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NOTES ON WAGE-PRICE GUIDEPOSTS

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Author's Preface

This is, as the cover indicates, a working paper. There has been considerable discussion at the Federal Reserve Bank of Minneapolis concerning appropriate post-freeze policies. These notes were put together with two purposes in mind: (1) to summarize some of the more popular guidepost proposals and (2) to compare these proposals in the context of a simple model of aggregate supply. We have found this exercise helpful and hope that its limited distribution might motivate someone to carry on the work in greater detail that is necessary for the proper formulation, and more importantly now, evaluation of guidepost policies.

The various proposals have been gathered either from the press or verbally. Each particular author may disagree completely with the kind of model I have chosen to analyze their proposals, and no attempt has been made to confirm the analysis with them.

Guidepost policies of the early sixties dealt basically with wages. Since wage increases were to be limited to productivity growth, prices, on average, were to be unchanged. Recognizing the futility of administering such a low rate of growth in wages in the present inflationary situation, most current proposals for post-freeze policies involve both a guidepost for wages based on inflation and productivity, and a guidepost for prices based on unit labor costs. For example, the Okun proposal involves a price guidepost wherein business absorbs the first one percent increase in unit labor costs while labor gives up one-half of the past year's increase in prices. This proposal raises the question of equity between the sacrifices of business and labor. The purpose of these notes is to evaluate these proposals in the context of a simple aggregate supply model.

Several elementary models are developed and examined from two points of view -- (1) how do the proposals affect the rate of change in prices (inflation), and (2) how do the proposals affect the relative shares of labor and capital (income distribution). The following notation is common to all the models presented below:

L_t = labor input in manhours

W_t = money wage rate

w_t = percent change in the money wage rate

P_t = price level

z_t = inflation, i.e. percent change in prices

X_t = real income (output)

N_t = nonlabor share of output

M_t = output per manhour, i.e. $M_t = \frac{X_t}{L_t}$

g_t = growth in productivity, i.e. percent change in output per manhour

U_t = unit labor cost, i.e. $\frac{L_t W_t}{X_t}$

h_t = percentage change in unit labor cost

We first present a basic wage-price model, and then discuss specific proposals as deviations from the basic model. Okun's proposal for price determination is given with two interpretations.

1. The Basic Model

The basic guidepost proposals involve a wage rule and a variant of a price markup equation. Without adjustment these may be written as:

$$w_t = z_{t-1} + g_{t-1} \tag{1}$$

and

$$P_t = c\ U_t \tag{2}$$

where $c > 1$ is the markup factor. Note that since wages are determined independently of the level of economic activity there is no tradeoff a' la Phillips in this model. Equation (1) may be interpreted as a labor supply function; and, with P fixed, equation (2) could be interpreted as a labor demand function since it is identical to the marginal productivity condition implied by a Cobb-Douglas production function.[1] From this point of view, it

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[1/] Write the production function as

$$X = A\ L^a\ K^{1-a}.$$

With the firm a price taker, labor will be demanded to the point where the marginal product equals the real wage, i.e.

$$\frac{aX}{L} = \frac{W}{P}\ .$$

Solving for P yields (2) with $c = \frac{1}{a}$.

is somewhat paradoxical that price markup equations are often justified in models because of monopolistic elements when they are implied by a competitive model. Of course, P_t is the decision variable in this model and is not fixed -- it is, however, independent of aggregate output if the labor/output ratio is constant.[2] Furthermore, in the context of a guidepost policy, the parameter c need not be interpreted as something derived from profit maximizing theory of the firm -- it is administered by the government to be unchanged from period to period.

Inflation: Using the definition of U_t, we may rewrite (2) as

$$(3) \qquad P_t = c\,\frac{W_t L_t}{X_t}.$$

Taking logarithms, and first differences we have

$$(4) \qquad \ln P_t - \ln P_{t-1} = \ln W_t - \ln W_{t-1} - \ln M_t + \ln M_{t-1}.$$

And recognizing that log-first-differences are approximations to percent changes,[3] we may rewrite (4) as

$$(5) \qquad z_t = w_t - g_t.$$

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[2]/Monetary and fiscal policies determine the level of aggregate demand. If the labor/output ratio is constant, then the aggregate supply curve is horizontal so that prices (as well as wages) in this model are independent of current output and policy. The price level is determined entirely by the supply sector.

