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Research Department

Policy Analysis in Business Cycle Models

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Working Paper 520

October 1993

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In this chapter we study the quantitative properties of fiscal and monetary policies in business cycle models. We set out a theoretical framework and use it to analyze two kinds of policies: exogenously given policies and optimal policies with commitment. We illustrate how this framework can be applied in practice by computing the quantitative properties of optimal policies.

Our framework builds on the primal approach to optimal taxation in the public finance literature (see, for example, Atkinson and Stiglitz 1980; Lucas and Stokey 1983; and Chari, Christiano, and Kehoe 1992). Using this approach we characterize the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by two simple conditions: the resource constraint and an implementability constraint. The implementability constraint is an infinite horizon version of either the consumer or the government budget constraint in which the consumer and firm first order conditions are used to substitute out the prices and policies. Thus, both constraints depend only on allocations. This characterization implies that optimal allocations are solutions to a simple programming problem. We use these solutions to compute the optimal policies and the equilibrium prices.

It is interesting to study the business cycle properties of economies with exogenously given policies. In the business cycle literature the quantitative properties of economies with exogenous tax systems have been studied by Braun (1992), Chang (1990), Greenwood and Huffman (1991), and McGrattan (1992). These authors use the consumer and firm first order conditions and the resource constraint to solve for the allocations given the exogenous policies. This procedure assumes either that lump sum taxes are available or leaves the initial debt obligations of the government as a free variable determined in equilibrium. We show how the first order conditions and the resource constraint can be used together with the implementability constraint to compute competitive equilibria within a class of policies. Ruling out lump sum taxes and fixing the initial debt are especially important when making welfare comparisons across alternative policies. We show by way of an

example that welfare rankings of alternative policies can be reversed when initial conditions are fixed.

Using the general approach we analyze fiscal and monetary policy in two related models. We specify the parameters for preferences and technology to be similar to those used in the public finance and business cycle literature. The stochastic processes for technology shocks and government consumption are chosen to mimic those in the postwar U.S. economy. With these specifications, we show that the optimal policies for our model economies have four properties:

- Tax rates on labor are roughly constant over the business cycle.
- Capital income taxes are close to zero on average.
- The *Friedman rule* is optimal: nominal interest rates are zero.
- Monetary policy responds to shocks: money is countercyclical with respect to technology shocks: the growth rate of money responds negatively to technology shocks and positively to government consumption shocks.

In terms of the properties of fiscal policy, optimal tax policies should smooth distortions over time and states of nature. This involves running a surplus in “good times” and a deficit in “bad times.” In our model, good times are associated with above-average technology shocks and below average government consumption; bad times, with the converse. For reasonable parameter values, smoothing tax distortions turns out to imply that tax rates on labor (or consumption) should be essentially constant. Smoothing tax distortions also implies that capital tax rates should be close to zero on average, a result reminiscent of one in the deterministic literature (Judd 1985 and Chamley 1986).

In terms of the properties of monetary policy, if the models had lump-sum taxes, then following the Friedman rule would be optimal. Phelps (1973) argues that in models with distorting

taxes, it is optimal to tax all goods, including the liquidity services derived from holding money. Hence, Phelps argues that in such models the Friedman rule is not optimal. In our monetary model, however, even though the government has distorting taxes, the Friedman rule turns out to be optimal. In our model, deviating from the Friedman rule amounts to taxing a subset of consumption goods, called *cash goods*, at a higher rate than other consumption goods. Optimality requires that all types of consumption goods be taxed at the same rate; thus, optimality requires following the Friedman rule.

The cyclical properties of optimal monetary policy amount to requiring that the government inflate relatively in bad times and deflate relatively in good times. In effect, then, such a policy allows the government to use nominal government debt as a shock absorber. In the model, the government would like to issue real state-contingent debt in order to insure itself from having to sharply raise and lower tax rates when the economy is hit with shocks. The government achieves this outcome by issuing nominal noncontingent debt and then inflating or deflating to provide the appropriate ex post real payments. In bad times, inflating is optimal, so the real debt payments are relatively small. In good times, deflating is optimal, so the real debt payments are relatively large.

We discuss some of the computational issues associated with the analysis of one of the models in the paper. We describe our basic procedure, which is based on the minimum weighted residual method described by Judd (1991). This procedure is more cumbersome to implement than procedures based on standard linearization techniques such as the one described in Christiano (1991). For this reason, it is useful to compare results based on the two methods. It turns out that both methods deliver essentially the same results for allocations but are quite different for policies. The reason is that the policies depend upon ratios of allocations and small errors in computing allocations turn out to make large differences for computed policies.

