

Costly Information and the Stock Market

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In a simple, coherent, general equilibrium model it is demonstrated why stock market prices do not reflect costly but socially useless information.

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## Costly Information and the Stock Market

by John Bryant

In a recent article, Sanford Grossman [5, p. 94] states, "It is clear that if information is costly, then no competitive equilibrium exists which reveals information . . . . It is very important to note that each trader assumes that the equilibrium price random variable will not be affected by his decision to buy information . . . ."

The seeming paradox implied by this statement can be described as follows. If traders gather costly information, then the stock market reflects that information, and there is no return to be made on the information. If traders do not gather information, then the stock market does not reveal the information, and there may be a profit to be made in gathering information. This paradox suggests that there may be no Nash equilibrium.

However, in reading Grossman's statement, it is unclear whether costly information implies a problem for the stock market or for the equilibrium concept used. We contend in this note that the problem lies with the equilibrium concept, or more accurately, in the description of the strategy space. In a perfect stock market there are no inherently small traders, and this is inconsistent with price-taking behavior. The relevant solution concept is, indeed, Nash equilibrium. But the trader should not take the price vector as given, but rather the strategies of the other traders, that is to say, the strategy of the auctioneer.<sup>1/</sup> This is argued in the context of a simple, coherent, general equilibrium model. To peek ahead, we conclude that individuals do not collect costly information for the purpose of investing in the stock market. Uninformed traders are a Nash equilibrium.

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<sup>1/</sup> This is analogous to the result that there is no pure strategy Nash equilibrium for price setting competitive firms. See Bryant [3].

### The Model

Now we turn to our simple model. It differs from standard models of the Finance literature in being a coherent general equilibrium model.

First, let us specify the structure of the economy. There are a continuum of individuals indexed by  $x \in [0,1]$ . They live two periods. Each individual is equally endowed with labor,  $L(x) = L$ , in her first period of life. Each is endowed with nothing in her second period of life. There exists, for each individual, a risky technology for transforming work this period into the single transferable consumption good next period. These technologies are as follows. With probability .5, the individual gets two units of goods for each unit of work; and with probability .5, the individual gets one unit. The drawings on individual technologies are independent. In addition, each individual can safely transform work this period into the consumption good this period one-for-one. The individual's utility is a function of her consumption of the consumption good in her two periods of life. All individuals have the same utility function, and it is strictly concave, differentiable, and additively separable.

$$U(C_1(x), C_2(x)) = U_1(C_1(x)) + U_2(C_2(x))$$

where  $C_1(x)$ ,  $C_2(x)$  are first- and second-period consumption of the individual. Moreover,  $U'_1(0) = U'_2(0) = \infty$ . Henceforth, we will deal with the representative individual and drop the index  $x$ .

Now let us add the stock market. In the stock market the risk of individual technologies is perfectly diversified against. Claims against individual technologies are costlessly bought and sold using the first-period consumption good in exchange, and each individual ends up with the safe portfolio. This market for claims is always open. The market is run by an auctioneer who

settles on the prices at which supply and demand are equated for each  $x$ . The auctioneer allows no trades to take place at a "disequilibrium" price. It is this imposed strategy of eliminating nonzero excess demand which the individual takes as given, not a particular price decision of the auctioneer.<sup>2/</sup>

Now let us analyze the model. First we consider the case with no information on the outcomes of individual technologies prior to realization. Let  $W$  be the individual's amount of work,  $S$  her holding of the safe portfolio, and  $P$  the goods price of the safe portfolio. The price of the safe portfolio is, of course, the price of claims on individual technologies. The individual's strategy is choice of  $W$  and  $S$ , taking as given that  $P$  is set to equate aggregate supply and demand. The individual's problem can be written:

$$\begin{aligned} & \max_{W, S} U_1(C_1) + U_2(C_2) \\ & \text{s.t. } C_1 = L - W + P(W-S) \\ & C_2 = 1.5 S \end{aligned}$$

with equilibrium condition  $W = S$ , as individuals are identical. It follows that the auctioneer chooses  $P = 1$ .

Now let us add free "inside" information. Suppose that after the stock market opens, every individual learns the outcome of her own technology. Note that individuals buy their portfolios before they learn their outcomes so that they can diversify. When individuals learn their outcomes, at unchanged prices they try to buy or to sell short claims on their own technologies. The market clearing price for a technology is then  $P(x) = 4/3$  if the outcome of the technology is good, or  $P(x) = 2/3$  if it is bad. The auctioneer's decision is to set  $P = 1$  before information is available, and  $P(x) = 4/3$  or  $2/3$  when it becomes available. Moreover, no trades take place.

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<sup>2/</sup> This standard formulation of competitive equilibrium is due to Arrow and Debreu [1].

