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AND HEDGING DECISIONS OF A COMPETITIVE FIRM

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The Role of Credit in Separating the Storage
and Hedging Decisions of a Competitive Firm

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Assuming that farm firms maximize not simply expected profits but rather the expected utility of profits seems to complicate analysis of their grain storage decisions, but this need not be true. Danthine and Holthausen have shown that, under certain circumstances, the existence of a forward grain market allows firms to separate the problem of maximizing their expected postharvest utility of profits into two independent problems, a storage decision and a hedging decision. The firm's optimal storage decision has a simple form analogous to the optimal decision of a firm maximizing profit in a world without uncertainty -- inventory will be chosen to equate the marginal cost of storing grain to the forward value of the marginal increment in next period's grain supplies. The shape of the firm's utility function and the firm's beliefs about the probability distribution of next period's spot price have no effect on the optimal storage decision; they affect only the firm's hedging decision.

Although Danthine and Holthausen's separation result is theoretically pleasing and empirically promising, they derived it (apparently independently) under fairly restrictive assumptions. In particular, they assumed that firms operate in only a two-period world and maximize the expected utility of total two-period profit. A more standard approach, even in a two-period world, is to assume that firms maximize the utility of first-period profit plus the (possibly discounted) expected utility of

second period profit. And, even for such relatively short horizon problems as a farmer's optimal postharvest storage strategy, it is more realistic to assume a many-period sequence of decisions. This paper employs dynamic programming to derive Danthine and Holthausen's separation result under these more standard and empirically relevant assumptions. It turns out that the key to extending their results is to allow firms to borrow and lend. This highlights the relevance of rural capital market conditions to the analysis of farmers' storage decisions.

Danthine and Holthausen's Separation Result

Danthine and Holthausen consider a two-period economy with forward trading and a random second-period spot price in which a firm maximizes its expected utility of total first- and second-period profits, subject to a concave, nonstochastic technology for transforming first-period input into second-period output.^{1/} That is, the firm's objective is

$$(1) \quad \max_{x \geq 0, f} EU[(q(x) - f)p + p^f f - c(x)],$$

where E denotes mathematical expectation conditioned on all information available when the input level is chosen; $U[\cdot]$ is a strictly concave function of profits; $q(x)$ is second-period output, a strictly concave function of x , the first-period input; f is the firm's forward sale ($f > 0$) or purchase ($f < 0$) of output; p^f is the forward price of output; $c(x)$ is the cost of procuring and using input x ; and p , the only random variable, is the spot price of output in the second period. Let $c'(x) > 0$ and $c''(x) < 0$ for all $x \geq 0$. While the nonstochastic technology makes this objective function inappropriate for many decisions involving crop production, it captures the essence of a farmer's postharvest grain storage decision.

Specifically, let x be grain stored, $q(x)$ be grain available in the next period, and $c(x)$ be the cost of procuring and storing grain. Assuming $x > 0$, the necessary and sufficient conditions for maximizing this utility function are

$$(2) \quad E\{U'[\cdot]_p\}q'(x) = E\{U'[\cdot]\}c'(x)$$

$$(3) \quad E\{U'[\cdot]_p\} = E\{U'[\cdot]\}p^f.$$

Substituting (3) into (2) gives

$$(4) \quad E\{U'[\cdot]\}p^fq'(x) = E\{U'[\cdot]\}c'(x),$$

which simplifies to the separation result

$$(5) \quad p^fq'(x) = c'(x).$$

According to (5), the firm equates marginal cost to the forward value of marginal product without regard to its attitudes towards risk or its beliefs about the probability distribution of the second-period spot price of output. These attitudes and beliefs do affect f , the firm's forward trading position (see Holthausen), but they are separated from the production or storage decision when the firm can engage in forward trade.

