Federal Reserve Bank of Minneapolis
Research Department

LAISSEZ-FAIRE BANKING AND CIRCULATING MEDIA OF EXCHANGE

Stephen D. Williamson*

Working Paper 382

Revised March 1988

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ABSTRACT

A model with private information is constructed that supports conventional arguments for a government monopoly in supplying circulating media of exchange. The model also yields predictions, including rate-of-return dominance of circulating media of exchange, that are consistent with observations from free banking regimes and fiat money regimes. In a laissez faire banking equilibrium, fiat money is not valued, and the resulting allocation is not Pareto optimal. However, if private agents are restricted from issuing circulating notes, there exists an equilibrium with valued fiat money that Pareto dominates the laissez faire equilibrium and is constrained Pareto optimal.

*Federal Reserve Bank of Minneapolis. I wish to thank Peter Howitt, Bruce Smith, Neil Wallace, and seminar participants at the Federal Reserve Bank of Minneapolis, the University of Pennsylvania, and the University of Western Ontario for their helpful comments and suggestions. Financial support from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged. Any remaining errors are my own.

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I. Introduction

The view that there is a legitimate basis for a government monopoly in supplying circulating media of exchange has long been a part of conventional wisdom in monetary economics. Milton Friedman gave some reasons to support this view in Friedman [1960] where he states, in outlining the ills of laissez faire monetary arrangements, that (Friedman 1960, p. 6):

> ". . . the [private] contracts in question are peculiarly difficult to enforce and fraud particularly difficult to prevent. The very performance of its central function requires money to be generally acceptable and to pass from hand to hand . . . in fraud as in other activities, opportunities for profit are not likely to go unexploited."

Thus, the information externalities inherent in monetary exchange imply, according to this view, that an arrangement with unfettered private intermediation can be dominated by another arrangement with: (1) a government-supplied, universally-recognizable medium of exchange and (2) legal restrictions on private intermediaries.

There is a growing literature that puts laissez faire monetary arrangements in a much more favorable light, thus challenging this conventional wisdom. For example, Rolnick and Weber [1983,1984] and King [1983] argue that the U.S. free banking era (1837-63), usually characterized as a laissez faire banking regime, was much less chaotic than once thought. In particular, fraudulent banking practices, counterfeiting, and below-par redemption of free bank notes appear to have been the exception rather than the rule in most states during this period. The nineteenth-century Scottish free banking system seems to have functioned with even fewer of the perceived problems of the U.S. free banks (see White 1984). informal theoretical reasoning, Hayek [1978] argues that efficiency gains would result from a move from current regimes to ones with unfettered financial arrangements, and Fama (1980, p. 47) argues that ". . . there is nothing in the economics of this (the banking) sector that makes it a special candidate for government control." More formally, in Sargent and Wallace [1982], a Pareto optimal equilibrium allocation exists without restrictions on financial intermediation. In their model, if there are restrictions that effectively prohibit private intermediaries from issuing close substitutes for fiat money, then an equilibrium is not Pareto optimal.

In this paper a model is constructed that provides support for the conventional wisdom of Friedman and others concerning government supply of circulating media of exchange. This model also yields predictions consistent with what is observed in laissez faire regimes with unrestricted private note issue, and with what is observed in fiat money regimes. In particular, (1) circulating media of exchange are dominated in rate of return by other assets; (2) in a laissez faire equilibrium, fraudulent note issue is nonexistent, in spite of the fact that there is a potential for such practices; (3) bank deposit liabilities coexist in equilibrium with valued fiat money (in a fiat money regime) or with circulating notes (in a laissez faire regime); (4) assets yielding relatively high (low) rates of return have relatively low (high) transactions velocities.

The model economy is populated by overlapping generations of consumers. Each consumer has an uncertain lifetime and uncertain preferences, which creates, as in Diamond and Dybvig [1983], an uncertain demand for liquid assets. Capital may be either good or bad, and its type is observable only to the agent who produces it. Bad capital is cheaper to produce, but yields an inferior return. In a laissez faire banking regime, capital serves as backing for circulating notes (good and bad) and for noncirculating notes (bank deposits). Because of asymmetric information in the market for circulating notes, there is an incentive for private agents to produce bad capital and to issue bad circulating notes, where no such incentive exists with perfect information.

In the laissez faire banking regime, bad circulating notes are much like the "lemons" in Akerlof [1970]. In this regime, there exists no stationary equilibrium with valued fiat money and, in the only stationary equilibrium that exists, a version of Gresham's law holds; the only notes that circulate are those backed by bad capital. In equilibrium, bad circulating notes (assets with a high transactions velocity backed by assets with a low rate of return) coexist with bank deposits (assets with a low transactions velocity backed by assets with a high rate of return). In equilibrium, there is no misrepresentation of the quality of circulating notes. That is, there is no fraudulent note issue, in spite of the existence of a potential for fraud. A stationary equilibrium may not exist, but existence is more likely the less costly is bad capital to produce and the higher is the probability of a state where circulating notes are required to finance consumption.

Most (if not all) fiat money regimes place restrictions on the kinds of liabilities private agents may issue. This model shows why these restrictions might be necessary to induce agents to hold fiat money, and why they may also be welfare-improving. If there is a legal restriction that bans the issue of private circulating notes, there exists a unique stationary equilibrium with valued fiat money in which all agents are better off than in the stationary laissez faire banking equilibrium. This monetary equilibrium is also constrained Pareto optimal, in a sense defined in the paper. These results hold in spite of the fact that there exist assets in the model that dominate fiat money in rate of return. In the equilibrium with legal restrictions, as in the laissez faire banking equilibrium, a circulating medium of exchange (fiat money) coexists with bank deposits, and transactions velocity is negatively correlated (across assets) with rate of return.

The remainder of the paper is organized as follows. In Section II the model is constructed, and in Section III a Pareto optimal allocation is defined for this environment with private information. Section IV examines the laissez faire banking regime. An equilibrium is characterized and shown to be nonoptimal. Next, in Section V it is shown that legal restrictions permit the existence of a monetary equilibrium which Pareto dominates the laissez faire banking equilibrium. The final section is a summary and conclusion.

II. The Model

This is an overlapping generations model, where private information is introduced to capture features of private transactions. In each period $t = 1, 2, 3, \ldots, \infty$, n agents are born, each of whom is endowed with y units of a consumption good when young. In middle age, agents learn their type, i, where $i = 1, 2$. Type and age are private information. A type 1 agent lives for two periods with utility given by $u(c_1,c_2)$, and type 2 agents live 3 periods with utility $v(c_1,c_3)$. Here, c_i denotes consumption in the i^{th} period of life. The functions $u(\cdot,\cdot)$ and $v(\cdot,\cdot)$ are twice continuously differentiable, increasing in both arguments, and strictly concave. It is assumed that

$$\lim_{c_1/c_2 \to \infty} \frac{D_1 u}{D_2 u} = \infty, \quad \lim_{c_1/c_2 \to \infty} \frac{D_1 u}{D_2 u} = 0,$$

$$\lim_{c_1/c_3 \to \infty} \frac{D_1 v}{D_2 v} = \infty, \text{ and } \lim_{c_1/c_3 \to \infty} \frac{D_1 v}{D_2 v} = 0.$$

The parameter λ will denote the fraction of agents in a generation who are type 1, and will also denote the probability of being type 1 for any agent. We have $0 < \lambda < 1$.

