B_t$ . The agent's allocation of these assets is then restricted by

$$M_t + B_t \leq \Psi_t. \quad (4)$$

The agent's wealth at the beginning of next period is given by

$$\Psi_{t+1} \leq M_t + B_t(1 + i_t) + [1 - \theta n_t \nu_t] z_t - x_t + \tau_{t+1},$$

where  $\tau_{t+1}$  is the monetary transfer the government makes to the representative agent.

Given the initial wealth  $\Psi_t$ , this agent chooses his consumption  $x_t$ ; the number of bank trips  $n_t$ ; the assets that he chooses to hold  $M_t$  and  $B_t$ ; and, implicitly, the wealth  $\Psi_{t+1}$  that he carries into the next period subject to (2), (3), and (4).

In Appendix A, we show that as long as the cash-in-advance constraint (2) is binding, the optimal solution for  $n$  can be well approximated by

$$\sqrt{\frac{i_t}{\theta_t \nu_t}} \simeq n_t. \quad (5)$$

This is the celebrated squared root formula derived by [Baumol \(1952\)](#) and [Tobin \(1956\)](#).

We can once again use the cash-in-advance constraint (2) to replace the variable  $n$  in the

last equation and obtain

$$\frac{m}{x} = \sqrt{\frac{\theta\nu}{i}}, \quad (6)$$

which delivers a relationship between real money balances as a proportion of output and the nominal interest rate in bonds.

Assuming that the cash-in-advance constraint is binding is quite reasonable for the countries in the period considered in [Figure 1](#), with the possible exception of Japan, which since 1995 has had near-zero interest rates.<sup>4</sup>

We also show in Appendix A that in equilibrium, it must be the case that

$$E \left[ \frac{1}{1+r'} \left( \frac{1+i}{1+\pi(s')} \right) \right] = 1, \quad (7)$$

where  $r(s')$  is a measure of the real interest rate.<sup>5</sup> This last expression is the well known Fisher equation relating the nominal interest rate with the real interest rate and the inflation rate.

Summarizing, the theory delivers two equilibrium relationships, (6) and (7), that involve three endogenous variables: the rate of inflation, the rate of money growth relative to output, and the nominal interest rate. These two conditions do not fully characterize the equilibrium of the model. Conspicuous by its absence is a description of how monetary policy is executed. This was a conscious choice, since according to the theory, the two implications ought to hold independently of how policy is executed.

It is very standard, particularly in the New Keynesian literature, to assume that the policy instrument is the nominal interest rate. And we will follow that tradition in Section 4, in which we estimate a fully specified model. But to validate the empirical performance of those two equations, as we do in the next section, we do not need to take a stand on

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<sup>4</sup>We discuss the case of Japan in a separate subsection in which we also study the period of very low interest rates that followed the financial crisis of 2008-09 in a few other countries.

<sup>5</sup>This real interest rate is measured in terms of marginal utilities of real wealth, using the indirect utility function. In Appendix A, we show how this relates to a real interest rate measured in units of consumption, rather than in wealth.

how monetary policy is executed.

### 3 Empirical Analysis

In order to obtain an explicit solution for inflation, we take logs in (6) and compute the difference over two consecutive periods to obtain

$$\ln \frac{P_{t+1}}{P_t} = \pi_{t+1} = \ln \frac{M_{t+1}}{M_t} - \ln \frac{x_{t+1}}{x_t} + \frac{1}{2} \ln \frac{i_{t+1}}{i_t} + \frac{1}{2} \ln \frac{\nu_{t+1}}{\nu_t}. \quad (8)$$

The left-hand side and the first three terms of the right-hand side are observable. They correspond to our measures of inflation, money growth, output growth, and the growth rate of the short-term interest rate. We treat the ratio  $\ln \frac{\nu_{t+1}}{\nu_t}$  as an unobservable, but as it was assumed to be stationary, it should have little effect on the low-frequency component we will focus on. This equation can be taken directly to the data.

Notice that our theoretical assumptions (Baumol and Tobin's assumptions, really) pin down the coefficients on the right-hand side, so there is no room for parameter estimation in this exercise. Equation (8) departs from most of the previous papers that focused on the low frequency, like [Lucas \(1980\)](#) and [Benati \(2009\)](#), which set the value of the interest rate elasticity to zero, rather than to 1/2 as the Baumol-Tobin model implies.<sup>6</sup>

Equation (7) requires some additional manipulation. First, we use a log-linear approximation to write it as

$$i_{t+1} = r_{t+1} + E_t \pi_{t+1},$$

which involves an expectation term. But we can write

$$\pi_{t+1} = E_t \pi_{t+1} + \xi_{t+1}^\pi,$$

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<sup>6</sup>Had we followed their strategy, the fit of the model to the data would worsen for most of the countries we analyze below.

where  $\xi_{t+1}^\pi$  is zero-mean shock, independent from any of the variables in the information set at time  $t$ , since they are expectational errors. Thus, for the empirical implementation, we use

$$i_{t+1} = \pi_{t+1} + r_{t+1} + \xi_{t+1}^\pi, \quad (9)$$

and we treat the  $\xi_{t+1}^\pi$  as unobservable. Being mean-zero shocks, they should also have little effect on the low frequency component. The nominal interest rate on left-hand side of equation (9) is observable. However, since the availability of index bonds is very limited in practice, we do not have direct observations on the real interest rate; this lack poses a problem in testing the empirical implications of this equation.

In order to proceed, we will make the following assumption.

**Assumption 1 (Integrated Capital Markets):** During the period under consideration and for the countries analyzed, there were no restrictions on capital movements, so real interest rates ought to be the same across countries.

Assumption 1 is clearly problematic, since it requires, among other things, the risk of default to be the same for all countries. It also requires differences in the treatment of capital income taxes across all these countries not to create wedges between the return to capital across countries. It is also particularly incorrect for the period before the '80s, when capital controls were the norm around the world.

In spite of its problems, Assumption 1 has a practical advantage: we can use data for the USA, assume that the Fisher equation holds, and use US data plus equation (9) to estimate a real interest rate. Our assumption implies that we can use that real interest rate to test the Fisher equation in all other countries. That will be our strategy. In fact, as we will focus on the low-frequency component, we need only to assume that deviations from perfect capital market integration are very short lived, which is a somewhat weaker assumption.

In studying particular countries it should clearly be possible to do better. For each country, one could try to estimate real interest rates using other data, like the return to

capital from national income accounts. But the purpose of this cross-country analysis is to see the extent to which these two laws emerge even when ignoring all specific details of the countries in our sample. Our hope is that despite this assumption, the analysis allows the reader to see the two equations emerge in the data. To a large extent, our conclusion will be that improvements in the fit of the theory, while worth making on a country-by-country case, will bring modest progress to our ability to understand the medium- and long-run behavior of inflation for this group of countries as a whole.

Our naive model, which abstracts from all sorts of imaginable plausible frictions, has no hope to match high-frequency data. Thus, following [Lucas \(1980\)](#), we abandon that specific quest at the outset and use a statistical filter to remove the high-frequency components in the data. In any event, we present below both the low-frequency component and the original data. Our eyes — and yours also, we hope — will see the original data in a different way after observing the low frequency component.

By construction, whatever one may learn from this strategy is of little use for quarter-to-quarter or year-to-year policy questions. However, as we argue at the end of the paper, our analysis is useful in providing answers to important policy questions, some of them are debated today. We believe that the lessons derived from this exercise are still somewhat ignored in those debates today, 40 years after the publication of Lucas’s (1980) analysis.

The discussion above highlights a key degree of freedom at this stage: the ability to split the data between a high-frequency component (the short run) and the low-frequency component (the long run). [Lucas \(1980\)](#) defends his filtering technique on theoretical grounds, and it has a remarkable advantage: the two illustrations emerge beautifully in Lucas’s figures as the parameter that controls how “long” the long-run increases are. Lucas’s paper is like a mystery movie. If you stare at the data, chaos prevails. But as the reader moves along the sequence of plots, each retaining more and more of the very low frequency, lights appear. By the time the reader arrives at the end, the two illustrations shine and order prevails over chaos. It’s just like the book of Genesis.

Our paper offers just a picture: we take a stand on a particular way to split the data. This, in turn, provides a specific definition of what we mean by short and long run. This definition clarifies for which policy questions our framework will not be useful and for which questions it may be. Our choice of filter is based on a common interpretation of recent US monetary policy and is discussed next.

### 3.1 The filter

To decompose the data, we use the Hodrick-Prescott (HP) filter, popularized by the real business cycle (RBC) literature. An advantage of that filter is that the decomposition made between the high-frequency and the low-frequency components is controlled by a single parameter, denoted by  $\lambda$ . The degree of freedom involves the choice of that parameter. By taking a stand on the value for  $\lambda$ , we take a stand on a particular way to decompose the data between the “cycle” (the high-frequency component) and the “trend” (the low-frequency component).

Below, we will estimate a structural monetary model subject to monetary policy regime changes that can shift the unconditional mean of nominal variables. Each regime is covariance stationary, and so oscillations of all frequencies are present in each regime. Although we label the extracted components from the HP filter as “cycle” and “trend,” terms that are commonplace, it is evident from the analysis of the structural model that regime changes that shift the unconditional mean get picked up by the low-frequency component of the HP filter.<sup>7</sup>

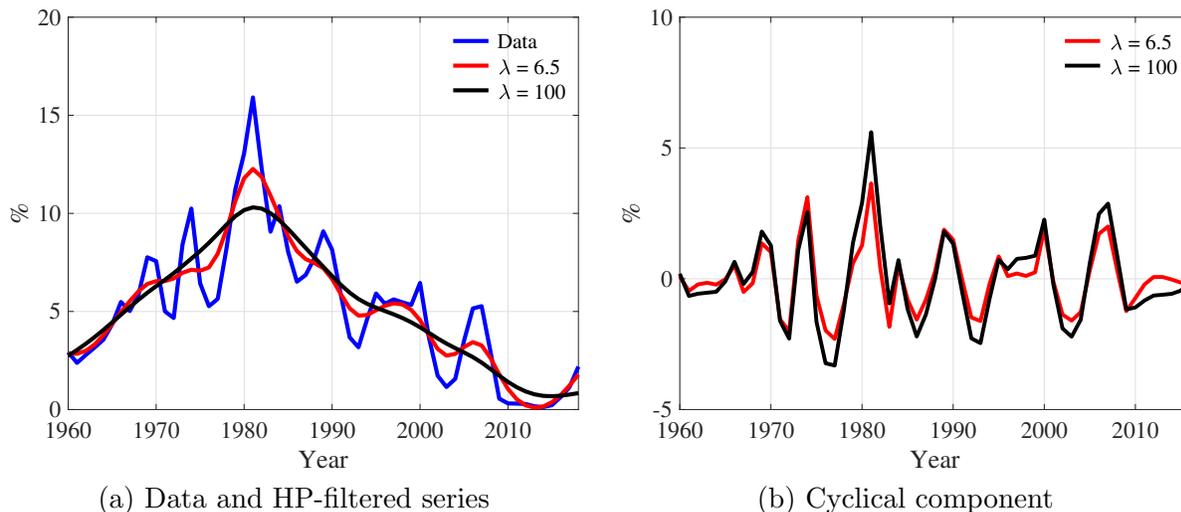
In order to discipline the choice of  $\lambda$ , we use a history of monetary policy in the USA. Specifically, we base our choice of  $\lambda$  on a particular narrative regarding the behavior of the short-term interest rate in the US since 1960. We believe it to be a widely accepted narrative among macroeconomists, particularly those within the Federal Reserve system. To describe it, it is useful to refer to [Figure 2\(a\)](#), which depicts the time path for the federal

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<sup>7</sup>See [Kulish and Pagan \(2019\)](#) for a discussion on the distinction between cycles and oscillations.

funds rate, as well as two computations of the low-frequency component extracted using two alternative values for  $\lambda$ . The relative merits of the two values for  $\lambda$  are discussed in detail below.

Figure 2: U.S. nominal interest rates



In [Figure 2\(b\)](#), we plot the two corresponding measures of the high-frequency component, obtained by subtracting from the original data the two measures of the low-frequency component in [Figure 2\(a\)](#). The key historical element to build the narrative is the notion of a “tightening cycle.” Any such cycle is defined as a series of consecutive periods exhibiting increasing values for interest rates. These are clearly visible in [Figure 2](#), more obviously so in panel (b). Particularly famous tightening cycles are the ones known as the Volcker stabilization — starting at the end of the seventies — and Greenspan’s conundrum — the one that starts in 2004.<sup>8</sup> In contrast, nobody interprets the increasing part of the low-frequency components in [Figure 2\(a\)](#) as a tightening cycle that started in 1960 and ended in 1980!

The narrative we adopt sees these cycles in the interest rate as the policy response to temporary shocks, in an attempt to stabilize the economy around certain desired values.<sup>9</sup>

<sup>8</sup>These two are the first results in a Google search for the term “tightening cycle.”

<sup>9</sup>The “tightening” cycles are followed by their corresponding “easing” period, in which the interest rate is decreasing.

This role of policy finds its strongest intellectual rationale in the New Keynesian literature, which emphasizes frictions in the setting of prices. These models have been specifically developed to study deviations from steady state values due to temporary shocks, which justifies their wide use of log-linearization methods to solve the models. In these models, price frictions generate only temporary effects on the equilibrium that vanish “in the long run.”

As emphasized above, we do not need to take a stand on how policy is executed to show the empirical performance of the two illustrations. However, in order to make these statement precise, it is convenient to consider an interest rate policy that follows a standard Taylor rule as in [Taylor \(1993\)](#). Thus, let the policy rate be given by

$$i_t = i^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y^*) + \varepsilon_t^i,$$

where  $i_t$ ,  $\pi_t$ , and  $y_t$  represent the policy interest rate, inflation, and output, respectively, and  $\varepsilon_t^i$  is a monetary policy shock. The triplet  $(i^*, \pi^*, y^*)$  is typically interpreted as the steady state values for the variables.

In the literature, the second and third terms on the right hand side of the Taylor rule are meant to capture the cycles described in [Figure 2\(b\)](#). They represent the attempt by the monetary authority to stabilize the equilibrium values of inflation and output around  $\pi^*$  and  $y^*$ . Most of the literature uses a variation of this Taylor rule, in which the triplet  $(i^*, \pi^*, y^*)$  is indeed assumed to be time invariant.<sup>10</sup> Under this interpretation, our separation of the data as done in [Figure 2](#) is incorrect, since the fluctuations in [Figure 2\(b\)](#) should be obtained by subtracting a constant from the data, not the low frequency in [Figure 2\(a\)](#), as we did.

Thus, our interpretation of policy, one that is consistent with our filter, amounts to allowing for slowly moving changes in the target for inflation, denoted by  $\pi_t^*$ . And, as the changes in the inflation target ought to be accompanied by the corresponding changes in

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<sup>10</sup>For exceptions, see [Ireland \(2007\)](#), [Cogley and Sbordone \(2008\)](#), and [Ascari and Sbordone \(2014\)](#).

the interest rate, owing to the Fisher equation, this amounts to letting the value for  $i_t^*$  also be time varying. We have in mind a policy rule better described by equation (3.1) but in which the deviations of inflation and the interest rate are made relative to values that are changing over time.<sup>11</sup> In deciding the best choice of our filter, we aim to capture the slowly moving term  $i_t^*$ , while we expect the filter to remove the second and third terms in the rule.

The distinction just made between deviations from a steady state — which imply a set of values that are constant over time — and deviations from a given trend — the low-frequency movements in Figure 2(a) — is key. We address this issue in detail in the next section, in which we estimate a small-scale New Keynesian model and allow for shocks to the targets  $i_t^*$  and  $\pi_t^*$ . We make very precise in the model this distinction between movements that capture the tightening cycles around a trend and the ones that explain the trend, and we let the data separate the two. We also defend our filter by evaluating its performance using simulated data from the estimated model. For the analysis of this section, we use our discussion above, plus the evidence in Figure 2 to justify our choice of  $\lambda$ .

The simple quantity theory model spelled out above, with all its simplifying assumptions, has no bearing on interest rate movements that correspond to the second and third terms in the Taylor rule.<sup>12</sup> This is so much so that the implied relationship between the nominal interest rate and inflation in our model — as described, for instance, in (9) — is positive and one to one. In contrast, the conventional wisdom in central banks, supported by the workings of New Keynesian models, is that increases in the nominal interest rate imply *reductions* of inflation.<sup>13</sup> The quest to understand the fluctuations depicted in Figure 2(b) is therefore abandoned at the start. On the other hand, we argue that in order to understand the remaining component, the quantity theory is almost all you need.

