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NOTES ON HUMAN CAPITAL, GROWTH, AND UNEMPLOYMENT

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The model

We use an overlapping generations model. The intent is to develop a simple abstraction, which is capable of generating unemployment as an equilibrium phenomenon, and embed it in a model where growth occurs as a consequence of technological changes which impact one industry at a time. In this environment, we examine the incentives individuals have to accumulate industry-specific human capital. The argument quite simply is that if technological changes cause industry-specific human capital to become obsolete, then higher growth causes less investment in specific human capital and therefore, possibly less unemployment.

It is useful in making this story precise to model the nature of unemployment. We focus on "wait" unemployment, rather than "search" unemployment. We model unemployment as an equilibrium phenomenon which arises because workers and firms are unable to make binding state-contingent contracts. Firms are assumed to be better informed than workers about technology shocks. More precisely, we assume that the information available to the state or other enforcers of contracts is limited and hence, restricts the set of enforceable contracts.

A firm is assumed to have a technology for producing a product given by

$x = f(\theta, n)$

x = output

θ = random variable with a continuous density $g(\theta)$ on $[0, \bar{\theta}]$

n = number of workers employed

$f = [0, \bar{\theta}] \times \mathbf{R}_+$ is increasing twice differentiable and concave in n.

We will also assume that the marginal productivity of labor is increasing in θ. There are y workers attached to the firm. Each worker has the option of supplying one unit of labor or not working. Workers derive no disutility from work and, for simplicity, are assumed to be risk neutral. The firm maximizes expected profits.

Prior to the realization of θ, firms offer a contract to workers which specifies their consumption (or wage) and the number of workers employed contingent upon the realization of θ. The contract must guarantee a minimal ex-ante level of utility T. The idea is that workers are perfectly mobile ex-ante and perfectly immobile ex-post. The realization of θ is private information to the firm.

In addition, we will assume that the number of workers actually employed ex-post is private information to the firm. This may seem unreasonable but in an enriched model where the work week can also be varied, it is more plausible to argue that the aggregate labor input is not observed. It is difficult for indi-

vidual workers to monitor the precise level of labor input to the firm when there are variations in the ability and efficiency of other workers. This kind of assumption is particularly plausible if the firm has many locations where production occurs. Given these assumptions, it follows that the contract cannot be contingent upon θ or on the number employed, which is n. The contract must therefore involve a wage payment to each worker who works, w^1 (observable by the worker himself) and a payment to those who do not work, w^2. Given these payments, the firm then chooses the ex-post level of employment efficiently. We will assume that the consumption sets of the owners of the firm are bounded below by zero. In this environment, the contract must involve a wage payment of zero to those who do not work and a constant wage to those who do. The optimal contract then solves the following programming problem:

$$(2.2) \qquad \max_{\{w,n(\theta)\}} \int_0^{\bar{\theta}} \{f(\theta,n(\theta))-wn(\theta)\}g(\theta)d\theta$$

$$\text{s.t.} \qquad w\int_0^{\bar{\theta}} \frac{n(\theta)}{y}\, g(\theta)d\theta \geq T$$

$$\text{and} \qquad f_2(\theta,n(\theta)) \geq w \qquad \text{all } \theta,$$

$$0 \leq n(\theta) \leq y \qquad \text{all } \theta.$$

In this simple model, unemployment is generated because of private information. It is also useful to note that the constancy of the wage rates across states is ensured <u>not</u> because of different attitudes towards risk on the part of firms and the workers, but because of the inability to make the wage payment

contingent upon the state of nature or upon employment--neither of which is observed by workers.

This model is now embedded into a growth economy. We use an overlapping generations model primarily because we need to develop a model with heterogeneous agents and it is easier to characterize the equilibrium in such an environment. At any point, there are old and young workers. Old workers have human capital which is specific to the industry in which they were trained when they were young. There are a large number of firms in each industry. Each firm has the same number of old workers and in equilibirum, will have the same number of young workers. In each period, a new industry is born. The technology is such that when an industry is born, workers must spend one period learning the new technology, during which time no production takes place. Production is possible only in those firms who belong to industries that were born in previous periods. New industries are more productive than old industries. Over time, therefore, the oldest industries will be unable to attract any young workers and will vanish.

