

Federal Reserve Bank of Minneapolis
Research Department

IDENTIFYING POLICY EFFECTS

Christopher A. Sims*

Working Paper 351

May 1987

NOT FOR DISTRIBUTION
WITHOUT AUTHOR APPROVAL

*Federal Reserve Bank of Minneapolis and University of Minnesota.
Prepared for the Brookings International Models Comparison Pro-
ject. Research Supported in part by NSF Grant SES-8608078.

The views expressed herein are those of the author and not neces-
sarily those of the Federal Reserve Bank of Minneapolis or the
Federal Reserve System. This paper is preliminary and is circu-
lated to stimulate discussion. It is not to be quoted without the
author's permission.

I. Introduction

A striking characteristic of the simulations reported at this conference was the diversity of sizes of responses to fiscal and monetary policy actions reported by the various modeling groups. This paper argues that the real range of uncertainty about policy effects is at least as great as the range between the most extreme reported results. As was pointed out in discussion at the conference, policy authorities seem in fact to recognize this uncertainty. The reluctance of European governments to undertake fiscal expansion probably stems in part from doubt that demand expansion has substantial or persistent real effects, despite the consensus of most of the models represented here that it does.

The results displayed at the conference fit a rough pattern, with models claiming to allow for rational expectations showing smaller fiscal policy effects than models not making that claim. It might seem then that the uncertainty about those effects is closely tied to the profession's uncertainty about the validity of the rational expectations assumption. But this pattern in the results is artificial. As will be explained more completely below, neither rational expectations nor Keynesian macroeconomic theory resolves the question of the strength of fiscal policy effects on demand a priori. Neither is it true that, given either rational expectations or Keynesian assumptions, the data then resolve the remaining uncertainty. The apparent consistency of results across models of a given type at the conference is an artifact of their all failing to confront directly

the implications for inference of policy endogeneity--the fact that changes in controllable variables do not arise from changes in policy alone.

We present an approach to identifying the effects of policy changes which does not require any claim that we know a priori whether expectations are rational. Results from applying this approach suggest that the main sources of uncertainty about fiscal and monetary policy effects are the difficulties in distinguishing movements in controllable variables generated deliberately from those occurring as passive responses to developments elsewhere in the economy. There is, for example, difficulty in distinguishing interest rate changes due to monetary policy from those due to changes in real productivity. There is also difficulty in distinguishing policy-generated changes in government expenditures from random fluctuations in the timing of budgeted expenditures across the fiscal year. It turns out therefore that the statistical evidence has multiple interpretations. The tendency of output to fall following a rise in the interest rate may reflect either the ability of monetary policy to influence real output or the anticipation by financial markets of other influences on the level of output. The large fiscal multipliers, uniform across categories of expenditure, in many of the conference models can be reproduced with flexibly identified VAR's which exclude prices and financial variables, but not in larger VAR's. This could either reflect misspecification of financial sectors of the models which show large multipliers, or a tendency of financial variables spuriously to absorb predictive power because of their quick response to new information.

Section II describes the paper's identification methodology. It is more abstract and technical than the rest of the paper and could be skipped by readers interested in results. Section III sketches the foundations in uncertainty about economic behavior of our uncertainty about the effects of policy. Section IV, which could be read independently, lays out the range of results emerging from varying plausible identifying restrictions.

II. Identifying Policy Disturbances

An econometric model has the general form

$$(1) \quad F(Y(t-s), s \geq 0; X(t-s), s \geq 0; \beta_F; u_F(t)) = 0$$

$$G(X(t-s), s \geq 0; Y(t-s), s \geq 0; \beta_G; u_G(t)) = 0,$$

where Y and X are the variables in the model, the a 's are unknown parameters of behavior, and the u 's are random disturbances. The model is ordinarily specified to be dynamically complete, in the sense that the equations in (1) can be solved for $Y(t)$ and $X(t)$ at each t given values for the other arguments and that in addition by successive substitution (1) can be used to define $Y(t)$, $X(t)$ as a function of $\{u_F(s), u_B(s); s \leq t\}$.¹

We distinguish G as the set of equations defining policy, and we interpret the path of the stochastic process u_G as representing changes in policy through time. Using the model to find the effects of a change in policy is then a simple numerical exercise if we have given forms for G and F and given values for their arguments. We make the changes in u_G corresponding to the change in policy under consideration, solve the model for new

values of Y and X , and interpret the changes in the solution as the effects of the change in policy.

While some economists will agree that this scheme is a reasonable way to approach quantitative policy analysis, it is nonetheless controversial. The controversy is symptomatic of the current absence of communication in the economics profession between members of the rational expectations school, who think they are building an entirely new approach to quantitative macroeconomics, and economists actually engaged in quantitative policy analysis.

A. The conventional objection

Some economists actually using large macroeconomic models find the scheme above pedantic. In practice some subset of variables in the model is almost always designated as a vector of controllable, or policy, variables. We will let X be the controllable variables. These are almost always taken to coincide with the list of dependent variables in the policy equations G .² Then any change in the values of $\{u_G(t+s), s=1, \dots, q\}$, while all variables dated t or earlier and $\{u_F(t+s), s=1, \dots, q\}$ remain fixed, will translate uniquely into a change in the values of $\{X(t+s), s=1, \dots, q\}$. The G equations serve no purpose but to make this translation; the effects on Y of the change in u_G are determined entirely by the F equations. Since variations in potential X paths are easier to interpret than variations in paths of the unobservable u_G disturbances, potential variations in policy are in practice almost always specified in terms of the X paths, with no reference to the G equations. In fact, the memoranda specify-

ing the simulation exercises for this conference are a case in point--they focused almost entirely on defining perturbations in the paths of policy variables.

The foregoing objection to defining policy changes in terms of disturbances to the policy equations is completely justified if we take the model structure (1), including the values of its parameters, as known without error and uncontroversial. But of course there is great uncertainty and dispute about the validity of the structure of any actual model. Any method for inferring from data the values of the parameters β_F which determine the response of the economy to policy must depend on knowing the form of G . The fact that G is not needed to carry out numerical exercises in policy analysis once a fixed form for F is accepted does not mean that our conclusions from the data about the response of the economy to policy do not depend on G .

Most economists have taken an econometrics course in which they learned that one mathematically convenient set of assumptions about G is that current values of private sector variables, $Y(t)$, do not appear in G and that u_F and u_G are mutually uncorrelated and serially uncorrelated. This set of assumptions is summarized as the assertion that controllable variables X are "predetermined" in the remainder of the model. If X is predetermined, standard procedures for estimating β_F apply without further need to use the structure of G . Most large macroeconomic models in use now pay little or no attention to the simultaneous equations methods logically required for estimation even if some elements of Y are not predetermined in F . Failure of X to be

predetermined, since it raises complex difficulties in simulating responses to policy changes as well as in estimation, is scarcely acknowledged as a possibility.³

Yet predeterminedness of X is obviously an unreasonable assumption. Taxes, expenditures, monetary aggregates, and interest rates all obviously respond systematically within a quarter to disturbances originating in the private sector. When modelers write down forms for G , they make such contemporaneous responses explicit. How can they then proceed as if endogeneity of controllable variables were not central to interpreting model results? The reason is that, in large models as currently implemented, the statistical methods for dealing with endogeneity of policy, indeed of simultaneity in general, have often seemed to make little difference to estimation results. For policy endogeneity itself, this point was documented by Goldfeld and Blinder [1972] in an influential article.

The standard econometric prescription for dealing with the possibility that X is endogenous is instrumental variable estimation. We deal with the possibility, say, that the interest rate $R(t)$ does not vary solely in response to changes in monetary policy by finding an instrumental variable $Z(t)$ which is correlated with changes in monetary policy but not correlated with any of the other influences on $R(t)$. $Z(t)$ is then used, essentially, as an index to separate episodes of movement in R which are likely to have been due to policy change from episodes which are likely not to have been. Goldfeld and Blinder's paper showed that use of instrumental variables to estimate parts of F containing X vari-

ables gave results very similar to those obtained treating X as predetermined.