A Problem in Generalizing the Separation Result

Firms facing multiperiod decision problems are commonly assumed to maximize the expected value of a discounted sum of utilities of profits. Dathine and Holthausen, on the other hand, assume that the firms maximize a utility function of the undiscounted sum of profits. The importance of this restrictive assumption in their derivation of the sep-

aration result is best seen in equations (4) and (5). Equation (5), the separation result, follows from (4) only because the marginal utility terms on both sides of equation (4) are equal.

If we attempt to generalize Danthine and Holthausen's result under the more conventional assumption that firms maximize the sum of first- and discounted expected second-period utility of profits, then the derivation of the separation result breaks down. Let the firm's objective be

$$(6) \quad \max_{x \geq 0, f} \{U[w_t - c_t(x_t)] + \beta E_t U[(q(x_t) - f_t)p_{t+1} + p_t^f f_t]\},$$

where w_t is initial wealth, β is a subjective discount factor, and all other operators, functions, and variables are as before except for time subscripts. Assuming $x_t > 0$, the necessary and sufficient conditions for a maximum are given by

$$(7) \quad U'[w_t - c_t(x_t)]c'_t(x_t) = \beta E_t \{U'[(q(x_t) - f_t)p_{t+1} + p_t^f f_t]p_{t+1}\} q'(x_t)$$

$$(8) \quad E_t \{U'[(q(x_t) - f_t)p_{t+1} + p_t^f f_t]p_{t+1}\} = E_t \{U'[(q(x_t) - f_t)p_{t+1} + p_t^f f_t]\} p_t^f.$$

Substituting (8) into (7) gives

$$(9) \quad U'[w_t - c_t(x_t)]c'_t(x_t) = \beta E_t \{U'[(q(x_t) - f_t)p_{t+1} + p_t^f f_t]\} p_t^f q'(x_t),$$

which, unlike (4), cannot be simplified because the current and expected marginal utilities it contains are not identically equal.^{2/} Even in this two-period economy, the assumption that the firm maximizes the expected value of a discounted sum of utilities implies that its storage decision is not independent of its attitudes toward risk and its beliefs about the probability distribution of the second-period spot price.

Introducing a Credit Market Resolves the Problem

If a firm's storage and hedging decisions could be separated only when it maximized the expected value of its utility of two-period total profits, then the separation result (5) would be of limited use in modeling farmers' optimal postharvest marketing strategy. However, if we add a credit market to the model so that firms can borrow or lend at an interest rate r_t , then the separation result can be derived for the typical multiperiod objective function as well. The key role that credit markets play in separating the firm's storage and hedging decisions in a multiperiod problem is to allow the firm to equate its current and expected marginal utilities by smoothing out its profits over time. In particular, firms will borrow or lend so as to equate terms in current and expected marginal utility analogous to those in equation (9) whose non-equality blocked the derivation of the separation result.

These claims can be established by solving the following multiperiod problem. Let the firm's objective be

$$(10) \quad \max_{\{\mu_t\}} E_0 \left\{ \beta^n U[w_n] + \sum_{t=0}^{n-1} \beta^t U[w_t - c_t(x_t) - s_t] \right\}$$

subject to a given w_0 and

$$(11) \quad w_{t+1} = (1+r_t)s_t + (q(x_t) - f_t)p_{t+1} + p_t^f f_t,$$

and

$$(12) \quad (x_t, s_t) = \mu_t(w_t),$$

where s_t is the firm's borrowing ($s_t \leq 0$) or lending ($s_t > 0$) at time t and $\mu_t(\cdot)$ is a contingency plan for selecting x_t and s_t based on w_t , r_t , $c_t(\cdot)$, and f_t . The firm's maximization problem can be solved by dynamic programming. According to this procedure (Bertsekas, pp. 50-51), the firm's choice of x_t , f_t , and s_t , for $t = 0, 1, \dots, n-1$, is determined as the solution of

$$(13) \quad \max_{x_t > 0, f_t, s_t} \{U[w_t - c_t(x_t) - s_t] + \beta E_t V[w_{t+1}]\},$$

subject to (11) and (12), where $V[w_{t+1}]$ is the optimized value of the problem from period $t + 1$ on; that is,

$$(14) \quad V[w_{t+1}] = \max_{\{\mu_t\}} E_{t+1} \left\{ \beta^n U[w_n] + \sum_{j=t+1}^{n-1} \beta^j U[w_j - c_j(x_j) - s_j] \right\}.$$