There are two sources of uncertainty: (1) there are firm-specific technology shocks which are independent across firms and last for only one period, and (2) the productivity of new industries is a random variable. At the time that an industry (or a new technology) is born, everybody knows the productivity of that industry. This level of productivity is then constant for all time.

As pointed out earlier, it is impossible for young workers may be attracted to a given firm. The technology we posit, therefore, has the characteristic that even if no young workers are employed by a particular firm, production is still possible. The production function of a typical firm is:

Technology

$$x_{kt} = \gamma(k)F(\theta z_{1kt}, \theta z_{2kt} + N_{kt}) \qquad k < t,$$

where

x_{kt} = the output at time t of a firm in an industry born at time k.

z_{1kt} = the number of old workers assigned to Task 1.

z_{2kt} = the number of old workers assigned to Task 2.

N_{kt} = the number of young workers employed by the firm; they can be assigned only to Task 2.

θ = firm-specific technology shock which is independent across firms and industries.

F = $\mathbf{R}_+ \times \mathbf{R}_+ \rightarrow \mathbf{R}$ is linear homogeneous, increasing in both arguments, concave and twice differentiable $F_1(x,x) > F_2(x,x)$.

$\gamma(k)$ = the productivity of industry k.

We assume a particular process on $\gamma(k)$. We will assume that

(2.3) $\gamma(k) = \gamma_k \gamma(k-1)$,

where γ_k follows a first order marker process with c.d.f. given by $F(\gamma_k/\gamma_{k-1})$.

We will attempt now to motivate the particular structure assumed on the industry productivity level $\gamma(k)$. Assume for the time being that γ_k is a constant equal to γ. Then we can interpret γ as the growth rate of the economy in the steady state. Presumably, if the economy is stationary, in each period that a new industry is born, it is more productive by a factor of γ than its predecessor. We allow this growth rate to be persistent over time.

We can interpret the firm-specific technology shock θ as some measure of the amount of human capital acquired by old workers. Crucial to our story, however, is that workers do not know the value of θ until production actually occurs. We will assume for convenience that the distribution of θ is uniform:

(2.4) $\theta \sim \text{Unif}\,[0,\bar{\theta}]$.

References

At each period, a constant population of workers is born and live for two periods. All individuals are risk neutral and have preferences over consumption given by

(2.5) $c_1 + \beta c_2$

where

c_1 = consumption when young.

c_2 = consumption when old.

Information

At time t, $\gamma(t)$--which is the productivity of an industry born at time t--is known. The firm firm-specific technology shock γ is not realized until production takes place. The fundamental nonconvexity in this model lies in our assumption that workers must choose to work in only one firm and are either employed or unemployed by that firm. Subsequent to the realization of γ, but prior to the realization of θ, each firm in the economy offers a "contract" to young workers. A contract is an enforceable promise to pay a given worker a wage, conditional on the realization of the firm-specific shock. The realization of γ is assumed to be private information to the owners of the firm. We assume that contracts cannot be made contingent upon the number of workers employed by the firm. This assumption can be justified on the grounds that it is difficult for those who must adjudicate contracts to verify the number of workers employed. Alternatively, any worker might not be able to observe the total labor input into the firm.

We have already shown in Section 2 that the optimal contract involves payment of a fixed wage to each worker if he is employed, and zero otherwise.

Endowments

Each individual is endowed with one unit of labor when young and one unit of labor when old. Old workers own the firm in which they were trained when young.

Optimal contracts and human capital accumulation

Subsequent to the birth of a new industry with its associated productivity level $\gamma(t)$ and prior to the realization of the firm-specific shocks θ, each firm offers a contract to young workers. The contract specifies a wage rate and a probability of employment. The firm also decides the number of young workers, Y, to whom the contract is to be offered.

All young workers who accept a contract with a given firm acquire human capital and will be the owners of the firm in the subsequent period. Essentially, workers accept a contract with a firm and are then trained. The productivity shock θ is realized and the firm decides how many workers to employ in production. Workers who accept a contract with a particular firm are perfectly immobile thereafter.

The two somewhat extreme assumptions made here require some justification. First, there were no choices made between consumption and accumulation of human capital. Second, we do not require that workers be actually employed in a given firm to become future owners of the firm. We merely require that the workers accept a contract to become eventual owners of the firm. As will become apparent, we do not believe that incorporating learning by doing radically alters our results. While individual agents do not make choices within a given firm over capital accumulation versus consumption, they do make choices of whether or not to enter a new-born industry or an existing firm. Entering a new industry involves no consumption when young. Of course, higher productivity in the new industry implies higher consumption

when old. As a consequence, individuals and the economy make decisions about current versus future consumption.