The plan of this chapter is as follows. In Section 1 we describe a version of a standard business cycle model with shocks to government consumption and technology and define a competitive equilibrium and a Ramsey equilibrium. In Section 2 we characterize the set of allocations that can be implemented as a competitive equilibrium with distorting taxes. We also show how a competitive equilibrium can be computed when policies are restricted to lie in a given class. In Section 3 we characterize Ramsey equilibria and compute the associated policies and allocations in a standard quantitative business cycle model. In Section 4 we compute Ramsey equilibria for a version of Lucas and Stokey's (1983) monetary economy. In Section 5 we discuss computational issues. Section 6 concludes.

1. A Real Economy

Consider a production economy populated by a large number of identical infinitely-lived consumers. In each period $t = 0, 1, \dots$, the economy experiences one of finitely many events s_t . We denote by $s^t = (s_0, \dots, s_t)$ the history of events up through and including period t . The probability, as of time 0, of any particular history s^t is $\mu(s^t)$. The initial realization s_0 is given. This suggests a natural commodity space in which goods are differentiated by histories.

In each period t there are two goods: labor and a consumption-capital good. A constant returns to scale technology is available to transform labor $\ell(s^t)$ and capital $k(s^{t-1})$ into output via $F(k(s^{t-1}), \ell(s^t), s_t)$. Notice that the production function incorporates a stochastic shock. The output can be used for private consumption $c(s^t)$, government consumption $g(s^t)$, and new capital $k(s^t)$. Throughout we will take government consumption to be exogenously specified. Feasibility requires

$$(1.1) \quad c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), \ell(s^t), s_t) + (1 - \delta)k(s^{t-1})$$

where δ is the depreciation rate on capital. The preferences of each consumer are given by

$$(1.2) \quad \sum_{t,s^t} \beta^t \mu(s^t) U(c(s^t), \ell(s^t))$$

where $0 < \beta < 1$ and U is increasing in consumption, decreasing in labor, strictly concave, and satisfies the Inada conditions.

Government consumption is financed by proportional taxes on the income from labor and capital and by debt. Let $\tau(s^t)$ and $\theta(s^t)$ denote the tax rates on the income from labor and capital. Government debt has a one-period maturity and a state-contingent return. Let $b(s^t)$ denote the number of units of debt issued at state s^t and $R_b(s^{t+1})b(s^t)$ denote the payoff at any state $s^{t+1} = (s^t, s_{t+1})$. The consumer's budget constraint is

$$(1.3) \quad c(s^t) + k(s^t) + b(s^t) \leq (1 - \tau(s^t))w(s^t)\ell(s^t) + R_b(s^t)b(s^{t-1}) + R_k(s^t)k(s^{t-1})$$

where $R_k(s^t) = [1 + (1 - \theta(s^t))(r(s^t) - \delta)]$ is the gross return on capital after taxes and depreciation and $r(s^t)$ and $w(s^t)$ are the before tax returns on capital and labor. Competitive pricing ensures that these returns equal their marginal products, namely

$$(1.4) \quad r(s^t) = F_k(k(s^{t-1}), \ell(s^t), s_t)$$

$$(1.5) \quad w(s^t) = F_l(k(s^{t-1}), \ell(s^t), s_t).$$

Consumers purchases of capital are constrained to be nonnegative and the purchases of government debt are bounded above and below by some arbitrarily large constants. We let $x(s^t) = (c(s^t), \ell(s^t), k(s^t), b(s^t))$ denote an allocation for consumers at s^t and let $x = (x(s^t))$ denote an allocation for all s^t . We let $(w, r) = (w(s^t), r(s^t))$ denote a price system.

The government sets tax rates on labor and capital income and returns for government debt to finance the exogenous sequence of government consumption. The government's budget constraint is

$$(1.6) \quad b(s^t) = R_b(s^t)b(s^{t-1}) + g(s^t) - \tau(s^t)w(s^t)\ell(s^t) - \theta(s^t)(r(s^t) - \delta)k(s^{t-1}).$$

We let $\pi(s^t) = (\tau(s^t), \theta(s^t), R_b(s^t))$ denote the government policy at s^t and let $\pi = (\pi(s^t))$ denote the policy for all s^t . The initial stock of debt, b_{-1} and the initial stock of capital, k_{-1} , are given.

Notice that for notational simplicity we have not explicitly included markets in private claims. Since all consumers are identical such claims will not be traded in equilibrium and hence their absence will not affect the equilibrium. Thus, we can always interpret the current model as having complete contingent private claims markets.

Given this description of an economy, we now define a competitive equilibrium. A competitive equilibrium is a policy π , an allocation x and a price system (w, r) such that given the policy and the price system, the resulting allocation maximizes the representative consumer's utility, (1.2), subject to the sequence of budget constraints (1.3); the price system satisfies (1.4) and (1.5); and the government's budget constraint (1.6) is satisfied. Notice that we do not need to impose the feasibility condition, (1.1), in our definition of equilibrium. Given our assumptions on the utility function, (1.3) is satisfied with equality in an equilibrium and this feature together with (1.6) implies (1.1).