[3]/For small y, it is approximately true that

$$\ln (1 + y) = y.$$

Exact percentage change may be written as

$$y = \frac{X - X_{-1}}{X_{-1}} = \frac{X}{X_{-1}} - 1.$$

Substituting into the approximation formula, we have

$$\ln X - \ln X_{-1} = y.$$

Substituting (1) into (5), we have

(6) $$z_t = z_{t-1} - (g_t - g_{t-1}).$$

Thus, according to this model inflation can be dampened only when there is an ever increasing rate of growth in productivity. This, in effect, is a kind of accelerator mechanism -- a slowdown in the rate of growth in productivity (not necessarily an outright decline in output per manhour) is sufficient to cause an acceleration in the rate of growth of prices. Assuming that productivity growth is constant, then the steady state rate of inflation is simply determined by the price initial condition.

Income distribution: In this model, capital gets the residual product after labor gets its share so that the distribution of income is determined entirely by the markup parameter c. Thus

(7) $$N_t = P_t X_t - W_t L_t.$$

Substituting for P_t from (3), capital's share is given by

(8) $$N_t = (c - 1) W_t L_t.$$

As long as c is unchanged from period to period, the ratio of capital income to labor income equals c - 1 in each period regardless of what happens to productivity.

Lagged pricing rule: The time path of factor shares is independent of productivity because prices are established on a current rather than a lagged unit labor cost basis. Perhaps a more relevant model is one wherein price guideposts are established with the same timing as are wage guideposts. To make the timing comparable, modify (2) to read

(9) $$P_t = c\ U_{t-1}.$$

This means that (5) must be modified to read

$$(10) \qquad z_t = w_{t-1} - g_{t-1}$$

and (6) then becomes

$$(11) \qquad z_t = z_{t-2} - (g_{t-1} - g_{t-2}).$$

Except for the technicality that the order of the difference equation has increased by one degree, the inflation implications are essentially the same as above.

This modification, however, does lead to a fundamentally different implication for income distribution. Substituting (9) into (7), and solving for the ratio of factor shares, we have

$$(12) \qquad f_t = c\,\frac{U_{t-1}}{U_t} - 1$$

where f_t is the ratio of nonlabor to labor income. The relative shares of capital and labor now depend on unit labor costs. If unit labor costs are declining ($U_t < U_{t-1}$), then capital's share is increasing, and vice versa.

If unit labor costs were to approach some equilibrium value, then the steady state distribution of income would be the same as that resulting from using current unit labor costs in the markup equation. However, the model does not guarantee an equilibrium value for unit labor costs. Using the log-first-difference procedure, the growth in unit labor costs may be written as

$$(13) \qquad h_t = w_t - g_t$$

Substituting from (1), we have

$$(14) \qquad h_t = z_{t-1} - (g_t - g_{t-1}).$$

Since there is nothing in the model to guarantee that h_t approach zero, there need not be a steady state distribution of factor shares. Indeed, if productivity growth is constant and there is inflation, the model implies that capital's share is continually below the level implied by c alone. If there is a constant rate of inflation as is implied by constant productivity growth, then U_{t-1}/U_t is constant, and capital's share is lower the higher the rate of inflation.

2. Lerner's rule

Lerner has proposed a wage guidepost that is essentially (1), but with some amount ($e > 0$) subtracted in order to achieve a decelerating rate of inflation. Together with the markup rule the Lerner model is

(15) $$w_t = z_{t-1} + g_{t-1} - e$$

(2) $$P_t = c\ U_t.$$

Inflation. Substituting (15) into (5), the rate of inflation in the Lerner model is given by

(16) $$z_t = z_{t-1} - (g_t - g_{t-1}) - e.$$

The rate of inflation is lower by e than in the basic model. In particular, if the rate of growth of productivity is constant through time, there is a continual slowdown in inflation which, after some point in time, becomes an actual deflation. No equilibrium z exists.

Income distribution: On the income distribution question, the Lerner model has precisely the same implications as the basic model. As long as current unit labor costs are used in the markup formula, the relative capital/ labor factor shares are equal to (c - 1).

Lagged pricing rule: Introducing the lagged pricing rule into the Lerner model by replacing (2) with (9), has the effect (as in the basic model) of shifting the order of the ~~differnece~~ equation that determines the inflation rate, i.e.

$$(17) \qquad z_t = z_{t-2} - (g_{t-1} - g_{t-2}) - e.$$

The formula for relative factor shares (12) also holds for the Lerner model, and the formula for the rate of growth in unit labor needs only to be modified slightly to read

$$(18) \qquad h_t = z_{t-1} - (g_t - g_{t-1}) - e.$$

As long as changes in the rate of growth of productivity don't dominate e in (17) and (18), there will be (on average at least) a continual slowdown in the rate of inflation and in unit labor costs -- both rates of growth becoming negative at some point in time. As long as h_t is positive, labor is benefiting relative to capital via relation (12), but when h_t turns negative, capital's share becomes larger than would be the case in the basic model or the Lerner model without lagged pricing.