Since, for $t = 0, 1, \dots, n-1$, the constraint set ($x_t > 0$, f_t and s_t unrestricted) is convex and U is strictly concave, the optimized value function V is strictly concave and the necessary and sufficient conditions for maximizing the firm's problem are

$$(15) \quad U'[w_t - c_t(x_t) - s_t] c'_t(x_t) = \beta E_t \{V'[w_{t+1}] p_{t+1}\} q'(x_t)$$

$$(16) \quad \beta E_t \{V'[w_{t+1}] p_{t+1}\} = \beta E_t \{V'[w_{t+1}]\} p_t^f$$

$$(17) \quad U'[w_t - c_t(x_t) - s_t] = \beta E_t \{V'[w_{t+1}]\} (1+r_t).$$

Substituting (16) into (17) gives

$$(18) \quad U'[w_t - c_t(x_t) - s_t] c'_t(x_t) = \beta E_t \{V'[w_{t+1}]\} p_t^f q'(x_t),$$

which is similar to (9). In this case, however, the differing marginal utility expressions on both sides of (18) do not block the separation result, for, according to (17), the firm borrows or lends so as to make the marginal utility of current consumption proportional to the expected marginal utility of next period's initial wealth.^{3/} From (17) and (18) we obtain

$$(19) \quad (1+r_t)c'_t(x_t) = p_t^f q'(x_t),$$

which is the separation result for the more general problem.

According to this version of the separation result, the forward marginal cost of output (costs incurred in the current period times the one-period rate of return on loans) equals the forward value of marginal product, a natural extension of (5) to the more general problem. In fact, if we explicitly introduce credit markets in the objective function (1) by charging for the interest forgone on resources devoted to storing grain, then (1) becomes

$$(20) \quad \max_{x \geq 0, f} E U[(q(x)-f)p + p^f f - (1+r)c(x)]$$

and the separation result (5) would be identical to (19). In that sense, Danthine and Holthausen's analysis of the two-period, utility of total profits problem is a shortcut method for deriving the optimal storage decision of a firm maximizing an n-period sum of discounted utilities of profit, provided the firm can borrow and lend at a fixed interest rate. Where this assumption holds at least approximately, their method or the dynamic programming procedure developed here can be used to generate testable hypotheses about farmers' grain storage decisions.

Conclusion

We have shown that the separation result obtained by Danthine and Holthausen for a firm maximizing the expected utility of its two-period total profit also holds for a firm maximizing the discounted n -period sum of its expected utilities of profits, provided the firm is allowed to borrow and lend as well as trade forward contracts. Under these assumptions, the two-period, utility of total profit analysis of Danthine and Holthausen can be thought of as a shortcut method for deriving the storage decisions of a firm facing a multiperiod problem. These results help justify the use of both the separation result and the two-period, utility of total profit method of analysis in studies of multiperiod agricultural decision problems such as farmers' optimal postharvest grain storage strategy. However, we have also shown that multiperiod problems of this type can be directly analyzed through dynamic programming. A logical next step would be to exploit the dynamic programming method to rigorously extend to an n -period storage and hedging problem the additional conclusions that Danthine and Holthausen derived from the separation result.

FOOTNOTES

1/As Danthine (p. 83) points out, the technology may be stochastic, but only if the realization of the error term does not affect the marginal productivity of the input (for example, if the error term is additive).

2/This difficulty is not caused by the new parameters w_t and β , as can be seen by setting $w_t = 0$ and $\beta = 1$.

3/The factor of proportionality, $\beta(1+r_t)$, equals one under the common assumption that $\beta = (1+r_t)^{-1}$.

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