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<sup>11</sup>We make no attempt at explaining why those values change as they did during the period.

<sup>12</sup>In some formulations, such as Woodford (2003), the term  $(y_t - y^*)$  in the Taylor rule is the difference between the equilibrium value for output and the one that would prevail under flexible prices - the output gap. In our simple quantity theory model, the output gap is by definition zero, so even that term disappears from the rule.

<sup>13</sup>See Uribe (2020) for a masterfull integration of these seemingly contradictory statements.

This discussion sets the stage to justify our preferred value for  $\lambda$ : the smallest value that eliminates from the data the tightening cycles. In [Figure 2\(a\)](#), we plot two alternative values for the low-frequency component, corresponding to values of 6.5 and 100. The first value, 6.5, is the one that the RBC literature suggests for yearly data. Its object of study is very different from ours (note the  $R$  in RBC), so there is no reason why what fits its objective should fit ours. And as can be seen in the figure, it does not: when using  $\lambda_1 = 6.5$ , the tightening cycles are still visible. On the other hand, when using  $\lambda = 100$ , the cycles are completely removed from the policy rate.<sup>14</sup> Therefore, in what follows, we set  $\lambda = 100$ . In [Appendix B.3](#), we also show the results when using  $\lambda = 6.5$ .

By taking a stand on a particular value for  $\lambda$ , we take a stand on our definition of “long run.” There seems to be a common wisdom in central banking that to see the mechanics of the quantity theory operating in the data, one needs to look at averages over decades. Our choice of filtering implies a much tighter definition of “long run.” We make this explicit in [Figure 2\(b\)](#), in which we plot the high-frequency component of the interest rate for the two values of the parameter in the HP filter. As expected, the fluctuations are higher when using our preferred parameter of 100. But the two series are very similar. Both identify the same number of cycles, defined as the time period contained between two consecutive crossings of the horizontal axis. Those would correspond to a tightening cycle when the curve is increasing, or an easing cycle when the curve is decreasing. For both measures, the average cycle is three years, with a maximum of six years and a minimum of one year, in 1967. One interpretation of the filter we use, which we adopt, is to leave out of the data all fluctuations that last less than three years on average, which amounts to the average duration of the monetary policy cycles in the United States.

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<sup>14</sup>The behavior of the low frequency obtained for values of lambda between 90 and 110 are indistinguishable to the eye, given the size of these figures. How could we resist the seductive power of using a round number like 100?

## 3.2 Preliminaries

We now take equations (6) and (7) to the data. We selected countries that are members of the OECD and for which we have complete data since 1960. These are the countries included in [Figure 1](#), plus Mexico and Turkey, both countries are members of the OECD, but they experienced substantially higher inflation rates than the rest. This set of countries provides enough variation of experiences and with the exception of Germany, all experienced an inverted- $U$  shape for inflation, like the one depicted in [Figure 1](#). Mexico and Turkey are included as examples of substantially higher inflation rates.

We use the short-term interest rate on government debt for  $i$ , gross domestic product for output, and the CPI for prices. For the monetary aggregate, we use  $M1$ , which is the sum of currency plus checkable deposits. For the United States,  $M1$  provides a misleading measure of total assets available for transactions, owing to regulatory changes that occurred in the early '80s. [Lucas and Nicolini \(2015\)](#) discuss this issue in detail and propose a new measure called NewM1, which adds the Money Market Demand accounts created in 1982 to the standard measure of  $M1$ . Thus, for the USA only, we use NewM1 rather than  $M1$ . Doing so raises the issue of whether the simple model described above could account for a regime change due to the regulatory changes in the middle of the sample.<sup>15</sup> Thus, for the USA only, we will also show the results using the currency component of  $M1$ , which according to [Lucas and Nicolini \(2015\)](#) ought to be relatively invariant to the regime change. In Appendix B.2, we discuss the data and their sources in detail.

The period we focus on is 1960-2005, consistent with the data in [Figure 1](#). There are a few exceptions. For the countries that joined the eurozone, accurate measures of  $M1$  are not available after 1999, since currency in circulation cannot be properly measured. Thus, for the exercise implied by the money demand equation (8), we end in 1999 for those countries.

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<sup>15</sup>The model in [Lucas and Nicolini \(2015\)](#) does imply, not surprisingly, that such a regime change ought to change the relationship between the nominal interest rate and the ratio of money to output.

The presence of very low interest rates presents additional theoretical considerations that are worth discussing separately: recall our assumption in Section 2 regarding a binding cash-in-advance constraint. The validity of that assumption at very low rates is questionable, as we discuss at the end of this section. Thus, for Japan, we end the sample in 1990, before the country lowered its interest rate to almost zero. In a final subsection, we discuss the policy implications of our analysis for Japan since 1990, as well as the evidence since 2005 for several other countries, which contains several years of very low interest rates. Finally, because of data availability, we start the analysis of Turkey only in 1970.

As we mentioned above, we have no independent estimate for the real interest rate in the USA. Therefore, in the case of that country, we simply plot the inflation rate and the nominal interest rate, so as to appreciate the positive correlation.

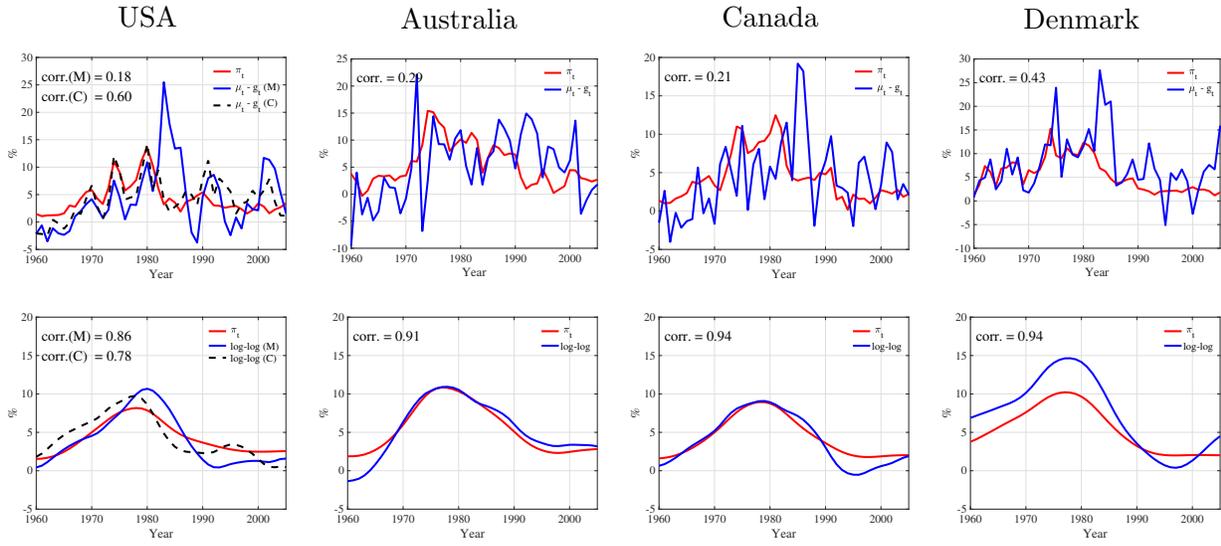
### 3.3 Results

In the top panels of [Figure 3](#) to [Figure 6](#), we show the data corresponding to the money demand equation (8). We first plot the raw data for the inflation rate and for the growth rate of nominal money over real output. In the plots of raw data for Illustration 1, we do not make the adjustment for changes in the nominal interest rate, as (8) implies. The reason is that this adjustment makes the theoretical prediction for inflation way more volatile than in the data, since the high-frequency movements in interest rates are very volatile and the value for the elasticity implied by the Baumol-Tobin formulation is quite high. This is consistent with the old empirical literature on money demand, which argued that the estimated “short-run” interest rate elasticity was much smaller than the “long-run” elasticity.<sup>16</sup> We then plot the low-frequency component for the theoretical inflation, as predicted by equation (8), together with the low-frequency component of inflation. The bottom panels of [Figure 3](#) to [Figure 6](#) show the data corresponding to the Fisher equation (9).

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<sup>16</sup>See [Lucas \(1988\)](#) for a discussion.

## Illustration 1



## Illustration 2

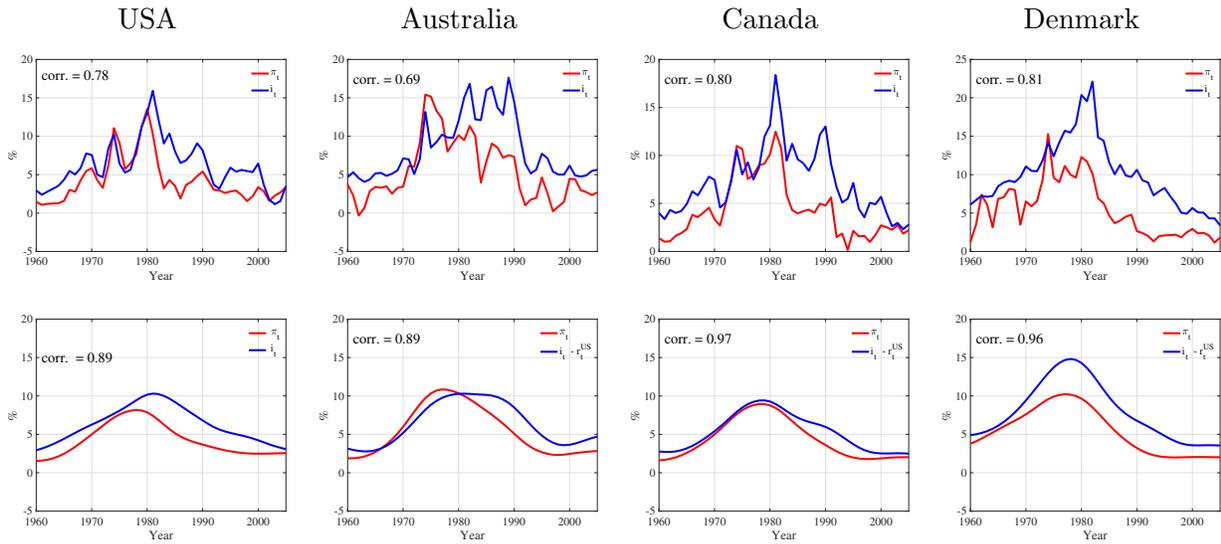
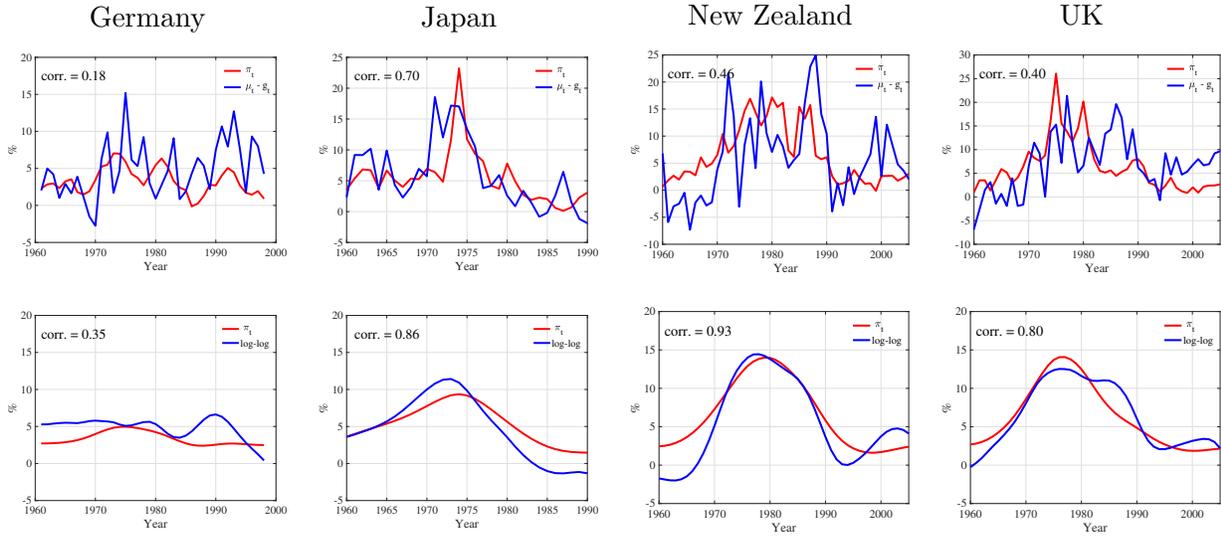


Figure 3: Countries in Group 1 (a)

## Illustration 1



## Illustration 2

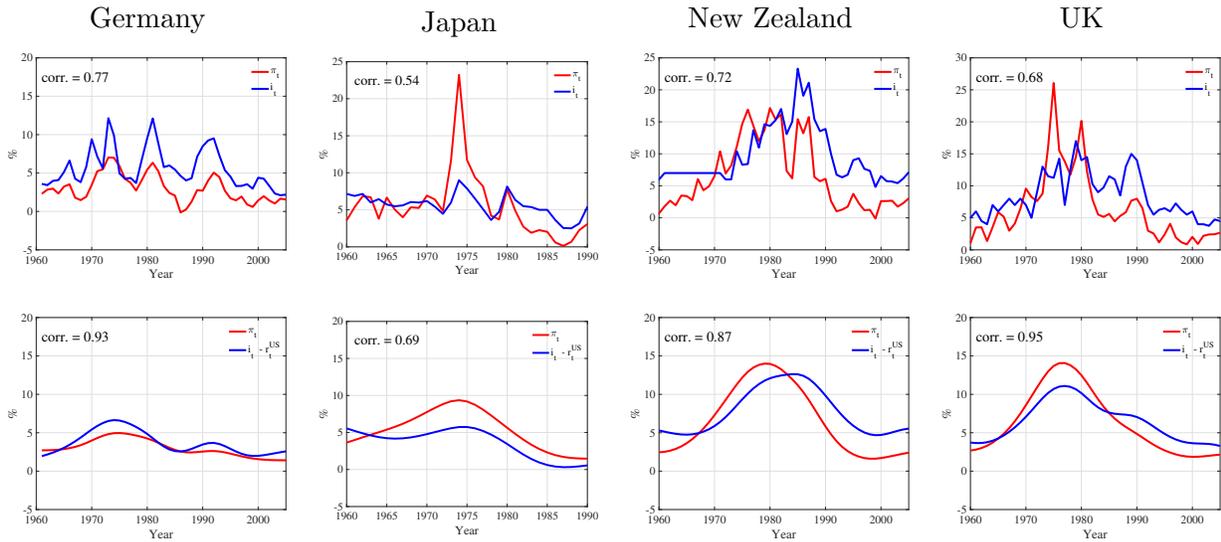
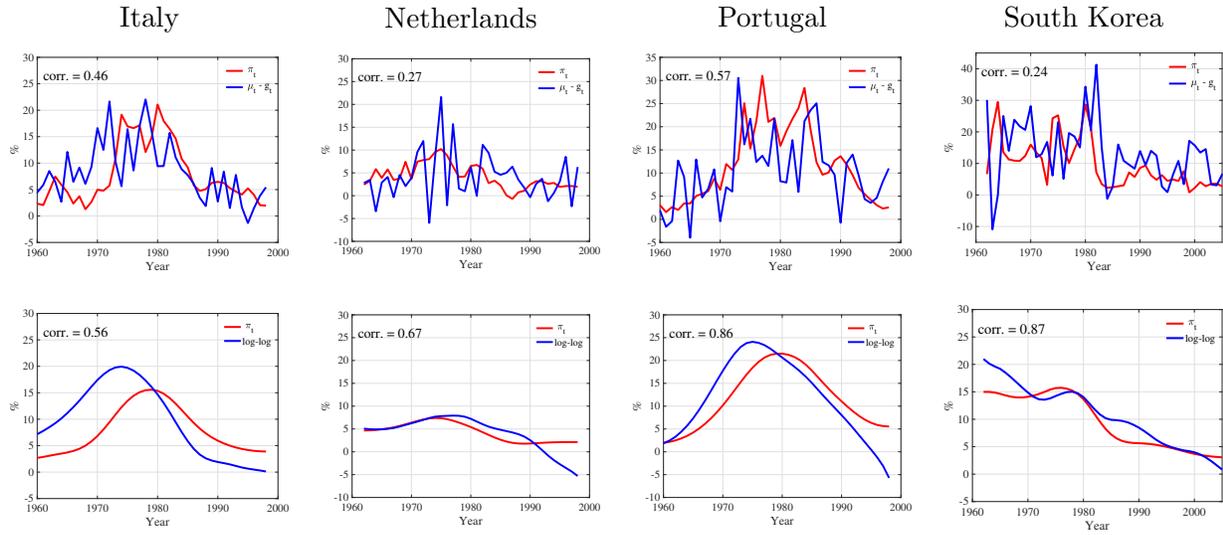