But stated informally, as a matter of substantive economics, the problem of distinguishing movements in interest rates or monetary aggregates (say) which are due to policy changes from movements which are responses to developments elsewhere in the economy appears very difficult. If reliable instrumental variables Z were available, the problem would not be difficult. Where did Goldfeld and Blinder get their Z variables? In conventional macroeconometric models, both when Goldfeld and Blinder wrote and as represented in the models displayed at this conference, the model structure implies that there are many candidate Z's and that they are highly correlated with the X's. It is the fact of their high correlation with the X's that makes results insensitive to whether instrumental variables are used or X's are treated as predetermined. But this means that the models' structures imply that there is in fact no serious policy endogeneity problem: essentially all the variation in the X's is attributable to variation in policy.

This apparent implication of the models flies in the face of common sense and is an indictment of the models' structures. Given a system like (1) and the assumption that u_F and u_G are uncorrelated, the potential Z's for handling possible endogeneity of X are just those variables which appear in G but not in F. That is, they are variables which have an important influence on policy, but no influence on the economy except through their influence on policy behavior. Such variables must be extremely

rare. Some model builders have worked under the illusion that the Z's may be variables like the Federal Reserve discount rate, which never change except by deliberate decision of a policy maker. But the fact that such variables only change deliberately does not make them potential instruments unless policy makers do not pay any attention to current economic conditions when they change the variables. Furthermore, given any Z variable which enters G, the rational expectations critique explains that if expected future values of X matter to the private sector, then observations on Z will affect private sector behavior via effects on forecasts of X even if the private sector is not influenced directly by Z.⁴

Goldfeld and Blinder's comforting result then depends on taking too seriously the structure of conventional macroeconomic simultaneous equations models. The apparent availability of many instrumental variables, highly correlated with controllable variables, is an illusion generated by the tightly restricted forms which are routinely imposed on such models to stabilize their behavior as forecasting aids and to make them convenient to manipulate in policy analysis exercises. The same criticism applies with equal force to models which claim to account for the rational expectations critique while allowing quantitative policy analysis. Such models have universally (as far as I know) taken predeterminedness of policy variables as given, without testing it.

Specifying policy changes as changes in the path for u_G is not pedantic. It forces us to confront the fact that not all changes in controllable variables are necessarily policy changes, that it is the effect of policy changes, not the effect of other changes in controllable variables, which we want to isolate.

B. The rational expectations objection

The rational expectations critique of econometric policy evaluation is sometimes interpreted as implying that attempts to project the effects on Y of changes in u_G are pointless or self-contradictory. The argument is that β_F , which determines the response of the private sector to u_G , is generally functionally dependent, via expectation-formation, on β_G . We can think of G as the stochastic mechanism by which random disturbances u_G generate X . The methods we have suggested for analyzing the effects of u_G on Y thus depend on our treating the paths of u_G we feed through the model as drawn from the historically given probability distribution for u_G paths. If we attempt to analyze u_G paths which cannot plausibly be taken as drawn from the historical distribution, we are in effect changing the stochastic mechanism generating X . That is, we are in effect changing β_G which thereby changes β_F . The change in β_F invalidates the analysis.

Yet, the argument goes, "real" policy changes are precisely changes in the stochastic mechanism generating policy. If G and the stochastic process generating u_G are left unchanged, the overall stochastic process generating Y and X remains fixed. The objective of policy should be precisely to change this stochastic process for the better. Policy analysis which must be conditioned on this process remaining unchanged is at best considering only trivial changes in policy, at worst internally inconsistent. What must be the object of our analysis is the effect on the stochastic process for Y and X of changes in β_G and in the parameters characterizing the u_G process. We can carry out such an analysis if we

correctly model the dependence of β_F on these parameters. This point of view is well explained by Sargent [1984].

The counterarguments to this rational expectations view have two main components:

- i) The fact that a potential policy change must be regarded as drawn from a given probability distribution does not mean it is trivial.
- ii) There is really no alternative to recognizing that policy changes are drawn from a given probability distribution.

Because I have argued against this rational expectations stance at length elsewhere ([1980], [1982], [1985], and [1986]), here I will only illustrate the counterarguments with examples. In a football game, the play called on a given down has its effect determined in part by what the defense expects the offense to do. Because of this, football teams develop game plans which specify the probability mix of plays. A game plan may call for passes 50 per cent of the time on first down, e.g. This does not make the choice of whether to pass or not on a particular first down trivially easy or unimportant. Analogously, the fact that we are choosing monetary and fiscal policies in a way which involves no fundamental shift in the nature of the public's uncertainties about likely future policies does not mean that the choices are trivially easy or unimportant.

Choice of the game plan itself presents the same need to recognize that opponents have a range of uncertainty about what that game plan will be. It could conceivably be useful to predict

the effects of a game plan while assuming that it will be a total surprise to the opposing team; but this is only because in practice opposing teams may not be fully rational. Correspondingly, rational expectations theorists may find it useful to replace the money illusion they banish from their models with a "greek alphabet illusion", which supposes that the public treats β_G as fixed even as policy authorities contemplate changing β_G . But this does amount to assuming irrationality on the part of the public. A rational public will have a probability distribution (or, more likely, many such distributions across individuals) over the range of possible policy actions. Ordinarily it will be unreasonable to suppose that policy actions can induce the public to abandon their previous models of policy behavior rather than simply updating their probability models with the new data provided by current policy actions.

The conclusion then is that there is no logical inconsistency or inherent triviality in attempting to determine the effects on Y of changes in the path of u_G . There are legitimate objections to the way conventional simultaneous equations models have been used to analyze the effects of policy changes, but these objections amount to asserting that those models do not correctly identify the responses of Y to u_G . Rational expectations does not in fact offer any valid alternative to the basic framework of econometric policy analysis: we attempt to quantify fluctuations in policy historically and then to infer the likely effects of future policy changes, which are also conceived as fluctuations in some quantitative measure of policy stance. Both past and future fluctuations in policy stance should be modeled as stochastic.

C. A new approach to identification

The conventional claim that policy variables are predetermined amounts to a claim of independence of certain disturbances (u_G and u_F) and minimum delay in the effects of some variables on others (a delay of one period in the effects of Y on X in the policy equations). The assertion of minimum delay in all effects of private sector variables on controllable variables is implausibly restrictive. The conventional response to this, as we have already pointed out, is to locate instruments for controllable variables. Candidates for instruments are variables appearing in the policy equations G which do not appear in the private sector equations F . Rational expectations implies that, via expectations formation effects, every variable appearing in G is likely to appear in F .

Nonetheless, there is some prospect of identifying the effects of policy by combining minimum-delay and independence restrictions in a somewhat unconventional way. We outline the approach here. The version we describe is especially convenient computationally and gives interesting results. In general, though, other types of identifying restrictions ought probably to be employed as well, even at some cost in convenience.

We begin by postulating a linear model in which the observable variables Y (we no longer need to distinguish X and Y) are determined by current and past values of a vector of underlying disturbances u according to

$$(2) \quad Y(t) = \sum_{s=1}^{\infty} A_s u(t-s) = A(L)u(t).$$

We suppose that the elements of $u(t)$ are mutually uncorrelated at a given date t and also that they are serially uncorrelated across t 's. This assumption is definitional: if the u 's are correlated, then the A 's can be redefined so that they are uncorrelated. We think of the u 's as the underlying, interpretable, determinants of the economy's behavior. Some of the elements of u are policy disturbances. The response of $Y_i(t+s)$ to a unit change in $u_j(t)$ is given by $A_{ij}(s)$. Thus the elements of A include the answer to the question posed by the policy analysis problem as we have formalized it.

