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POLICY ANALYSIS UNDER
STOCHASTIC REPLANNING

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1. Introduction

To begin, consider the simple dynamic game depicted in Figure 1. In this game, there are two players (P1 and P2) and two time periods (0 and 1). Payoffs occur at the end of period 1 and each player seeks to maximize his payoff. The players must choose either decision 1 or 0 at the beginning of each period. P1 is dominant in the sense that he is first to announce his strategies, although decisions are taken simultaneously by both players in each period.

The "open loop" (or "precommitment") solution to this game can be found using the payoff matrix in Figure 2. In equilibrium, each player plays the sequence of strategies (1,1), yielding payoffs (10,5). The time inconsistency of this solution is evident: P1 clearly has an incentive to change his period 1 strategy to zero, once period 0 has passed.

The game in Figure 1 also admits a time consistent solution. That solution may be found by backward induction, as outlined in Figure 3. In equilibrium, P1 plays (0,0) and P2 plays (1,1). By construction P1 has no incentive to change strategies once period 0 has passed.

If one wishes to describe policymaking in terms of a dynamic Stackelberg game, the consistent solution seems the more realistic of the two solutions outlined above. The intuitive appeal of the consistent solution is increased if that solution is viewed as the outcome of a noncooperative game with three players: P2, representing private agents of an economy, and two

policy "administrations," one acting at time 0 and one acting at time 1. While the time 0 administration can predict what the time 1 administration will do, it cannot control the future administration's actions, precluding any precommitment.

In what follows, I would like to suggest a class of positive models of policymaking that build on the preceding insight in a somewhat ad hoc, but perhaps revealing way. In particular, I wish to consider cases intermediate to the precommitment and time consistent solutions, where there is some exogenously specified probability α ($1 > \alpha > 0$) of policy remaining on its precommitted path. Viewed in another way, there is a probability α that the current "administration" will not be voted out of office next period. Conditional on this last event, the probability of the current administration sticking to its preannounced policy is assumed to be one. These models will be called "stochastic replanning" models.

As an example, consider the dynamic game of Figure 1 with $\alpha = 1/2$. Casting the government in the role of the dominant player, P1 is now split into two administrations. The period 1 administration, if it comes to power, chooses its (period 1) strategies as in the consistent solution (see Figure 3). The period 0 administration, seeking to maximize its expected payoff, must take into account the possibility that its period 1 strategies may not come to pass, as must private agents (P2). Accordingly, the expected payoff matrix corresponding to that of Figure 2 will be the matrix in Figure 4. Inspection of that matrix

reveals that the equilibrium outcome of this new game occurs when the period 0 administration plays (0,0) and P2 plays (1,1), i.e., at the consistent solution. For sufficiently large $\alpha (> \frac{4}{7})$, however, the period 0 administration would find it optimal to announce the precommitment strategies (1,1).

In the next section, the idea of stochastic replanning is extended to a more complex dynamic game.

2. Stochastic Replanning with an Infinite Horizon

An example will now be considered where private agents and governmental administrations have (potentially) infinite planning horizons. Analysis of this example is greatly simplified by introducing the state variable

$$S_t = \begin{cases} 1 & \text{If replanning occurs (a new administration comes} \\ & \text{into office);} \\ 2 & \text{otherwise.} \end{cases}$$

It might seem at first that a countably infinite state space is needed for S_t , with S_t indexing the number of periods since replanning (administration turnover). Fortunately, this turns out not to be the case for the example considered below.

There is always probability α that the current administration will continue in power next period. Hence $\{S_t\}$ is simply a sequence of independent Bernoulli trials. It follows that

$$(1) \quad \Pr\{S_{t+n}=j | S_t\} = \Pr\{S_{t+n}=j\} = \begin{cases} 1-\alpha & \text{for } j = 1, \\ \alpha & \text{for } j = 2; \end{cases}$$

- (2) The probability of any current administration being in power n periods in the future is α^n .

The above facts will be useful for the analysis that follows.^{1/}

The example to be considered will be the "generic" one considered by Whiteman (1984) in deriving expressions for the open loop and time consistent solutions to policy games with linear rational expectations models. The example is simple enough to be tractable, yet captures the essential features of more complex models. In this example, policymakers are confronted with

$$E_t y_{t+1} - \rho y_t = x_t + e_t. \quad (1)$$

Here y_t is an "endogenous" stochastic process reflecting decisions of private agents, E_t is the conditional expectations operator, ρ is a parameter with $|\rho| > 1$, x_t is a variable controllable by the current administration, and $\{e_t\}$ is a sequence of shocks. Initially, it is convenient to assume that $\{e_t\}$ is nonstochastic and known to all private agents and administrations.