Figure 4: Countries in Group 1 (b)

## Illustration 1



## Illustration 2

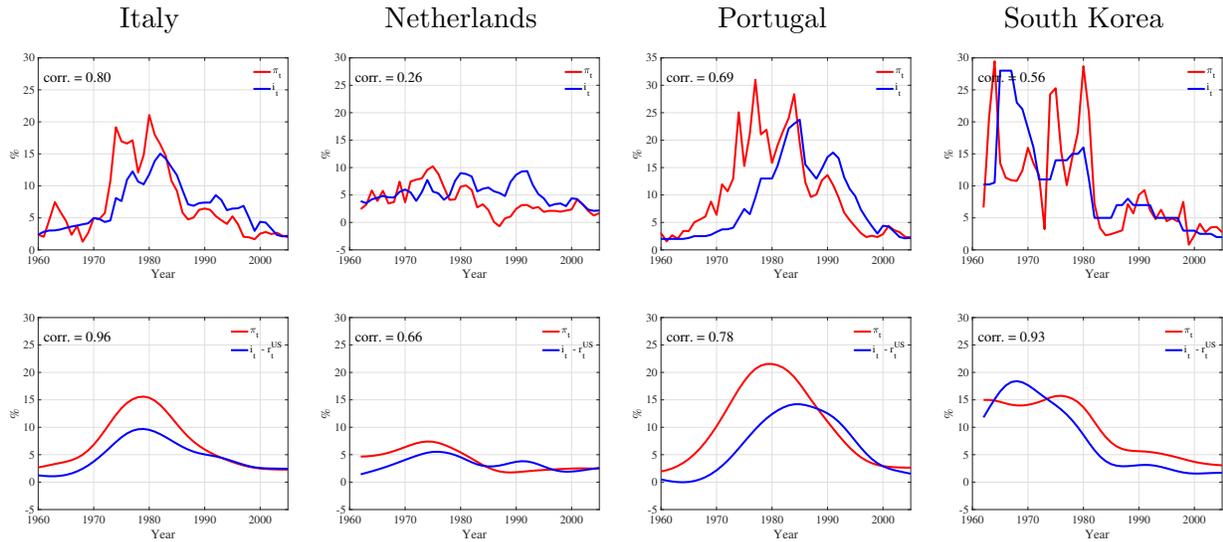
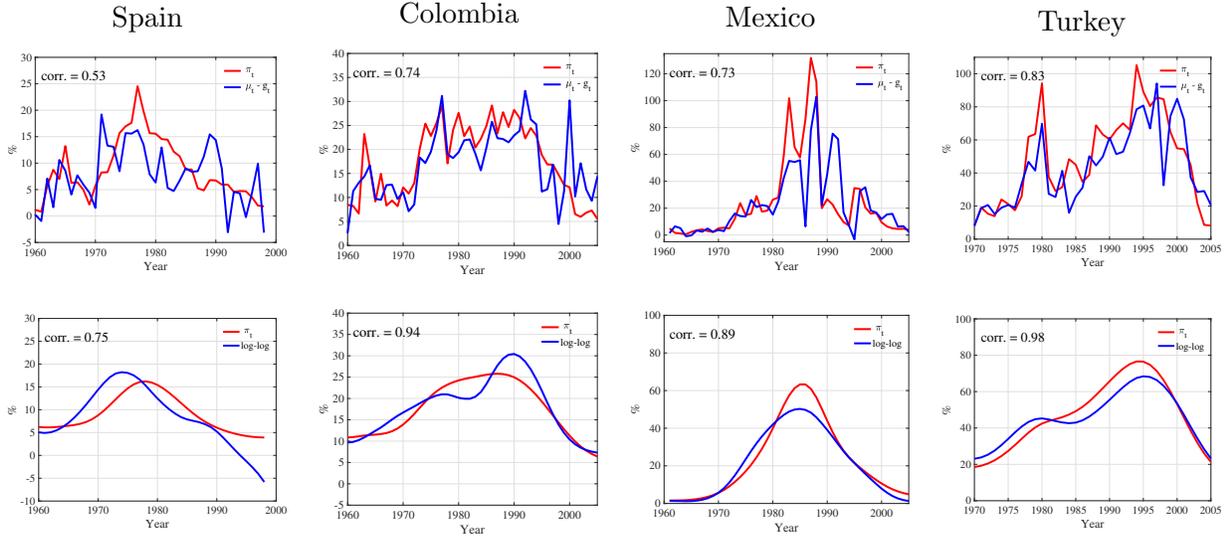


Figure 5: Countries in Group 2 (a)

## Illustration 1



## Illustration 2

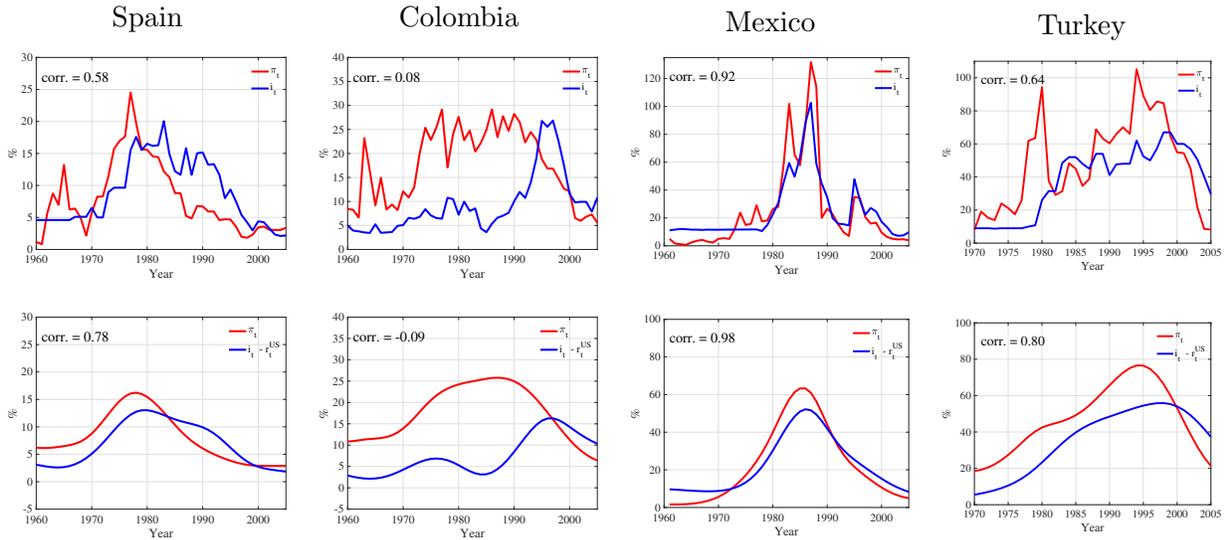


Figure 6: Countries in Group 2 (b)

The first column of [Figure 3](#) presents the results for the United States. As mentioned above, in the case of the United States, we use both Cash and NewM1 for Illustration 1. The yearly data do not make apparent the relation between money growth and inflation. However, once the low-frequency movements are isolated and the effect of changes in the low-frequency component of the interest rate is taken into account, as equation (8) implies, the match between the theory and the data is quite notable, in spite of the regime change.

This reasonable match with the theory offers an alternative interpretation besides the one proposed by [Sargent and Surico \(2011\)](#) for the experience in the United States. They replicate the analysis in [Lucas \(1980\)](#), using the same filter he does. They extend the sample in [Lucas \(1980\)](#) to include data from 1980 till 2005. They use a monetary aggregate that is very close to M1 and show — as we do in Appendix B.1 — that the data do not align well with the theory.<sup>17</sup> They propose a model with regime changes in the monetary policy rule to account for that failure. In using either Cash or NewM1, we show that no puzzle arises. As we show next, this phenomenon is specific to the United States. In the analysis for all the other countries that follows, we use M1 as the measure of money.

We separate the countries in two groups. The first group, shown in [Figure 3](#) and [Figure 4](#), includes the countries for which we do not find any particular behavior that makes our assumptions of the model especially suspicious. This group contains the single clear failure we can identify: Germany. It is also the only country in which the low-frequency component of inflation barely moves, which transforms it into an outlier in light of the evidence of [Figure 1](#). The second group, shown in [Figure 5](#) and [Figure 6](#), includes a set of countries for which the nominal interest rate is lower than the inflation rate for several years in the first two decades of the period analyzed. To us, this behavior suggests government intervention in the credit market, relatively common in the '60s and '70s, so that the observed interest rate may not be a market-determined price, as our model implies. If this is the case, the observed interest rate may not be the true opportunity cost of money, as

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<sup>17</sup>They also ignored — as [Lucas \(1980\)](#) did — the effect of the movements in the interest rate, which are important. But the main difference is the monetary aggregate they use.

seen from the point of view of the agents in the model. This would clearly impose a bias in our two theoretical predictions. This group also includes cases with higher inflation rates.

Each picture is worth a thousand words. As we provide plenty of pictures, words will be kept to a minimum. We read the sequence of plots as an affirmation of the success of our simple theory, particularly when compared with other theories in social sciences. To support the visual inspection, in all cases, we compute the simple correlation between the series and report it in the corresponding plot. We mostly let the readers evaluate the pictures themselves and emphasize just a few features of the plots.

Firstly, while the correlation between the data and the theory is very high, there are sizable differences in some cases, of up to a few percentage points; these do matter for policy. A 2% or larger difference between observed inflation and the theoretical counterpart, as observed in many cases, is an important difference that can and should be further studied on a case-by-case basis. It is most likely that in order to understand those differences, country-specific features ought to be brought to the policy debate table. We purposely ignored those details in our exploration, since our objective is to evaluate how far one can go with our simple theory.

Secondly, for the group of countries in [Figure 5](#) and [Figure 6](#), for which we guess that financial repression was prevalent in the first decades of the sample, the evidence is worse, particularly when evaluating the second implication (the Fisher equation). A poster child of this issue is Colombia, where financial repression was the norm till the reforms of the early '90s.

Finally, a clear failure case is Germany. We provide an interpretation for that case in the next section, in which we estimate a standard small-scale model. But before doing so, we address one additional issue, which has important policy implications: the liquidity trap.

### 3.4 The near-zero nominal interest rates periods

The evidence analyzed so far covers a period in which, with the exception only of Japan since the mid-'90s, nominal interest rates remained substantially above zero.<sup>18</sup> This is important from the point of view of the theory, since a positive interest rate is a necessary condition for the cash-in-advance constraint to be binding in equilibrium. When this is not the case, real money demand is not uniquely determined. In our simple representative agent economy, the results are stark: as long as the interest rate is positive, the constraint is binding and the equilibrium of the model is uniquely pinned down. However, sensitive modifications that allow for some minor heterogeneity, like agent-specific borrowing limits or heterogeneous access to credit markets that imply heterogeneous returns on nominal assets, would affect these implications of the simple model when the nominal interest rate is positive but very close to zero.<sup>19</sup>

To further clarify this discussion, consider the solution of our simple model, given by (6). Notice that the solution for real money balances as a fraction of output goes to infinity when the nominal interest rate goes to zero. How can that be a solution for agents that have finite wealth? The answer is that in equilibrium, the private sector's borrowing from the government is also going to infinity, keeping the wealth of the private sector bounded. While this is mathematically correct for any positive interest rate, it is of little, if any, applied interest.<sup>20</sup>

To illustrate the difficulties in using real money demand theory at very low interest rates, we now compute the theoretical inflation. Besides the solution in (6), we use an alternative functional form proposed by [Selden \(1956\)](#) and [Latané \(1960\)](#) and explored in

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<sup>18</sup>This is the reason we ended the analysis for Japan in 1990 in the analysis of previous section.

<sup>19</sup>For an analysis with heterogeneous borrowing constraints, see [Benati et al. \(2020\)](#). An alternative model that delivers similar results is analyzed in [Alvarez and Lippi \(2009\)](#).

<sup>20</sup>The question of the behavior of money demand at very low rates has been the subject of a debate — see [Lucas \(2000\)](#) and [Ireland \(2009\)](#). To settle the debate has proven difficult, given the lack of evidence during the 20th century. The recent evidence could be used to shed light into the issue, but we leave that for future research.

detail in [Benati et al. \(2020\)](#). The specific functional form is given by

$$\frac{M_t}{P_t x_t} = \frac{A}{1 + b i_t}. \quad (10)$$

Notice that when  $i_t = 0$ , real money demand - as a fraction of output - is finite. Thus, it departs from the Baumol-Tobin specification at very low interest rates. On the other hand, the parameters  $A, b$  can be chosen such that (6) and (10) are very close to each other for interest rates above 2% and all the way up to 30%, which is a range that includes most of the experiences analyzed in this section — Mexico and Turkey in Group 2 are the two exceptions.<sup>21</sup>

In [Figure 7](#), we extend the analysis presented in [Figure 3](#) and [Figure 4](#) to several countries in Group 1 that maintained the interest rate very close to zero for several periods. As mentioned above, in computing theoretical inflation, we present both the log-log case implied by our theory and the Selden-Latané case, in which the parameters have been chosen to match as much as possible the solution in (6).<sup>22</sup>

As the figure shows, when using the log-log specification, the implications of money demand theory become much worse in the periods of very low interest rates. The Selden-Latané alternative specification does substantially better but still fails to perform as it did in the previous years. One clearly could do better by trying to estimate the value for real money demand at zero interest rates using country-specific data. However, the brief theoretical analysis just discussed suggests that inference about the behavior of real money demand at very low rates using evidence of periods with relatively higher rates, when the cash-in-advance constraint can be safely assumed to be uniformly binding, could be misleading. This evidence suggests that monetary aggregates may be particularly uninformative at very low rates.

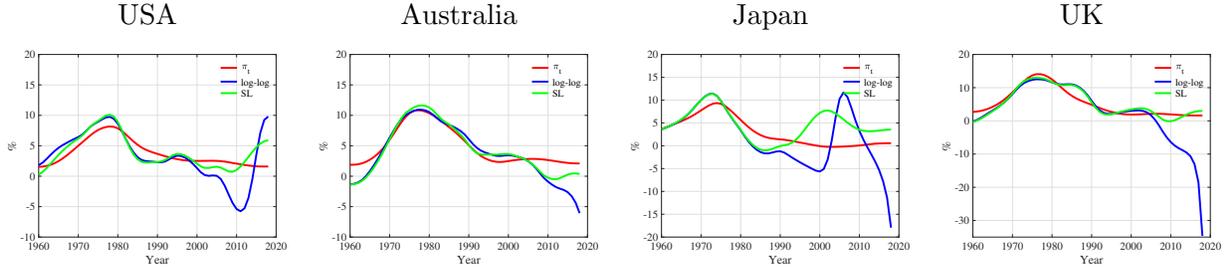
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<sup>21</sup>A detailed comparison of the two formulations and a modification of the theory that can generate a money demand equation that resembles (10) can be found in [Benati et al. \(2020\)](#).

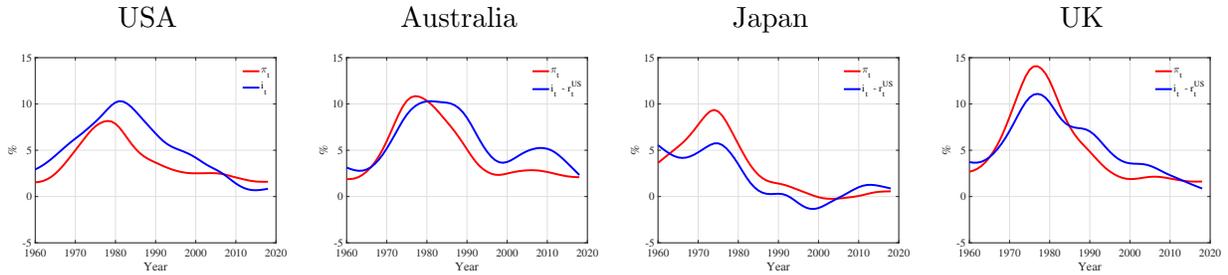
<sup>22</sup>Specifically, we choose parameter  $b = 0.14$  for the Selden-Latané specification. The value for  $A$  is irrelevant for computing growth rates, as we do in this section.