A typical firm faces a market determined parameter, T, which is the minimum expected utility from current consumption that the firm must pay its workers. The value of this parameter T will, in general, be different for different firms since any worker compares the present value of the expected utility associated with accepting a contract with a given firm.

We now characterize the optimal contract. The programming problem faced by a firm is simplified by adopting some new notation. Let

$$(2.6) \qquad y = \frac{Y}{z\bar{\theta}}$$

$$n = \frac{N}{z\bar{\theta}}$$

$$w = \frac{W}{\gamma(k)}$$

where

Y = the number of young workers offered a contract.

N = the number of young workers employed ex-post.

W = wage offered a young worker.

$\bar{\theta}$ = maximum value of θ.

Since the number of old workers in a firm is a fixed input, we can use the number of young workers per old worker as a decision variable. We have assumed that no old workers choose to work in other firms as young workers. In a stationary equilibrium, we will show that they have no incentive to join other firms.

We start by solving the decision problem of old workers given a contract, and subsequent to the realization of θ. Old workers must make decisions on how many young workers to employ and the number of old workers to assign to Tasks 1 and 2. The probelm faced by old workers is then given by

$$(2.7) \quad \max_{z_1,N} \quad \gamma(k)F\big(\theta z_1, \theta(z-z_1)+N\big) - WN$$

$$\text{s.t.} \quad 0 \le N \le Y,$$

$$0 \le Z_1 \le Z.$$

Using the constant returns to scale assumption on the production function F, letting $x = \frac{z_1}{z}$ and using equation (2.6), the solution to this problem can be obtained by solving

$$(2.8) \quad \max_{x,n} \quad \theta\{F\big(x, 1-x+\tfrac{n}{\theta}\big) - w\,\tfrac{n}{\theta}\}$$

$$\text{s.t.} \quad 0 \le x \le 1,$$

$$0 \le n \le y.$$

The necessary and sufficient first order conditions are

$$(2.9) \quad F_1\big(x, 1-x+\tfrac{n}{\theta}\big) = F_2\big(x, 1-x+\tfrac{n}{\theta}\big) \qquad \text{if } 0 < x < 1.$$

and

$$(2.10) \quad F_2\big(x, 1-x+\tfrac{n}{\theta}\big) = w \qquad \text{if } 0 < n < y.$$

We use Euler's theorem to show that both equation (2.9) and (2.10) cannot hold. We have that profits are given by

$$(2.11)\quad F\left(x,1-x+\frac{n}{\theta}\right) - w\,\frac{n}{\theta} = xF_1\left(x,1-x+\frac{n}{\theta}\right) + \left(1-x+\frac{n}{\theta}\right)F_2\left(x,1-x+\frac{n}{\theta}\right) - w\,\frac{n}{\theta}.$$

If both equations hold then we have

$$(2.12)\quad \text{PROFITS} = F_2\left(x,1-x+\frac{n}{\theta}\right).$$

The right side of equation (2.12) is increasing in x. Hence, it cannot be that $x < 1$. Inspection of equations (2.9) and (2.10) reveals that for sufficiently low values of θ, equation (2.10) will be binding. For sufficiently high values of θ, it must be that $n = y$, and

$$(2.13)\quad f\left(\frac{n}{\theta}\right) = F\left(1,\frac{n}{\theta}\right).$$

The critical value of θ, such that employment equals the number of contracted workers, is denoted by θ^* and is given implicitly by

$$(2.14)\quad f'\left(\frac{y}{\theta^*}\right) = w.$$

For values of θ less than θ^*, we therefore have the implied demand function for young workers given by

$$(2.15)\quad n(\theta) = \theta f'^{-1}(w).$$

For values of θ larger than θ^* but with $x = 1$, of course, $n(\theta) = y$.