The identification procedure we will use assumes that A in (2) has a one-sided inverse, meaning that (2) can be rewritten as

$$(3) \quad \sum_{s=1}^{\infty} B_s Y(t-s) = B(L)Y(t) = u(t).$$

This assumption is not special to this approach to identification. A standard simultaneous equations model begins with a system in the form (3).⁵ If $u_i(t)$ is, say, the monetary policy disturbance, then the i 'th equation of (3) defines the normal reaction of monetary policy to developments in the economy. If (2) is not invertible, then either some policy reaction function or some of the behavioral equations of the private sector cannot be written as functions of observable variables. Nonetheless the assumption is not innocuous. It fails obviously if u and Y are not of the same length. There is no reason in general that we should have data on a number of time series which exactly matches the number of underlying causal mechanisms in the econ-

omy. On the other hand, we may hope that the u 's can be ordered in importance. If so, the effect of having kY 's and $m > ku$'s may be small if k is large enough. The true A matrix would be $k \times m$, but the entries in the right-hand $m - k$ columns would be small. If we pretend A is actually $k \times k$, the resulting estimated A will then be close to the first k columns of the true A .

A conceptually more difficult problem is that even if A is square, it may fail to be invertible. For example suppose that real defense expenditures were determined entirely by fluctuations in defense policy unrelated to other influences on the economy. If real defense spending is Y_j and the underlying disturbance is u_j , then the j 'th row of $A(L)$ is zero except for the diagonal element $A_{jj}(L)$. But suppose further that defense policy decisions are known several quarters before they impact defense spending in the national income accounts. Then $u_j(t)$, the current disturbance to defense policy, will affect current values of interest rates, asset prices, and investment immediately, as investors anticipate the effects of the future change in defense spending. But A_{jjs} will be zero for $s = 1, \dots, q$, where q is the delay between a policy change and the effect on spending. If $A_{jj0} = 0$, then $A_{jj}(L)$ does not have a one-sided inverse. If $A_{ji}(L) = 0$, all i not equal to j for some one j , then $A(L)$ has no one-sided inverse if $A_{jj}(L)$ has none.

The reason this defense policy example raises a difficulty is that the only variables in the system which respond quickly to u_j also respond more strongly to other influences. There will be no way to construct current $u_j(t)$ from data on $Y(s)$

for $s \leq t$. A similar problem will arise whenever variables included because they define policy actually respond weakly in the short run to policy changes. Because revenues and expenditures are determined by budget decisions which are debated and decided well in advance of their effects on the national income accounts, it is easy to imagine that this kind of problem could be important in practice. We provide some evidence below consistent with this possibility.

Equation (3) is not far from the form of a vector autoregression (VAR). Observing that

$$(4) \quad B_0^{-1} = A_0,$$

we can multiply (3) through by A_0 to obtain

$$(5) \quad Y(t) = \sum_{s=1}^{\infty} C_s Y(t-s) + v(t) = C(L)Y(t) + v(t),$$

where $C_s = -A_0 B_s$, all s , and $v(t) = A_0 u(t)$. We have made assumptions adequate to insure that $v(t)$ is uncorrelated with $Y(s)$ for $t < s$ and thus that (5) meets the defining conditions for a VAR representation of Y . The system (5) is a reduced form model and relatively easy to estimate. Once we have estimates of $C(L)$, all we need to obtain the $A(L)$ matrix is A_0 , since

$$(6) \quad A(L) = B^{-1}(L) = A_0 [I - C(L)]^{-1}.$$

We have begun by assuming that the variance matrix for $u(t)$, call it Φ , is diagonal. We can estimate the variance matrix Σ for $v(t)$. By construction

$$(7) \quad \Sigma = A_0 \Phi A_0^{-1}.$$

We can expect to determine A_0 from observation of Σ if we can put enough a priori restrictions on A_0 so that the total of $k^2 + k$ free parameters in A_0 and Φ is reduced to no more than the $(k^2+k)/2$ distinct entries in Σ . Since k elements of A_0 can be set as normalizations, we must find at least $(k^2-k)/2$ restrictions, leaving no more than $(k^2-k)/2$ parameters free in A_0 itself.⁶

Restrictions which set some elements of $B_0 = (A_0)^{-1}$ to zero are minimum-delay restrictions. They are likely to be justified by claims about which agents in the economy can see and react to which variables with small delay. For example, we might suppose that policy authorities cannot react to income and the price level within the quarter because data on those variables is available only with a delay of close to a quarter. And we might suppose that demand for money does not react within the quarter to levels of government spending, because demand for money is not directly responsive to the composition of output and because the forecasting value of GNP components cannot affect the public's behavior before publication, which again has a delay of close to a quarter.

Minimum delay restrictions, justified as suggested here by assumptions about information flow, are quite consistent with rational expectations assumptions. Rational expectations implies

that any variable affecting government behavior is likely to enter the behavioral equations of the private sector, but such effects will be delayed as long as it takes for the public to receive data on the variables. It should be clear that these methods allow at least in principle the possibility that we can accurately estimate the effects of policy changes on the economy, in the presence of rational expectations, without even explicitly modeling expectations formation behavior.

Besides being consistent with rational expectations, this approach is convenient. If we impose restrictions on B_0 only, they imply no restrictions on C in (5). Therefore the VAR reduced form can be estimated by simple standard techniques, ignoring the identifying restrictions. Maximum likelihood estimation of B_0 can then be undertaken at a second stage, treating the \hat{y} estimated from the VAR residuals as data, with no loss of efficiency.

In section IV below we will apply these ideas to several models of differing size. We will see that they can give interesting and plausible results, but also that they suggest that disparate views of the effects of policy may remain consistent with the data.

First, though, we discuss briefly what a priori theory ought to lead us to expect from such a look at the data.

III. Ambiguities in the Theory

A. Ricardian ambiguity

That Keynesian macroeconomics allows the effects of fiscal and monetary policy to be relatively strong or weak depend-

ing on parameters of money demand, investment demand, labor supply and the like is well understood. The recent attention to Ricardian equivalence propositions in the rational expectations literature may suggest that there is no such uncertainty under the rational expectations hypothesis.

But rational expectations equilibrium need not imply the naive Ricardian conclusion that fiscal policy has no effect on demand. In fact, it is consistent with strong effects on demand from fiscal policy and absence of such affects from monetary policy. This range of possibilities is displayed in a particular rational expectations overlapping generations model by Gertler and Aiyagari [1985]. They point out that, depending on the way fiscal and monetary policy react in the long run to interest rates prices and budget deficits, the economy can be either "monetarist/Ricardian", with fiscal policy variations irrelevant to the price level while monetary policy variations affect it strongly, or quite different, with the price level dependent directly on fiscal policy and only weakly and indirectly on monetary policy. Though their model maintains the assumption of instantaneous price adjustment and market clearing, I believe that in a model with realistic short run frictions in nominal price adjustment the effects on prices they find for monetary and fiscal policy would translate into corresponding temporary effects on real variables.

Their result is robust to deviations from their simple equilibrium model. It arises from the interaction of interest rate and tax policy in any model in which government debt is backed by commitments to future taxation. We sketch the argument

heuristically here, and provide in an appendix a more thorough discussion for an explicit equilibrium model simplified along different lines from that of Gertler and Aiyagari.

The monetarist/Ricardian approach focuses on the demand for money. From a variety of possible underlying theoretical models one arrives at a liquidity preference relation of the form

$$(8) \quad g(M/P, y, R) = 0$$

as an equilibrium condition, with M the money stock, y a vector of real variables, P the price level, and R the nominal interest rate. If one supposes that y is little affected by government demand management and that R does not systematically permanently change its level in responses to changes in the level of M , then the level of M determines the price level through the liquidity preference relation g . The rational expectations equilibrium school is distinguished from the Keynesian school in being more willing to assume the relative independence from demand policy of y . But the assumption that R does not move permanently in response to changes in the level of M depends on assumptions about fiscal policy. A contraction in M with y fixed allows P to remain fixed in (8) if R rises. If there is no change in taxes or expenditures, the nominal deficit will then increase because of the higher R , and will grow exponentially. One way to describe this situation is to say that the fiscal authorities must increase taxes to prevent exponentially exploding debt if there is a permanent increase in the interest rate. If the fiscal authorities act as if they feel such a compulsion, then the monetary authority can set the money stock where it pleases and thereby control the price level.