Figure 7: Illustrations for countries with periods of low interest rates (1960–2018)

### Illustration 1



### Illustration 2



Note, on the other hand, that the evidence regarding the second illustration is as good for the low interest rate period as it is for the rest of the sample.

Two policy implications follow. First, the effect of expansions of the the central bank’s balance sheet on the real side of the economy — the so called “unconventional policies” — when interest rates are very low are hard to predict, since it is hard to estimate the demand for those assets. In particular, the effect of “helicopter drop”-type policies in the liquidity trap is very hard to evaluate given the existing evidence. Second, in exiting a period of zero policy rates, increasing the low-frequency component of the the policy rate in a way that resembles a positive shock to the target — as the Federal Reserve did between 2015 and 2018 — can act as an effective tool to fight persistently low inflation.

## 4 Analysis of simulated data

Our choice of  $\lambda = 100$  for the HP filter parameter was justified by our desire to remove the tightening cycles from the observed series of the short interest rate. The low-frequency component so extracted allows for the two quantity theory predictions to emerge, as the analysis of the previous section shows — except in the case of Germany.

In this section, we estimate a small-scale New Keynesian model using data for the USA. We depart from the literature in that we allow for policy regime changes that could potentially account for the low-frequency movements in the data. We then let the data inform us about to what degree the movements in inflation are accounted for by these shocks.

We also use the estimated model to simulate data and filter them the same way we filtered the data in the previous section. We then compare the results of the same model but with the new policy shock we introduce shut down, and we again filter the simulated data, as we did with the true data. The comparison makes clear that those policy regime shocks are essential in explaining what appears as low-frequency movements in inflation, interest rates and money growth.

We then simulate the estimated model varying the degree of price frictions and again using the same filter we applied to the data. We show that the price frictions barely change the implications regarding the low-frequency behavior of inflation, interest rate, and money growth, which in all cases are explained by the policy regime shocks. We interpret these exercises as evidence that the strength of the price frictions in the model does not change the medium-run implications of the simulated data. We see all this evidence as a validation of our filtering choice, since it is the case that when applying the same filter to simulated data, we always remove the tightening cycles.

Overall, the evidence provided in this section is fully consistent with the notion that the data in [Figure 2\(b\)](#) are what the New Keynesian literature is all about. The conventional wisdom of central banks that, at that frequency, increases in the policy rate bring about

declines in inflation may well be consistent with the behavior of the data, once the low-frequency component has been extracted using our preferred filter. However, to understand the low-frequency component of inflation, the one-to-one relationship between the short-term nominal interest rate and inflation described by our version of the Fisher equation (7) suffices, and the degree of price frictions is largely irrelevant for understanding that behavior. Our analysis is consistent and complementary to the one in [Uribe \(2020\)](#), who masterfully integrates the two effects within a single theoretical model, considering both temporary and permanent shocks to the policy rate.

We now briefly describe the model, which follows [Ireland \(2004\)](#) very closely. We then discuss the estimation strategy and the results before analyzing the simulated data. A full description of the estimation and a detailed analysis of the simulation exercises is in Appendix C.

## 4.1 The Model

As mentioned above, for the analysis that follows, we use an extension of the New Keynesian model of [Ireland \(2004\)](#).<sup>23</sup> Our main departure is allowing for shocks to the inflation target.

These equations are the familiar Euler equation (which is the Fisher equation of our simple model of Section 2), the New Keynesian Phillips curve, and the Taylor rule shown below.

$$x_t = (z - \ln \beta) - (i_t - \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t x_{t+1} + (1 - \omega)(1 - \rho_a)a_t \quad (11)$$

$$\pi_t = (1 - \beta)\pi^s + \beta \mathbb{E}_t \pi_{t+1} + \psi x_t - e_t \quad (12)$$

$$i_t = i_t^* + \rho_i (i_{t-1} - i_{t-1}^*) + \phi_\pi (\pi_t - \pi_t^*) + \phi_x x_t + \varepsilon_{i,t}. \quad (13)$$

In the equations above,  $x_t$  is the output gap,  $\pi_t$  is the log of the gross rate of inflation, and  $i_t$  is the log of the gross nominal interest rate.

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<sup>23</sup>Ireland's model is also used by [Sargent and Surico \(2011\)](#). Details of the non-linear model can be found in [Ireland \(2004\)](#).

We differ from [Ireland \(2004\)](#) in that we allow for the inflation target,  $\pi_t^*$ , to depend on time, as shown in equation (13). This therefore implies that  $i_t^*$  must also be time dependent, as we make explicit below. In particular, we assume that the inflation target,  $\pi_t^*$ , and the implied target nominal interest rate,  $i_t^*$ , follow the processes below:

$$\pi_t^* = (1 - \rho_\pi)\pi^s + \rho_\pi\pi_{t-1}^* + \mathbb{I}^s\varepsilon_{\pi,t} \quad (14)$$

$$i_t^* = z - \ln\beta + \pi_t^*. \quad (15)$$

According to equation (15), the implied target for the nominal interest rate,  $i_t^*$ , is determined by the steady state real interest rate,  $z - \ln\beta$ , and the inflation target,  $\pi_t^*$ . The variable  $z$  is the steady state growth rate of labor augmenting productivity,  $Z_t$ , which follows in logs a unit root with drift  $z$  and  $\beta$  is the household's discount factor. The inflation target,  $\pi_t^*$ , follows a regime dependent AR(1) process where  $\mathbb{I}^s$  is an indicator variable that is turned on at  $T^{\text{on}}$  and then turned off at  $T^{\text{off}}$ ; that is,

$$\mathbb{I}^s = \begin{cases} 1 & \text{for } t \in [T^{\text{on}}, T^{\text{off}}) \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

As we explain below, the dates of regime change,  $T^{\text{on}}$  and  $T^{\text{off}}$ , are estimated alongside the structural parameters following the method outlined by [Kulish and Pagan \(2017\)](#). In estimation, we set  $\mathbb{I}^s = 0$  at the start of the sample and  $\pi^s = 0.005$ , equivalent to an inflation target of 2% in annualized terms. Before  $T^{\text{on}}$ , shocks to the inflation target are turned off and the model is a standard New Keynesian model with a constant inflation target.

At  $T^{\text{on}}$ , the inflation target changes in two ways. First,  $\mathbb{I}^s = 1$  and  $\varepsilon_{\pi,t}$  now affect the inflation target,  $\pi_t^*$ . Second, we allow in estimation, but do not require, the long-run inflation target — that is,  $\pi^s$  — to change from  $\pi^s = 0.005$  to  $\pi^s = 0.005 + \Delta_\pi$ , and  $\Delta_\pi$  is estimated. Thus, at time  $T^{\text{on}}$ , the inflation target is subject to a permanent shock,  $\Delta_\pi$ , and

to persistent but temporary shocks,  $\varepsilon_{\pi,t}$ , until  $T^{\text{off}}$ . Notice that because of the persistence of the inflation target process,  $\rho_{\pi}$  in equation (14), the long-run inflation target is reached gradually. Finally, at time  $T^{\text{off}}$  policy reverts to its original regime; that is,  $\mathbb{I}^s = 0$  and  $\pi^s = 0.005$ .

This choice allows for a potentially slow-moving component that pushes up inflation during the first two decades, capturing the rise in inflation post-Bretton Woods, with a slow reversion to the original 2% per year inflation rate observed since the '80s. In estimating the model, we let the data choose the values for the key five parameters,  $\{\Delta_{\pi}, \rho_{\pi}, T^{\text{on}}, T^{\text{off}}, \sigma_{\varepsilon_{\pi}}\}$ , where  $\sigma_{\varepsilon_{\pi}}$  is the standard deviation of the shock  $\varepsilon_{\pi t}$  in (15).

The model is estimated on five observable series: real GDP per capita growth; the federal funds rate; core inflation as measured by the CPI, excluding food and energy; the Michigan survey measure of inflation expectations; and money growth. For the United States, as discussed above, we use NewM1, the monetary aggregate proposed in [Lucas and Nicolini \(2015\)](#).

The equations linking the observable variables; output growth,  $g_t$ ; and money growth,  $\mu_t$ , to the endogenous variables are given by

$$g_t = \hat{y}_t - \hat{y}_{t-1} + z_t \quad (17)$$

$$x_t = \hat{y}_t - \omega a_t \quad (18)$$

$$\mu_t = m_t - m_{t-1} + \pi_t + g_t \quad (19)$$

$$m_t = \bar{m} + \rho_m m_{t-1} - (1 - \rho_m) \eta \left( \frac{1 + i^s}{i^s} \right) i_t + \xi_t \quad (20)$$

$$\mathbb{E}_t^{\text{obs}} \pi_{t+1} = \frac{1}{4} \left( \sum_{j=1}^4 \mathbb{E}_t \pi_{t+j} \right) + v_t, \quad (21)$$

where  $\hat{y}_t$  is the percentage deviation of stochastically detrended output,  $Y_t/Z_t$ , from its steady state;  $\mu_t$  is money growth; and  $m_t = \ln(M_t/P_t Y_t)$  is the log of real money balances to output. The constant  $\bar{m}$  pins down real money balances to output in steady state. We

use the Surveys of Consumers from the University of Michigan as the measure of inflation expectations,  $\mathbb{E}_t^{\text{obs}}\pi_{t+1}$ , and allow for measurement error,  $v_t$ .

The economy is subject to the following non-policy shocks: a preference shock,  $a_t$ ; a markup shock,  $e_t$ ; a money demand shock,  $\xi_t$ ; and a technology shock,  $z_t$ . These are governed by the equations below:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \quad (22)$$

$$e_t = \rho_e e_{t-1} + \varepsilon_{e,t} \quad (23)$$

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{m,t} \quad (24)$$

$$z_t = z + \varepsilon_{z,t}. \quad (25)$$

In steady state,  $\pi_t = \pi_t^* = \pi^s$ ,  $i_t = i^s$ ,  $g_t = z$ , and  $i^s = \pi^s + z - \ln \beta$ ; all other variables (including the output gap) settle on zero. The reason nominal variables are left in levels, as opposed to percentage deviations from steady state, is that in estimation we allow for changes in the steady states of these variables.

We estimated the model treating the regime changes as unanticipated. This seems to us a reasonable choice, particularly for the shock  $T^{\text{on}}$ : it is conceivable that the breakdown of the Bretton Woods system and the inflation that ensued took most by surprise. It is less plausible that the disinflation shock,  $T^{\text{off}}$ , was a complete surprise. However, we do not believe this to be very critical, since we allow the change in target to be very slowly moving, by allowing for the autoregressive component in (14). Note that a very high value for  $\rho_\pi$  implies that the economy very slowly approaches the new long-run target. So although  $\Delta_\pi$  is unanticipated, the transition path that it triggers for  $\pi_t^*$  towards its new steady state is anticipated. The estimation does indeed deliver a very high value for  $\rho_\pi$ , so we do not believe that allowing for additional anticipation of the policy shift, that is of  $\Delta_\pi$ , would much change the results.

## 4.2 Estimation

We calibrate the parameters that determine the steady state,

$$\beta = 0.9975, \quad z = 0.0044, \quad \pi^s = 0.005, \quad \bar{m} = 1,$$

before estimation. Jointly, they imply a mean growth rate of real GDP per capita of 1.76% in annual terms, a mean nominal interest rate of 4.75%, a steady state inflation rate of 2% in annual terms, and a ratio of money to output of about 25% in annual terms. We set the Calvo price parameter to be 0.6, consistent with the findings of [Fitzgerald et al. \(2020\)](#). This value implies a slope of the NK Phillips curve  $\psi = 0.3$ .

To guard against the possibility that our proposed policy regime change captures the higher macroeconomic volatility before the Great Moderation, we use a parsimonious specification and introduce the parameter  $\kappa$ , which multiplies the standard deviations of all structural shocks, except that of money demand, before  $T_\kappa$ . That is, we assume that the standard deviations of all structural shocks, except for money demand, shift in the same proportions. In other words, the estimation is not forced to rely on shocks to the inflation target to account for the increased volatility in the earlier part of the sample. At the mode, we estimate a value for  $\kappa$  of 2, implying that the volatility of structural shocks halved after  $T_\kappa$ , which at the mode is estimated precisely around 1985q1.

We use priors that are, for the most part, standard in the literature. Below, we discuss the key ones, that characterize the policy regime change,  $\rho_\pi$  and  $\Delta_\pi$ . In the case of  $\Delta_\pi$ , which captures the long-run change of the inflation target, we use a wide uniform prior that ranges from -8% to 24% in annual terms. At the mode,  $\Delta_\pi$  is estimated at roughly 0.01, which in annual terms amounts to a jump in the target of about 4% per year. The estimated value for  $\rho_\pi = 0.98$ , implying a very slow adjustment of the target to its newer, higher value.

For the date breaks,  $T^{\text{on}}$  and  $T^{\text{off}}$ , we use uniform priors but restrict  $T^{\text{off}}$  to lie between

1979Q4 and 1983Q4, the quarters corresponding to the Volcker disinflation. In turn,  $T^{\text{on}}$  is restricted simply to take place before 1979Q4. Importantly, while the estimation allows for changes in the policy regime, these changes are not imposed. The estimation is free to choose  $\Delta_\pi = 0$  and  $\sigma_\pi = 0$ , if it so desires.

The data strongly favor a specification in which the increase in inflation in the '70s is in large part interpreted as permanent, with  $\pi^s$  smoothly increasing from 2% to roughly 6% at the mode and with negligible mass for  $\Delta_\pi < 0$ . Most of the remaining variation is explained by temporary shocks to the inflation target. The date breaks are precisely estimated with the inflationary regime beginning in the late '60s and ending in the early '80s. The estimates of the policy rule parameters are in line with those found in the literature. In the interest of space, the full set of estimates of the structural parameters and date breaks appears in Appendix C.

The main difference across regimes is that once  $\mathbb{I}^s = 1$ , shocks to the inflation target can have an impact on endogenous variables. To gauge how the contribution of structural shocks changes with the policy regime, we compare variance decompositions for Regime 1, for which  $\mathbb{I}^s = 0$ , with those for Regime 2, for which  $\mathbb{I}^s = 1$ .<sup>24</sup> Table 1 shows that shocks to the inflation target,  $\varepsilon_\pi$ , account for the bulk of fluctuations in inflation and the nominal interest rate in Regime 2. Interestingly, the variance decomposition for real GDP growth,  $g_t$ , is essentially the same for the two regimes, with productivity shocks accounting for around three-fourths of its variance across regimes and the target shocks accounting for just 0.5% of its volatility. Thus, not accounting for these monetary policy regime changes will wrongly assign these fluctuations to the other shocks and will therefore give rise to biases in the estimates.

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<sup>24</sup>This decomposition of the unconditional variance is due to the structural shocks alone, capturing what the unconditional variance would be if the regime were to prevail indefinitely. It does not account for the fraction of the variance in the data that results from permanent changes of the inflation target from  $\Delta\pi$ .

Table 1: Variance decomposition

	Regime 1: $\mathbb{I}^s = 0$					Regime 2: $\mathbb{I}^s = 1$				
	$i_t$	$\pi_t$	$g_t$	$\mu_t$	$x_t$	$i_t$	$\pi_t$	$g_t$	$\mu_t$	$x_t$
<i>Shocks</i>										
$\varepsilon_i$	2.2	6.2	4.0	3.7	22.6	0.4	0.8	4.0	2.5	21.2
$\varepsilon_a$	94.5	15.6	23.2	19.4	70.8	18.1	1.9	23.1	11.1	66.5
$\varepsilon_e$	3.3	78.2	0.4	1.2	6.6	0.6	9.7	0.4	1.6	6.2
$\varepsilon_z$	0.0	0.0	72.4	20.7	0.0	0.0	0.0	72.0	18.6	0.0
$\varepsilon_\pi$	0.0	0.0	0.0	0.0	0.0	80.8	87.6	0.5	16.6	6.0
$\varepsilon_\xi$	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0	49.5	0.0

### 4.3 Simulation analysis

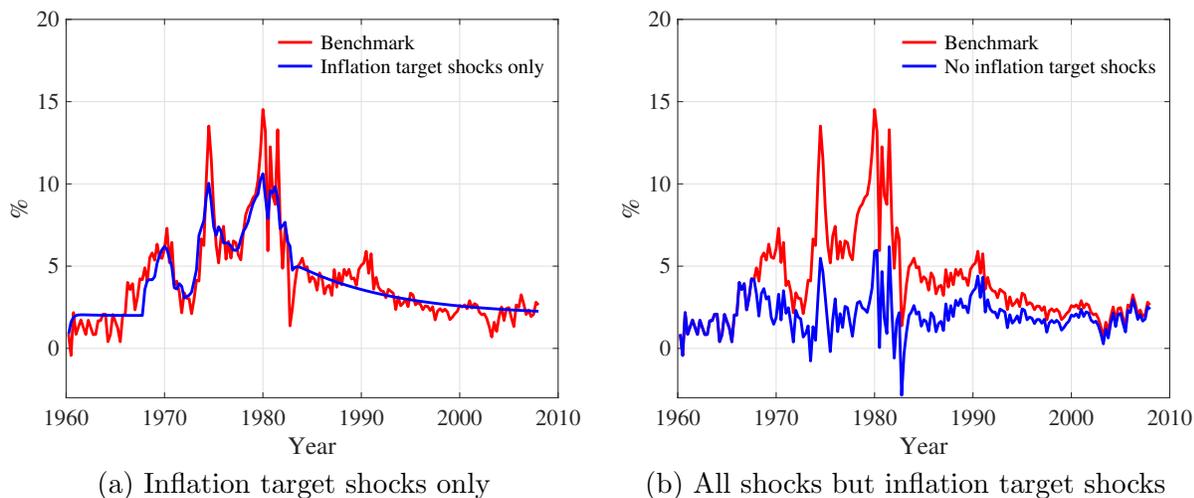
We now use the estimated model to run several simulations that show the importance of the estimated shocks to the inflation target.<sup>25</sup> In addition, we validate the filter we applied to the data by treating the simulated data with the same filter.