The consumption good can either be consumed or used to produce capital, and all agents have access to the technology for producing capital. Capital produced at time t gives a return at time t + 2, when the capital depreciates completely. The good which is produced from capital is equivalent to the endowment good in consumption, but it cannot be used to produce more capital. It might be useful to think of capital as fruit trees which require some resources to plant, yield fruit in two periods, and then die.

Capital may be of two types: good and bad. It requires $z(\beta z)$ units of the endowment to produce z units of good (bad) capital, where $\beta < 1$. Good (bad) capital yields a return of $\alpha_1(\alpha_2)$ units of the consumption good, where $\alpha_1 > 1$ and $\alpha_2/\beta < 1$. Thus, bad capital is less costly to manufacture but yields an inferior return. Capital type is observable only to the agent who produces it; to other agents a unit of good capital is indistinguishable from a unit of bad capital. However, agents can at any time verify the existence and age of a given lump of capital, without verifying its type.

At time 1, there are λn middle-aged type 1 agents, $(1-\lambda)n$ middle-aged type 2 agents, and $(1-\lambda)n$ old agents, each of whom is endowed with H units of fiat money. Fiat money consists of unbacked, perfectly divisible, noncounterfeitable notes issued by the government. Middle-aged type 1 agents and old agents supply fiat money inelastically at time 1 so as to maximize time 1 consumption, while middle-aged type 2 agents maximize period 2 consumption by selling their endowment at that date.

The trading technology is such that exchange and production occur in the following fashion. At the beginning of a period, agents trade endowment goods and financial claims in a market location, and consume endowment goods. They then move, with whatever endowment goods they wish to transport, to a production location where, at the end of the period, capital is produced and the returns from previously-produced capital appear and are consumed. Endowment goods transported from the market location to the production location cannot be consumed, and capital is immobile at the production location. At the market location, agents are indistinguishable; their types and ages are private information. However, at the production location there is a technology which can distinguish individual agents, in the sense that agents can be associated with previous actions at the production location. Agents can choose whether or not they submit to this technology, so that 'hiding' is possible.

As in Diamond and Dybvig [1983], agents' random preferences imply that they have an uncertain future requirement for liquid assets. Here, it is asymmetric information which can make assets illiquid, rather than the nature of short-term and long-term production technologies, as in Diamond and Dybvig's model. An agent who produces capital when young, and subsequently learns she is type 1, cannot trade with any other agent who knows the type of this capital. Therefore, an adverse selection problem might arise in which securities backed by good capital and by bad capital are offered on the market and are indistinguishable to potential buyers of these securities.

III. Pareto Optimal Allocations

In this environment, it is assumed that it is infeasible for the social planner to confiscate endowments, to produce capital, or to transport goods from the market location to the production location. The planner cannot observe trades or consumption at the market location, but she can direct and observe production and can observe consumption at the production location. She cannot observe the type of capital produced. Agents of the same type and age are treated identically by the planner.

So that young agents will give part of their endowment willingly to the planner, at each time t at the market location the government offers one identity badge, stamped with the date, in exchange for x_t units of the endowment good. A time t identity badge is a claim either to c_{t+1}^t units of the consumption good at the market location at time t + 1, or c_{t+2}^t units at the market location at t + 2. The technology is such that an agent can hold only one identity badge at a time, and identity badges must be stored at the market location. Any identity badges exchanged with the planner for consumption are destroyed.

At the production location, young agents identifying themselves are directed to produce k_t^g units of good capital and k_t^b units of bad capital, and they receive z_t^t units of consumption. If middle-aged (old) agents identify themselves (ages are observable if these agents produced capital when young), they receive $z_t^{t-1}(z_t^{t-2})$ units of consumption at the production location.

The planner then faces the following resource constraints.

(1) $$c_t^t \leq y - k_t^g - \beta k_t^b - x_t, \quad t = 1, 2, 3, \ldots$$

(2) $$\lambda c_t^{t-1} + (1-\lambda)c_t^{t-2} \leq x_t, \quad t = 1, 2, 3, \ldots$$

(3) $$z_t^t + \lambda z_t^{t-1} + (1-\lambda)z_t^{t-2} \leq \alpha_1 k_{t-2}^g + \alpha_2 k_{t-2}^b, \quad t = 3, 4, 5, \ldots$$

(4) $$z_t^t = z_t^{t-1} = z_t^{t-2} = 0, \quad t = 1, 2.$$

In (1), (2), and (3), $c_t^s(z_t^s)$ denotes consumption at the market location (production location) in period t by an agent born at time s. Note that all quantities are measured in units per young agent. Consumption has been set to zero where agents derive no utility from it, which allows us to omit notation distinguishing consumption by agent type. Inequality (1) states that the consumption of young agents is bounded above by their endowment net of goods exchanged for identity badges and goods transported to the production location for capital production. The left-hand side of (2) is the consumption per young agent of middle-aged and old agents in the market location, and the right-hand side is the quantity of the endowment good acquired by the planner in exchange for identity badges. Inequality (3) states that the returns from capital produced two periods previously constrains the consumption of young, middle-aged, and old agents in the production location. There is no pre-existing capital at t = 1, so that consumption at the production location must be zero for t = 1, 2 (see (4)).

Attention is confined here to allocations for which it is in each agent's interest to report her true type and age at the market location. The allocation should also be such that it is in each agent's interest to participate by acquiring an identity badge when young and identifying herself at the production location in each period of life. These restrictions on the allocation are in part implicit in (1)-(4), and also imply that the allocation must satisfy certain incentive-compatibility constraints. First, it must be in a young agent's interest to acquire an identity badge.

$$(5) \qquad \lambda u(c_t^t+z_t^t, c_{t+1}^t+z_{t+1}^t) + (1-\lambda)v(c_t^t+z_t^t, c_{t+2}^t, z_{t+2}^t) \geq \lambda u(c_t^t+x_t+z_t^t, z_{t+1}^t)$$
$$+ (1-\lambda)v(c_t^t+x_t+z_t^t, z_{t+2}^t).$$

On the left-hand side of (5) is the expected utility of an agent born at time t given the allocation. On the right-hand side is the agent's expected utility if she does not acquire an identity badge. Second, a young agent must have the incentive to transport the endowment good to the production location when young, identify herself, and produce capital.