But the situation is in fact more symmetric than this. Someone must prevent the exponential explosion of the debt. However one might as easily say that, if the fiscal authority refuses to adjust taxes to pay for increased interest payments on the debt, then the monetary authority must limit its attempts to control the level of M and hence of the price level. Contrary to what might be the intuition of some economists, such a situation does not result in an indeterminate price level. It only makes the price level depend on the aggregate volume of nominal debt as determined by the fiscal authorities. This is explained more fully by Gertler and Aiyagari and in the Appendix.

Another way to look at this same point is to shift focus from the liquidity preference relation to the forward-integrated government budget constraint. The current budget constraint can be taken as

$$(9) \quad B_t = R_{t-1}B_{t-1} + P_t(x_t - \tau_t),$$

where B is the outstanding amount of one-period bonds, R_t is the gross interest rate quoted at t on bonds to be redeemed at t + 1, P_t is the price level (the rate of exchange between goods and newly issued bonds), x_t is real government expenditures, and τ_t is real taxes. We ignore the possibility that high powered money enters the budget constraint. This is reasonable if all money is inside money or if high powered money is a small fraction of the government debt. Assuming that the nominal government debt cannot grow indefinitely at a rate which on average exceeds R, (9) can be solved forward to yield the identity

$$(10) \quad B_t = \sum_{s=1}^{\infty} (R_t R_{t+1}, \dots, R_{t+s-1})^{-1} (\tau_{t+s} - x_{t+s}) P_{t+s},$$

which implies

$$(11) \quad B_t/P_t = \sum_{s=1}^{\infty} (\theta_t \theta_{t+1}, \dots, \theta_{t+s-1})^{-1} (\tau_{t+s} - x_{t+s}),$$

where $\theta_t = R_t P_t / P_{t+1}$. Equation (11) states that the current real value of the government debt is the present value of future real budget surpluses net of interest, discounted at the ex post real rate of return on government debt, θ . A rational expectations equilibrium approach is likely to assume that the variations in θ are serially uncorrelated and not systematically related to policy. Under those assumptions, (11) asserts that, so long as its future real tax and expenditure choices are fixed, the fiscal authority will make the current price level move in proportion to the current level of nominal debt through its current choices between tax and debt finance.⁷

But of course this conclusion is dependent on side assumptions, just as is the conclusion from the liquidity preference relation that the monetary authority controls the price level. If the monetary authority insists on holding M fixed while the fiscal authority increases B , it will in general thereby increase the interest rate. Then the fiscal authority cannot keep its future real net surpluses fixed without generating explosive growth in the debt. If the monetary authority insists on holding M fixed (or more generally on increasing the nominal interest rate by more than the rate of increase in the price level), then the fiscal authority must accommodate by increasing taxes in reaction

to increased debt or interest expenses. But if the fiscal authority insists on holding real net surpluses fixed when it increases the nominal debt, then the monetary authority must accommodate by keeping nominal interest rates stable.

Referring to equation (11), we see that anticipated contraction in real expenditures would, *ceteris paribus*, tend to lead to current deflationary pressure. Suppose that system stability is ensured by a positive relation between real taxation and the gross deficit, including interest payments. If the monetary authorities react to current deflationary pressure by expanding the money stock and lowering nominal interest rates they will generate expectations of lower future taxation, offsetting some of the deflationary pressure. If this pattern of reactions is regular, unanticipated increases in money stock and declines in nominal interest rates will predict future budget surpluses. As pointed out in the appendix, such a pattern of higher future net surpluses following monetary expansion would be expected if fiscal policy is fixing the price level, while it would not be expected if monetary policy were fixing it. Furthermore, if high future output *ceteris paribus* generates high future taxes, then anticipations of high future output, through the same pattern of reactions, will be associated with current monetary expansions to offset the current deflationary pressure.

We can conclude, then, that a tendency for low interest rates to precede high future output and net surpluses could arise in a model in which expansionary monetary policy had no influence on output and no immediate impact on prices.

B. Financial market rationality in a Keynesian framework

Frictionless financial markets will make asset prices and interest rates quickly incorporate information relevant to predicting the future of the economy. In the simplest Keynesian model, autonomous expenditure fluctuations determine the level of income and employment with no feedback from price variables. Nonetheless, if asset prices quickly react to new information, they may have strong predictive value even in an economy accurately described by a pure multiplier-accelerator model. One expects, in this case, that introducing financial variables into an econometric model of the economy will obscure the true Keynesian dynamics.

Fiscal actions are ordinarily embodied in legislation before they affect revenues and expenditures. They are thus to a considerable extent known in advance, and might easily affect financial market variables before they affect revenues and expenditures. This is exactly the situation, discussed in section II above, in which any method of identification based on minimum-delay and orthogonality-of-disturbance assumptions⁸ will fail. We would expect models including financial variables to be biased toward showing weak effects from shifts in fiscal policy.

IV. Some Empirical Explorations

A. Fiscal policy effects

Table 1 shows the range of fiscal multiplier results produced by the models reported at the conference. The three rational expectations models (Taylor, Liverpool, and McKibbin-Sachs) produce 8 quarter multipliers ranging from 0.61 to 1.1.

The two models with the longest records and widest application as forecasting tools, DRI and Wharton, produce 8 quarter multipliers of 2.08 and 1.61.⁹ To gain some insight into whether the high or low end of this range is doing violence to the data, we present some identifications of VAR systems of varying size.

On the hypothesis that low multipliers might emerge when financial variables are allowed to absorb predictive power from fiscal variables, we first estimate a 7 variable VAR in nominal GNP components alone, for the U.S. The equations relating innovations (one-step-ahead forecast errors) in the identified version of this model are displayed in Table 2. (The data and VAR modeling methods for each model we will be discussing are summarized in the Model Notes section.) As can be seen from the upper panel of Table 5, this small VAR model with simple Keynesian identification produces implied 8 quarter multipliers for Defense Purchases and for State and Local Purchases that are well over one. The less than unity multiplier for nondefense purchases is not consistent with the larger multipliers for the other kinds of government expenditure under a simple Keynesian interpretation, but this component of spending is much smaller than the other two.

The lower panel of Table 5 shows that introduction of interest rates, money and prices changes this picture substantially. Nondefense purchases now have a negative 8-quarter multiplier and the 8-quarter multiplier on defense purchases has dropped below one. Note that the multipliers in the two models are conceptually comparable. Each is interpreted as the response of the economy to an unpredicted deviation of government expendi-

ture from its usual passive response to surprise movements in GNP. In each case monetary policy is presumed to react in its usual fashion to the disturbance--implicitly in the first model, explicitly via the money supply equation in the second model.

The last panel of Table 5 shows the multipliers from a particular identification of the world VAR model prepared for this conference. Here state and local expenditures are lumped with federal nonmilitary expenditures, and both are in real terms. The multipliers on military expenditure start smaller than in the middle panel, but do not shrink, staying close to one. The state and local multipliers remain large.

Two interpretations of these patterns suggest themselves. One is that one of the possibilities suggested in the preceding section is realized--Keynesian multipliers substantially greater than one and fairly uniform across expenditure categories are obscured when financial variables enter the system. The other is that the apparent large multipliers are misleading, that the world is non-Keynesian.

The Keynesian interpretation must confront the discrepancy in estimated multipliers for different kinds of government spending, even in the model excluding prices and financial variables. However, since we have omitted because of time constraints computing standard errors on these multipliers, the discrepancies may not be significant. And the identification scheme used here is not ideal; the failure to distinguish investment and interest payments on the debt in the variable list may be crucial.

If the Keynesian interpretation is correct, it suggests that Keynesian econometric modeling which includes financial variables in the system must take explicit account of rational expectations in financial markets and the difficulty of measuring fiscal policy changes if it is to produce accurate results. If the tendency of interest rates, money and prices to absorb predictive power from fiscal variables is spurious, standard simultaneous equations methodology applied to systems with money supply and demand relations will be distorted just as is the lower part of Table 5.

Conventional equilibrium business cycle models cannot easily explain the large "multiplier" on state and local expenditures which appears so consistently in the exercises reported here. Garcia [1987] has confronted that large observed multiplier with an equilibrium growth model and found that the multiplier can be explained--but only by allowing for the possibility of persistent underinvestment in state and local public capital.