In our first exercise, we simulate the estimated model, setting the shocks to the target equal to their estimated values while setting the value for all other shocks to zero. We repeat the exercise but set the target shocks to zero and set all other shocks to their estimated values. In [Figure 8](#), we show the values for inflation in the United States during the period, as well as the two alternative simulations. In panel (a), we show the first case, in which only the target shocks are set to their estimated values and all other shocks are set to zero. In panel (b) we show the second case, in which the trend shocks are set to zero and all other shocks to their estimated values. The figure makes clear that the shocks to the target alone do a much better job at tracking the evolution of the low-frequency component of inflation in the data.

As additional evidence, we simulate the model 40 times, setting the shocks to the target equal to the mode of the estimated posterior distribution and drawing all other

<sup>25</sup>In the text we focus the analysis on the evolution of inflation. In Appendix C, we show the results for the nominal interest rates and for money growth.

Figure 8: Role of inflation target shocks



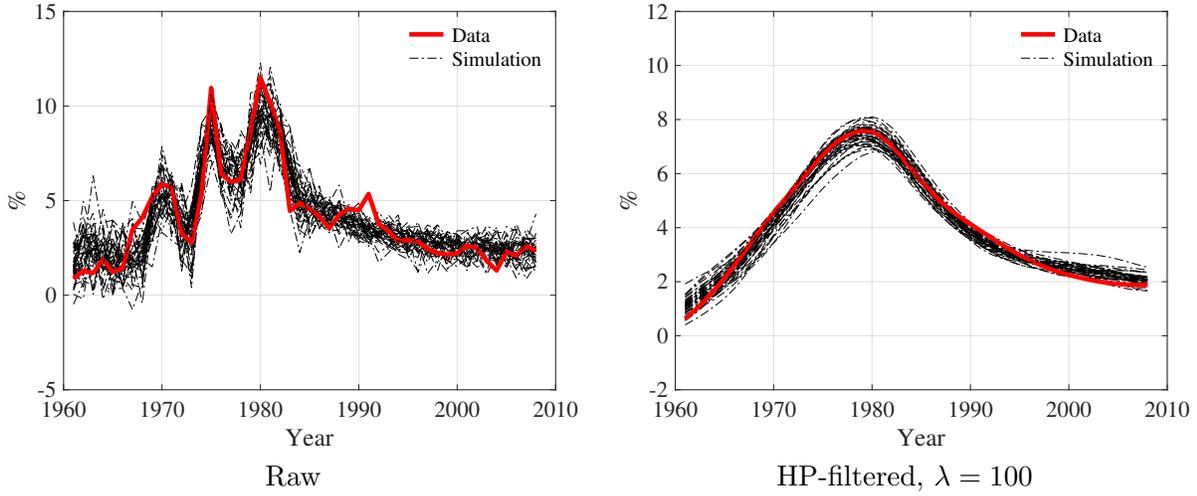
shocks randomly from their posteriors. In the left panel of [Figure 9\(a\)](#), we plot the 40 simulations, together with the inflation data. The right panel of the same figure shows the filtered version of the series in the left panel. We next re-estimate the model but calibrated all the shocks to the target to be zero, which is the standard procedure in the New Keynesian literature. We then simulate the model 40 times, drawing all shocks from their posterior distributions. The left panel of [Figure A7\(b\)](#) shows the 40 simulations plus the data for the period, and the right panel shows the filtered version of the series in the left panel. As the figures make clear, the model without the shocks to the target cannot reproduce the low-frequency movements detected in the data.

The preceding exercises all point in the same direction: the low-frequency movements in the data that we discussed in Section 3 are well captured by the shocks to the target, while all the other shocks typically used in the literature have a very hard time accounting for them, even if we do not allow for the shocks to the target in the estimation.

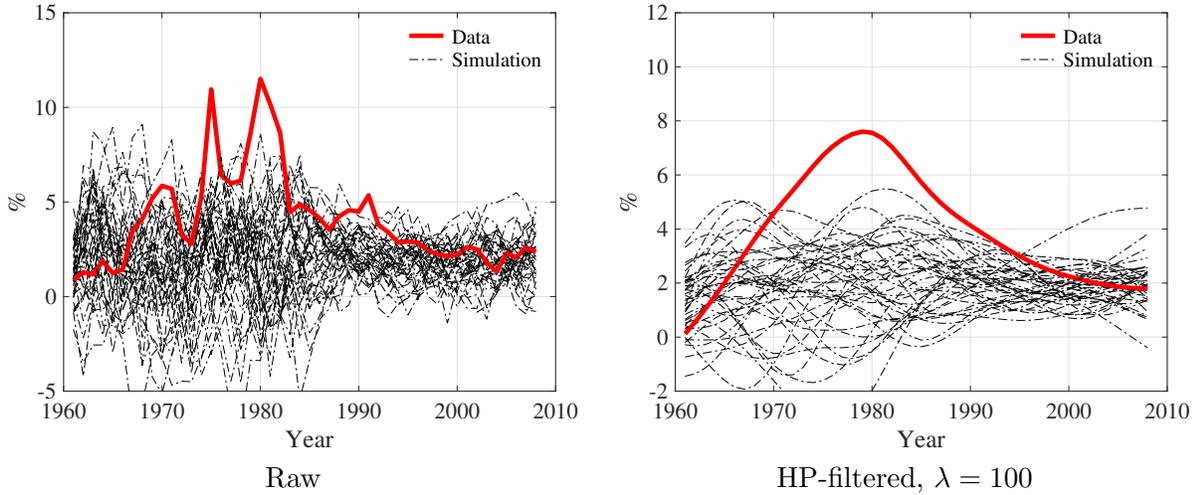
**Sensitivity to price-setting frictions** In order to explore the role of the strength of the price friction mechanism, we simulated the model by setting all shocks equal to their estimated values, but varied the value for the Calvo parameter. We used the values 0.9, 0.6, and 0.1. Recall that we calibrated the Calvo parameter to be 0.6 in the estimation, so

Figure 9: Model fitness of inflation rates

(a) Estimation with inflation target shocks



(b) Estimation without inflation target shocks

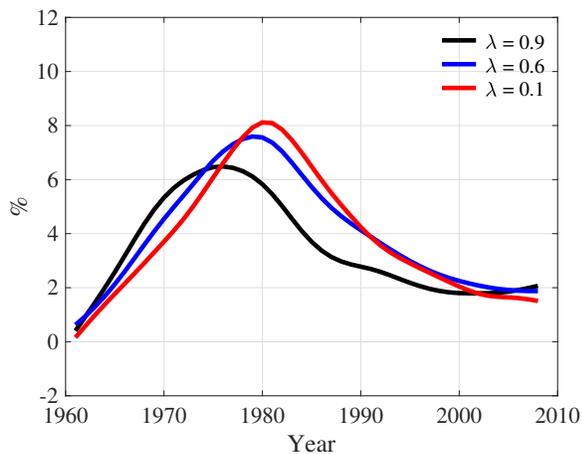


that case corresponds to the true data. Then, we filtered the simulated data. We present the results for inflation and the interest rate in [Figure 10\(a\)](#).

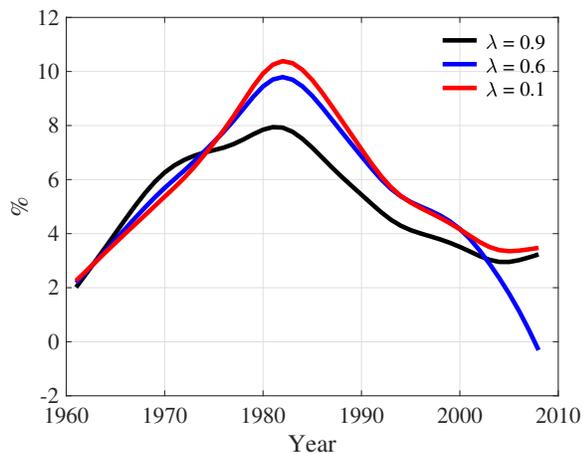
As the figure shows, while the specific value for that parameter does change both the maximum inflation attained and the date at which it occurs, the differences are relatively small, even though the variations on the Calvo parameter are very large. In [Figure 10\(b\)](#), we report the results of the same exercise, with the values of the estimated shocks to the target equal to zero. As in [Figure 10\(a\)](#), the differences across regimes with different price

Figure 10: Sensitivity to price stickness

(a) Simulation with inflation target shocks

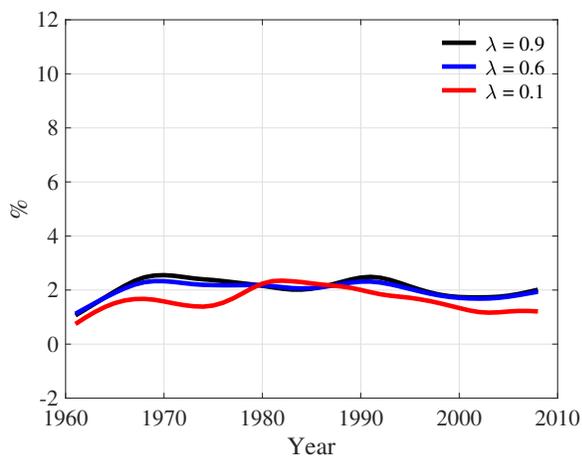


Inflation rates, HP filtered  $\lambda = 100$

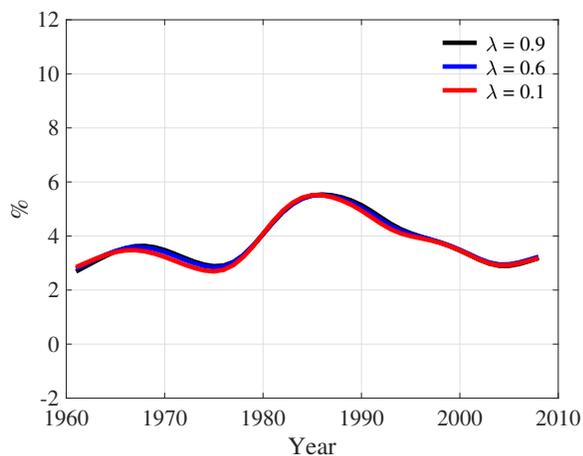


Nominal interest rates, HP filtered  $\lambda = 100$

(b) Simulation without inflation target shocks



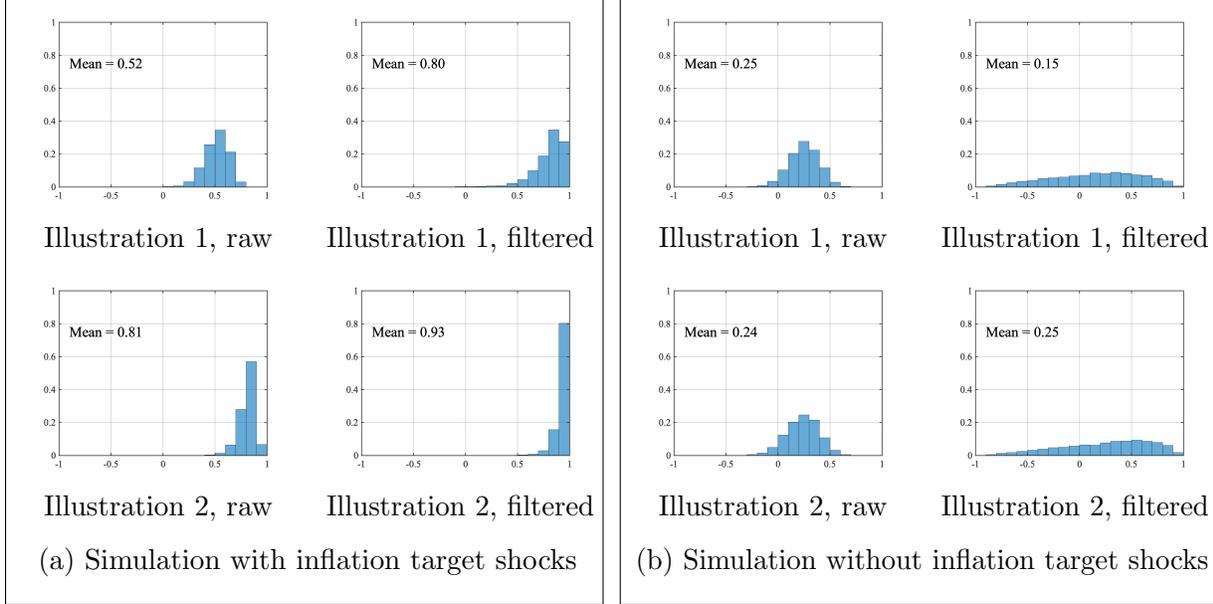
Inflation rates, HP filtered  $\lambda = 100$



Nominal interest rates, HP filtered  $\lambda = 100$

frictions is negligible, and in no case is it possible to reproduce the rise and subsequent fall in inflation that characterized the data in the United States between 1960 and 1990. These results reinforce the notion that the strength of the monetary transmission mechanism is not crucial for understanding the main trend observed in the inflation rates of the OECD countries presented in [Figure 1](#).

Figure 11: Correlation of series in the simulated data



**Correlation of illustrations** In [Figure 3](#) to [Figure 6](#), we report the correlation between the variables involved in the two illustrations, which according to our theory ought to all be equal to 1. A common feature in all cases is that the correlation increases substantially when using filtered data.

In order to evaluate the role of the shocks to the target in those correlations, we simulate the estimated model 10,000 times, drawing all shocks from their posterior distributions, and treat the data the same way we treat the true data in [Section 3](#). As in that section, we compute the correlation between the inflation rate and the two theoretically computed inflation rates — our two illustrations. We do both for the simulated data and for the filtered version. We then compute the distribution of the correlation in all cases. The distributions obtained through this process are depicted in the left panel of [Figure 11\(a\)](#), in which we also report the mean of each distribution. Figures on the top of the panel show the first illustration, while figures on the bottom of the panel show the second.

As a comparison, we repeat the exercise but set the shocks to the target equal to zero and draw all other shocks from their posterior distributions, as depicted in [Figure 11\(b\)](#).

Even though the two quantity theory predictions hold in the model by construction, the lack of shocks to the target implies that the correlations are lower and that filtering the data actually worsens the fit. This may be the reason why the country with the worst fit in our empirical analysis of Section 3 was Germany, for which there is very little evidence of an important low-frequency component.