Consider now the policy problem faced by the government. We suppose there is an institution or *commitment technology* through which the government can bind itself to a particular sequence of policies once and for all at time zero. We model this by having the government choose a policy π at the beginning of time and then having consumers choose their allocations. Formally, allocation rules are sequences of function $x(\pi) = (x(s^t | \pi))$ that map policies π into allocations $x(\pi)$. Price rules are sequences of functions $w(\pi) = (w(s^t | \pi))$ and $r(\pi) = (r(s^t | \pi))$ that map policies π into price systems. Since the government needs to predict how consumer allocations and prices will respond to its policies, consumer allocations and prices must be described by rules that associate government policies with allocations. We will impose a restriction on the set of policies that the government can choose. Since the capital stock at date 0 is inelastically supplied the government has

an incentive to set the initial capital tax rate as high as possible. To make the problem interesting, we will require that the initial capital tax rate, $\theta(s_0)$, is fixed at some rate. For similar reasons we also require that the initial return on debt, $R_b(s_0)$, be fixed at some rate.

A Ramsey equilibrium is a policy π , an allocation rule $x(\cdot)$, and price rules $w(\cdot)$ and $r(\cdot)$ such that:

(i) The policy π maximizes

$$\sum_{t,s^t} \beta^t \mu(s^t) U(c(s^t | \pi), \ell(s^t | \pi))$$

subject to (1.6) with allocations and prices given by $x(\pi)$, $w(\pi)$, and $r(\pi)$.

(ii) For every π' , the allocation $x(\pi')$ maximizes (1.2) subject to (1.3) evaluated at the policy π' and the prices $w(\pi')$ and $r(\pi')$,

(iii) For every π' , the prices satisfy

$$(1.7) \quad w(s^t | \pi') = F_l(k(s^{t-1} | \pi'), \ell(s^t | \pi'), s_t)$$

and

$$(1.8) \quad r(s^t | \pi') = F_k(k(s^{t-1} | \pi'), \ell(s^t | \pi'), s_t).$$

Notice that we require optimality by consumers and firms for *all* policies that the government might choose. This requirement is analogous to the requirement of subgame perfection in a game. To see why this requirement is important, suppose we had not imposed it. That is, suppose we required optimality by consumer and firms only at the equilibrium policies but allowed allocation and price rules to be arbitrary elsewhere. Then, it is possible to show that the set of equilibria is much larger. For example, allocation rules that prescribe zero labor supply for all policies other than some particular policy would satisfy all the equilibrium conditions. Since the government's budget constraint is then satisfied only at the particular policy, the government optimally chooses that policy. We think that such equilibria do not make any sense. That is, we think the requirement that

consumers and firms behave optimally for all policies is the sensible way to solve the government's forecasting problem.

2. Characterization of Competitive Equilibrium

We now turn to characterizing the equilibrium policies and allocation. In terms of notation, it will be convenient here and throughout the paper to let $U_c(s^t)$ and $U_\ell(s^t)$ denote the marginal utilities of consumption and leisure at state s^t and let $F_k(s^t)$ and $F_\ell(s^t)$ denote the marginal products of capital and labor at state s^t . We will show that a competitive equilibrium is characterized by two fairly simple conditions: the resource constraint

$$(2.1) \quad c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}), \ell(s^t), s^t) + (1 - \delta)k(s^{t-1})$$

and the implementability constraint

$$(2.2) \quad \sum_{t, s^t} \beta^t \mu(s^t) [U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)] = U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]$$

where $R_k(s_0) = (1 + (1 - \theta(s_0))(F_k(s_0) - \delta))$. The implementability constraint should be thought of as an infinite horizon version of either the consumer or the government budget constraint where the consumer and firm first order conditions have been used to substitute out the prices and policies.

We have

PROPOSITION 1 (Competitive Equilibrium Allocations). The consumption, labor, capital allocations, and the capital tax rate and return on debt in period 0 in a competitive equilibrium satisfy (2.1) and (2.2). Furthermore, given allocations and date zero allocations, which satisfy (2.1) and (2.2) we can construct policies, prices and debt policies which together with the given allocations and date zero allocations constitute a competitive equilibrium.