The objective of the administration in power at time t will be taken as to minimize

$$\lim_{J \rightarrow \infty} \frac{1}{2} E_t \sum_{j=0}^J \beta^j [y_{t+j}^2 + \lambda x_{t+j}^2], \quad \lambda > 0, \quad 1 > \beta > 0,$$

by choice of a plan for $\{x_{t+j}\}_{j=0}^{\infty}$, denoted $\{x_{t+j}^t\}$. The current (time t) administration must take $\{x_{t+j+k}^{t+k}\}$ as given for $j \geq 0$ and $k \geq 1$. That is, it cannot control the actions of future administrations.

For computational convenience, the discount factor β is assumed to equal 1. Hence the results derived below may be thought of as applying to the transformed sequences $\{x_t^*\}$ and $\{y_t^*\}$, where $x_t^* = \beta^{t/2} x_t$ and $y_t^* = \beta^{t/2} y_t$. Alternatively, one could think of the results below applying to the case where each administration has the "steady state" objective

$$\lim_{J \rightarrow \infty} \frac{1}{2J} E_t \sum_{j=0}^J [y_{t+j}^2 + \lambda x_{t+j}^2].$$

This interpretation is somewhat problematic, however, because each administration (when $1 > \alpha > 0$) is optimizing along a path for $\{S_t\}$ that ultimately has probability zero, which causes its optimization problem to be ill-defined.

For the special cases $\alpha = 1$ and $\alpha = 0$ (corresponding to the open loop and time consistent solutions of the above policy model), one can solve for x_t using the methods discussed in Hansen, Epple, and Roberds (1985), or Sargent (1984).^{2/} First consider the case of $\alpha = 1$. For the nonstochastic sequence $\{e_t\}$ equation 1 may be rewritten in "feedforward" form:

$$y_t = -\rho^{-1}(1-\rho^{-1}L^{-1})^{-1}(x_t + e_t), \quad (2)$$

where L^{-1} is the lead operator.

Dropping the superfluous superscript on $\{x_{t+j}^t\}$, the time $t = 0$ policy problem can be solved using the Lagrangian

$$\sum_{t=0}^{\infty} \{1/2[y_t^2 + \lambda x_t^2] + \theta_t [y_t + \rho^{-1}(1-\rho^{-1}L^{-1})^{-1}(x_t + e_t)]\} \quad (3)$$

where θ_t is a sequence of Lagrange multipliers. Differentiating (3) with respect to x_t and y_t yield

$$\lambda x_t + \rho^{-1}(1-\rho^{-1}L)^{-1}\theta_t = 0; \quad (4)$$

$$y_t + \theta_t = 0 \quad (5)$$

Equations (4) and (5) hold for $t \geq 0$, subject to the initial conditions that $\theta_t = 0$ for $t < 0$. Solving for x_t , we obtain

$$x_0 = (\lambda\rho)^{-1}y_0; \quad (6)$$

$$x_t = \rho^{-1}x_{t-1} + (\lambda\rho)^{-1}y_t, \quad t \geq 1. \quad (7)$$

To find the consistent policy, note that with $\alpha = 0$, the choice of x_t is always made by the time t administration, i.e., $x_t = x_t^t$ with probability 1. Hence the impact of x_t on y_s , $s < t$ is never taken into account when choosing x_t , and all lagged θ_t 's vanish from equation (4). Solving for x_t , one obtains

$$x_t = (\lambda\rho)^{-1}y_t, \quad t \geq 0. \quad (8)$$

Now consider the policy problem for $1 > \alpha > 0$, for the time zero administration. Since there is now uncertainty, rewrite (2) as

$$\begin{aligned}
 y_t &= -\rho^{-1}(1-\rho^{-1}L^{-1})^{-1}E_t(x_t + e_t) \\
 &= -\rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} (E_t x_{t+j} + e_{t+j}) \\
 &= -\rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} E_t x_{t+j} - \rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} e_{t+j} \quad (9)
 \end{aligned}$$