## 5 Conclusions: Policy implications

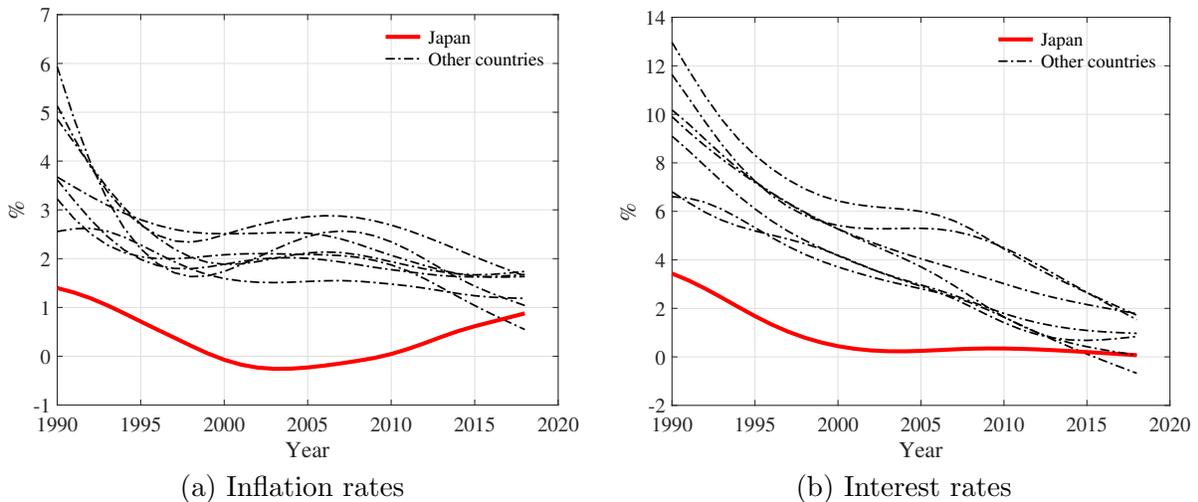
Are there direct policy implications that come out of our analysis? We believe so, but it depends on the case. We illustrate this by addressing three very topical policy issues.

We start with the one to which we have nothing to contribute. By the second half of 2016, the yearly inflation rate in the USA, as measured by the core personal consumption expenditures index, was gradually going up, to the point of getting very close to its target of 2%. However, at the beginning of 2017, the behavior reverted and inflation started falling to below 1.3% by August of that year, raising concerns regarding the optimal future path for the policy rate. The analysis of this paper is helpless in trying to understand and amend that event. No useful policy advice derives from our analysis.

On the other hand, another longer-run issue has also been a source of ample debate. The Federal Reserve announced an official target of 2% in January 2012, a time in which inflation, again measured with the core personal consumption index, had finally exceeded 2% for the first time since the eruption of the 2008 financial crisis. Inflation then remained above the 2% target till April of the same year, fell to 1.5% by the end of 2012, and has always been below the 2% target ever since. Thus, to the extent that the index we are using is the relevant one, inflation in the USA has been below its target for more than seven years by now. Our analysis suggests that had the nominal interest rate been 50 basis points higher than what it was during those years, inflation would had been closer to its target, on average, during the last seven years.

A final and probably more dramatic case is that of Japan. For over two decades, the Bank of Japan has been concerned about the low inflation rates in its country. This can be seen in panel (a) of Figure 12, which plots the low-frequency component of inflation in Japan — the solid red line — together with the equivalent measure for the other seven countries in Group 1. Japan appears as the clear outlier with substantially lower inflation, all the way till the end of the sample, at which point its inflation rate seems to converge with the group. Does the policy followed by the Bank of Japan explain this fact? We believe so. Panel (b) of Figure 12 plots the low-frequency components of the policy rates, where, Japan is again the clear outlier, with interest rates systematically lower than the rest, except at the end of the sample. In the natural counterfactual where Japan had maintained permanently higher interest rates — say, at the average value for the other countries — the inflation rate in Japan would have also been higher — say, at the average value of the other countries.

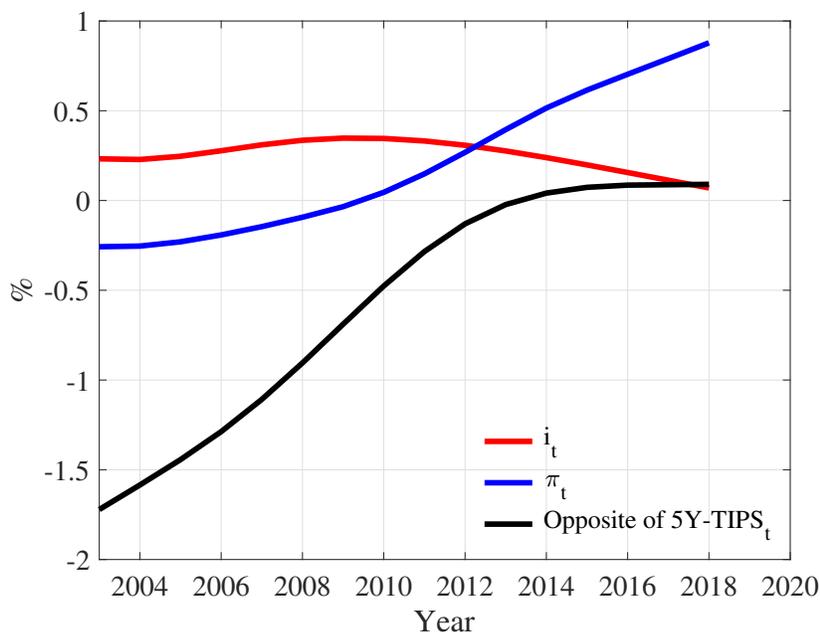
Figure 12: Low-frequency movements of Group 1 countries since 1990



The figure also hints at the reason why inflation in Japan started to increase somewhat over a decade ago. Notice that the negative trend in nominal interest rates of all the other countries in panel (b) is not followed by the inflation rates in those same countries since the year 2000 or so in panel (a). According to the model, this is possible only if the real interest

rate also falls during the period by a magnitude similar to that of the fall in the nominal interest rates. Under this interpretation, inflation went up marginally in the second decade of this century in Japan, due solely to the lower real rates that have been observed globally, since there is no movement in the low-frequency component of the policy rate in Japan. This implies that if global real rates start returning to positive values once the pandemic is over, even lower inflation readings in Japan become more likely, unless the nominal interest rate in Japan goes up in the medium term.<sup>26</sup>

Figure 13: Illustration 2 for Japan



The theoretical implication of declining real rates is corroborated by the data. In [Figure 13](#), we plot the low-frequency component of inflation and of the nominal interest rates in Japan, as presented in [Figure 12](#). We also plot the negative of the low-frequency movements of the real interest rate in the United States, as measured by the interest rate on five-year TIPS, its shortest maturity index bond. The plot starts in 2003, the year they were first issued in the United States.<sup>27</sup> According to the theory, inflation — the blue line — ought to be equal to the sum of the nominal interest rate — the solid red line — plus the

<sup>26</sup>See [Uribe \(2020\)](#) for a complementary analysis that points towards similar conclusions.

<sup>27</sup>We compare this measure with the one obtained using 10-year TIPS and also subtract the low-frequency of inflation from the low-frequency of the nominal interest rate, as suggested in Assumption 1.

negative of the real rate. This clearly does not hold exactly, but it becomes apparent in the figure that the upward trend in inflation ought to come from the upward trend in the negative of the real rate, since the nominal interest rate barely moves.

This analysis also sheds light on possible future scenarios in the United States, following the policy decisions made after the onset of the 2020 pandemic.<sup>28</sup> We make reference to two different decisions. The first was the one to set the interest rate at its effective zero lower bound. Given the current negative real rates of around  $-1.5\%$  exhibited by indexed bonds in the United States, that situation is compatible with inflation rates around  $1.5\%$ , relatively close to the current  $2\%$  target.

However, once the economic effect of the pandemic recedes, one could reasonably expect the real rate to return to positive values. Once that happens, and to the extent that the policy rate remains at zero, our theory and our data imply a decreasing trend for inflation, possibly to negative territory. Is it reasonable to expect the policy rate to remain at zero for a long period? This brings us to the second important policy change made in 2020: the new monetary policy framework announced by the Fed in August 2020 and the FOMC statement that followed in September. These decisions make a prolonged period of policy rates very close to zero quite likely. In a nutshell, they imply that the Fed will refrain from increasing the nominal interest rate until inflation is on track to moderately exceed  $2\%$  for some time.

Sure enough, high-frequency movements of the kind we ignored here may generate inflation rates above  $2\%$  for “some time,” in which case the policy rate will increase. Our paper has nothing to contribute to that debate. However, keeping the interest rate at zero for long periods may become a “target” shock with deflationary pressures like the ones described above. If this happens and inflation tends to negative values, the new framework implies that the nominal interest rate will remain at zero, forcing the trend inflation to remain in negative territory if the real rates become positive again. In this case,

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<sup>28</sup>The analysis also applies to several other central banks of developed economies.

low inflation and low interest rates reinforce each other. This low-frequency component of policy risks bringing about a convergence of the United States inflation rate and the Japanese experience since the mid-'90s.<sup>29</sup>

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<sup>29</sup>The analysis in [Uribe \(2020\)](#), which distinguishes temporary from permanent changes in the policy rate leads to similar conclusions.

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## A Algebra

The Bellman equation describing the decision problem is

$$V(\omega) = \max_{x,n,m,b} U(x) + \beta E \left[ V\left(\frac{m + b(1+i) + (1-\theta\nu n)z - x}{1 + \pi(s')} + \tau(s')\right) \right] - \varepsilon [m + b - \omega] - \delta [x - mn],$$

where for simplicity, we omitted the dependence of current variables on the state of the economy  $s$ . The first order conditions are

$$x : U'(x) = \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] + \delta \quad (\text{A1})$$

$$n : \delta m = \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] \theta \nu z \quad (\text{A2})$$

$$m : \delta n + \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] = \varepsilon \quad (\text{A3})$$

$$b : \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] (1 + i) = \varepsilon, \quad (\text{A4})$$

and the envelope condition is

$$V'(\omega) = \varepsilon. \quad (\text{A5})$$

In what follows, we focus the analysis on circumstances in which the nominal interest rate is bounded away from zero, so the cash in advance constraint (2) is binding. Note that (A3) and (A4) imply

$$\delta n + \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] = \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] (1 + i),$$

which, combining with (A2), yields

$$\frac{m}{n} i = \theta \nu z.$$

Replacing the equilibrium conditions (2) and (3), we obtain

$$i = n^2 \frac{\theta \nu}{(1 - \theta \nu n)}.$$

Note that  $\theta \nu n$  represents the welfare cost of inflation. Estimates of this cost for relatively low values of the nominal interest rates like the ones we will consider in the empirical section are relatively small, on the order of less than 2% of output. That means  $\gamma n$  ranges between 0 and 0.02. We then approximate the solution by

$$\sqrt{\frac{i}{\theta \nu}} \simeq n,$$

which is the celebrated squared root formula derived by [Baumol \(1952\)](#) and [Tobin \(1956\)](#). We can once again use the cash-in-advance constraint (2) to replace the variable  $n$  in the last equation and obtain

$$\frac{m}{x} = \sqrt{\frac{\theta\nu}{i}},$$

which delivers a relationship between real money balances as a proportion of output and the nominal interest rate in bonds.

In addition, we can use (A5) and (A4) to obtain

$$E \left[ \frac{\beta V'(\omega')}{V'(\omega)} \frac{1}{\pi(s')} \right] (1+i) = 1,$$

which can be written as

$$E \left[ \frac{(1+i)}{1+r(s')} \frac{1}{\pi(s')} \right] = 1,$$

where  $r(s')$  is a measure of the real interest rate. This last expression is the well known Fisher equation relating the nominal interest rate with the real interest rate and the inflation rate. This real interest rate is measured in terms of marginal utilities of real wealth, using the indirect utility function. In order to obtain a real interest rate in terms of the utility function, which is the usual way to measure it, note that (A3) and (A4) imply

$$\delta n = \beta E \left[ \frac{V'(\omega')}{1+\pi(s')} \right] i.$$

Replacing in (A1) delivers

$$U'(x) = \beta E \left[ \frac{V'(\omega')}{1+\pi(s')} \right] \left(1 + \frac{i}{n}\right).$$

Using (A4), we can write it as

$$\frac{U'(x)}{\left(1 + \frac{i}{n}\right)} = \frac{\varepsilon}{i}.$$

But (A4), together with the envelope conditions, implies

$$\beta E \left[ \frac{\varepsilon'}{1+\pi(s')} \right] (1+i) = \varepsilon.$$

so, using the previous equation, we obtain

$$E \left[ \left[ \frac{\beta U'(x')}{U'(x)} \frac{\frac{1+i'}{\left(1+\frac{i'}{n}\right)}}{\frac{1+i}{\left(1+\frac{i}{n}\right)}} \right] \left( \frac{1+i}{1+\pi(s')} \right) \right] = 1,$$

which implies that the expectation of the inverse of the real interest rate times the ratio of the nominal interest rate divided by the inflation rate must be equal to 1.

## B Data

### B.1 The United States

The series of nominal GDP, the three-month Treasury bill rate, currency in circulation, and “standard” M1 are collected from FRED.<sup>30</sup> Currency and the three-month T-bill rate are used as the measures of cash and the interest rate associated with it.

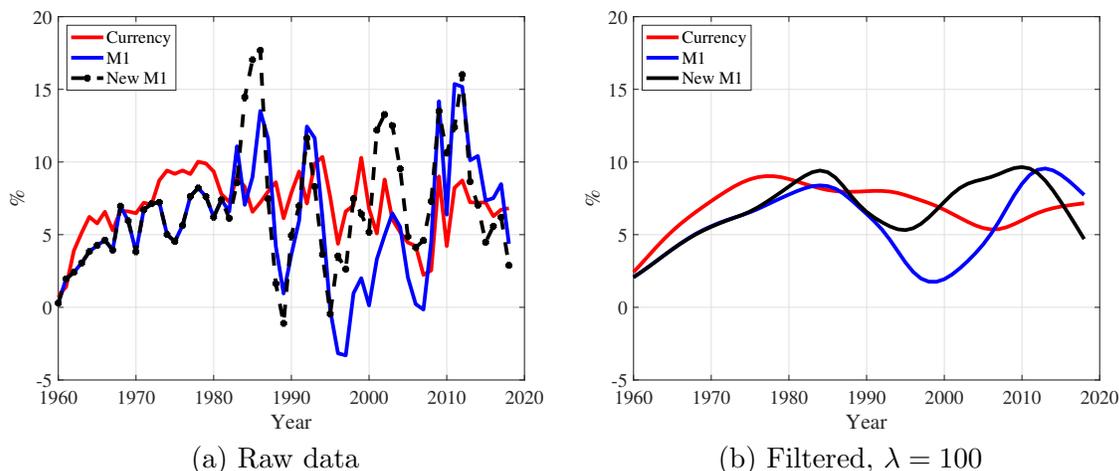
**NewM1** The construction of NewM1 follows [Lucas and Nicolini \(2015\)](#):

$$\text{NewM1} = \text{M1} + \text{MMDAs}.$$

Money Market Demand Accounts (MMDAs) series are constructed by aggregating term RCON6810 under Schedule RC-E from individual banks’ call reports. The original data are publicly available at the Central Data Repository Public Data Distribution website of Federal Financial Institutions Examination Council.<sup>31</sup>

The MMDAs series have been issued since 1982Q3 but the data are available only after 1984Q2. We apply a linear interpolation of money growth rates for the periods in between. [Figure A1](#) depicts the money growth rates of cash, the “standard” M1, and the New M1 series since 1960.

Figure A1: Money growth in the United States



**Imputed interest rate** We impute the interest rate associated with the New M1 by subtracting the fraction of interests paid by Deposits and by MMDAs from the three-month T-bill rate; that is,

$$\tilde{r} = r^{3m} - s_d i^d - s_a i^a,$$

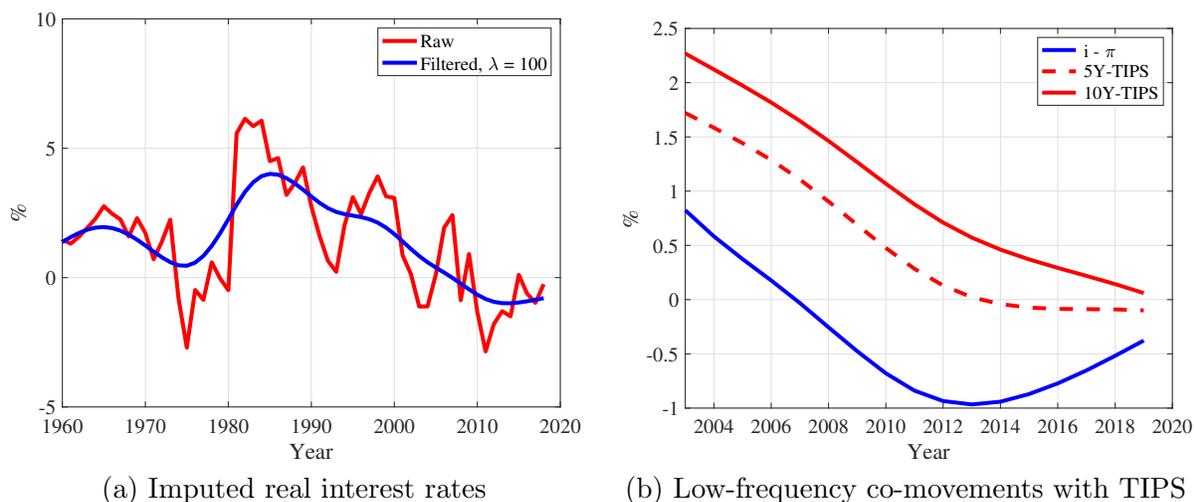
<sup>30</sup>FRED: <https://fred.stlouisfed.org/>.