(6) $$\lambda u(c_t^t+z_t^t, c_{t+1}^t+z_{t+1}^t) + (1+\lambda)v(c_t^t+z_t^t, c_{t+2}^t, z_{t+2}^t) \geq \lambda u(c_t^t+k_t^g+\beta k_t^b, c_{t+1}^t)$$

$$+ (1-\lambda)v(c_t^t+k_t^g+\beta k_t^b, c_{t+2}^t).$$

Third, there must be no unexploited gains from private trading of identity badges.

(7) If $x_{t+1} \leq c_{t+1}^t$, then $c_{t+2}^t \geq c_{t+2}^{t+1}$.

(8) If $x_{t+1} \geq c_{t+1}^t$, then $c_{t+2}^t \leq c_{t+2}^{t+1}$.

If $x_{t+1} \leq c_{t+1}^t$, then agents born at time t who learn they are type 2 in middle age can exchange their identity badge with the planner for c_{t+1}^t units of the consumption good and then acquire an identity badge dated t + 1. This arbitrage is profitable if $c_{t+2}^t < c_{t+2}^{t+1}$. Similarly, if $x_{t+1} \geq c_{t+1}^t$ then, as an alternative to acquiring an identity badge dated t + 1, an agent born at t + 1 could purchase the badge of a type 1 middle-aged agent for c_{t+1}^t units of the consumption good. If $c_{t+2}^t > c_{t+2}^{t+1}$, then this arbitrage is profitable, since if the agent learns she is type 1 she can consume more, and if she is type 2 she can exchange the identity badge with the planner for the consumption good in middle age and purchase the badge of a middle-aged type 1 agent.

Last, it must be in the interest of a young agent to produce capital as directed by the planner. That is,

$$(9)\qquad \lambda u(c_t^t+z_t^t, c_{t+1}^t+z_{t+1}^t) + (1-\lambda)v(c_t^t+z_t^t, c_{t+2}^t, z_{t+2}^t)$$
$$\geq \lambda u\left(c_t^t+(1-\beta)k_t^g+z_t^t, c_{t+1}^t+z_{t+1}^t\right) + (1-\lambda)v\left(c_t^t+(1-\beta)k_t^g+z_t^t, c_{t+2}^t\right).$$

Inequality (6) requires some comment. The planner imposes the most severe penalty possible for any deviation from directed production. However, deviations cannot be detected until the capital yields a return, and the best the planner can do in the case of a deviation is to set $z_{t+2}^t = 0$, which minimizes the ex ante gain from cheating. Clearly there is no gain to producing good capital if directed to produce bad capital, as this only reduces utility. However, the agent gains $1 - \beta$ units of consumption for each unit of bad capital misrepresented as good capital. It is in the interest of the agent to cheat to the maximum if she cheats at all, producing only bad capital.[1]

Definition 1: A constrained Pareto optimal allocation $\{\bar{c}_t^t, \bar{c}_t^{t-1}, \bar{c}_t^{t-2}\}$, $\{\bar{k}_t^g, \bar{k}_t^b\}$, $\{\bar{x}_t\}$, satisfies (1)-(9) and has the property that there is no alternative allocation (ˆ) satisfying (1)-(9) and

$$(10)\qquad \lambda u(\hat{c}_t^t+\hat{z}_t^t, \hat{c}_{t+1}^t+\hat{z}_{t+1}^t) + (1-\lambda)v(\hat{c}_t^t+\hat{z}_t^t, \hat{c}_{t+2}^t+\hat{z}_{t+2}^t) \geq \lambda u(\bar{c}_t^t+\bar{z}_t^t, \bar{c}_{t+1}^t+\bar{z}_{t+1}^t)$$
$$+ (1-\lambda)v(\bar{c}_t^t+\bar{z}_t^t, \bar{c}_{t+2}^t+\bar{z}_{t+2}^t),\ t = 1, 2, 3, \ldots$$

$$(11)\qquad \hat{c}_1^0 \geq \bar{c}_1^0$$

$$(12)\qquad \hat{c}_1^{-1} \geq \bar{c}_1^{-1}$$

$$(13)\qquad \hat{c}_2^0 \geq \bar{c}_2^0$$

(14) At least one of (10)-(13) with strict inequality (for any t in the case of (10)).

This definition of Pareto optimality is standard, as in Wallace [1980], except that the allocation must also satisfy incentive-compatibility constraints in this environment with private information.

IV. Laissez Faire Banking Equilibrium

The purpose of this section is to consider a particular private market mechanism in this environment and to ask whether this mechanism can support a constrained Pareto optimal allocation. If attention is restricted to equilibria that are stationary (in a sense to be defined later), then the answer is no.

With this private arrangement, there are two kinds of securities; one may circulate and the other does not. A circulating security, or circulating note, is a claim on a specified fraction of the return on a specified lump of capital. Circulating notes are initially issued and held by the agent who produces the capital backing the notes, but the sale of a circulating note transfers the claim. To normalize, let one circulating note be backed by one unit of capital. Therefore, if a circulating note is good (bad), it can be redeemed two periods from its date of issue for $\alpha_1(\alpha_2)$ units of consumption. Given the information structure, good and bad circulating notes are indistinguishable to a buyer when traded, but the existence and age of the capital backing a given note can be verified.

Circulating notes in this model share some of the features associated with the bank notes issued in laissez faire regimes such as the U.S. free banking period. In particular, they can be issued in small denominations, and redemption value does not depend on who holds the note. Unlike the securities issued by U.S. free banks, though, these notes are not redeemable at any time at their par value, but look more like shares in a mutual fund. However, this is not out of line with the views of Rolnick and Weber [1987], who argue that free bank notes were priced to reflect their perceived backing, and that free banks thus functioned much like mutual funds. With regard to the fixed redemption period of the notes in this model, it is useful from an analytical point of view to abstract from agents' redemption decisions. Also, this will show that problems can arise with laissez faire banking that have nothing to do with "overissue" of circulating notes or stochastic redemption.

The other type of security is a noncirculating note, which is intermediated by a bank. Banks at the production location issue one noncirculating note in exchange for each unit of capital deposited with the bank. Noncirculating notes can be redeemed only by the initial noteholder. By issuing notes to many agents the bank can predict with virtual certainty that the fraction of agents who will return in two periods to redeem notes will be $1 - \lambda$. Agents returning noncirculating notes for redemption receive nothing if the capital they deposited initially is found to be bad, and they receive a share in the return on the bank's assets if they deposited good capital. Thus, agents will exchange only good capital for noncirculating notes, and each note can be redeemed in two periods for $\alpha_1/(1-\lambda)$ units of consumption.

Noncirculating notes have several features associated with bank deposits subject to withdrawal or to payment by check. First, there are contingencies under which these assets cannot be used to carry out transactions. If an agent learns she is type 1 in middle age, then a noncirculating note is worthless but a circulating note or fiat money will not be, provided that some agent is willing to accept these other assets in exchange. Second (by definition), these notes do not change hands, just as, under normal circumstances, checks are redeemed immediately and do not circulate. Third, there is a need here, as in the case of deposit banking, for the bank to diversify across depositors to assure a predictable pattern of withdrawals (i.e., redemptions).