The appropriate Lagrangian expression is now, for the time zero administration,^{3/}

$$\begin{aligned}
 E_0 \sum_{t=0}^{\infty} \{ & \frac{1}{2} E_t [y_t^2 + \lambda x_t^2] + E_t \theta_t [E_t y_t + \rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} E_t x_{t+j} \\
 & + \rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} e_{t+j}] \} \\
 = \sum_{t=0}^{\infty} \{ & \frac{1}{2} [E_0(E_t y_t^2) + \lambda \alpha^t (x_t^0)^2 + \lambda E_0(x_t^2 | S_n=2, t \geq n > 0)(1-\alpha^t)] \\
 & + E_0(E_t \theta_t)(E_t y_t) + (E_0 \theta_t) \rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} e_{t+j} \\
 & + E_0(E_t \theta_t) \rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} \alpha^{t+j} (x_{t+j}^0) \\
 & + (E_0 \theta_t) \rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} E_0(x_{t+j} | S_n=2, t+j \geq n > 0)(1-\alpha^{t+j}) \} \quad (10)
 \end{aligned}$$

Assuming certainty equivalence, differentiating (10) with respect to x_t^0 and $E_t y_t$ yields

$$\lambda x_t^0 + \rho^{-1}(1-\rho^{-1}L)^{-1}E_t \theta_t = 0; \quad (11)$$

$$E_t y_t + E_t \theta_t = 0; \quad (12)$$

for $t \geq 0$. Proceeding as in the case $\alpha = 1$ yields

$$x_0^0 = (\lambda\rho)^{-1}y_0; \quad (13)$$

$$x_t^0 = (\lambda\rho)^{-1}y_t + \rho^{-1}x_{t-1}^0, \quad t \geq 1. \quad (14)$$

Similarly, for any time t administration

$$x_t^t = (\lambda\rho)^{-1}y_t; \quad (15)$$

$$x_{t+j}^t = (\lambda\rho)^{-1}y_{t+j} + \rho^{-1}x_{t+j-1}^t, \quad j \geq 1. \quad (16)$$

It follows that, as of time t , for $j \geq 1$

$$x_{t+j} = (\lambda\rho)^{-1}y_{t+j} + \rho^{-1}x_{t+j-1} \quad (17)$$

with probability α ; and

$$x_{t+j} = (\lambda\rho)^{-1}y_{t+j} \quad (18)$$

with probability $1 - \alpha$. (A more rigorous derivation of equations (17) and (18) is given in Appendix A.) To solve for the sequence $\{E_t x_{t+j}\}_{j=0}^{\infty}$, use (17) and (18) to eliminate the y_t 's from (2):

$$\begin{aligned} E_{t+j} y_{t+j+1} - \rho y_{t+j} &= x_{t+j} + e_{t+j} \\ \Rightarrow (\lambda\rho) E_{t+j} x_{t+j+1} - \lambda\alpha x_{t+j} - \rho y_{t+j} &= x_{t+j} + e_{t+j} \\ \Rightarrow (\lambda\rho) E_t x_{t+j+1} - (1+\lambda\alpha+\lambda\rho^2) E_t x_{t+j} + \alpha\lambda\rho E_t x_{t+j-1} &= e_{t+j} \end{aligned} \quad (19)$$

for $j \geq 1$. For $j = 0$, it must hold that

$$(\lambda\rho) E_t x_{t+1} - (1+\lambda\alpha+\lambda\rho^2) x_t + \alpha\lambda\rho x_{t-1} = e_t \quad (20)$$

if $S_t = 2$ (no replanning), or

$$(\lambda\rho)E_t x_{t+1} - (1+\lambda\alpha+\lambda\rho^2)x_t = e_t$$

if $S_t = 1$ (replanning occurs). In other words, equation (20) always holds subject to the pseudo-initial condition $x_{t-1} = 0$, when $S_t = 1$.