<sup>31</sup>FFIEC: <https://cdr.ffiec.gov/public/>.

where  $s_d$  and  $s_a$  are the ratio of deposits to NewM1 and the ratio of MMDAs to NewM1, and  $i^{3m}, i^d$ , and  $i^a$  are the interest rates on three-month T-bills, deposits, and MMDAs, respectively.

**Real interest rates** The real interest rate is constructed by subtracting the three-month T-bill rate by inflation. In view of the lack of real interest rates for other countries, we use the real rates of the United States as the proxy of real rates in other countries for the quantitative illustration of Fisher Equation. Figure A2(a) plots the constructed raw series of US real interest rates since 1960 and the HP-filtered series using smoothing parameter 100. Figure A2(b) compares the imputed real interest rates with interest rates on Treasury Inflation-Indexed Securities (TIPS) at the five- and ten-year maturities. As can be seen from Figure A2(b), the difference between our imputed real interest rates and interest rates on long-term TIPS is very stable over time.

Figure A2: Imputation of US real interest rates



## B.2 Other OECD countries

We need data for prices, money stock M1, GDP, and interest rates for each country. In view of the lack of real GDP, we collect data of nominal GDP in local currency and impute real GDP with prices. The main source for nominal interest rate and M1 is the OECD data website and the main source for nominal GDP is the International Financial Statistics (IFS) of the International Monetary Fund (IMF).<sup>32</sup> We collect data for all countries starting from 1960 as long as there is availability. For countries with missing values up till 1960, we splice the series from the OECD and the IFS with data constructed in Benati et al. (2020).<sup>33</sup> Money data for countries in eurozone (Germany, Italy, Netherlands, Portugal, and Spain) are available only up till 1998.

<sup>32</sup>OECD data: <https://data.oecd.org/>; IFS data: <https://data.imf.org/>.

<sup>33</sup>See Benati et al. (2019) for more details about the original data sources.

We finally have data for 16 OECD countries other than the United States and break them into two groups based on the similarity of inflation movements:

1. USA, Australia, Canada, Denmark, Germany, Japan, New Zealand, and the UK;
2. Italy, Netherlands, Portugal, South Korea, Spain, Colombia, Chile, Mexico, and Turkey.

The following list details special issues in the construction of the dataset.

**Australia** Interest rates in 1960–1967 and M1 in 1960 are spliced with [Benati et al. \(2020\)](#).

**Canada** Nominal GDP in 1960 is spliced using [Benati et al. \(2020\)](#). Between 1982 and 2005, M1 in the OECD dataset has faster growth at the beginning and lower growth in later years compared with the M1 data in [Benati et al. \(2020\)](#), which results in a similar cumulative growth across these two sources.

**Chile** We use data for Chile after 1985 because Chile had several years of hyperinflation over 100% in 1970s. Interest rates in 1985–1997 are spliced with [Benati et al. \(2020\)](#).

**Colombia** The OECD provides nominal interest rates only after 1986. Interest rates in [Benati et al. \(2020\)](#) and the OECD behave similarly after 1995 but are significantly higher in OECD than in [Benati et al. \(2020\)](#). We use [Benati et al. \(2020\)](#) for interest rates in all periods for consistency.

**Denmark** Interest rates between 1960 and 1986 are spliced using [Benati et al. \(2020\)](#).

**Germany** The IFS provides nominal GDP only after 1992. We use [Benati et al. \(2020\)](#) for nominal GDP in all periods for consistency.

**Italy** Interest rates before 1979 are spliced using [Benati et al. \(2020\)](#). The IFS provides nominal GDP only after 1995 and the OECD does not have data for M1. We use [Benati et al. \(2020\)](#) for nominal GDP and M1 in all periods.

**Japan** Interest rates before 2003 are spliced using [Benati et al. \(2020\)](#).

**Mexico** Interest rates before 1997, prices before 1969, and M1 before 1977 are spliced using [Benati et al. \(2020\)](#). Nominal GDP for all years is taken from [Benati et al. \(2020\)](#).

**Netherlands** Interest rates before 1982 and nominal GDP before 1995 are spliced using [Benati et al. \(2020\)](#). M1 for all years is taken from [Benati et al. \(2020\)](#).

**New Zealand** Interest rates before 1974, nominal GDP before 1970, and M1 before 1978 are spliced using [Benati et al. \(2020\)](#).

**Portugal** Interest rates before 1986 and nominal GDP before 1995 are spliced using [Benati et al. \(2020\)](#). M1 for all years is taken from [Benati et al. \(2020\)](#).

**South Korea** Interest rates are taken from [Benati et al. \(2020\)](#).

**Spain** Interest rates before 1976 are spliced using [Benati et al. \(2020\)](#). Note that interest rates in the OECD dataset are higher than those in [Benati et al. \(2020\)](#) between 1977 and 1981. The IFS provides nominal GDP since 1995. We use [Benati et al. \(2020\)](#) for nominal GDP in all periods for consistency.

**Turkey** Data for Turkey are available from 1969 onwards. Nominal GDP before 1987 is spliced using [Benati et al. \(2020\)](#). Interest rates for all years are taken from [Benati et al. \(2020\)](#).

**The UK** We use all variables for all years from [Benati et al. \(2020\)](#).

[Table A1](#) provides the summary statistics of mean and standard deviation of inflation  $\pi$ , nominal interest rate  $i$ , money growth  $\mu$ , and real GDP growth  $g$  by country.

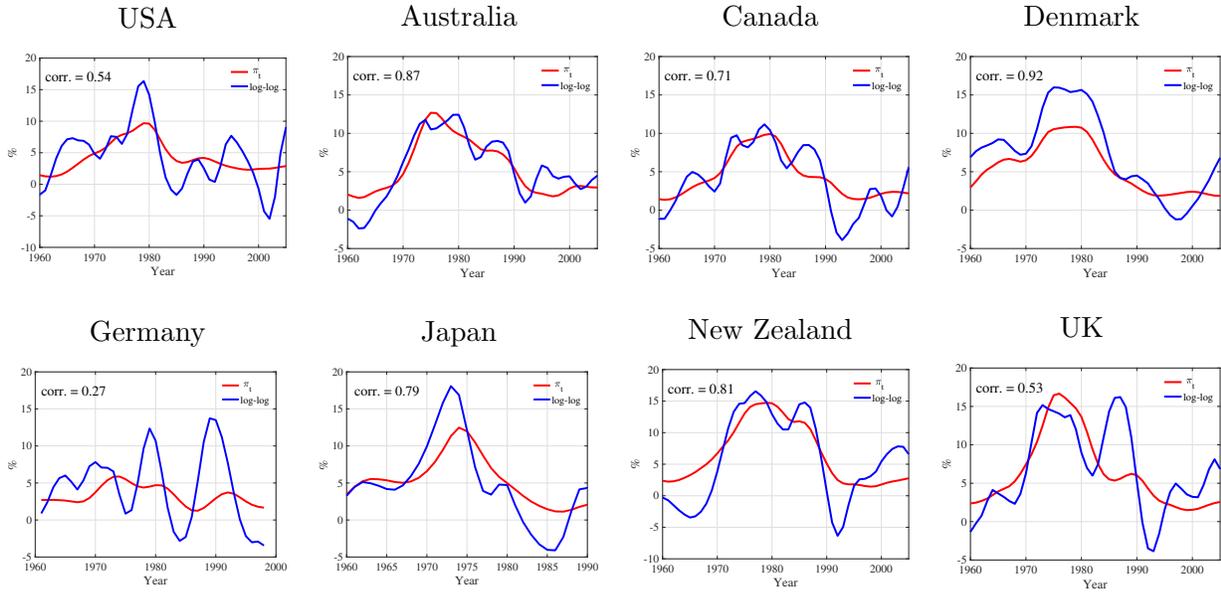
### B.3 Additional results

In [Figure A3](#) and [Figure A3](#), we report the two illustrations when we filter series using smoothing parameter  $\lambda = 6.5$ .

Table A1: Mean and standard deviation of main variables

Country	Periods	$\pi$	$i$	$\mu$	$g$
USA - Currency	1960–2005	4.26	6.16	7.25	2.94
		(2.91)	(3.13)	(2.34)	(2.43)
USA - Standard M1	1960–2005	4.26	6.16	5.14	2.94
		(2.91)	(3.13)	(3.71)	(2.43)
USA - New M1 - Interp1	1960–2005	4.26	4.97	7.32	2.94
		(2.91)	(2.57)	(6.16)	(2.43)
USA - New M1- Interp2	1960–2005	4.26	4.97	6.50	2.94
		(2.91)	(2.57)	(4.11)	(2.43)
Australia	1960–2005	5.48	8.28	9.01	3.73
		(4.03)	(4.06)	(6.21)	(2.74)
Canada	1960–2005	4.36	7.18	8.06	3.70
		(3.18)	(3.50)	(4.58)	(2.95)
Denmark	1960–2005	5.45	9.89	10.79	2.73
		(3.55)	(4.39)	(6.26)	(2.39)
Germany	1961–2005	3.00	5.61	8.18	3.16
		(1.80)	(2.53)	(3.51)	(2.95)
Japan	1960–2005	3.85	4.22	11.36	4.51
		(4.37)	(2.60)	(7.07)	(4.94)
New Zealand	1960–2005	6.56	9.65	8.82	2.88
		(5.38)	(4.50)	(7.62)	(2.35)
The UK	1960–2005	6.38	8.35	9.72	2.75
		(5.44)	(3.57)	(6.13)	(2.04)
Italy	1960–2005	7.14	6.56	13.14	4.26
		(5.63)	(3.64)	(6.58)	(2.79)
Netherlands	1962–2005	3.96	5.29	7.72	3.42
		(2.59)	(1.96)	(4.98)	(2.88)
Portugal	1960–2005	10.28	8.35	12.99	4.87
		(8.13)	(6.62)	(6.64)	(3.32)
South Korea	1962–2005	9.73	10.06	24.47	9.88
		(7.57)	(7.18)	(12.39)	(6.21)
Spain	1960–2005	7.92	8.85	12.99	4.87
		(5.57)	(5.08)	(6.64)	(3.32)
Chile	1980–2005	13.39	26.24	22.01	5.03
		(9.70)	(18.00)	(14.04)	(5.84)
Colombia	1960–2005	17.59	9.20	21.70	4.15
		(7.65)	(6.11)	(7.66)	(2.15)
Mexico	1960–2005	24.43	24.32	28.22	5.02
		(31.02)	(21.68)	(24.00)	(5.10)
Turkey	1969–2005	46.80	36.52	48.36	4.75
		(27.75)	(20.98)	(22.46)	(7.14)

## Illustration 1



## Illustration 2

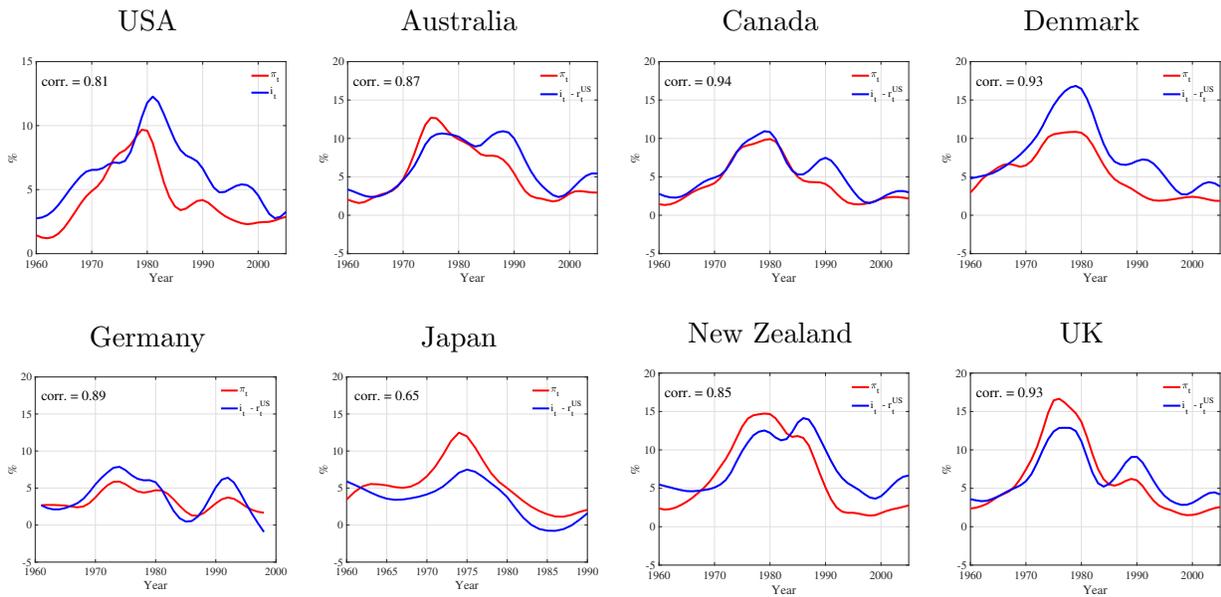
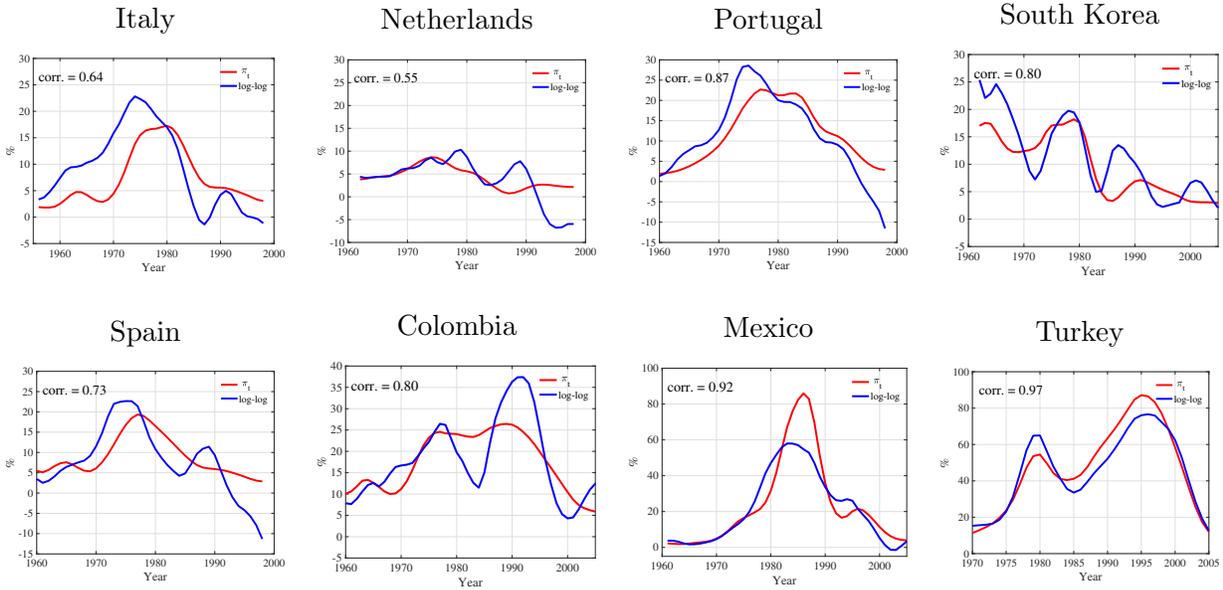