We let p_t denote the price of fiat money in terms of the consumption good. Note that fiat money is traded at the market location. Circulating notes issued at time t can be traded at t + 1 and at t + 2 at the market location. In the first instance, such a note will be called a secondhand circulating note and in the second a thirdhand circulating note. Secondhand circulating notes, which potential buyers cannot identify as good or bad, trade at price q_t.

Circulating notes will not be issued at time t if $q_{t+1} = 0$, since in this case they are dominated in rate of return by noncirculating notes. If $q_{t+1} > 0$, then the one-period return to holding a newly-issued good circulating note, q_{t+1}, is less than the corresponding return for a newly-issued bad circulating note, q_{t+1}/β. Thus, agents who learn they are type 2 in middle age will not sell their good circulating notes, and will not issue new good circulating notes. However, agents who learn they are type 1 in middle age will be forced to sell their good circulating notes.

The fraction of traded secondhand circulating notes that are good is denoted by γ_t, where t is the date when these notes were newly-issued. Agents who buy secondhand circulating notes diversify across these notes so as to obtain the mean redemption value, which is $\gamma_t\alpha_1 + (1-\gamma_t)\alpha_2$. Note that there is no incentive to trade a good circulating note thirdhand. At time t + 2, such a note trades for its expected redemption value to other agents, which is less than or equal to α_1, its actual redemption value. Thus, good circulating notes are issued only by young agents, and are held until redemption unless the agent learns she is type 1 in middle age.

All bad circulating notes issued at time t will be traded in the secondhand market at t + 1. To see this, note first that bad circulating notes will be issued at t only if $q_{t+1} > 0$. Agents in the third period of life will not issue these notes, and neither will a type 1 agent in middle age. A type 2 middle-aged agent acquiring one of these notes at time t will sell it at t + 1 in order to consume, as will a type 1 middle-aged agent who issued the notes when young. For an agent who issues bad circulating notes when young and then learns she is type 2 in middle age, consider the following three strategies. First, the agent could hold the note until maturity, earning a two-period gross rate of return of α_2/β. Second, the note could be traded on the secondhand market at t + 1, other secondhand notes could be purchased with the proceeds, and these notes could be held until maturity, earning a two-period gross rate of return of $[\gamma_t\alpha_1+(1-\gamma_t)\alpha_2]/\beta$. Third, the second strategy could be duplicated, except that the secondhand notes carried into period t + 2 would be sold in the thirdhand market. This third strategy must yield the same two-period gross rate of return as the second due to arbitrage. Thus, the second and third strategies are weakly-preferred to the first and, without loss of generality, we can assume that all bad circulating notes issued at t are traded secondhand at t + 1. Also without loss of generality, we assume that thirdhand notes traded consist of the entire stock of circulating notes which were traded secondhand in the previous period.

The preceding reasoning helps to simplify notation. An agent born at time t chooses nominal fiat money balances, m_t^t and m_{t+1}^t, noncirculating notes, d_t, issues of good circulating notes when young, g_t, issues of bad circulating notes, b_t^t and b_{t+1}^t, and secondhand circulating notes, s_t^t and s_{t+1}^t, to maximize expected utility. Here, a superscript denotes the agent's date of birth, and a subscript the date when an asset is acquired. An agent born at time t then solves

(15) $\max \lambda u(f_t^t, f_{t+1}^t)+(1-\lambda)v(f_t^t, f_{t+2}^t)$

subject to:

$$(16) \qquad f_t^t = y - d_t - p_t m_t^t - g_t - \beta b_t^t - q_t s_t^t$$

$$(17) \qquad f_{t+1}^t = p_{t+1} m_t^t + q_{t+1}(g_t + b_t^t) + [\gamma_{t-1}\alpha_1 + (1-\gamma_{t-1})\alpha_2] s_t^t$$

$$(18) \qquad p_{t+1} m_{t+1}^t + \beta b_{t+1}^t + q_{t+1} s_{t+1}^t = p_{t+2} m_t^t + [\gamma_{t-1}\alpha_1 + (1-\gamma_{t-1})\alpha_2] s_t^t + q_{t+1} b_t^t$$

$$(19) \qquad f_{t+2}^t = p_{t+2} m_{t+1}^t + q_{t+2} b_{t+1}^t + [\gamma_t \alpha_1 + (1-\gamma_t)\alpha_2] s_{t+1}^t + \alpha_1 g_t + [\alpha_1/(1-\lambda)] d_t.$$

In (15)-(19), to economize on notation we have dropped explicit treatment of the market for thirdhand circulating notes and the transfer of goods from the market location to the production location. Here, f_t^s denotes consumption at the market and production locations at time t by an agent born at time s.

Type 1 agents liquidate all assets in middle age, while type 2 agents shuffle their portfolios in middle age and then liquidate in old age.

Definition 2. A laissez faire banking equilibrium (LE) is a nonnegative sequence of prices $\{\bar{p}_t, \bar{q}_t\}$, a nonnegative sequence of asset quantities $\{\bar{a}_t\} = \{\bar{m}_t, \bar{m}_{t+1}^t, \bar{s}_t^t, \bar{s}_{t+1}^t, \bar{b}_t^t, \bar{b}_{t+1}^t, \bar{d}_t, \bar{g}_t\}$, a nonnegative sequence of consumption quantities $\{\bar{f}_t\} = \{\bar{f}_t^t, \bar{f}_{t+1}^t, \bar{f}_{t+2}^t\}$, and a sequence of γ's, $\{\bar{\gamma}_t\}$, such that:

(i) $\bar{f}_t$ and $\bar{a}_t$ are chosen to solve (15) subject to (16)-(19), given $\{p_t, q_t\} = \{\bar{p}_t, \bar{q}_t\}$.

(ii) The money market clears.

$$(20) \qquad \bar{p}_t\{\bar{m}_t^t + (1-\lambda)\bar{m}_t^{t-1}\} = \bar{p}_t(2-\lambda)H, \; t = 2, 3, 4, \ldots$$

(21) $\bar{p}_1\bar{m}_1^1 = \bar{p}_1 H.$

(iii) The market for secondhand circulating notes clears.

(22) $\bar{s}_t^t + (1-\lambda)\bar{s}_t^{t-1} = \lambda\bar{g}_{t-1} + \bar{b}_{t-1}^{t-1} + (1-\lambda)\bar{b}_{t-1}^{t-2}$, $t = 3, 4, 5, \ldots$

(23) $\bar{s}_2^2 + (1-\lambda)\bar{s}_2^1 = \lambda\bar{g}_1 + \bar{b}_1^1$

(24) $\bar{s}_1^1 = 0.$

(iv) The sequence of γ's is correctly anticipated. That is, if any one of $\bar{g}_t > 0$, $\bar{b}_t^t > 0$, or $\bar{b}_t^{t-1} > 0$ holds, then

$$\gamma_t = \frac{\lambda\bar{g}_t}{\lambda\bar{g}_t + \bar{b}_t^t + (1-\lambda)\bar{b}_t^{t-1}}.$$

Otherwise, equilibrium prices and quantities must be consistent with any $\gamma_t \in [0,1]$.