Equation (19) is a nonstochastic difference equation in $E_t X_{t+j}$ that can be solved by traditional methods. The characteristic polynomial of equation (19) is equal to

$$\begin{aligned} c(z) &= (\lambda\rho)z^{-1} - (1+\lambda\alpha+\lambda\rho^2) + \alpha\lambda\rho z \\ &= (-\lambda\rho)(-z^{-1} + (\lambda^{-1}\rho^{-1} + \alpha\rho^{-1} + \rho) - \alpha z). \end{aligned} \quad (21)$$

Let $a(z) = -z^{-1} + (\lambda^{-1}\rho^{-1} + \alpha\rho^{-1} + \rho) - \alpha z$. Then $\lim_{z \rightarrow 0} a(z) = -\infty$ and $\lim_{z \rightarrow \infty} a(z) = -\infty$. Now $a(1) = \lambda^{-1}\rho^{-1} + \alpha\rho^{-1} + \rho - 1 - \alpha$. Defining $g(\alpha) = a(1)$, $g(\alpha)$ has a minimum on $[0,1]$ at $\alpha = 1$, where $g(\alpha) = \lambda^{-1}\rho^{-1} + \rho - 2 > 0$. It follows that $c(z)$ always admits a factorization ^{4/}

$$c(z) = c_0(1-c_1z^{-1})(1-c_2z)$$

where $c_1, c_2 \in (0,1)$. (For $\alpha = 0$, $c_2 = 0$, and for $\alpha = 1$, $c_1 = c_2$.)

The solution for $E_t x_{t+j}$ may thus be written in feedback-feedforward form as

$$E_t x_{t+j} = c_2 E_t x_{t+j-1} + c_0^{-1} \sum_{k=0}^{\infty} c_1^k e_{t+j+k}, \quad (22)$$

for $j \geq 1$. To solve for x_t , I will now assume that e_t follows the first order difference equation $e_t = \gamma e_{t-1}$, where $|\gamma| \leq 1$. Then equation (22) reduces to

$$E_t x_{t+j} = c_2 E_t X_{t+j-1} + c_0^{-1} \gamma^j e_t / (1 - c_1 \gamma). \quad (23)$$

Suppose now that $S_t = 2$. Using equations (23) and (17), equation (9) may be rearranged to yield the following feedback-feedforward law for x_t :

$$x_t = f_0 e_t + f_1 x_{t-1} \quad (24)$$

(A complete derivation of equation (24), along with expressions for f_0 and f_1 , can be found in Appendix B.) In the case that $S_t = 1$, one can show, using equations (18), (9) and (23), that

$$x_t = f_0 e_t. \quad (25)$$

In other words, x_t will have two representations, depending on the current value of S_t .

Finally, note that having solved for $\{x_t\}$, the equilibrium values of $\{y_t\}$ may be obtained either by using equation (9), or equations (17) and (18).

3. Interpretations of the Results

Equation (22), given knowledge of the $\{e_t\}$ sequence, can be used to generate time t forecasts of x_{t+j} and y_{t+j} . Equation (22) can also be used to generate what I will term "impulse response functions," by which is meant the difference between two hypothetical forecasts of x_{t+j} or y_{t+j} , the first given a hypothetical path for $\{e_t\}$ of the form $\{\dots, e_{t-2}, e_{t-1}, \bar{e}_t, 0, 0, \dots\}$, where \bar{e}_t is specified by the forecaster, and the second given a path $\{\dots, e_{t-2}, e_{t-1}, 0, 0, 0, \dots\}$ for $\{e_t\}$.

What is unusual about such "impulse response functions," in the case that $1 > \alpha > 0$, is that these functions will vary randomly over time. While these impulse responses will be identical for $S_t = 2$, in the case that $S_t = 1$ the scaling of the impulse response will be determined by the magnitude of x_{t-1} . Such random impulse response functions are evocative of Sims' (1982) random coefficient VAR methods for forecasting series. While the degree of coefficient randomness generated by this simple example does not begin to capture the complexity of Sims' specification, this might not be the case for an example with more complex policy dynamics.

The above example also yields an insight concerning policy analysis not entirely inconsistent with Sims' (1982) views on the subject.^{5/} In the example above, to evaluate the performance of a policy $\{x_{t+j}^t\}_{j=0}^{\infty}$, one needs to know the probability α . Knowing the sequence $\{x_{t+j}^t\}$ is not sufficient, for policies are as in Sims' Weltanschauung, regularly recomputed subject to the evolution of $\{S_t\}$. Of course, it is extremely doubtful that Sims' world view included policy replanning only driven by some exogenous process such as $\{S_t\}$. The exogeneity assumption maintained in the examples above, however, may represent a useful first step in constructing positive models of policymaking.