Proof. We first show that a competitive equilibrium must satisfy (2.1) and (2.2). To see this, note that we can add (1.3) and (1.6) to get (2.1) and thus feasibility is satisfied in equilibrium. Next, consider the allocation rule $x(\pi)$. The necessary and sufficient conditions for c , ℓ , b , and k to solve the consumer's problem are given as follows. Let $p(s^t)$ denote the Lagrange multiplier on constraint (1.3), then by Weitzman's (1973) and Ekeland and Scheinkman (1986) theorems these conditions are constraint (1.3) together with first order conditions for consumption and labor

$$(2.3) \quad \beta^t \mu(s^t) U_c(s^t) \leq p(s^t) \quad \text{with equality if } c(s^t) > 0$$

$$(2.4) \quad \beta^t \mu(s^t) U_\ell(s^t) \leq -p(s^t)(1 - \tau(s^t))w(s^t) \quad \text{with equality if } \ell(s^t) > 0$$

first order conditions for capital and bonds

$$(2.5) \quad \left[p(s^t) - \sum_{s^{t+1}} p(s^{t+1}) R_b(s^{t+1}) \right] b(s^t) = 0$$

$$(2.6) \quad \left[p(s^t) - \sum_{s^{t+1}} p(s^{t+1}) R_k(s^{t+1}) \right] k(s^t) = 0$$

and the two transversality conditions. These conditions specify that for any infinite history s^∞ ,

$$(2.7) \quad \lim p(s^t) b(s^t) = 0$$

$$(2.8) \quad \lim p(s^t) k(s^t) = 0$$

where the limits are taken over sequences of histories s^t contained in the infinite history s^∞ .

We claim that any allocation which satisfies (1.3) and (2.3)–(2.8) must satisfy (2.2). To see this multiply (1.3) by $p(s^t)$, sum over t and s^t and use (2.5)–(2.8) to give

$$(2.9) \quad \sum_{t, s^t} p(s^t) [c(s^t) - (1 - \tau(s^t))w(s^t)\ell(s^t)] = p(s_0) [R_k(s_0)k_{-1} + R_b(s_0)b_{-1}].$$

Using (2.3) and (2.4) and noting that interiority follows from the Inada conditions, we can rewrite equation (2.9) as

$$(2.10) \quad \sum_{t, s^t} \beta^t \mu(s^t) [c(s^t) U_c(s^t) + \ell(s^t) U_\ell(s^t)] = U_c(s_0) [R_k(s_0)k_{-1} + R_b(s_0)b_{-1}].$$

Thus, (2.1) and (2.2) are necessary conditions that any Ramsey equilibrium must satisfy.

Next, suppose that we are given allocations and date 0 policies satisfying (2.1) and (2.2). We construct the competitive equilibrium as follows. First, note that for an allocation to be part of a competitive equilibrium it must satisfy (1.3) and (2.3)–(2.8). Multiplying (1.3) by $p(s^t)$ and summing over all dates and states following s^t and using (2.3)–(2.8) we get

$$(2.11) \quad b(s^t) = \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \mu(s^t | s^r) [U_c(s^t) c(s^t) + U_\ell(s^t) \ell(s^t)] / U_c(s^r) - k(s^r).$$

Thus any competitive equilibrium debt allocation must satisfy (2.11) and hence (2.11) defines the unique debt allocations given consumption, labor, and capital allocations. The wage rate and the rental rate on capital are determined by (1.4) and (1.5) from the capital and labor allocations. The labor tax rate is determined from (1.5), (2.3), and (2.4) and is given by

$$(2.12) \quad - \frac{U_\ell(s^t)}{U_c(s^t)} = (1 - \tau(s^t)) F_\ell(s^t).$$

We can use (1.3), (2.3), (2.5), and (2.6) to construct the capital tax rate and the return on debt. From these conditions it is clear that given the allocations the tax rate on capital and the return on debt satisfy

$$(2.13) \quad \mu(s^t) U_c(s^t) = \sum_{s^{t+1} | s^t} \beta \mu(s^{t+1}) U_c(s^{t+1}) R_k(s^{t+1})$$

$$(2.14) \quad \mu(s^t) U_c(s^t) = \sum_{s^{t+1} | s^t} \beta \mu(s^{t+1}) U_c(s^{t+1}) R_b(s^{t+1})$$

$$(2.15) \quad c(s^t) + k(s^t) + b(s^t) = (1 - \tau(s^t)) w(s^t) \ell(s^t) + R_b(s^t) b(s^{t-1}) + R_k(s^t) k(s^{t-1})$$

where $R_k(s^t) = [1 + (1 - \theta(s^t))(r(s^t) - \delta)]$. It turns out that these conditions do not uniquely determine the tax rate on capital and the return on debt. To see this suppose that s_t can take on one of N values. Then counting linearly independent equations and unknowns in (2.13)–(2.15) gives $N + 1$ equations and $2N$ unknowns at each date and state. Thus, the tax rate and capital and the

return on debt are not uniquely determined if there is uncertainty. One particular set of policies supporting a competitive equilibrium has the capital tax rate not contingent on the current state. That is suppose for each s^t ,

$$(2.16) \quad \theta(s^t, s_{t+1}) = \bar{\theta}(s^t) \quad \text{for all } s_{t+1}.$$