For the most part, definition 2 is a standard definition of rational expectations equilibrium. However, there is a difference here, in that agents' expectations of quantities traded matter. Part (iv) of the definition specifies that agents correctly anticipate the quantities of good and bad circulating notes in equilibrium. However, if no circulating notes are issued at time t in equilibrium, the definition states that agents' beliefs about γ_t must be irrelevant. That is, equilibrium prices and quantities must in this case be consistent with any $\gamma_t \in [0,1]$.

Define r_t, the maximum one-period gross rate of return at time t, by

$$r_t \equiv \max\{p_{t+1}/p_t, q_{t+1}/\beta, [\gamma_{t-1}\alpha_1 + (1-\gamma_{t-1})\alpha_2]/q_t\}.$$

Then, from (15)-(19) and the Kuhn-Tucker conditions, optimization implies

(25) $$-\lambda D_1 u - (1-\lambda)D_1 v + \lambda q_{t+1} D_2 u + (1-\lambda)\alpha_1 D_2 v \le 0,$$

(26) $$-\lambda D_1 u - (1-\lambda)D_1 v + \lambda r_t D_2 u + (1-\lambda) r_t r_{t+1} D_2 v \le 0,$$

(27) $$-\lambda D_1 u - (1-\lambda)D_1 v + \alpha_1 D_2 v \le 0.$$

The left-hand side of (25) is the marginal expected utility from issuing good circulating notes when young, the left-hand side of (26) is the marginal expected utility from acquiring the asset with the highest one-period return when young, and the left-hand side of (27) is the marginal expected utility from acquiring a noncirculating note when young.

Preferences and technology are further-restricted as follows:

(28) $$D_2 u(x,y) + y D_2 D_2 u(x,y) \ge 0, \ x, \ y \ge 0.$$

(29) $$\alpha_1^{\frac{1}{2}} D_2 v(x, \alpha_1^{\frac{1}{2}} y) \ge D_2 u(x,y), \ x, \ y \ge 0.$$

Inequality (28) states that the coefficient of relative risk aversion of $u(x,y)$, for any fixed $x \ge 0$, is less than or equal to unity. To obtain some notion of the implications of (29), suppose that $u(x,y) = x^{1-\mu} + \delta y^{1-\mu}$ and $v(x,y) = x^{1-\mu} + \delta^2 y^{1-\mu}$, with $0 < \delta < 1$, and $\mu > 0$. Thus, δ is a discount factor and μ is the coefficient of relative risk aversion of the period utility function. Then (29) implies that $\alpha_1^{(1-\mu)/2} \ge 1/\delta$. From (28), $\mu < 1$, so that (29) puts a lower bound on α_1.

<u>Proposition 1</u>: In a LE, if good circulating notes are issued in period t, then bad circulating notes are also issued in period t.

Proof: Suppose that good circulating notes are issued in a LE at time t, but bad circulating notes are not. Then, the two-period gross rate of return to acquiring newly-issued bad circulating notes at time t, selling them at time t + 1, and using the proceeds to acquire secondhand notes which are redeemed at time t + 2 is $\alpha_1/\beta > \alpha_1$. Also, the one-period rate of return to issuing bad circulating notes at t and selling them at t + 1 is $q_{t+1}/\beta > q_{t+1}$. Thus, bad circulating notes dominate good circulating notes in rate of return, a contradiction. Q. E. D.

Proposition 1 points out an important effect of private information in the model. If all information were public, then good and bad circulating notes would sell at different prices. Therefore, bad circulating notes could not co-exist with good circulating notes in equilibrium, since they yield an inferior return when held from time of issue to maturity.

In what follows, attention is restricted to a particular class of equilibria.

Definition 3: A stationary laissez faire banking equilibrium (SLE) is an equilibrium satisfying definition 2 with prices and consumption allocations which are time-stationary. That is, $(p_t,q_t) = (p,q)$ and $(f_t^t,f_{t+1}^t,f_{t+2}^t) = (f_1,f_2,f_2)$ for all t = 1, 2, 3, ..., where p, q, f_1, f_2, and f_3 are nonnegative constants.

Nonmonetary Equilibrium

We first examine SLE's where fiat money is not valued; that is $p_t = 0$ for all t. This requires some notation for characterizing equilibria. Let A denote the set of assets that are held in a SLE, where $A \subset \{G,B,F\}$. If $G \in A$, then good circulating notes are held in equilibrium; if $B \in A$ then bad circulating notes are held; if $D \in A$ then noncirculating notes are held. Given proposition 1, the only possibilities in a nonmonetary equilibrium are:

Case 1. $A = \{G,B,D\}$

Case 2. $A = \{B\}$

Case 3. $A = \{D\}$

Case 4. $A = \{G,B\}$

Case 5. $A = \{B,D\}$.

We will proceed to show that Case 5 is the only possibility. First, in a Case 3 SLE, we must have $q = 0$, since otherwise there would be excess demand for circulating notes to be traded for consumption by middle-aged type 2 agents. However, if $q = 0$, then there is an excess demand for secondhand circulating notes for $t = 2, 3, 4, \ldots$. Therefore Case 3 is not a possibility. In the remaining cases, bad circulating notes are held, which implies, given definition 2, that

$$(30) \qquad r_t = q/\beta,$$

for all t. For cases 1 and 4, where good circulating notes are held, we must have $r_t r_{t+1} < \alpha_1$, otherwise good circulating notes are dominated in rate of return by bad circulating notes. Therefore, from (30),

$$(31) \qquad q < \beta\alpha_1^{\frac{1}{2}}.$$

In cases 1 and 4, (25) and (26) hold with equality, so that (27) and (30) imply

$$(32) \qquad -\alpha_1\beta^2 + (1-\lambda)q^2 + \lambda\alpha_1\beta \leq 0.$$

Suppose that, facing prices constrained by (30), (31), and (32), an agent held only good circulating notes, so that (25) held with equality, that is

$$(33)\qquad -\lambda D_1u(y-x,qx) - (1-\lambda)D_1v(y-x,\alpha_1x) + \lambda qD_2u(y-x,qx) + (1-\lambda)\alpha_1D_2v(y-x,\alpha_1x) = 0,$$

for some $x > 0$. Then, (33) implies that the left-hand side of (26) equals

$$\lambda q(1/\beta-1)D_2u(y-x,qx) + (1-\lambda)(q^2/\beta^2-\alpha_1)D_2v(y-x,\alpha_1x)$$

$$\leq \lambda(1/\beta-1)\left[qD_2u(y-x,qx)-\alpha_1D_2v(y-x,\alpha_1x)\right]$$

$$\leq \lambda(1/\beta-1)\left[\alpha_1^{\frac{1}{2}}D_2u(y-x,\alpha_1^{\frac{1}{2}}x)-\alpha_1D_2v(y-x,\alpha_1x)\right] \leq 0.$$

The first inequality follows from (32), the second is implied by (28) and (31), and the third follows from (29). This then implies that (28) holds. Thus, we have a contradiction, and can therefore rule out cases 1 and 4.