Finally, the example above may shed some light on the "rules versus discretion" debate. Suppose that $\{e_t\}$ is some easily forecastable process, e.g., a Gaussian ARMA process, and that $\{e_t\}$ and $\{S_t\}$ are independent at all leads and lags. This

modification has the effect of making the policy problem of the example less trivial for the cases $\alpha = 0$ and $\alpha = 1$. Appealing once again to certainty equivalence, equation (22) becomes

$$E_t x_{t+j} = c_2 E_t x_{t+j-1} + c_0^{-1} \sum_{k=0}^{\infty} c_1^k E_t e_{t+j+k}. \quad (26)$$

Under an AR(1) specification for $\{e_t\}$, the solution for $\{x_t\}$ and $\{y_t\}$ will then be the same as given for the certainty case above (see eqs. (25), (26), (17), and (18)).

Assume that initially $\alpha = 0$, so that precommitment is impossible. Then it is intuitively clear that the performance of "policy" might not be linearly increasing in α . For example, increasing the probability of precommitment from zero to 1/10 may result in only a slight improvement of policy performance. If one imagines the recent "monetarist experiment" in the U.S. as such a change from $\alpha = 0$ to a small value of α , the results of this experiment are perhaps not surprising.

4. Numerical Examples

In this section, numerical examples are presented that illustrate properties of the model presented in Sections 2 and 3.

Example 1. In this example, it is assumed that $\rho = 2$, $\lambda = .1$, and that $e_t = 1$ for all t . Equilibrium values of x_t and y_t are plotted in Figures 5 and 6, respectively. Values are plotted for $\alpha = 0, 1$, and $.5$ (for one realization of $\{S_t\}$).

Examples 2-10. In these examples, e_t is stochastic, and is assumed to follow the process

$$e_t = .9e_{t-1} + u_t,$$

where $\{u_t\}$ is Gaussian white noise independent of $\{S_t\}$, and $E u_t^2 = 1$.

In each of the examples, the parameter ρ is equal to 1.1. The parameter λ takes on the values 1, 10, and 0.1, and α takes on the values 0, 1, and .5. A random number generator was used to construct artificial e_t and S_t (for the $\alpha = .5$ case) time series of length 500. The same artificial series were used for all simulations. For every example, the statistic $S(\alpha, \lambda) \equiv$

$$\text{svar}(y) + \lambda \text{svar}(x)$$

was calculated, where "svar" means sample variance. A sample performance index was then calculated as

$$P(\alpha, \lambda) = 100(S(\alpha, \lambda)/S(0, \lambda)).$$

The index P gives the sample performance of a policy as a percentage of the sample performance of the best consistent policy.

The results of the simulations are reported in Table 1. These results suggest that performance improves (i.e. P falls) as the probability of precommitment α rises. Evidently, for the example considered the advantage of precommitment increases with the weight λ attached to policy fluctuations. Also increasing with λ is the degree of nonlinearity in the improvement of policy performance due to increasing α . The reader should note that the generality of the last two effects is far from obvious.

Appendix A. Derivation of Equations (17) and (18).

Let the random process $\{w_t\}$ be defined as

$$w_t = \min_j \{t-j\},$$

$$\text{s.t. } j \leq t, S_j = 1.$$

That is, w_t equals the number of periods since the last administration change. The process w_t is a Markov chain with state space $= \{0,1,2,\dots\}$, and transition matrix T given by^{6/}

$$\begin{bmatrix} (1-\alpha) & \alpha & 0 & 0 & \dots \\ (1-\alpha) & 0 & \alpha & 0 & \dots \\ (1-\alpha) & 0 & 0 & \alpha & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} .$$

Now consider the period 0 administration's problem. Since it is assumed that $w_0 = 0$, for $t > 0$ it must be true that

$$w_t \in \{0,1,\dots,t\}.$$

Assuming x_t and y_t to be w_t -measurable, and from the definition of w_t , one can define

$$x_t = x_t(w_t);$$

$$y_t = y_t(w_t);$$

$$x_t^0 = x_t(t);$$

$$x_{t+j}^t = x_{t+j}(j).$$

Under this notation, equation (9) takes the form

$$y_t(w_t) = -\rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} e_{t+j}$$

$$-\rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} \sum_{w_{t+j}=0}^{t+j} x_{t+j}(w_{t+j}) \Pr\{w_{t+j}|w_t\}. \quad (\text{A.1})$$

The time 0 administration's problem is to maximize, by choice of $\{x_t(t)\}_{t=0}^{\infty}$, the expression