We can then use (2.13) to define $\bar{\theta}(s^t)$ and use the date $t + 1$ version of (2.15) to define $R_b(s^{t+1})$. It is straightforward to check that the constructed return on debt satisfies (2.14). Another set of policies supporting the same competitive equilibrium has the return on debt not contingent on the current state. (For details see Chari, Christiano, and Kehoe 1991a.) \square

From the proof of the proposition it is clear that certain policies are uniquely determined by the theory while others are not. Specifically, the labor tax rate is determined while the state-by-state capital tax rate and return on debt are not. From (2.13), however, it is clear that the value of revenues from capital income taxation at date $t + 1$ in terms of the date t good are uniquely determined. To turn this variable into a tax rate, consider the ratio of the value of these revenues to the value of capital income, namely,

$$(2.17) \quad \theta^e(s^t) = \frac{\sum q(s^{t+1})\theta(s^{t+1})(F_k(s^{t+1}) - \delta)}{\sum q(s^{t+1})(F_k(s^{t+1}) - \delta)}$$

where $q(s^{t+1}) = \beta\mu(s^{t+1}|s^t)U_c(s^{t+1})/U_c(s^t)$ is the price of a unit of consumption at state s^{t+1} in units of consumption at s^t . We refer to $\theta^e(s^t)$ as the ex ante tax rate on capital income.

Next, in defining the last variable that is uniquely determined by the theory it is useful to proceed as follows. Imagine that the government promises a state noncontingent rate of return on government debt $\bar{r}(s^{t-1})$ and levies a state-contingent tax $\nu(s^t)$ on interest payments from government debt. That is, ν and \bar{r} satisfy

$$(2.18) \quad R_b(s^t) = [1 + \bar{r}(s^{t-1})(1 - \nu(s^t))]$$

and $\sum q(s^t)\nu(s^t) = 0$, where $q(s^t)$ is the price of a unit of consumption at state s^t in units of consumption at state s^{t-1} . Thus, $\bar{r}(s^{t-1})$ is the equilibrium rate of return on a unit purchased in period $t - 1$ at s^{t-1} which yields a noncontingent return $\bar{r}(s^{t-1})$ in all states s^t . It is clear from (2.15) that the theory pins down $R_b(s^t)b(s^{t-1}) + R_k(s^t)k(s^{t-1})$. Given our definition of ν it is clear that the theory pins down the sum of the tax revenues from capital income and the interest on debt which is given by

$$(2.19) \quad \theta(s^t)[F_k(s^t) - \delta]k(s^{t-1}) + \nu(s^t)\bar{r}(s^{t-1})b(s^{t-1}).$$

We transform these revenues into a rate by dividing by the income from capital and debt to obtain

$$(2.20) \quad \eta(s^t) = \frac{\theta(s^t)(F_k(s^t) - \delta)k(s^{t-1}) + \nu(s^t)\bar{r}(s^{t-1})b(s^{t-1})}{(F_k(s^t) - \delta)k(s^{t-1}) + \bar{r}(s^{t-1})b(s^{t-1})}.$$

Next, we turn to discussing how Proposition 1 can be used in practice. Proposition 1 completely characterizes the competitive equilibrium allocations as long as there are sufficient degrees of freedom in setting policies. In some situations it may be reasonable to restrict the set of policies. For example, it may be reasonable to restrict tax rates to be less than 100 percent. Such restrictions can be easily imposed. We illustrate how to impose them in these examples.

Example 1 (Capital Tax Rates Bounded by Unity). Suppose that capital tax rates are at most 100 percent. Then in addition to (2.1) and (2.2) an allocation must satisfy an extra condition to be part of a competitive equilibrium. Rewrite (2.13) as

$$(2.21) \quad \mu(s^t)U_c(s^t) = \sum_{s^{t+1}|s^t} \beta\mu(s^{t+1})U_c(s^{t+1}) + \sum_{s^{t+1}|s^t} \beta\mu(s^{t+1})(1 - \theta(s^{t+1}))(F_k(s^{t+1}) - \delta).$$

Then if an allocation satisfies

$$(2.22) \quad F_k(s^{t+1}) \geq \delta$$

and $\theta(s^{t+1}) \leq 1$, (2.21) implies

$$(2.23) \quad \mu(s^t)U_c(s^t) \geq \sum_{s^{t+1}|s^t} \beta\mu(s^{t+1})U_c(s^{t+1}).$$

If an allocation satisfies (2.1), (2.2), (2.22), and (2.23) then there exists a set of policies, prices, and debt allocations which together with the given allocation constitute a competitive equilibrium which satisfies the restrictions on capital tax rates.