We also need to define another critical value of θ: that at which old workers get assigned to Task 2, or when equation (2.9) becomes binding. This is given by

$$(2.16) \qquad F_1\left(1,\frac{y}{\theta^{**}}\right) = F_2\left(1,\frac{y}{\theta^{**}}\right).$$

For all values of θ greater than θ^{**}, it also follows from the fact that the partial derivatives of F are homogeneous of degree zero that

$$(2.17) \qquad F\left(x,1+\frac{y}{\theta}-x\right) = \left(1+\frac{y}{\theta}\right)F\left(x^*,1-x^*\right).$$

Where x^* solves

$$F_1\left(x,1-x\right) = F_2\left(x,1-x\right),$$

let $F(x^*,1-x^*) = K$.

We are now in a position to define the profit function of the firm.

$$(2.18) \qquad \pi(w,y) = \int_0^{\theta^*}\theta\left[f(f'^{-1}(w)-wf'^{-1}(w)\right]d\theta + \int_{\theta^*}^{\theta^{**}}\theta\left[f\left(\frac{y}{\theta}\right)-w\,\frac{y}{\theta}\right]d\theta$$
$$+ \int_{\theta^{**}}^{1}\theta\left[\left(1+\frac{y}{\theta}\right)k-w\,\frac{y}{\theta}\right]d\theta,$$

where the cutoff points γ^* and γ^{**} are defined in equations (2.14) and (2.16) respectively. Recall also the assumption that γ is drawn from a uniform distribution.

The constraint facing the firm requires it to pay a minimum expected consumption to young workers by choosing a wage rate and the number of young workers. Implicit in the decision wage rate is an ex-post employment decision. This, combined with the number of young workers offered a contract, yields a probability of employment for each worker. Using equation (2.15), the programming problem faced by a firm is given by

$$(2.19)\qquad v(\gamma(k),T) = \underset{w,y}{\text{Max}}\ \gamma(k)\pi(w,y)$$

$$\text{s.t.}\qquad w\Big[\int_0^{\theta^*} \frac{\theta f'^{-1}(w)d\theta}{y} + \int_{\theta^*}^1 d\theta\Big] \geq \frac{T}{\gamma(k)}.$$

Obviously, the solutions to this program depend only upon the ratio of T to $\gamma(k)$. Denote these solutions by $w(\frac{T}{\gamma(k)})$ and $y(\frac{T}{\gamma(k)})$ respectively. Let $v(1,T) = v(T)$. It then follows that

$$v(\gamma(k),T) = \gamma(k)v(\frac{T}{\gamma(k)}).$$

We now characterize the solutions to this problem. We prove, as might be expected, that $v(T)$ is convex and $y(T)$ is decreasing in T and $w(T)$ is increasing in T:

$$\text{Max}\ \int_0^{\theta^*}\theta\big[f(f'^{-1}(w))-wf'^{-1}(w)\big] + \int_{\theta^*}^{\theta^{**}}\big[\theta f(\tfrac{y}{\theta})-wy\big]$$

$$+ \int_{\theta^{**}}^1\big[k(\theta+y)-wy\big]$$

$$\text{s.t.}\qquad w\big[1-\frac{y}{2f'^{-1}}(w)\big] = T$$

$$\text{or}\qquad y = 2f'^{-1}(w)\big[1-\frac{T}{w}\big].$$

$$\frac{Ty}{w} = \Big[\int_{\theta^*}^{\theta^{**}}[f(\tfrac{y}{\theta})-w] + \int_{\theta^{**}}^1[k-w]\Big]\Big\{2f'^{-1'}(w)\big(1-\frac{T}{w}\big)+2f'^{-1}\frac{(w)T}{w^2}\Big\}$$

$$\text{or}\qquad \frac{\frac{T}{w}\big(1-\frac{T}{w}\big)}{\frac{wf'^{-1'}(w)}{f'^{-1}(w)}\big(1-\frac{T}{w}\big)+\frac{T}{w}} = \Big[\int_{\theta^*}^{\theta^{**}} f'\frac{(\frac{y}{\theta})}{w} - \int_{\theta^{**}}^1\big(\frac{k-w}{w}\big)\Big]$$

$$\text{or}\qquad \frac{1}{\dfrac{\frac{wf'^{-1'}(w)}{f'^{-1}(w)}}{\frac{T}{w}} + \dfrac{1}{1-\frac{T}{w}}} = \Big[\int_{\theta^*}^{\theta^{**}}[f'(\tfrac{y}{\theta})-w] + \int_{\theta^{**}}^1(k-w)\Big].$$