To summarize, these fiscal multiplier exercises show that there is substantial uncertainty about the size of multiplier effects of fiscal policy changes. With different, apparently Keynesian, identification schemes one can obtain multipliers ranging from negative numbers to over 2, while maintaining essentially the same reduced form probability model of the data. A non-Keynesian interpretation of the data, in which the apparent multiplier effects are not demand effects at all, is probably as consistent with the data as one in which real Keynesian multiplier effects are obscured in large models containing financial variables.

B. Monetary policy effects

Next we will examine responses to monetary policy disturbances. In particular, we will ask whether expansion of the money supply and decline in interest rates is followed by a rise in the surplus net of interest payments or by a decline. In the model of the appendix, this would distinguish between a model where fiscal policy sets the price level (if low interest predicts lower net surplus or leaves it unaffected) or where instead monetary policy sets the price level.

Unfortunately we cannot resolve this question very precisely, because projections of net surplus or government interest payments are not reported in conference tables. Nonetheless we can see projected effects on interest rates themselves and the Congressional Budget Office [1985]¹⁰ provided projections of the direct effects of interest rate changes on the gross deficit via effects on interest payments. Those projections were that a sustained one percentage point increase in the interest rate would increase the deficit by 3, 9, 14, and 21 billion dollars in the first through fourth years, respectively. Using this lag distribution of effects and the reported model projections, we can determine whether they imply positive or negative effects on the net deficit. The results are displayed in Table 6.

The results are as diverse as for the fiscal multiplier analysis. The McKibbin/Sachs model, embodying rational expectations, shows a strong negative effect of monetary expansion on the net surplus, apparently fitting the idea that low interest rates from monetary expansion must translate into reduced future net

surplus if monetary policy determines the price level. The fit is only apparent, however, since the monetary expansion in this model produces only a short-lived effect on the interest rate. The Liverpool rational expectations model shows negligible effects of the monetary expansion on the net surplus, consistent with the negligible effects it finds on the interest rate and with an underlying theory that monetary policy determines price level. The VAR shows small effects of oscillating sign of the monetary expansion on the surplus, despite persistent large effects on the interest rate. Though the effect is small and the sign therefore uncertain, the sum over the four years is consistent with monetary determination of the price level. Surprisingly, this is even more true of the Wharton model: it is the only one which implies that a monetary expansion has substantial effects on the interest rate which lead to relaxed fiscal policy as the gross deficit decreases.

The remaining models all imply that monetary expansion substantially increases net surplus. Presumably the rationale for such an effect is that the expansion of output and rise in prices, with fixed fiscal policy, generate considerable additional revenue without corresponding increases in expenditures. The VAR model holds expenditures constant during the monetary expansion simulation, but holds the historical pattern of actual relations of taxes to prices and output constant rather than holding tax rate schedules constant. Its weaker positive response of revenues to expansion may therefore simply reflect a different specification of "constant fiscal policy." The implication would be that in

fact legislators tend to return most of the fiscal dividend of expansions to the taxpayers, cutting rates to reflect the reduced interest expenditures and increased tax base. It would be interesting to investigate whether the Wharton model explicitly includes a legislative tax rate response function or, like the VAR, uses a statistical average relation of revenues to other variables in generating its result.

The identification of the monetary shock in the VAR model for the conference simulations was based on hypotheses directly restricting the contemporaneous responses of variables in the system to money supply disturbances. We can obtain a check on the robustness of conclusions from that identification by comparing results from it to those from an identification in the style used for the small VAR's discussed in this paper, using minimum delay restrictions. The U.S. policy equations in innovations for this identification are displayed in Table 4. Comparing the money demand identifications in Tables 3 and 4, we see that the negative (large but insignificant) dependence of money demand innovations on real GNP innovations in Table 3 is replaced in Table 4 by a more plausible pattern of coefficients. This is probably because of the large number of foreign financial variables which are postulated to be orthogonal to U.S. money demand in the international model and which thereby strengthen identification of the demand equation.

The responses of five U.S. variables to shocks in money demand, money supply, and the "tax/surplus" equation are displayed in the Chart. Since the money supply does not move to a fixed

level and stay there in this simulation, it is not directly comparable to the simulations reported for the conference. However the money stock does follow a fairly flat path despite not being constrained. It moves to a level about 0.66 percent below the undisturbed path. The net surplus increases by 3.2 billion dollars after 8 quarters and 7.6 billion after 16 quarters. Scaled up to the 4 percent change in money stock of the conference simulations, the implied effects would be 19.2 and 45.6 billion dollars, larger than those in the Wharton model.

Note also that the monetary contraction produces a substantial contemporaneous decline in price level, as would occur in the version of the Appendix model in which monetary policy determines the price level. If one accepts that the identification scheme for this model has managed to separate monetary and fiscal policy effects, then this contemporaneous response of prices and the delayed response of net surplus are both in line with a view that treats fiscal policy as accommodating monetary policy rather than vice versa. Furthermore, the response of real output to monetary contraction in this identification is smaller than in the conference simulations. The small response of output is consistent with the smaller revenue response. This identification shows that, by attributing much of the predictive value of interest rates for output changes to other disturbances, one can conclude that the real effects of monetary policy changes are smaller than most of the conference models suggest.

Most of the response of the net surplus is directly attributable to the CBO estimates of interest rate impacts on the

budget, because the model implies small effects of money supply disturbances on the conventional budget surplus including interest payments. As noted below Table 4, the specification does not allow for a contemporaneous negative response of real government spending to the price level, and is in this respect unrealistic. This might have produced an unreasonably small response of the nominal deficit to the price decline generated by the monetary contraction.

Both VAR identifications agree that expansive monetary policy, as it reduces the interest component of government expenditure, is likely to generate offsetting changes in taxation. The apparent conclusion from the majority of the models presented at the conference, that monetary expansion reduces the deficit by more than its effects on interest expenditures, is thus shown to be suspect, because it does not allow for the endogenous reaction of fiscal policy.

V. Conclusion

The approaches displayed in this essay are only starts toward identifying the effects of macroeconomic policy. Because the methods used in this paper are new, we have not been able to fully exploit them in the time available. The responses to disturbances should be accompanied by standard errors. The identifications of the VAR system should have been explored more systematically--the small model identifications conflict too strongly with the data and the international model identification needs to be explored within each region, not just the U.S. Going beyond this convenient identification scheme could probably produce more

definite results. For example, if fiscal multipliers look spuriously weak in models including financial variables, this has implications about VAR impulse responses to financial variable innovations which we have not tried to check. It should be clear, though, that the conventional econometric modeler's assumption of convenience that controllable variables are predetermined simply papers over the recesses containing the real sources of our uncertainties in projecting the effects of policy changes. It should be clear also that the prominent debate over rational expectations is to a considerable degree irrelevant to the important questions at issue here. We need to pay more attention to stochastic specification and to modeling responses of policy to the private economy.

Table 1

Fiscal Multipliers for Conference Models

Model quarter	DRI	EEC	MCM	EPA	LINK	LIVERPOOL
1	1.53	----	1.26	1.14	----	----
4	2.05	1.18	1.56	1.57	1.24	.65
8	2.08	1.16	1.70	1.64	1.23	.61
12	1.86	1.10	1.61	1.63	1.14	.58

Model quarter	MINIMOD	OECD	TAYLOR	WHARTON	MCKIBBIN/SACHS
1	1.09	----	1.26	----	----
4	1.11	1.53	1.64	1.78	.97
8	1.03	1.30	1.10	1.61	.95
12	.94	1.07	.93	1.56	.85

Table 2

Innovation Equations for Pure GNP Components Model

$X = 1.32 Y$ (.33)	Exports
$Im = .91 X + 1.52 Y$ (.09) (.55)	Imports
$GMil = 0$	Military Expenditures Autonomous
$GSL = .013 Y$ (.056)	State and Local Purchases
$GNMil = -3.46 Y$ (.86)	Federal NonMilitary Purchases
$Y - X + Im - GMil - GN = .49 Y$ (.19)	Rest of GNP Response
$Surp + GMil + GNMil = 8.28 Y$ (.34)	Federal Tax + Transfers + Interest Response

Ratio of chi-squared to degrees of freedom (=14) for goodness of fit to covariance matrix is 6.06, indicating rejection by the Akaike criterion (which makes the critical value of this ratio 2.0) and even by the Schwarz criterion (which in this case makes the critical value $\log(145) = 4.98$).