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{\infty} \sum_{w_t=0}^t \frac{1}{2} y_t(w_t)^2 + \frac{\lambda}{2} x_t(w_t)^2 \Pr\{w_t|w_0\}$$

subject to (9), and $w_0 = 0$. To solve this problem, use the Lagrangian $L =$

$$\sum_{t=0}^{\infty} \sum_{w_t=0}^t \left(\frac{1}{2} y_t(w_t)^2 + \frac{\lambda}{2} x_t(w_t)^2 \right) + \theta_t(w_t) y_t(w_t)$$

$$+ \theta_t(w_t) \rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} \sum_{w_{t+j}=0}^{t+j} (x_{t+j}(w_{t+j}) \Pr\{w_{t+j}|w_t\})$$

$$\times \Pr\{w_t|w_0\}.$$

Differentiating L with respect to $y_t(t)$ and $x_t(t)$ yields

$$\frac{\partial L}{\partial y_t(t)} = [y_t(t) + \theta_t(t)] \alpha^t; \quad (\text{A.2})$$

$$\frac{\partial L}{\partial x_t(t)} = [\lambda x_t(t) + \rho^{-1} \sum_{j=0}^t (\rho^{-1})^j \theta_{t-j}(t-j)] \alpha^t, \quad (\text{A.3})$$

using the fact that $\Pr\{w_t=t|w_0=0\} = \alpha^t$. Setting (A.2) and (A.3) equal to zero and solving yield

$$x_0(0) = (\lambda\rho)^{-1}y_0(0)$$

and

$$x_t(t) = (\lambda\rho)^{-1}y_t(t) + \rho^{-1}x_{t-1}(t-1)$$

Solving similar optimization problems for administrations $t = 1, 2, \dots$ yields

$$x_t(0) = (\lambda\rho)^{-1}y_t(0); \tag{A.4}$$

$$x_t(w_t) = (\lambda\rho)^{-1}y_t(w_t) + \rho^{-1}x_{t-1}(w_{t-1}),$$

$$\text{for } w_t \geq 0, \text{ and } w_{t-1} = w_t - 1. \tag{A.5}$$

Equations (A.4) and (A.5) correspond to equations (17) and (18) of the text. To justify equation (19), note that (A.4) and (A.5) imply that for $k = 0, \dots, t$,

$$\begin{aligned} E(x_{t+j}(w_{t+j}) | w_{t+j}=0, w_t=k) &= \\ (\lambda\rho)^{-1}E(y_{t+j}(w_{t+j}) | w_{t+j}=0, w_t=k) &= (\lambda\rho)^{-1}y_{t+j}(0); \\ E(x_{t+j}(w_{t+j}) | w_{t+j}>0, w_t=k) &= (\lambda\rho)^{-1}E(y_{t+j}(w_{t+j}) | w_{t+j}>0, w_t=k) \\ &+ \rho^{-1}E(x_{t+j-1}(w_{t+j-1}) | w_{t+j}>0, w_t=k). \end{aligned}$$

Now use the following facts:

$$\begin{aligned} (1) \quad E(x_{t+j}(w_{t+j}) | w_t=k) &= \\ E(x_{t+j}(w_{t+j}) | w_{t+j}=0, w_t=k)Pr\{w_{t+j}=0 | w_t=k\} & \\ + E(x_{t+j}(w_{t+j}) | w_{t+j}>0, w_t=k)Pr\{w_{t+j}>0 | w_t=k\} & \end{aligned}$$

$$(2) \quad \Pr\{w_{t+j}=0 \mid w_t=k\} = 1 - \alpha \text{ for all } k, \text{ and}$$

$$\Pr\{w_{t+j}>0 \mid w_t=k\} = \alpha.$$

$$(3) \quad E(x_{t+j-1}(w_{t+j-1}) \mid w_{t+j}>0, w_t=k) = E(x_{t+j-1}(w_{t+j-1}) \mid w_t=k).$$

That is, simply knowing $w_{t+j} > 0$ does not provide any information about w_{t+j-1} .

It follows from facts (1)-(3) that

$$\begin{aligned} E(x_{t+j}(w_{t+j}) \mid w_t=k) &= (\lambda\rho)^{-1} E(y_{t+j}(w_{t+j}) \mid w_t=k) \\ &\quad + \alpha\rho^{-1} E(x_{t+j-1}(w_{t+j-1}) \mid w_t=k), \end{aligned}$$

or in the shorthand notation of the text,

$$E_t x_{t+j} = (\lambda\rho)^{-1} E_t y_{t+j} + \alpha\rho^{-1} E_t x_{t+j-1}.$$

Appendix B. Solution for $\{x_t\}$.