<u>Theorem</u>: $y(t)$ is decreasing in T and $\frac{T}{w}$ is increasing in T if

$$\frac{d\left(\frac{wf'^{-1'}(w)}{f'^{-1}(w)}\right)}{dw} \leq 0.$$

<u>Proof</u>: Recall

$$y = 2f'^{-1}(w)\left[1 - \frac{T}{w}\right].$$

Furthermore, (downward sloping demand) implies that $f'^{-1'}(w) \leq 0$. Hence, if $w(T)$ is <u>decreasing</u> in T, then $y(T)$ is decreasing <u>and</u> $\frac{T}{w}$ is increasing. It is also obvious that <u>if</u> $y(T)$ is increasing, then it <u>must</u> be that $\frac{T}{w(T)}$ is decreasing in T.

Suppose, in fact, that $y(T)$ is increasing in T. Let $x = \frac{T}{w}$ and $E(w) = w\,\frac{f'^{-1'}(w)}{f'^{-1}(w)}$, then we have

$$\frac{x(1-x)}{E(w)(1-x) + x} = \int_{\theta^*}^{\theta^{**}}\left[\frac{f'\left(\frac{y}{\theta}\right)}{w} - 1\right] + \int_{\theta^{**}}^{1}\left[\frac{k}{w} - 1\right].$$

We note that $\frac{1}{2} \leq x \leq 1$, $E(w) < 0$, and $E'(w) \leq 0$. Then $x(1-x) = x - x^2$ is decreasing in x because $\frac{\partial(x-x^2)}{\partial x} = 1 - 2x < 0;\ x > \frac{1}{2}$. Note that $\frac{\partial\left(E(w)(1-x)+x\right)}{\partial x} = -E(w) + 1 > 0$. Hence, the left side is decreasing in x. It is also increasing in w because $E'(w) \leq 0$ and $x \leq 1$. The right side is decreasing in w and decreasing in y.

Suppose x decreased in T. As we have shown, this implies y increases. Obviously $\left(x = \frac{T}{w}\right)w$ must increase. Therefore, the right side unambiguously falls. The left side, which is decreasing in x and increasing in w, rises. We have a contradiction. Hence, x must increase and y must decrease.

Now suppose $w(T)$ is decreasing in T. Then $x = \frac{T}{w}$ must be increasing in T and $y(T)$ must be increasing in T. Recall that the right side is decreasing in w <u>and</u> in y. Subsequently, the right side must increase. The left side is decreasing in x and increasing in w. Hence, the left side must fall and $w(T)$ must be increasing in T.

<u>Proof of convexity of $v(T)$ if $E(w)$ is constant</u>

Recall $v(T) = \pi[w(T), y(w(t),T)]$, where $w(T)$ solves $\pi_1(w,y(w,T)) + \pi_2(w,y(w,T))y_1(w,T) = 0$. By the envelope theorem, we have

$$v'(T) = \pi_2(w,y(w,T))y_2 = \left[\int_{\theta*}^{\theta**}[f'(\tfrac{y}{\theta})-w] + \int_{\theta**}^{1}(k-w)\right] \left(\frac{-2f'^{-1}(w)}{w}\right).$$

Therefore

$$R'(T) = \frac{-2f'^{-1}(w)x(1-x)}{\frac{wf'^{-1'}(w)}{f'^{-1}(w)}(1-x) + x},$$

and

$$R'(T) = \frac{-2f'^{-1}(w)x(1-x)}{E(1-x) + x}.$$

Now x is increasing in T and $x(1-x)$ is decreasing in T. Also, $w(T)$ is increasing while $f'^{-1}(w)$ is decreasing. Thus, $R'(T)$ is increasing in T.

Markov (2 chain)

(1) Output $V(T) + T$

(2) Unemployment (1 and 2)

(3) Average 2

(4) Sample paths for 2, output, unemployment. At each stage, check $y(\gamma T) = 0$ and $y(T) < 1$. For varying:

(a) $p = q > \frac{1}{2}$ prs $p = \frac{1}{2}$ (11D)

$\bar{p} = 3/4$ $p = .95$

$p = 1$

(b) $\gamma(1.25)$ Shift orst by prop. constant.

Shrink distribution.

(c) $\beta \in [.98^5]$

(d) $\delta \in [2/3, 3/4]$

NOTE: Benchmark: $\gamma = 1$

$y(T)$ $V(T)$ $w(T)$