The original system was estimated in current dollar levels at annual rates. The reported coefficients have been scaled, using 1986:1 levels of variables, to become elasticities.

Table 3

Innovation Equations for Model with R, M, P Added

$R = 10.34 M$ (8.42)	M supply
$M = -.261 R + 1.99 P - .817 Y$ $(.338) \quad (1.26) \quad (1.01)$	M demand
$Y = .801 P + .522 R$ $(.491) \quad (.281)$	IS
$X = -.826 R - .60 P + .501 Im + .497 Y$ $(.103) \quad (.25) \quad (.076) \quad (.303)$	Foreign Sector
$Im = -.439 R - .287 P + 2.03 Y$ $(.127) \quad (.326) \quad (.41)$	
$GMil = 0$	Military Exp. Autonomous
$GSL = .0162 Y$ $(.062)$	Response of State & Local
$GNMil = -4.223 Y$ $(.928)$	Response of Federal NonMil.
$Y - X + Im - GMil - GN = .615 Y$ $(.158)$	Rest of GNP Response to GNP
$Surp + GNMil + GMil = 8.76 Y$ (1.03)	Federal tax + Transfers + Interest Response to Y

The ratio of chi-squared to degrees of freedom (=28) is 5.74, larger than the Schwarz criterion which here is $\log(145) = 4.98$, thus indicating that the model is not a good fit.

The original system was estimated in current dollar levels at annual rates, except that interest rate entered as a per cent. The reported coefficients have been scaled, using 1986:1 levels of variables, to become elasticities, except that interest rate has been in effect converted to natural units (divided by 100) rather than in effect converted to a logarithm.

Because of the absence of an exchange rate and the importance of exchange rate movements in the sample period, the foreign sector equations are simply triangularized, with no attempt to interpret them separately as distinct behavioral relations. The reader must bear in mind that, despite their resemblance to a standard simultaneous equations model, these equations describe only within-quarter responses to forecast errors.

Table 4

Some U.S. Equations from the International Model

$GSurp = 1.316 Y - .609 P - .430 GNmil - .133 GMil$	Tax/Surplus
$R = -.0366 SP + 1.403*M + .455 GSurp - .0745 JPar$ $+ .0358 Epar$	Money Supply
$M = .241 Y + .298 P - .256 R$	Money Demand
$GNmil = -1.310 Y$	Expenditure Response to GNP
$GMil = 0$	Autonomous Military Expenditure

Y: real GNP;

GMil: real national defense purchases of goods and services;

GNMil: All government (including non-Federal) real purchases of goods and services other than national defense and Commodity Credit Corporation components;

GSurp: Federal government deficit; R: 3 month treasury bill rate.; M: M1;

Ratio of chi-squared to degrees of freedom (=852) is 1.895, acceptable by the Akaike criterion and a fortiori the Schwarz criterion. It should be noted, however, that some of the equations not reported here were difficult to interpret, and another specification, significantly better by the Akaike but not the Schwarz criterion, was fit as well. That specification in particular showed that allowing for a contemporaneous negative response of real government spending to inflation is important. The displayed identification is thus far from a definitive interpretation.

Standard errors not presented here because convergence was too rapid to allow the numerical optimization routine's approximate Hessian to become plausibly accurate. Of course ideally we should proceed to numerical differentiation in this case.

Table 5
Pure GNP components model multipliers

Variable shocked	Defense Purchases	Nondefense Purchases	State & Local	Rest of GNP
-quarters				
1	1.24	1.36	1.24	.82
4	1.98	.56	1.46	1.02
8	1.59	.72	2.50	1.32
12	1.46	1.50	4.34	1.44

R, M, P added to U.S. GNP component model

Variable shocked	Defense Purchases	Nondefense Purchases	State & Local	Rest of GNP
-quarters				
1	1.40	1.60	1.40	.82
4	1.24	.77	1.54	1.00
8	.78	-1.99	2.42	1.38
12	.82	-2.06	3.46	1.56

International Model

Variable shocked	Defense Purchases	Nondefense Purchases (All govt.'s)
quarters		
1	1.15	2.47
4	1.11	2.46
8	1.17	2.29
12	1.30	2.25

Table 6

Effects on net surplus of monetary expansion

year	DRI	EEC	MCM	EPA	LINK
1	8.6	.6	3.0	22.7	0.0
2	37.6	1.2	25.7	36.0	7.9
3	38.3	16.1	38.4	28.9	21.4
4	18.3	22.9	32.3	10.6	24.

year	LIVPL	MINIMOD	VAR	OECD	WHARTON
1	-1.9	3.0	-2.2	17.9	-2.7
2	1.0	19.1	5.9	44.3	-3.9
3	.8	23.8	0.8	34.8	-10.2
4	1.3	28.0	-13.5	25.8	-20.5

year	MCK/SACHS
1	-5.4
2	-22.6
3	-27.6
4	-18.0

Computed from conference volume tables for simulation D according to the formula:

$$NSurp(j) = -GDEF(j) + 3 RS(j) + 6 RL(j-1) + 5 RL(j-2) + 7 RL(j-3),$$

where NSurp(j) is the year j number in the columns above, RS(j) is the year j effect on the short rate, RL(j) is the year j effect on the long rate, and GDEF(j) is the year j effect on the gross deficit. The lag distribution is based on the CBO table cited in the text. However that CBO analysis applied to a permanent change in "the interest rate", not to dynamic responses when both short and long run interest rates change. Our choice of how to extrapolate this to a situation with both short and long run rates changing is obviously somewhat arbitrary. For the VAR model there are no long rate forecasts, so the lag distribution is applied directly to short rates.

Notes to the Figure

Each column of plots shows the responses to a one standard deviation disturbance in the equation named at the bottom of the column. The interest rate is measured in percentage points, the net surplus in 100 dollars per dollar of nominal GNP, and the remaining variables in logarithmsX100 (so they have the same percentage point units as the interest rate). The number at the right of each row of plots gives the size of the maximum absolute deviation from the horizontal axis shown in that row. Each plot shows responses over 16 quarters.

Appendix

We consider a simple model in which the real interest rate is constant and in which we suppress the explicit analysis of money supply and demand--we treat monetary policy as fixing the interest rate. It is a model with no quantity variables. Within it we discuss how changes in policy contemporaneously affect the price level. In a more realistic model there would be stickiness in prices and quantity effects from the policy changes which affect the price level in this model.

We suppose the government issues one-period bonds, the total stock of which outstanding issued at t are B_t , with gross interest rate quoted at t given by R_t . The price level (the rate of exchange between goods and newly issued bonds) is P_t and the real value of taxes at t is τ_t . Real government expenditures, which we take as fixed exogenously, are given by x_t . The budget constraint for the government then is

$$(A1) \quad B_t = R_{t-1}B_{t-1} + P_t(x_t - \tau_t).$$

We ignore the appearance of money in the budget constraint; as stated in the text, this is a reasonable approximation if all money is inside money or if outside money is a small fraction of total government debt.

We make the simplifying assumption that agents choosing portfolios discount at a nonrandom rate equal to the fixed real rate of return on real assets. This is exactly justified if output and consumption are constant, but is approximately correct whenever fluctuations in consumption are small relative to the

degree of curvature in agents' consumption functions. Under these simplifying assumptions the rate of time discount β must match the real rate of return and, portfolio equilibrium requires

$$(A2) \quad R_t^{-1} = \beta E_t [P_t/P_{t+1}].^{11}$$

We postulate two simple policy rules. Monetary policy fixes R according to

$$(A3) \quad R_t^{-1} = a + bP_{t-1}/P_t + v_t,$$

and fiscal policy fixes τ according to

$$(A4) \quad \tau_t = c + dB_{t-1}/P_{t-1} + w_t.$$

Equations (A2) and (A3) together imply

$$(A5) \quad P_t/P_{t+1} = \beta a + \beta bP_{t-1}/P_t + \beta v_t + u_{t+1},$$

where $E_t[u_{t+1}] = 0$. Where w , v and x are the only sources of stochastic disturbance to the system, u_t will be determined by $\{w_s, v_s, x_s; s \leq t\}$.