From equation (24), it follows that

$$E_t x_{t+j} = c_2^j [x_t - \tilde{c} e_t] + \gamma^j \tilde{c} e_t,$$

for $j \geq 0$, where

$$\tilde{c} = [c_0(1-c_1\gamma)(1-c_2\gamma^{-1})]^{-1}.$$

It then follows that

$$\sum_{j=0}^{\infty} \rho^{-j} E_t x_{t+j} = a_0 e_t + a_1 x_t,$$

where

$$a_0 = \tilde{c} [(1-\gamma\rho^{-1})^{-1} - (1-c_2\rho^{-1})^{-1}]$$

and

$$a_1 = (1-c_2\rho^{-1})^{-1}.$$

Substituting the above into equation (9), one obtains

$$y_t - \rho^{-1} a_1 x_t + [-\rho^{-1} a_0 + (\gamma - \rho)^{-1}] e_t,$$

abbreviated as

$$y_t = b_0 e_t + b_1 x_t. \tag{B.1}$$

Finally, equation (17) implies, for $S_t = 2$,

$$y_t = \lambda \rho x_t - \lambda x_{t-1} \tag{B.2}$$

Equating (B.1) and (B.2) yields

$$x_t = \rho^{-1}x_{t-1} + (\lambda\rho)^{-1}b_1x_t + (\lambda\rho)^{-1}b_0e_t,$$

which in turn implies equation (24), where

$$f_0 = (1 - (\lambda\rho)^{-1}b_1)^{-1}(\lambda\rho)^{-1}b_0,$$

and

$$f_1 = \rho^{-1}(1 - (\lambda\rho)^{-1}b_1)^{-1}.$$

Similar substitution when $S_t = 1$ yields equation (25).

Table 1

Example	λ	α	Performance Index P(%)
2	1	0	100.0
3	1	1	58.6
4	1	0.5	76.9
5	10	0	100.0
6	10	1	44.0
7	10	0.5	74.6
8	0.1	0	100.0
9	0.1	1	91.2
10	0.1	0.5	95.8

Footnotes

1/It is crucial to note that fact (1) does not yield information about the probability of the current administration remaining in power. Instead, it gives the probability that some administration will be keeping or losing its mandate at various times in the future.

2/An alternative methodology for solving these problems is presented by Whiteman (1984).

3/In the following expressions, note x_t^0 is nonstochastic, although y_t , x_t , and θ_t are stochastic.

4/Since $a(z)$ is continuous on $(0, \infty)$, approaches $-\infty$ on the endpoints of that interval, and is positive at $z = 1$, $a(z)$ and hence $c(z)$ must have one root in $(0, 1)$ and another in $(0, \infty)$.