Our result is that new information does not stimulate new trade. This result is not an artifact of the fiction of an auctioneer, as we now demonstrate. Suppose, for the moment, that there is no auctioneer. Instead we consider core solutions. It is clear from the symmetry of the problem that any core solution implies that the economy can be divided into two groups, each containing equally well-off people--those learning of a good technology, and those learning of a bad one. But these groups must be equally well off, too, because no trade is always an option. We conclude that the core solution is no trade following the disclosure of the information. Does the market reflect the information? As in practice, prices are only registered when a trade occurs, it seems a reasonable interpretation that prices do not reflect the information.

We have treated free information, but costly information is our subject. Naturally, if free information is not reflected in the stock market prices, costly information is not either. Clearly, then, costly information is not gathered, for if free information is not exploitable, neither is costly information. Suppose to the contrary that a deviant individual does gather information. As she is the only person desiring to trade, her supply or demand is market excess supply or demand. As a result her information is revealed, and no trades take place. Therefore, she gets no return on her incurred cost. In the model, no one wants to trade with an informed person, and only the informed want to trade.

Our model of the stock market has a glaring flaw which renders our results suspect. Ours is the limiting version of a "thin" market, no trades at all take place after the market first clears. We now make the market "thick," and show how this influences our results. To look ahead, in this circumstance prices reflect free information, but not costly information, which is still not gathered.

First, let us alter the model to generate a "thick" stock market, one with active trading. Consider first the case without information. To generate trade we suppose that after individuals make their work decisions and buy their portfolios of stocks,  $\alpha \cdot 100$  percent of them learn they are going to die between periods.<sup>3/</sup> This knowledge comes only to the affected individual, and is not verifiable. Production of goods next period is unaffected by death. The "early diers'" utility function becomes  $U_1(C_1)$  (dropping the constant  $U_2(0)$ ). They sell their portfolio of stocks for goods to healthy individuals. Let  $P'$  be the price that the "early diers" get for their portfolio, and  $S'$  be the purchase of additional stock by the healthy individuals. The individual's problem now can be written

$$\begin{aligned} \max_{W,S} \{ & (1-\alpha) [\max_{S'} [U_1(L-W+P(W-S)-P'S') + U_2[1.5(S+S')]] \\ & + \alpha U_1[L-W+P(W-S)+P'S] \} \end{aligned}$$

with equilibrium condition  $W = S = \frac{(1-\alpha)}{\alpha} S'$ . It is easily seen that this implies  $P = P' = 1$ . The "early diers" make neither capital gain nor loss. Because being an early dier is not verifiable, the market cannot share the risk of being an "early dier," however.<sup>4/</sup>

Now let us suppose that, once again, individuals learn the outcome of their own technology. Individuals get this information at the same time that they learn whether or not they will die early. Once again, the market clearing prices settled on by the auctioneer reveal the information. The sales by the "early diers" are known beforehand, and there is an equal demand for their

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<sup>3/</sup>This device was introduced in Bryant [4].

<sup>4/</sup>This requires that early death is not verifiable before second-period consumption. Otherwise a false claim could be penalized in second-period consumption.

diversified portfolios. Only their identity is not known. The information on individual technologies alone impacts prices. The early diers are no better or worse off because of the revelation of the information, as they hold diversified portfolios. However, as the early diers enter sales in all stocks, the observed prices do reflect the information. Moreover, if information is costly, it is not collected, as any informed deviant impacts price completely. Therefore, costly information is not reflected in observed prices.

Once again, let us demonstrate that our result need not depend upon the fiction of an auctioneer. Let us suppose that the market does not act as if there is an auctioneer, and costly information is available. Suppose that irreversible sales are made before supplies and demands are equated (no recontracting). Then one might guess that, depending on the queueing of orders, the informed can buy and sell at unchanged prices until the normal value of "early diers" transactions are met. Then prices change to reflect the information. The informed make some profit, and the early diers take some capital loss. Further, one might guess that the number of information gatherers would rise until the value of the capital gain from information just equals the cost of information.

The guesses of the previous paragraph are likely wrong, however. Previously, in the "thin" market, we argued that a group of individuals could guarantee itself no capital losses by refusing to trade when information became available. This is not possible for the "early diers," however; they must trade. Nevertheless, there may still be a strategy which the uninformed "early diers" can use to protect themselves. They hold diversified portfolios of stocks. Rather than selling the stocks individually, suppose they can refuse to disbundle, and just sell shares in their portfolios.<sup>5/</sup> If they do so, the portfolios sell at unchanged prices, and any information gatherers cannot use their

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<sup>5/</sup> Are these demand deposits? See Bryant [4].

information to advantage. Therefore, the costly information is not gathered by anyone.

We conclude that costly information is not gathered. In the model, the information has no social value. The market does not, then, produce the distortion of resources wasted on the gathering of socially useless information.

## References

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