3. Nelson's rule

Because of the belief that it is a more marketable product, it has been suggested that workers be granted all of the past increase in prices plus some part (k) of the past increase in productivity. The Nelson rule together with the markup formula might read as:

(19) $$w_t = z_{t-1} + k\, g_{t-1}, \quad 0 < k < 1$$

and

(2) $$P_t = c\, U_t.$$

Inflation: Substituting (19) into (5), we have the rate of inflation given by

(20) $$z_t = z_{t-1} - (g_t - k\, g_{t-1}).$$

The factor k serves to increase the probability that inflation is decelerating. In particular, if the rate of growth of productivity is constant, there will be a continual slowdown in the rate of inflation with eventual deflation.

Income distribution: Formula (19) in no way modifies the income distribution conclusions of the basic model as given in equations (7) and (8).

Lagged pricing rule: In the inflation context, appending the lagged pricing rule (9) to the Nelson rule simply shifts the order of the difference equation determining the inflation rate so that

(21) $$z_t = z_{t-2} - (g_{t-1} - k\, g_{t-2}).$$

From the income distribution point of view, formula (12) still holds, but the rate of growth in unit labor cost is now

(22) $$h_t = z_{t-1} - (g_t - k\, g_{t-1}).$$

The implications are basically the same as for the Lerner rule with lagged pricing.

It might be noted that we have run experiments imposing the Nelson wage rule on the Wharton model. Those experiments indicate that when k is close to zero, inflation moderates and there is a substantial shift in favor of capital's share. Considering productivity changes, these results seem generally consistent with equations (21) and (22) -- which should not be too surprising since the Wharton price equations are basically forms of (9) with more complex lag structures.

4. Okun's wage rule

Okun has proposed guideposts for both prices and wages. In this section we consider only the wage guidepost which says that wages should be permitted to grow at a rate equal to productivity growth plus some part (k) of past inflation. Together with the markup equation the model is

$$(23) \qquad w_t = g_{t-1} + k\, z_{t-1}, \quad 0 < k < 1$$

and

$$(2) \qquad P_t = c\, U_t.$$

<u>Inflation</u>: Substituting (23) into (5), the rate of inflation in this model is given by

$$(24) \qquad z_t = k\, z_{t-1} - (g_t - g_{t-1}).$$

The rate of inflation again depends critically on productivity; but assuming a constant trend rate of growth for this factor, inflation will continually decay, but will not become an actual deflation. The steady state rate of inflation is zero.

<u>Income distribution</u>: Income distribution is unchanged by this wage adjustment formula.

<u>Lagged pricing rule</u>: The inflation formula resulting from replacing (2) with (9) is now

(25) $$z_t = k\, z_{t-2} - (g_{t-1} - g_{t-2})$$

and unit labor costs grow at a rate of

(26) $$h_t = k\, z_{t-1} - (g_t - g_{t-1}).$$

With productivity growing at a constant trend rate of growth, inflation approaches a steady state of zero while capital's share via relation (12) approaches the steady state value of the current pricing rule from below.

5. Okun's price rule I

Okun has suggested that business "absorb" some part (b) of the increase in unit labor costs. Different interpretations of "absorb" lead to different formulations of the markup rule. In this section I discuss my interpretation of "absorb," and in the next section I present an alternative interpretation.

The markup rule (2) in the basic model implies that the percent change in prices is exactly the same as the percent change in unit labor costs. I interpret Okun to be suggesting that under the guideposts the percentage change in prices is b less than the percentage change in unit labor costs, i.e.

(27) $$z_t = h_t - b.$$

Substituting (13) and (23) into (27) we have the rate of inflation in the Okun model given by

(28) $$z_t = k\, z_{t-1} - (g_t - g_{t-1}) - b$$

which is simply b lower than the Okun wages only guidepost model. However, in the case of constant productivity growth, the addition of the "absorption" rule eventually leads to deflation rather than to a steady state of zero inflation.

Because the nonlabor share is a residual, we must have the price level to determine income distribution. To maintain the spirit of the markup model, we assume that firms observe the markup rule by adjusting the markup factor in each period, i.e. we now date the c factor of (2) so that

$$(29) \qquad P_t = c_t \, U_t .$$

But to have a time path of c_t's that makes (29) consistent with (27), we first use the log-first-difference approximation of (29) to establish

$$(30) \qquad z_t = C_t + h_t$$

where C_t is the approximation to the percent change in c_t. Since the right hand side of this equation must also satisfy (27) we have that

$$(31) \qquad C_t = -b.$$

That is, the markup factor is declining at a constant rate.