In cases 2 and 5, we have

$$(34)\qquad q_t = (\alpha_2\beta)^{\frac{1}{2}}.$$

Therefore (31) holds, since $\alpha_2/\beta < 1$ and $\alpha_1 > 1$. For case 2, (26) holds with equality. That is,

$$(35)\qquad -\lambda D_1u(y-x,qx/\beta) - (1-\lambda)D_1v(y-x,q^2x/\beta^2) + \lambda(q/\beta)D_2u(y-x,qx/\beta) + (1-\lambda)(q^2/\beta^2)D_2v(y-x,q^2x/\beta^2) = 0,$$

for some $x > 0$. Then (35) implies that the left-hand side of (27) equals

$$-\lambda(q/\beta)D_2u(y-x,qx/\beta) - (1-\lambda)(q^2/\beta^2)D_2v(y-x,q^2x/\beta^2)$$

$$+ \alpha_1 D_2v(y-x,q^2x/\beta^2)$$

$$> \lambda[-(q/\beta)D_2u(y-x,qx/\beta)+\alpha_1 D_2v(y-x,q^2x/\beta^2)]$$

$$> \lambda[-\alpha_1^{\frac{1}{2}}D_2u(y-x,\alpha_1^{\frac{1}{2}}x)+\alpha_1 D_2v(y-x,\alpha_1 x)] > 0.$$

Here, the first inequality follows from (31), the second inequality is implied by (28) and the fact the $v(\cdot,\cdot)$ is increasing in both arguments, and the third inequality follows from (29). Thus, (27) does not hold, and case 2 is not a possibility.

Proposition 2: A unique nonmonetary SLE exists if and only if

(36) $\quad \alpha_1(\beta-\lambda)-\alpha_2(1-\lambda) \le 0.$

Proof: Necessity. We have shown that if a SLE exists then it is a case 5 equilibrium, so that (26) and (27) hold with equality. But then (34) implies that, if (25) holds, as it must, then (36) holds.

Sufficiency. Suppose that (36) holds. Then, since $u(\cdot,\cdot)$ and $v(\cdot,\cdot)$ are concave, there is a unique $w > 0$ and a unique $d > 0$ such that, with $f_1 = y - d - w$, $f_2 = w(\alpha_2/\beta)^{\frac{1}{2}}$, $f_3 = \alpha_1 d/(1-\lambda) + (\alpha_2/\beta)w$, agents facing prices $p_t = 0$ and $q_t = (\alpha_2\beta)^{\frac{1}{2}}$ for all t choose $f_t^t = f_1$, $f_{t+1}^t = f_2$, $f_{t+2}^t = f_3$, and $d_t = d$ for all t. Here, w is the quantity of savings by each young agent in the form of new issues of bad circulating notes and secondhand circulating notes. The number of bad circulating notes issued in period t (per young agent) is k_t^b, where

$$k_t^b = (w/\beta)\sum_{i=0}^{t-1}[-\lambda(\alpha_2/\beta)^{\frac{1}{2}}]^i, \; t = 1, 2, 3, \ldots .$$

Therefore, $k_t^b > 0$ for all t, and $\lim_{t\to\infty} k_t^b = w/\beta[1+\lambda(\alpha_2/\beta)^{\frac{1}{2}}]$. Thus, there exists a SLE, and, given the uniqueness of d and w, it is unique. Q.E.D.

Note, in spite of the fact that consumption and prices are stationary in the SLE equilibria, that the number of circulating notes issued in each period, k_t^b, is not a constant. However, k_t^b converges to a steady state.

Monetary Equilibrium

Still restricting attention to SLE's, we now look for monetary equilibria, where $p_t > 0$ for all t. Therefore,

(37) $r_t = 1,\ t = 1, 2, 3, \ldots .$

Proposition 3: In a monetary SLE, if it exists, either good circulating notes are issued or no circulating notes are issued.

Proof: Suppose that bad circulating notes are issued in a stationary monetary equilibrium, but good circulating notes are not. Then, given (37), the one-period return to issuing a circulating note must be unity, or $q_t = \beta$, and the one-period return to acquiring a secondhand circulating note must also be unity, or $q_t = \alpha_2$. But $\alpha_2 \neq \beta$. Q.E.D.

Proposition 1 and Proposition 3 then imply that, in a monetary SLE, either circulating notes are not issued or good and bad circulating notes coexist with valued fiat money. A monetary SLE, if it exists, will then be one of the following cases, where A is now the set of assets held in equilibrium in addition to fiat money.

Case 1. $A = \{G,B,D\}$

Case 2. $A = \{D\}$

Case 3. $A = \{G,B\}$

Case 4. $A = \{\ \}$.

Proposition 4: A monetary SLE does not exist.

Proof: For cases 1 and 3 we have

(38) $$q_t = \beta$$

and

(39) $$\gamma_t = (\beta-\alpha_2)/(\alpha_1-\alpha_2).$$

Through an argument essentially identical to the one which eliminated cases 1 and 4 for the nonmonetary SLE, we can rule out cases 1 and 3 here. With cases 2 and 4, no circulating notes are issued, and therefore these notes do not trade on the secondhand market. For there to be no incentive to issue bad circulating notes, we must have

(40) $$q \leq \beta.$$

Also, zero excess demand for secondhand circulating notes implies

(41) $$q \geq \gamma_t\alpha_1 + (1-\gamma_t)\alpha_2.$$

Therefore, an equilibrium does not exist if

(42) $$\gamma_t > (\beta-\alpha_2)/(\alpha_1-\alpha_2)$$

for any t. Since $(\beta-\alpha_2)/(\alpha_1-\alpha_2) < 1$, and because case 2 and case 4 equilibria must be consistent with any $\gamma_t \in [0,1]$ for all t, an equilibrium does not exist. Q.E.D.

Essentially, the nonexistence of a monetary SLE arises because there exists an underlying asset which must dominate fiat money in rate of return in equilibrium. There does not exist a monetary SLE where fiat money and notes backed by good and bad capital circulate, but if good and bad notes do not circulate in equilibrium, there always exist some conjectures for which there is either an excess demand for secondhand circulating notes or there is an incentive to issue bad circulating notes, either of which involves a contradiction. The existence of such conjectures relies on the restriction that $\alpha_1 > 1$.