Example 2 (Noncontingent Capital Taxes and Return on Debt). Suppose that neither capital tax rates nor the return on debt can be made state-contingent. Then the additional restriction that the allocation must satisfy so that we can construct a competitive equilibrium is given as follows.

Substituting (2.11) and (2.12) into the consumer's budget constraint yields after some simplification,

$$(2.24) \quad \frac{\sum_{t=r} \sum_{s^t} \beta^{t-r} \mu(s^t | s^r) [U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)]}{U_c(s^r)} - (1 + (1 - \bar{\theta}(s^{r-1}))(F_k(s^r) - \delta))k(s^{r-1}) \\ = \bar{R}_b(s^{r-1})b(s^{r-1})$$

where $\bar{\theta}(s^{r-1})$ satisfies

$$(2.25) \quad U_c(s^{r-1}) = \sum_{s^r} \beta\mu(s^r | s^{r-1})U_c(s^r)[1 + (1 - \bar{\theta}(s^{r-1}))(F_k(s^r) - \delta)].$$

The requirement that the debt is not state-contingent is then simply the requirement that the left side of (2.24) with $\bar{\theta}(s^{r-1})$ substituted from (2.25) be equal for all s_r . We then have that if an allocation satisfies this requirement together with (2.1) and (2.2), a competitive equilibrium can be constructed which satisfies the restriction that neither the capital tax rate nor the return on debt be state-contingent. Clearly, computing equilibria with noncontingent capital taxes and return on debt is a computationally difficult exercise.

Example 3 (Exogenous Tax Policies). Suppose next that we are given a policy for tax rates on capital and labor income and we want to ask whether there exists a state-contingent return on debt

and allocations which together with the tax policies constitutes a competitive equilibrium. Then, the additional restrictions are simply the relevant first order conditions

$$(2.26) \quad -\frac{U_\ell(s^t)}{U_c(s^t)} = (1 - \tau(s^t))F_\ell(s^t)$$

$$(2.27) \quad U_c(s^t) = \sum_{s^{t+1}} \beta \mu(s^{t+1} | s^t) U_c(s^{t+1}) [1 + (1 - \theta(s^{t+1}))F_k(s^{t+1}) - \delta].$$

Thus, if an allocation and the given tax policy satisfies (2.1), (2.2), (2.26), and (2.27) then there exist state-contingent returns on debt and debt allocations which constitute a competitive equilibrium.

The general approach to characterizing competitive equilibria with distorting taxes described thus far is known as the primal approach to taxation in the public finance literature (see Atkinson and Stiglitz 1990). The basic idea is to characterize an equilibrium in terms of allocations as far as possible. This approach can be useful when we study optimal taxation. This approach is also useful in studying exogenous tax systems.

We find it useful to first describe the traditional approach to studying the business cycle implications of exogenous tax systems followed by Braun (1992), Chang (1990), Greenwood and Huffman (1991), and McGrattan (1992), among others. In this approach, stochastic processes for the labor tax rate and the capital tax rate are chosen to mimic those in the postwar U.S. data. The consumer's first order conditions (2.26) and (2.27) and the resource constraint (2.1) are used to compute the stochastic processes for the allocations. This procedure is widely used in public finance. Of course, for a variety of reasons including problems in measuring and estimating tax processes, the allocations and policies computed this way will not typically satisfy the government budget constraint, or its analog, the implementability constraint. One interpretation of the procedure is that

lump sum taxes are available and are appropriately used to ensure that the implementability constraint is satisfied.

Here we outline how this procedure can be modified to compute a competitive equilibrium when there are no lump sum taxes. Such a modification is especially important when making welfare compensation across alternative policies. The basic idea is to consider a class of tax policies with enough degrees of freedom in specifying the stochastic processes on tax policies so that by varying policies within this class the implementability constraint is satisfied. For simplicity, consider Example 3 and assume the state follows a Markov chain. Suppose that the capital and labor tax rates depend only on the inherited capital stock k and the current state s . Suppose also the capital tax rates are a given function $\theta(k,s)$ and the class of labor tax rates are given by $\tau(k,s;\alpha) = \alpha + \phi(k,s)$ where ϕ is a given function and α is a parameter which will be varied to satisfy the implementability constraint. Notice that by varying α we are varying the mean of the labor tax while keeping its other moments unchanged. Given this class of policies and the Markov assumption on the state, it follows from the resource constraint (2.1) and the first order conditions (2.26) and (2.27), that the consumption, labor, and capital allocation rules can be described as stationary functions of the inherited capital stock and the current state. Denote these functions as $c(k,s;\alpha)$, $l(k,s;\alpha)$, and $k'(k,s;\alpha)$, respectively. Notice that in order to compute these functions, we do not need to know the debt allocation rule or the return on debt. Next we verify (2.2). One way of verifying this condition is to stimulate a large number of sufficiently long strings of the state s^t . Obviously, this procedure is cumbersome. An alternative procedure is to use (2.11) to define a recursive law of motion for the debt allocation. Notice that given that consumption, labor, and capital are stationary functions of (k,s) , (2.11) makes it clear that the debt allocations are also stationary functions. From (2.11), the debt allocations can be recursively written as follows