Substituting (29) into the distribution formula (7), relative shares are given by

$$(32) \qquad f_t = c_t - 1.$$

And, utilizing (31), we conclude that the Okun proposal involves an ever declining capital share of income.

Lagged pricing rule: To impose the lagged pricing rule on the Okun model, we rewrite (29) as

$$(33) \qquad P_t = c_t \, U_{t-1} .$$

Similarly the price guidepost (27) must be rewritten as

$$(34) \qquad z_t = h_{t-1} - b.$$

Substituting (13) and (23) into (34), the inflation rate is

$$z_t = k\, z_{t-2} - (g_{t-1} - g_{t-2}) - b \tag{35}$$

which again is simply the previous result with a higher order difference equation.

Substituting (33) into (7) we have the ratio of factor shares given by

$$f_t = c_t \frac{U_{t-1}}{U_t} - 1. \tag{36}$$

Equation (31) still holds in this case by virtue of (33) and (34). Equation (26) also still holds, but its interpretation is altered by the fact that now because of (28) the inflation rate becomes negative after some point in time (assuming changes in the growth of productivity do not confound the issue). Thus, in the early stages of this model capital's relative share is declining; but once deflation sets in, unit labor costs are acting to raise the relative share of capital. Without completely solving these difference equations, my guess is that the markup effect dominates the unit labor cost effect so that capital's relative share is continually declining.

6. Okun's price rule II

An alternative interpretation of Okun's price rule is to make the "absorption" factor multiplicative rather than additive. The model, with current pricing, is then

$$w_t = g_{t-1} + k\, z_{t-1} \tag{23}$$

and

$$z_t = d\, h_t, \quad 0 < d < 1. \tag{37}$$

Substituting (13) and (23) into (37), the rate of inflation in this model is given by

$$(38) \qquad z_t = d\ (k\ z_{t-1} - (g_t - g_{t-1}))$$

which is proportional to the rate of inflation in the wages only Okun model. Thus the factor d serves to damp the difference equation, but does not alter the steady state when productivity growth is constant.

To examine the income distribution question, we again must find a time path for the c_t's which makes (29) consistent with (37). Combining (30) and (37) we see that C_t is no longer a constant but is given by

$$(39) \qquad C_t = (d - 1)\ h_t$$

where the sign of C_t is opposite to that of h_t since $d < 1$. Since relative factor shares are determined by (32), capital's relative share declines over time when unit labor costs are rising and rises when unit labor costs are falling.

<u>Lagged pricing rule</u>: To impose the lagged pricing rule we use equation (33), and rewrite the price guidepost as

$$(40) \qquad z_t = d\ h_{t-1}.$$

Substituting (13) and (23) unto (40), we have inflation given by

$$(41) \qquad z_t = d\ (k\ z_{t-2} - (g_{t-1} - g_{t-2}))$$

which has basically the same implications as (38).

Utilizing (33) and (40) the growth rate of c_t which makes these two equations consistent is given by

$$(42) \qquad C_t = (d - 1)\ h_{t-1}.$$

Relative factor shares are given by (36), and unit labor cost growth is determined by (26). When unit labor costs are rising, capital's relative share is declining and vice versa. With constant rates of productivity growth, the model implies a steady state zero rate of inflation, so that the relative share of capital approaches a new equilibrium value from above. At the new equilibrium, capital's share is lower than it was initially.

7. Summary

On inflation: Although the focus of this paper has been on wage and price guidelines, the implications of the models depend critically on productivity. If declines in productivity growth are serious enough, inflation can still accelerate in spite of guidepost policies short of a freeze.

Assuming that productivity follows a constant trend rate of growth, then all of the guidepost models entail decelerating rates of inflation. In models 4 and 6, the steady state rate of inflation is zero; while models 2, 3, and 5 imply a continual reduction in inflation, and, eventually, actual deflation.

On income distribution: In the first four models with prices based on current labor costs, relative factor shares remain constant no matter what happens to productivity. In the last two models plus the first four with prices based on lagged unit labor costs, relative factor shares depend critically on productivity growth. Generally speaking, increases in the rate of growth of productivity shift the distribution of income in favor of the nonlabor factors.