Because of a "lemons" problem (see Akerlof 1970), a version of Gresham's law holds in equilibrium; the only private notes that circulate are those backed by bad capital, in spite of the fact that there are underlying assets that yield a higher rate of return. Bank deposits (noncirculating notes) are also held in equilibrium, and the assets backing deposits dominate the assets backing circulating notes in rate of return. The transactions velocity of circulating notes is higher than for bank deposits, since the entire stock of circulating notes is traded each period while bank deposits are held to maturity.

In spite of the fact that there exists the potential for 'fraudulent' circulating note issue, and though this potential for fraud is important in producing the results, there is no fraud in a SLE equilibrium. All circulating notes are recognized as being backed by bad capital, and agents cannot credibly claim otherwise. This is consistent with Rolnick and Weber's [1983, 1984] reinterpretation of the evidence from the U.S. free banking era. Rolnick and Weber argue that fraudulent behavior in the issue of free bank notes was not as important a feature of the free banking period as once thought.

As noted earlier, a SLE equilibrium may not exist, given proposition 2. That is, in a SLE equilibrium only bad notes circulate, but if (36) does not hold then there is an incentive to issue good circulating notes in equilibrium, and the equilibrium unravels. Note that a SLE is more likely to exist the smaller is β and the larger is λ. That is, if β is smaller, then there is a larger difference between the one-period return on newly-issued good and bad circulating notes, and that difference is more important the larger is λ, the probability of an agent being type 1 and having to liquidate in middle age.

Nonoptimality of the SLE

It is clear that the SLE allocation would not be optimal with full information. There exist preferred feasible allocations where agents produce only good capital. However, it may be the case that there is production of bad capital in a constrained Pareto optimal allocation when information is private, though it will be shown in what follows that the SLE allocation is not constrained Pareto optimal.

Consider a (*) allocation, implemented by the planner as follows:

(43) $$c_t^t = y - c^* - k^*,$$

(44) $$c_{t+1}^t = c_{t+2}^t = c^*,$$

(45) $$z_t^t = z_t^{t-1} = z_t^{t-2} = 0,$$

(46) $$z_{t+2}^t = \alpha_1 k^*/(1-\lambda),$$

(47) $$k_t^g = k^*,$$

(48) $$k_t^b = 0,$$

(49) $\quad x_t = c^*,$

for $t = 1, 2, 3, \ldots$, where $c^* > 0$ and $k^* \geq 0$ solve:

(50) $\quad \max_{c,k}\{\lambda u(y-c-k,c)+(1-\lambda)v(y-c-k,c+\alpha_1 k/(1-\lambda))\}.$

Also, we have:

(51) $\quad c_1^0 = c_1^{-1} = c_2^0 = c^*.$

By construction, the (*) allocation satisfies (1)-(4). Since agents cannot consume in middle age in the (*) allocation without an identity badge, (5) holds, and (6) holds by virtue of the fact that c^* and k^* are chosen to solve (50). Also, since $x_{t+1} = c_{t+1}^t$ for all t, and $c_{t+2}^t = c_{t+2}^{t+1}$ for all t, (7) and (8) are satisfied. Inequality (9) holds since

$$\lambda u(y-c^*-\beta k^*,c^*) + (1-\lambda)v(y-c^*-\beta k^*,c^*) \leq \lambda u(y-c^*,c^*) + (1-\lambda)v(y-c^*,c^*)$$

$$\leq \lambda u(y-c^*-k^*,c^*) + (1-\lambda)v(vy-c^*,c^*+\alpha_1 k^*/(1-\lambda)).$$

The first inequality follows since $u(\cdot,\cdot)$ and $v(\cdot,\cdot)$ are increasing in both arguments, and the second inequality is an implication of the fact that c^* and k^* are chosen to solve (50). Thus, the (*) allocation is feasible and incentive compatible.

<u>Proposition 5</u>: The SLE allocation is not constrained Pareto optimal.

<u>Proof</u>: Let a (^) denote the SLE allocation. We need only show that all agents weakly prefer the (*) allocation to the (^) allocation, and that there

are some agents who strictly prefer it. First, agents who are old and middle-aged at $t = 1$ are better off with the (*) allocation, since

$$\hat{c}_1^0 = \hat{c}_2^0 = \hat{c}_1^{-1} = 0 < c_1^{*0} = c_2^{*0} = c_1^{*-1} = c^*.$$

For the (^) allocation, let $\hat{k}$ denote the quantity of bad capital that is produced each period to back noncirculating notes, and let $\hat{s}$ denote the quantity of the consumption good exchanged by each young agent for newly-issued and secondhand circulating notes. The expected utility of each agent born at $t = 1, 2, 3, \ldots,$ with the (^) allocation is then

$$\lambda u(y-\hat{s}-\hat{k},(\alpha_2/\beta)^{\frac{1}{2}}\hat{s}) + (1-\lambda)v\left(y-\hat{s}-\hat{k},(\alpha_2/\beta)\hat{s}+[\alpha_1/(1-\lambda)]\hat{k}\right)$$

$$< \lambda u(y-\hat{s}-\hat{k},\hat{s}) + (1-\lambda)v\left(y-\hat{s}-\hat{k},\hat{s}+[\alpha_1/(1-\lambda)]\hat{k}\right)$$

$$\leq \lambda u(y-s^*-k^*,s^*) + (1-\lambda)\left(y-c^*+[\alpha_1/(1-\lambda)]k^*\right).$$

The first inequality follows from the fact that $u(\cdot,\cdot)$ and $v(\cdot,\cdot)$ are increasing in both arguments and $\alpha_2/\beta < 1$. Since c^* and k^* are chosen to solve (50), we get the second inequality. Therefore, agents born at $t = 1, 2, 3, \ldots,$ strictly prefer the (*) allocation to the (^) allocation. Q.E.D.

In this environment, laissez faire banking arrangements do not lead to a stationary equilibrium allocation that is constrained Pareto optimal. The reader might ask at this point whether there might be other arrangements that also exhibit observable features of real-world banking systems, but support a constrained Pareto optimal allocation. In answer to this question, note that other banking-type arrangements in this environment must necessarily involve trading between asymmetrically-informed agents, and will thus lead to the same type of lemons problem encountered here with this laissez faire banking arrangement. An important feature of the environment is that type 2 agents can mimic type 1 agents in middle age and thus avoid incurring any ex post penalty for producing bad capital when young.