$$(2.28) \quad U_c(k,s)b(k,s) = \sum_{s'} \beta \mu(s'|s) [U_c(k',s')c(k',s') + U_\ell(k',s')\ell(k',s') + U_c(k',s')b(k',s') \\ + U_c(k',s')k'] - U_c(k,s)k'$$

where $k' = k'(k,s)$ and $U_c(k,s)$, $U_\ell(k,s)$ are marginal utilities of consumption and labor and where we have suppressed the dependence on α . Notice that (2.28) defines a linear operator mapping bond allocation rules into themselves. Thus, from a computational perspective, once we have the consumption, labor, and capital allocations, it is relatively straightforward to compute the bond allocation rules. This procedure gives the end-of-period debt for all periods. Next, given k_{-1} and the date 0 policies, the allocation rules can be substituted into the consumer's budget constraint (1.3) evaluated at date 0 to obtain the value of the initial debt \hat{b}_{-1} . Of course, given an arbitrary α , the value of this debt \hat{b}_{-1} will not equal the given debt b_{-1} . The parameter α is then adjusted until $\hat{b}_{-1} = b_{-1}$.

Next we illustrate this procedure in an example where we compute welfare under alternative cyclical policies. We begin with a naive procedure which compares welfare across alternative policies without requiring that the initial debt be the same across these policy experiments. We show that imposing the government budget constraint can have sizeable effects on the welfare comparisons.

Example 4 (Cyclical Policies and Welfare). Consider an economy with preferences of the form

$$U(c,\ell) = (1-\gamma)\log c + \gamma\log(1-\ell)$$

where L is the endowment of labor. The production function is of the form

$$F(k,\ell,z,t) = k^\alpha(e^{\rho t} + z\ell)^{(1-\alpha)}.$$

The parameter ρ incorporates deterministic growth and the variable z is a technology shock that follows a symmetric two-state Markov chain with states z_1 and z_2 and transition probabilities

$\text{Prob}(z_{t+1} | z_t = z_i) = \pi$, for $i = 1, 2$. Government consumption is given by $g_t = ge^{at}$ where g is a constant. The parameters for preferences and technology are given in Table 1.

We begin by considering a baseline policy which has a constant capital tax rate $\theta(s^t) = 0.27$ and a constant labor tax rate $\tau(s_t) = 0.24$. The implied value of initial debt is $R_0 b_{-1} = 0.2$. We consider a countercyclical policy experiment. In this experiment, the tax rate is low when the current realization of the shock is low and high when the realization of the shock is high. This policy is intended to stimulate employment in the low state and reduce employment in the high state relative to our baseline policy and thus stabilize output. A naive approach to policy evaluation is to set the tax rate on labor in the higher state at its baseline level and reduce the tax rate on labor in the low state by a given amount. We conducted such an experiment and set the tax rate in the low state to 0.018, a decrease of 0.06 from its baseline level.

To evaluate welfare we compute the constant percentage change in consumption relative to the baseline levels in all dates and states, keeping labor at its baseline levels, required to yield a utility level equal to that in the policy experiment. In Table 2 we report the results of this exercise. Note that welfare increases by 0.65 percent, the standard deviation of output falls to 31 percent of its baseline level, and the standard deviation of employment rises to 153 percent of its baseline level. Next we consider a similar policy exercise except that we require the implementability constraint to be satisfied. The class of policies considered in this experiment is described as follows. We set the tax rate in the high state at τ , a number to be determined. The tax rate in the low state is set at $\tau - 0.06$ and thus the difference in taxes between the high and low states is the same as in the naive experiment. Here, however, we adjust τ so that the initial debt is the same as in the baseline setting. The equilibrium value of τ is 27 percent. In terms of the volatility measures we find that we get results very similar to the naive case. The standard deviation of output falls to 27 percent of its baseline level and the standard deviation of employment rises to 164 percent of its baseline level.

More interestingly, we find that welfare decreases by 0.1 percent relative to its baseline level. Thus requiring budget balance reverses the welfare ranking of the policy experiment relative to the baseline policy. To see why this reversal occurs, notice that in the naive approach the labor tax rates are lower in both states than in the policy requiring budget balance and thus the present value of tax revenue is lower in the naive approach. It follows that if the naive policies are part of a competitive equilibrium either the initial debt must be lower than under the baseline or lump sum taxes must be used to make up the lost revenues.