Figure A3: Countries in Group 1,  $\lambda = 6.5$

## Illustration 1



## Illustration 2

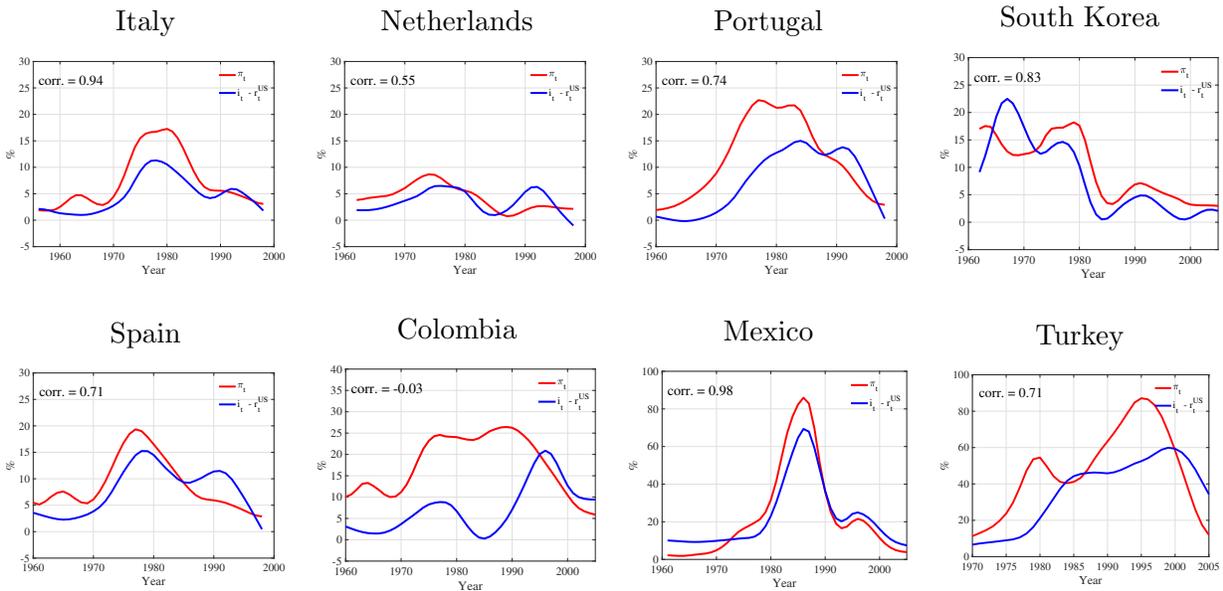


Figure A4: Countries in Group 2,  $\lambda = 6.5$

## C Estimation details

The model of Section 4 is estimated using Bayesian methods. We jointly estimate the structural parameters,  $\vartheta$ , and the dates of regime changes,  $\mathbf{T}$ . We estimate,  $T^{\text{on}}$ ,  $T^{\text{off}}$  and  $T_\sigma$ . The first two correspond to the dates of the high inflation regime for which  $\mathbb{I}^s = 1$ . In the case of  $T^{\text{off}}$  we sample from a uniform distribution over 1979q4 and 1983q4, which corresponds to the Volcker disinflation.  $T_\Omega$  is the date break for the variance of all structural shocks except for that of the money demand shock. The variance of the remaining structural shocks shifts proportionally at  $T_\Omega$  by a factor of  $\kappa$ , so that the variance covariance matrix shifts from  $\kappa\Omega$  to  $\Omega$ . This specification serves two purposes: first, it helps the model capture the decrease in volatility associated with the Great Moderation. Second, and more important for our purposes, is that it guards against the possibility that the estimation relies on shocks to the inflation target to account for the increased volatility of the 1970's. For the variance of shocks to money demand,  $\sigma_\xi$ , the volatility shifts in 1982q4 to  $\kappa_m\sigma_\xi$  which, as explained above, lines up with the regime change in the measurement of M1 explained in [Lucas and Nicolini \(2015\)](#).

The model is estimated on real GDP per capita growth, the Federal Funds rate, core inflation as measured by the CPI excluding food and energy, the Michigan survey measure of inflation expectations, and money growth.

To construct the likelihood of the model under regime changes, we use the method outlined in [Kulish and Pagan \(2017\)](#). That method deals with a more general case than the application we are considering, so we provide a brief discussion of the case we deal with here.

Let  $t = 1, 2, \dots, T$  index the observations in the sample. From period  $t = 1, 2, \dots, T^{\text{on}} - 1$ , the steady state level of inflation is  $\pi$ . The first-order approximation to the equilibrium conditions around this initial steady state is given by the linear rational expectations system of  $n$  equations that we write as:

$$A_0\mathbf{y}_t = C_0 + A_1\mathbf{y}_{t-1} + B_0\mathbb{E}_t\mathbf{y}_{t+1} + D_0\varepsilon_t \quad (\text{A6})$$

where  $A_0$ ,  $C_0$ ,  $A_1$ ,  $B_0$  and  $D_0$  are the structural matrices of the initial steady-state,  $\mathbf{y}_t$  is a  $n \times 1$  vector of state and jump variables and  $\varepsilon_t$  is an  $l \times 1$  vector of exogenous *i.i.d* shocks. The unique rational expectations solution to (A6) is

$$\mathbf{y}_t = C + Q\mathbf{y}_{t-1} + G\varepsilon_t \quad (\text{A7})$$

For  $t = T^{\text{on}}$  until  $T^{\text{off}} - 1$  the steady state level of inflation increases to  $\pi + \Delta\pi$  and  $\mathbb{I} = 1$  so the structural equations are given by

$$\bar{A}_0\mathbf{y}_t = \bar{C}_0 + \bar{A}_1\mathbf{y}_{t-1} + \bar{B}_0\mathbb{E}_t\mathbf{y}_{t+1} + \bar{D}_0\varepsilon_t \quad (\text{A8})$$

with solution

$$\mathbf{y}_t = \bar{C} + \bar{Q}\mathbf{y}_{t-1} + \bar{G}\varepsilon_t \quad (\text{A9})$$

At  $T^{\text{off}}$  the economy reverts to (A6) with steady state  $\pi$ . These structural changes imply

that the reduced form are time-varying over the sample. In general,

$$\mathbf{y}_t = C_t + Q_t \mathbf{y}_{t-1} + G_t \varepsilon_t. \quad (\text{A10})$$

With a sample of data,  $\{y_t^{obs}\}_{t=1}^T$ , where  $y_t^{obs}$  is a  $n_{obs} \times 1$  vector of observable variables that relates to the model's variables through the measurement equation below:

$$y_t^{obs} = H_t \mathbf{y}_t. \quad (\text{A11})$$

$H_t$  is time-varying to account for the fact that the Michigan measure of inflation expectations only becomes available after 1978. The observation equation, Equation (A11), and the state equation, Equation (A10), form a state-space model. The Kalman filter can be used to construct the likelihood function for the sample  $\{y_t^{obs}\}_{t=1}^T$ , given by  $\mathcal{L}(Y|\vartheta, \mathbf{T})$  as outlined in [Kulish and Pagan \(2017\)](#).

Given the joint posterior of the structural parameters and the date breaks,  $p(\vartheta, \mathbf{T}|Y) = \mathcal{L}(Y|\vartheta, \mathbf{T})p(\vartheta)p(\mathbf{T})$ , we simulate from this distribution using the Metropolis-Hastings algorithm as used by [Kulish and Rees \(2017\)](#). As we have continuous and discrete parameters, we separate them into two blocks: one for date breaks and one for structural parameters. The sampler delivers draws from the joint posterior of both sets of parameters.

Below we report results from our baseline estimation. The Online Appendix contains additional estimation results for different measures of money growth, and other specifications including estimating the slope of the NK Phillips curve.

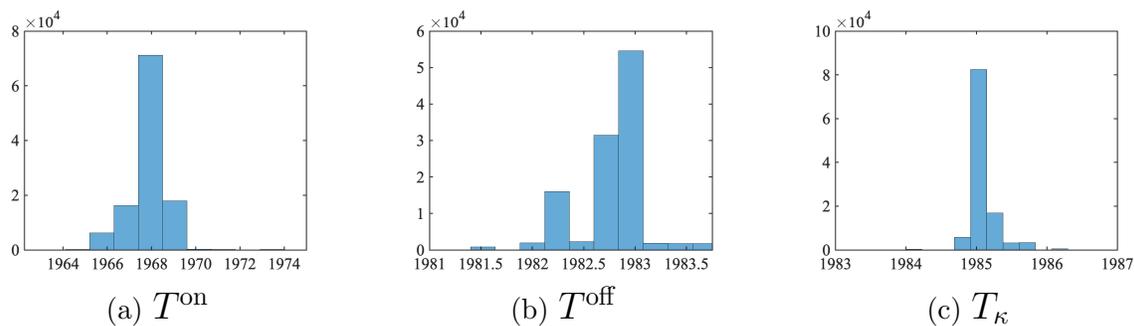
## C.1 Prior and posteriors of the structural parameters

Table A2: Baseline estimates

	Prior distribution			Posterior distribution			
	Shape	Mean	Std Dev.	Mode	Mean	5 %	95 %
<i>Standard Deviations</i>							
$100 \times \sigma_i$	Inv. Gamma	1	2	0.08	0.08	0.06	0.10
$100 \times \sigma_a$	Inv. Gamma	1	2	1.45	1.33	1.17	1.84
$100 \times \sigma_e$	Inv. Gamma	1	2	0.11	0.11	0.09	0.14
$100 \times \sigma_z$	Inv. Gamma	1	2	0.47	0.47	0.42	0.53
$100 \times \sigma_\pi$	Inv. Gamma	1	2	0.10	0.10	0.09	0.12
$100 \times \sigma_\tau^{obs}$	Inv. Gamma	1	2	0.16	0.15	0.12	0.20
$100 \times \sigma_\xi$	Inv. Gamma	2	3	1.25	1.25	1.11	1.41
$\kappa$	Normal	2	0.3	2.00	1.99	1.82	2.20
$\kappa_m$	Normal	2	0.3	1.37	1.32	1.14	1.61
<i>Structural parameters</i>							
$\rho_i$	Beta	0.5	0.2	0.90	0.90	0.86	0.93
$\phi_\pi$	Normal	2	0.5	2.06	2.12	1.40	2.74
$\phi_x$	Normal	0.125	0.05	0.37	0.37	0.31	0.42
$10 \times \omega$	Normal	0.5	0.1	0.52	0.53	0.37	0.69
$\eta$	Normal	0.5	0.05	0.46	0.46	0.38	0.55
$\rho_m$	Beta	0.5	0.2	0.95	0.96	0.91	0.98
$\rho_a$	Beta	0.5	0.2	0.88	0.88	0.84	0.91
$\rho_e$	Beta	0.5	0.2	0.46	0.47	0.34	0.56
$\rho_\tau$	Beta	0.5	0.2	0.79	0.80	0.68	0.89
$\rho_\pi$	Beta	0.5	0.2	0.97	0.98	0.97	0.98
$100 \times \Delta_\pi$	Uniform	[-2 , 6 ]		1.03	0.98	0.18	1.95
$\rho_\xi$	Beta	0.5	0.2	0.60	0.59	0.46	0.74

## C.2 Posteriors of date breaks

Figure A5: Posterior distribution of date breaks

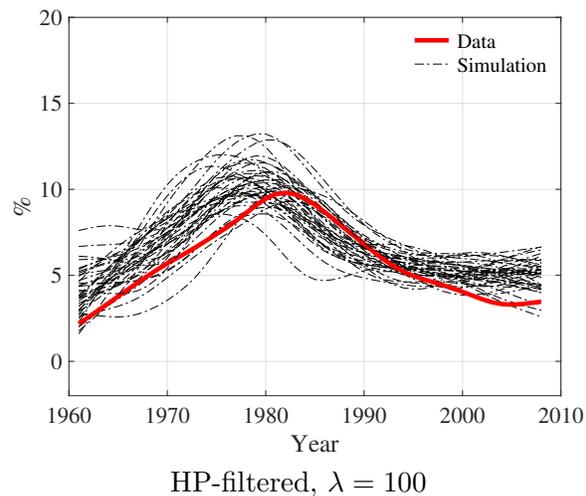
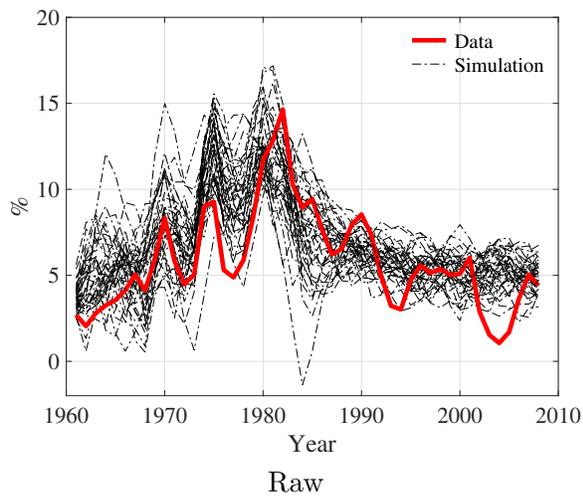


### C.3 Additional model fitness

Figure A6 and Figure A7 report model fitness of interest rates and money growth rates.

Figure A6: Model fitness of nominal interest rates

(a) Estimation with inflation target shocks



(b) Estimation without inflation target shocks

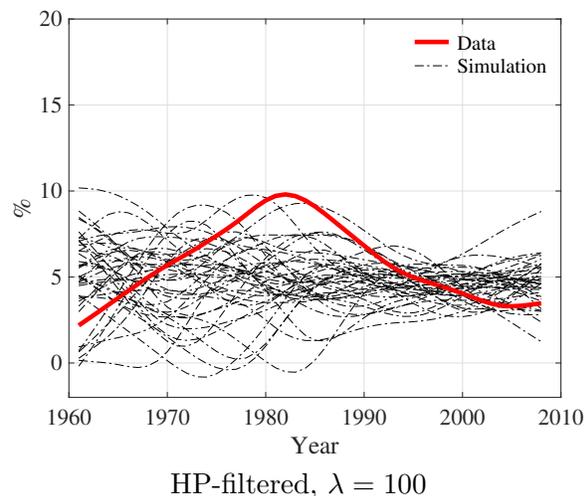
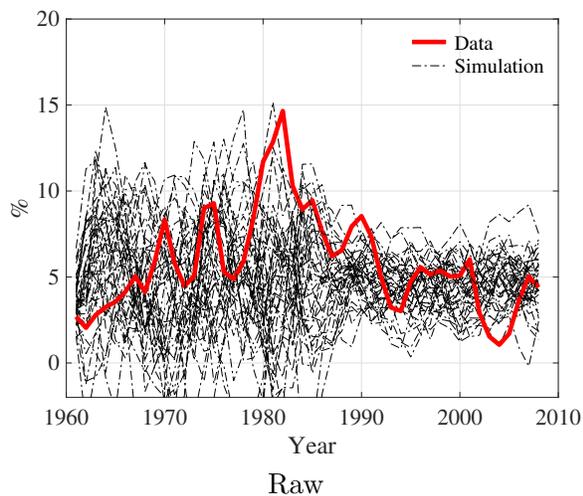
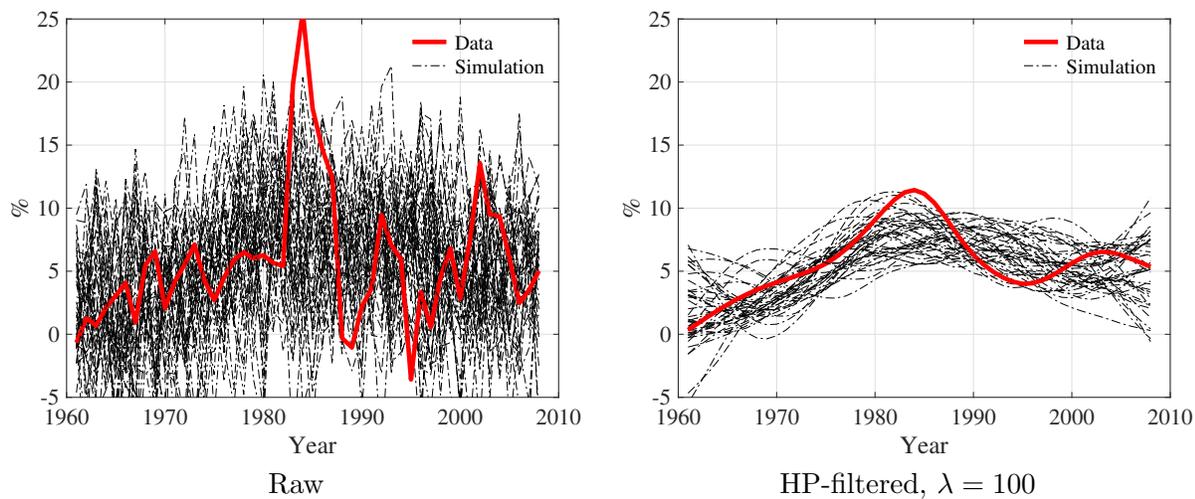


Figure A7: Model fitness of money growth rates

(a) Estimation with inflation target shocks



(b) Estimation without inflation target shocks