Most macroeconomists would expect that monetary policy can stabilize the price level only if it moves interest rates more than in proportion to the rate of inflation, that is, if b exceeds one in (A3), with the price level being more stable the bigger is b . From (A5) we see that P_t/P_{t+1} , the inverse of the inflation rate, satisfies an unstable stochastic difference equation if $b > \beta^{-1}$. This is not a real paradox, however. An "unstable"

stochastic difference equation may have a stationary solution. In this case, the stable solution for large b is given by

$$(A6) \quad P_{t-1}/P_t = -\beta a/(\beta b - 1) - \beta E_t \left[\sum_{s=0}^{\infty} v_{t+s} (\beta b)^{-s} \right].$$

Assuming that an equilibrium with negative prices or with the inflation rate increasing exponentially is infeasible,¹² (A6) gives the only possible stochastic process for prices when $\beta b > 1$. Note that this implies that the price level is entirely determined by the discount factor β , the parameters (a and b) of the monetary policy equation (A3), and the disturbance v to that equation.

If, on the other hand, $\beta b < 1$, (A5) has many stationary solutions (assuming v is stationary). If we think of the economy as beginning at some initial time 0, each initial P_{-1}/P_0 generates a different time path for prices. By not responding strongly enough to inflation, monetary policy fails to fix the price level. As we shall now see, this does not imply that the price level is necessarily indeterminate. Since the price level appears also in the government budget constraint, the price level can be determinate despite the monetary authorities' failure to respond to the interest rate if fiscal policy is appropriate. Formally, the problem is that the system has a loose initial condition; determining the initial price level requires one unstable root in the system which, via a feasibility constraint or transversality condition, eliminates the indeterminacy. The unstable root can be provided by an inflation-sensitive interest rate, but it also can be provided by fiscal policy.

For fiscal policy to take on the burden of fixing the price level, it must behave in a way which, in the presence of a strongly inflation-sensitive monetary policy, would be destabilizing. That is, it must not make taxes respond strongly to the real value of the government debt.

Using equations (A1) and (A2), taking a conditional expectation of the budget constraint (A1), gives us

$$(A7) \quad E_{t-1}[B_t/P_t] = \beta^{-1}B_{t-1}/P_{t-1} + E_{t-1}[x_t - \tau_t]$$

substituting (A4) into (A7) produces

$$(A8) \quad E_{t-1}[B_t/P_t] = (\beta^{-1} - d)B_{t-1}/P_{t-1} + E_{t-1}[x_t - c - w_t].$$

If $d < \beta^{-1} - 1$, (A8) generates an unstable stochastic difference equation in B/P . Assuming that equilibria in which the real government debt grows faster than the rate of time discount β are infeasible,¹³ however, we can as before solve forward in this case to obtain the unique stable solution to the "unstable" equation. With $d = 0$, for example, we obtain

$$(A9) \quad B_t/P_t = E_t \left[\sum_{s=1}^{\infty} \beta^s (w_{t+s} + c - x_{t+s}) \right].$$

This equation makes the real value of the government debt the discounted present value of government surpluses net of interest payments. Combining (A9) with the budget constraint (A1) gives us an expression for P_t :

Observe that (A10) allows no contemporaneous effect on the price level of the monetary policy disturbance v_t . By changing the interest rate R_t , monetary policy can change anticipated

(and thus actual) inflation, but the only effect is that higher interest rates produce higher future prices. The price level is determined by fiscal policy, with higher current or expected future taxes reducing the price level.

Equation (A10) was derived assuming $d = 0$, but with no assumptions about a or b ; equation (A6) was derived assuming $b\beta > 1$, but with no assumptions about c or d . Yet the two equations are clearly in general inconsistent. For example, if w , v , and x are serially and mutually independent, (A6) makes P_t dependent only on v_t conditional on data up through $t - 1$, while (A10) makes P_t dependent only on w_t and x_t . This is a symptom of the fact that the model has no solution if both fiscal and monetary authorities attempt to take charge of the price level. If d is small and b is large, the system has two unstable eigenvalues, while there is only one undetermined initial condition--the initial price level. Either the monetary authority controls the price level by moving the interest rate more than in proportion to inflation, or the fiscal authority controls the price level through the relation of taxes to expenditures. When the monetary authority controls the price level, the fiscal authority has no effect on the price level or on inflation, except that it must choose d large enough so that equation (A8) above is stable. When the fiscal authority controls the price level, the monetary authority controls the rate of inflation. The monetary authority has no effect on unanticipated inflation in that case, and must choose b small enough to make (A5) above a stable equation.

Equation (A10) together with the monetary policy equation (A3) imply that expected future net surpluses affect current interest rates, via the reaction of monetary policy to deflationary pressure. In particular, lower interest rates and expansions in money stock will be associated with higher expected future net surpluses. Thus in an equilibrium where fiscal policy is fixing the price level, it may appear that expansionary monetary policy increases government surpluses or reduces deficits. A true monetary expansion, an increase in v_t in (A3), has no effect on the net surplus or deficit in this kind of equilibrium. In an equilibrium where monetary policy sets the price level (large b in (A3), small d in (A4)), v_t increases reduce the real value of the debt and thereby reduce future net surpluses. This suggests a way of distinguishing the two kinds of equilibrium even when we find it hard to disentangle v_t and w_t in the data: we look to see whether periods when money stock increases and interest rates drop tend to precede fiscal contractions or fiscal expansions.

There remains a fourth case we have not yet considered, in which the solution is indeterminate. The monetary authority might keep the interest rate unresponsive to inflation, while the fiscal authority kept taxes responsive to the real value of the debt. In this case the model has a continuum of solutions because it has no unstable root and the price level is therefore indeterminate.

The message of this model might be summarized as: fiscal policy can affect nominal aggregate demand provided it is supported by an accommodative monetary policy; monetary policy can

affect nominal demand provided it is supported by a fiscal policy which converts high interest rates into high future taxes.

Model Notes

This paper discusses models based on three vector autoregressions--two small VAR's fit to U.S. data only and the large three-region international VAR whose simulations were presented at the conference. The simulations at the conference were prepared with a different identification scheme from that reported in Table 4, though the same estimate of the same VAR was used. We summarize here the data, the VAR estimation method, and the identification schemes used for each model specification.

A. Data

The two U.S. models used quarterly data from 1948:1 through 1986:1, with the estimation period beginning in 1949:1. The variables in the smaller model were the five nominal GNP components exports, imports, federal national defense purchases, other federal purchases, state and local purchases, plus total GNP and the federal government surplus. The larger model added the three month treasury bill rate (secondary market), M1 (a splicing of M1 definitions to cover the whole postwar period), and the GNP deflator.

Each of the three regions in the international model used data on real GNP, GNP deflator, short interest rate, stock prices, unemployment rate, wage rate, money stock, government purchases, government surplus, imports, exports, import prices, export prices, and current account balance. In addition the U.S. sector distinguished defense and nondefense spending. U.S. exchange rates with Japan and Europe and a commodity price index entered each regional system.

It is planned that a discussion paper documenting the world model will eventually become available separately, giving more detailed data definitions and sources and a more complete explanation of modeling methods. Data for all three models are available on diskette at cost.

B. VAR Estimation

The two smaller models were estimated in two steps. Initial ordinary least squares estimates were generated from the first part of the sample. Using the estimated disturbance variances from this stage, the remaining part of the sample was used with a Kalman filter procedure, one observation at a time. This procedure allowed for conditional heteroskedasticity by making the disturbance variance depend on actual squared lagged one-step-ahead forecast errors. Space limitations preclude description of the procedure or its underlying probability theory in detail here, but the input files used are available on diskette.