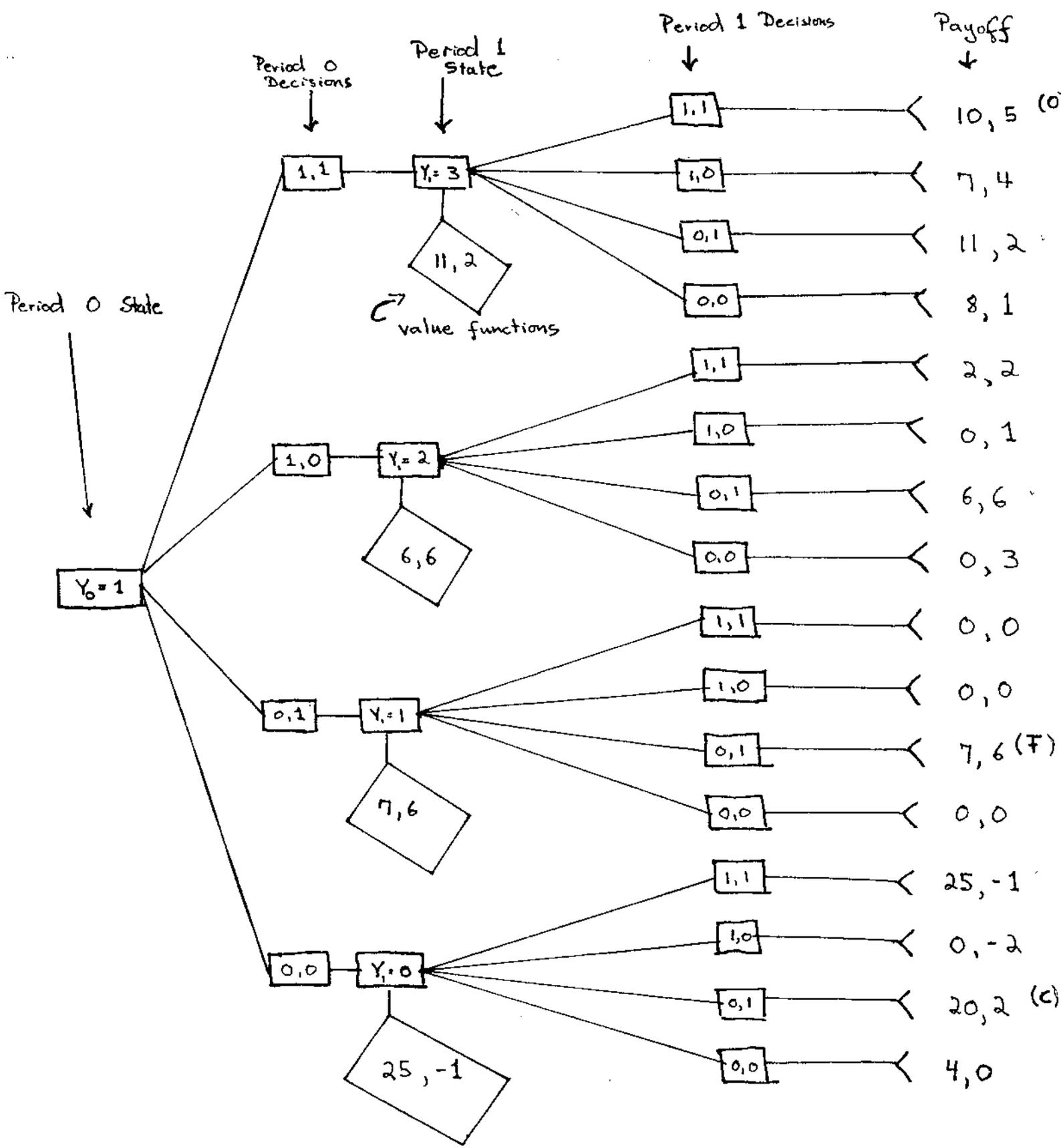
5/For this paragraph, I plead the customary disclaimer issued when attempting to interpret any paper by Sims.

6/Markov chains are extensively discussed in Chung (1975) and Hoel, Port, and Stone (1972).

References

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Dominant Player Example



F = Outcome under feedback solution
 C = Outcome under closed loop solution
 O = Outcome under open loop solution

Figure 1

Open Loop (Precommitment)
Solution

P1 \ P2	0,0	0,1	1,0	1,1
0,0	4,0	20,2	0,0	7,6
0,1	0,-2	25,-1	0,0	0,0
1,0	0,3	6,6	8,1	11,2
1,1	0,1	2,2	7,4	* 10,5

* = Equilibrium Outcome

P1 is dominant.

Figure 2

Consistent Solution

At t = 1 :

	P2		
P1		0	1
0		4,0	20,2
1		0,-2	25,-1*

$$Y_1 = 0$$

	P2		
P1		0	1
0		0,0	7,6*
1		0,0	0,0

$$Y_1 = 1$$

	P2		
P1		0	1
0		0,3	6,6*
1		0,1	2,2

$$Y_1 = 2$$

	P2		
P1		0	1
0		8,1	11,2*
1		7,4	10,5

$$Y_1 = 3$$

At t = 0 :

	P2		
P1		0	1
0		25,-1	7,6*
1		6,6	11,2

$$Y_0 = 1$$

P1 is dominant.

Figure 3

Replanning Solution
 $\alpha = \frac{1}{2}$

P1 \ P2	0,0	0,1	1,0	1,1
0,0	14.5, -.5	22.5, -.5	3.5, 3	7, 6 *
0,1	12.5, -1.5	25, -1	3.5, 3	3.5, 3
1,0	3, 4.5	6, 6	9.5, 1.5	11, 2
1,1	3, 3.5	4, 4	9, 3	10.5, 3.5

* = Equilibrium Outcome

Figure 4