Assuming that productivity follows a constant trend rate of growth, then models 2 and 3 imply a shift in favor of capital (in the long run at least); but model 4 implies a shift against the relative share of capital which diminishes as the steady state is approached. Model 5 implies a declining relative share of capital as long as there is positive inflation -- after that the result is not clear. And model 6, under this productivity assumption, implies that there is a continuing shift against capital's relative share which diminishes as the steady state zero rate of inflation is approached.

Which to choose? While there have been many forms of guideposts suggested, there has been little discussion of what specific criteria to use to choose among them. There has, of course, been a great deal of rhetoric -- largely centering on the nebulous concepts of "workability" and "equity." The guideposts of the early sixties were formulated on what is essentially model 4 with $k = 0$. Under this condition, productivity considerations aside, this model is unique among those presented here in that the rate of inflation is reduced to zero, and relative factor shares remain unchanged. But, it is argued, for any $k < 1$, rule 4 is not equitable and hence not workable because labor is giving up something while business is giving up nothing. If relative factor shares is not an appropriate equity criterion, then what is? It appears that it is this kind of criticism which has led to price guidepost proposals such as models 5 and 6 -- which, ironically, have the greatest impact on income distribution. The point is that there seems to be little economic reason to clutter model 4 with price guideposts as long as there is assurance that markup factors will not be altered.

Put another way, abstracting from productivity changes, the equity question seems to be whether or not there exists a set of values for k and b (in model 5) which causes the inflation rate to be lowered but does not change the income distribution. This, unfortunately, is not possible.

Certainly one factor that must be considered in choosing among models is the length of time that the guidepost policy will be in effect. If it is not too long, then the fact that certain models have undesirable steady state properties is not especially relevant.

Finally, it might be argued that while the government has certain responsibilities regarding income distribution, this need not be a major consideration in every action. What matters is the totality of monetary-fiscal-guidepost policies. On these grounds, since any of the models discussed is likely to reduce inflation, the choice may be properly left to the political scientist. This seems to suggest adoption of model 5 or 6.

Addendum A: Trend Rules

Since many of the guidepost proposals are based on historical averages of productivity growth, it is important to examine such proposals in the context of the basic models. We now assume that an average historical growth rate (g^*) of output per manhour is given. Thus, at any point in time, for purposes of calculating guideposts, computed output per manhour is given by

$$M_t^* = g_0 e^{g^* t} \tag{43}$$

where g_0 is also known by decree. The basic trend model then becomes

$$w_t = z_{t-1} + g^* \tag{44}$$

and

$$P_t = c \frac{W_t}{M_t^*}. \tag{45}$$

Note that once relation (43) is established, prices and wages are independent of output -- both current and past.

Substituting (44) into the log-first-difference form of (45), we have the rate of inflation given by

$$z_t = z_{t-1}. \tag{46}$$

Inflation is determined entirely by initial conditions. This result differs from the earlier basic model only because the effect of actual productivity growth is removed.

Factor shares, however, depend also on the actual outcome of productivity and not simply on the trend extrapolated rate used in the guidepost calculation. Substituting (45) into (7), the ratio of factor shares is given by

$$f_t = c \frac{M_t}{M_t^*} - 1. \tag{47}$$

As long as actual output per manhour is on its trend line, relative factor shares are constant. When productivity is high (low) relative to trend, capital's share increases (decreases) relative to labor's share.

<u>Lagged pricing rule</u>: The meaning of a lagged pricing rule in a trend rules context is less then clear. Simply lagging (45) seems to require that prices be based on what unit labor costs should have been rather than what they were. Also, from an informational point of value, lagging is not necessary.

<u>Models 2 through 6</u>: Imposing the trend rule on models 2 through 6 has essentially the same impact as it had on the basic model. Formulae for inflation in models 2, 3, and 4 are unchanged except for the substitution of g^* for g_t. Also, in these models, relative shares are determined by relation (47).

Imposing the trend rule on models 5 and 6 requires another definition. Define the percent change in unit labor costs under the trend rule as

$$h_t' = w_t - g_t' \tag{48}$$

where g_t' is the logarithm approximation to the percent change in productivity assuming that current output per manhour is determined by (43), i.e. $g_t' = \ln M_t^* - \ln M_{t-1}$. Now inflation is given by

(49) $$z_t = k\, z_{t-1} + (g^* - g_t') - b$$

and

(50) $$z_t = d\, [\, k\, z_{t-1} + (g^* - g_t')\,]$$

in models 5 and 6 respectively. Income distribution in both models is determined by

(51) $$f_t = c_t \frac{M_t}{M_t^*} - 1$$

where c_t is established by (31) or (39) with h_t' substituted for h_t.