The international model, because of its size, would not have produced easily interpreted impulse responses if estimated by least squares. Furthermore, the data series for the various regions started up at scattered dates. We devised a procedure based on the Kalman filter to allow model size to expand as additional series became available. There was no allowance for heteroskedasticity. The prior distribution on the parameters was similar to that described in Doan, Litterman and Sims [1984]. Each region's equations were estimated separately. The estimates were carried out with a suite of FORTRAN programs rather than with a standard packaged program because of the scale of the computa-

tions. Documentation of the estimation methods will not be available until the separate discussion paper on the model appears.

C. Identification

Identification for the two smaller models followed the methods described elsewhere in this paper. For the international model one identification, that underlying the Table 4 procedure, also followed those methods. The identification procedure for the conference simulations differed. To understand what was done, consider equation (4) in the text, which relates the matrix B_0 of contemporaneous coefficients on Y in the equations (3) defining the underlying behavioral disturbances u to the matrix A_0 of coefficients on u in the equations (2) defining how $Y(t)$ is determined by current and past u . The identification procedure used elsewhere in this paper imposed zero restrictions on elements of B_0 , which can be interpreted as minimum delay restrictions. However to reconstruct the system's response to one underlying u_i , we actually require the i 'th column of $A_0 = B_0^{-1}$. Every column of A_0 in general depends on all the elements of B_0 . It is not unreasonable that we might have beliefs, based on knowledge of economic behavior, directly about the responses of the economy to disturbances in policy--that is about the columns of A_0 themselves. For the conference simulations, we postulated forms for the money supply and fiscal policy columns of A_0 directly, rather than postulating minimum-delay restrictions on B_0 and deriving A_0 indirectly. We made a very simple specification of the fiscal policy column: it involved contemporaneous movements in the government expenditure variable and the government surplus vari-

able only. The monetary policy shock was defined by a more subtle process, based on Robert Litterman's [1984] analysis of the weekly pattern of interest rate, exchange rate and money stock movements. The money shock column of A_0 was specified to involve an upward movement in interest rate accompanied by contemporaneous downward movement in money stock and upward movement in exchange rates. Taking explicit account of time aggregation, other variables also were allowed to move contemporaneously with the money shock in the quarterly model.

Footnotes

¹Conventional treatments usually distinguish exogenous variables from endogenous variables, with completeness meaning only that endogenous variables can be expressed in terms of current and past random disturbances and current and past exogenous variables. But here we have not imposed any conditions on the stochastic properties of the u 's. If there is an exogenous variable Y_i , we simply include a trivial equation of the form $Y_i(t) = u_{Fi}(t)$ to "determine" it.

²In a multivariate economic model the dependent variables are determined by essentially arbitrary normalizations, from the point of view of the statistical theory. What is really asserted by the statement that the X 's are the dependent variables in G is little more than that the dimensions of X and G match.

³It should be noted that most early rational expectations macroeconomics was equally cavalier about treating controllable variables as predetermined. The hypothesis that one-step ahead prediction errors in money stock are entirely generated by random disturbances to policy, essential to much rational expectations monetarist empirical work, amounts to assuming money stock is predetermined.

⁴Ironically, this aspect of the rational expectations critique, implying the need for renewed attention to careful treatment of policy endogeneity, has generated little response. Indeed we find a proliferation of rational expectations models, like Sargent's [1973] influential Brookings paper and the rational expectations models presented at this conference, which maintain

the hypothesis that controllable variables are predetermined, apparently with convenience and convention as the only justification.

⁵This is a slight exaggeration. Though (3) is in the form of a standard linear simultaneous equations model, textbook simultaneous equations methods do not impose uncorrelatedness across i on the $u_i(t)$'s.

⁶The condition for identification given here is only a rank condition, necessary but not sufficient. I have not found a conveniently checked sufficient condition.

⁷Songdal Shim [1984] has tested the implications of (3) for time series data on a number of countries. Using VAR methodology, he shows that it holds remarkably well (with the side condition that θ is fixed) when the economy is disturbed in ways that do not create surprise changes in R . Surprise changes in R do seem to be associated with changes in θ , and thereby to violate the normal relation of nominal debt to current price level and future net surpluses. Hamilton and Flavin [1987] have tested (3) for U.S. data.

⁸Recall from section II that this class of identification methods includes standard simultaneous equations methods, those which will be applied in the next section of this paper, and most applied rational expectations modeling.

⁹The VAR fiscal multipliers are not reported because of an error we made in specifying the fiscal shocks for the conference simulations. We had meant that all government expenditure variables should be in nominal terms, and for Europe and Japan

they were. However we inadvertently used data on real government expenditures for the U.S. Thus the fiscal shocks specified for the conference, which required that the ratio of expenditures to the price level follow a given path, with no initial contemporary movement in other variables, were an unlikely and nearly uninterpretable disturbance. The results from the VAR model reported in this paper correct for the real U.S. fiscal variables, and the responses to other policy disturbances in the conference simulations are probably not much distorted by the misspecification of fiscal shocks.

¹⁰Table II-11, p. 75.

¹¹Note that because agents know that consumption is constant, they know that the random real return on their holdings of nominal bonds are in equilibrium exactly offset by fluctuations in their savings and in taxes. However, they assume that this would not remain true if they individually deviated from the equilibrium level of holdings of government debt.

¹²Infeasibility could stem either from the technology, with money essential for production and production driven to zero as the cost of holding money increases, or from a commitment to some discontinuous change in policy when inflation reaches a certain rate.

¹³In a representative agent model this is guaranteed by transversality conditions. In overlapping generations models it is likely to be guaranteed by technical feasibility conditions.

References

- Aiyagari, S. Rao and Mark Gertler [1985]. "The Backing of Government Bonds and Monetarism," *J. Monetary Economics* 16, 19-44.
- Doan, Thomas, Robert B. Litterman and Christopher A. Sims [1984]. "Forecasting and Conditional Projection Using Realistic Prior Distributions," *Econometric Reviews* 3, 1-100.
- Garcia, Teresa [1987]. *Government Purchases and Real Output: Empirical Evidence and an Equilibrium Model with Government Capital*, University of Minnesota Ph.D. Thesis (title of this reference may not be quite correct).
- Goldfeld, Stephen and Alan S. Blinder [1972]. "Some Implications of Endogenous Stabilization Policy," *Brookings Papers on Economic Activity*, number 3, 585-640.
- Hamilton, James and Majorie Flavin [1987]. "On the Limitations of Government Borrowing: A Framework for Empirical Testing," *American Economic Review* 76, 808-819.
- Litterman, Robert B. [1984]. "Forecasting and Policy Analysis with Bayesian Vector Autoregression Models," *Quarterly Review of the Minneapolis Federal Reserve Bank* 8, 30-41.
- Sargent, Thomas J. [1984]. "Autoregressions, Expectations and Advice," *American Economic Review* 74, 408-15.
- _____. [1973]. "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment," *Brookings Papers on Economic Activity*, number 2, 429-80.
- Sims, Christopher A. [1980]. "Macroeconomics and Reality," *Econometrica* 48, 1-48.

- _____. [1982]. "Scientific Standards in Econometric Modeling," in Current Developments in the Interface: Economics, Econometrics, Mathematics, M. Hazewinkel and A.H.G. Rinnoy Kan (Dordrecht, Boston, London: D. Reidel), 317-337.
- _____. [1985]. "A Rational Expectations Framework for Policy Analysis," Minnesota Center for Economic Research Discussion Paper Number 223. Also forthcoming in the proceedings of the Second Texas Symposium on Economics held at Austin, Texas in May 1985.
- _____. [1986]. "Are Forecasting Models Usable for Policy Analysis," Quarterly Review of the Federal Reserve Bank of Minneapolis, Winter, 2-16.
- Shim, Songdal [1984]. Inflation and the Government Budget Constraint: International Evidence, University of Minnesota Ph.D. thesis.