IV. Restrictions On Private Note Issue

In most (if not all) regimes with government-issued fiat money, there are legal restrictions on the types of liabilities that private agents may issue. A common restriction is the prohibition of at least some types of circulating notes. Suppose, then, that the government bans the issue of circulating notes, which is feasible in this environment given that the government can observe the production of capital and observes trading at the production location. Otherwise, trading possibilities are identical to those specified in the previous section. A young agent born at time t then chooses fiat money balances $m_t(=m_t^t=m_{t+1}^t)$ and nontransferable note acquisitions, d_t, to solve:

$$\max\{\lambda u(f_t^t,f_{t+1}^t)+(1-\lambda)v(f_t^t,f_{t+2}^t)\} \tag{51}$$

subject to:

$$f_t^t = y - p_t m_t - d_t \tag{52}$$

$$f_{t+1}^t = p_{t+1} m_t \tag{53}$$

$$f_{t+2}^t = p_{t+2} m_t + [\alpha_1/(1-\lambda)]d_t,\ m_t,\ d_t \geq 0. \tag{54}$$

The first order conditions for an optimum, provided that $p_t > 0$ for all t, are

$$-\lambda D_1 u - (1-\lambda)D_1 v + \lambda p_{t+1} D_2 u/p_t + (1-\lambda)p_{t+2} D_2 v/p_t = 0, \tag{55}$$

$$-\lambda D_1 u - (1-\lambda)D_1 v + \alpha_1 D_2 v \leq 0, \tag{56}$$

where (56) holds with equality if $d_t > 0$.

In a stationary monetary equilibrium (SME), $p_t = p > 0$ for all t, so that

(57) $$p_{t+1}/p_t = p_{t+2}/p_t = 1,$$

for all t. There exists a unique SME (given the concavity of $u(\cdot,\cdot)$ and $v(\cdot,\cdot)$) and the SME allocation is the (*) allocation of the previous section, given (43)-(50), (51)-(54), and (57). Also, note that bank deposits (noncirculating notes) are held in the SME. That is, suppose that noncirculating notes are not held in a SME. Then (55) holds or, using (57),

$$\lambda D_1u(y-x,x) - (1-\lambda)D_1v(y-x,x) + \lambda D_2u(y-x,x) + (1-\lambda)D_2v(y-x,x) = 0,$$

for some $x > 0$. Therefore, the left-hand side of (56) equals

$$-\lambda D_2u(y-x,x) + (\alpha_1-1+\lambda)D_2v(y-x,x) > \lambda[-D_2u(y-x,x)+\alpha_1D_2v(y-x,x)]$$
$$> \lambda[-\alpha_1^{\frac{1}{2}}D_2u(y-x,\alpha_1^{\frac{1}{2}}x)+\alpha_1D_2v(y-x,\alpha_1x)] > 0.$$

Here, the first inequality follows from the fact that $\alpha_1 > 1$, the second is implied by (28) and the fact that $v(\cdot,\cdot)$ is increasing in both arguments, and the third follows from (29).

Since the SME allocation is the (*) allocation, it is immediate from Proposition 5 that this allocation Pareto dominates the SLE allocation. It is also the case that the SME allocation is constrained Pareto optimal, at least within a certain class of allocations.

Definition 4: A production stationary (PS) allocation is an allocation where $c_t^t = c_1$, $c_{t+1}^t = c_2$, $c_{t+2}^t = c_3$, $z_t^t = z_1$, $z_{t+1}^t = z_2$, $z_{t+2}^t = z_3$ $k_t^g = k^g$, $k_t^b = k^b$, for all t, where $c_i \geq 0$, $z_i \geq 0$, i = 1, 2, 3, and $k^g \geq 0$, $k^b \geq 0$.

Proposition 6: The SME allocation with legal restrictions is constrained Pareto optimal within the class of PS allocations.

Proof: A PS allocation necessarily has $z_1 = z_2 = 0$, $(1-\lambda)z_3 = \alpha_1 k^g + \alpha_2 k^b$, $c_1 = y - \lambda c_2 - (1-\lambda)c_3$, and $x_t = \lambda c_2 + (1-\lambda)c_3$ for all t. Conditions (7) and (8) then imply that $c_2 = c_3$ in any PS allocation. Given this, the allocation given by $k^b = 0$, $k^g = k^*$, $c_1 = y - c^*$, $c_2 = c_3 = c^*$, is, in the set of PS allocations, the unique allocation which maximizes the expected utility of agents born at t = 1, 2, 3, ... Therefore, there is no other allocation in the set of PS allocations which could make at least one agent better off without making another worse off. Q.E.D.

Legal restrictions that ban circulating notes thus assure: (1) that a constrained Pareto optimal SME always exists and (2) that this SME Pareto dominates a SLE if the SLE exists. There are several features of the SME that are consistent with what is observed in real-world fiat money regimes. In particular, valued fiat money coexists with bank deposits (noncirculating notes), and the assets backing bank deposits dominate fiat money in rate of return. Also, fiat money has a higher transactions velocity than does the alternative asset.

The model provides an explanation for the observation that fiat money regimes tend to be supported by legal restrictions on the issue of private circulating media of exchange. Here, a stationary equilibrium with valued fiat money exists with legal restrictions where such an equilibrium does not exist without these restrictions. In spite of this, the imposition of legal restrictions brings about a welfare improvement. The SME allocation is constrained Pareto optimal while the SLE allocation is not, and the first allocation Pareto dominates the latter. These results can be contrasted to

Sargent and Wallace [1982], where restrictions on private intermediation make some agents better off and some worse off, and where a Pareto optimal equilibrium always exists under laissez faire.

V. Summary and Conclusions

The model constructed here confirms the conventional wisdom that the provision of circulating media of exchange should not be left to the private sector. The model delivers this result while yielding predictions consistent with what is observed in free banking regimes and in fiat money regimes.

In a stationary laissez faire banking equilibrium, fiat money is not valued, and the resulting allocation is not constrained Pareto optimal. However, if the government bans the issue of private circulating notes, there exists a stationary equilibrium with valued fiat money that Pareto dominates the laissez faire equilibrium and is constrained Pareto optimal. These results stem from features of the model intended to capture the nature of decentralized exchange. In particular, some financial claims are illiquid because of the fact that, under some contingencies, potential buyers of these claims have less information about future asset returns than do the sellers. With no restrictions on private financial arrangements, private agents can exploit this informational asymmetry by issuing notes backed by inferior assets, and a lemons problem results. Thus, there are advantages to having the government be a monopoly supplier of a universally-recognizable medium of exchange.

What is not studied in this paper is the possibility that the government might misuse its power to extract seignorage, once private circulating notes are prohibited, and thus inflict costs on the private sector through anticipated and unanticipated inflation. These costs are emphasized in Friedman's more recent writings. Friedman and Schwartz [1986, p. 59] even go so far as to argue that "leaving monetary and banking arrangements to the market would have produced a more satisfactory outcome (over the last quarter century) than was actually achieved through government involvement." Perhaps binding restrictions on government behavior are needed in regimes with fiat currency to generate preferred outcomes. However, this is a topic for further research.

Footnotes

[1]There are two alternative actions which have been omitted in (5)-(9). The first is the case where the agent does not acquire an identity badge and does not produce capital, and the second is the case where a badge is not acquired and the agent produces only bad capital. In both of these alternatives, the agent consumes zero in middle and/or old age, so that conforming to the planner's allocation is preferred.

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