3. Ramsey Equilibrium

Given our characterization of a competitive equilibrium, the characterization of the Ramsey equilibrium is immediate. We have

PROPOSITION 2. The allocations in a Ramsey equilibrium solve the following programming problem

$$(3.1) \quad \max \sum_{s^t} \sum_t \beta^t \mu(s^t) U(c(s^t), \ell(s^t))$$

subject to (2.1) and (2.2).

It will be convenient to write the Ramsey allocation problem in Lagrangian form:

$$(3.2) \quad \max \sum_{t, s^t} \beta^t \mu(s^t) W(c(s^t), \ell(s^t), \lambda)$$

subject to (2.1). The function W simply incorporates the implementability constraint into the maximand. For $t \geq 1$, we have

$$(3.3) \quad W(c(s^t), \ell(s^t), \lambda) = U(c(s^t), \ell(s^t)) + \lambda [U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)]$$

and for $t = 0$, W equals the right side of (3.3) evaluated at s_0 , minus $\lambda U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]$. Here, λ is the Lagrange multiplier on the implementability constraint, (2.2). The first order conditions for this problem imply, for $t \geq 0$

$$(3.4) \quad -\frac{W_\ell(s^t)}{W_c(s^t)} = F_\ell(s^t)$$

and

$$(3.5) \quad W_c(s^t) = \sum_{s_{t+1}} \beta \mu(s^{t+1} | s^t) W_c(s^{t+1}) [1 - \delta + F_k(s^{t+1})], \quad t = 0, 1, 2, \dots$$

A useful property of the Ramsey allocations is the following. If the stochastic process on s follows a Markov process then from (3.4) and (3.5) it is clear that the allocations from date 1 onwards can be described by time invariant allocation rules $c(k,s;\lambda)$, $\ell(k,s;\lambda)$, $k'(k,s;\lambda)$, and $b(k,s;\lambda)$. The date 0 first order conditions include terms related to the initial stocks of capital and bonds and are therefore different from the other first order conditions. The date 0 allocation rules are thus different from the stationary allocation rules which govern behavior from date 1 on.

We begin our analysis of optimal fiscal policy for this model by considering a nonstochastic version of the model in which the stochastic shock in the production function is constant. Government consumption is also constant, so $g(s^t) = g$. Suppose that under the Ramsey plan the allocations converge to a steady state. In such a steady state, W_c is constant. Thus, from (3.5),

$$(3.6) \quad 1 = \beta[1 + F_k - \delta].$$

The consumer's intertemporal first order condition is

$$(3.7) \quad U_{ct} = \beta U_{ct+1} [1 + (1 - \theta_{t+1})(F_{kt+1} - \delta)].$$

In a steady state, U_c is a constant, so (3.7) reduces to

$$(3.8) \quad 1 = \beta[1 + (1 - \theta)(F_k - \delta)].$$

Comparing (3.6) and (3.8), we can see that in a steady state the optimal tax rate on capital income, θ , is zero. This result is due to Chamley (1986).

In Chari, Christiano, and Kehoe (1991a), we show that an analogous result holds in stochastic economies; namely, the value of tax revenue across states of nature is approximately zero in a stationary equilibrium. However, the state-by-state capital taxes are not uniquely determined and can be quite different from zero. In Chari, Christiano, and Kehoe (1991a), we explore the quantitative properties of optimal policy in a parameterized version of the model. We consider preferences of the form

$$(3.9) \quad U(c, \ell) = [c^{1-\gamma}(L-\ell)^\gamma]^\psi/\psi$$

where L is the endowment of labor. This class of preferences has been widely used in the literature (Kydland and Prescott 1982; Christiano and Eichenbaum 1992; and Backus, Kehoe, and Kydland 1992). The production technology is given by

$$(3.10) \quad F(k, \ell, z, t) = k^\alpha [e^{\rho t + z} \ell]^{(1-\alpha)}.$$

Notice that the production technology has two kinds of labor augmenting technological change. The variable ρ captures deterministic growth in this change. The variable z is a technology shock that follows a symmetric two-state Markov chain with states z_ℓ and z_h and transition probabilities $\text{Prob}(z_{t+1} = z_i | z_t = z_i) = \pi$, $i = \ell, h$. Government consumption is given by $g_t = g e^{\rho t}$, where ρ is the deterministic growth rate and g follows a symmetric two-state Markov chain with states g_ℓ and g_h and transition probabilities $\text{Prob}(g_{t+1} = g_i | g_t = g_i) = \phi$, $i = \ell, h$. Notice that without technology or government consumption shocks, the economy has a balanced growth path along which private consumption, capital, and government consumption grow at rate ρ and labor is constant.

We consider two parameterizations of this model. (See Table 3.) Our *baseline* model has $\psi = 0$ and thus has logarithmic preferences. Our *high risk aversion* model has $\psi = -8$. The remaining parameters of preferences and the parameters for technology are the annualized versions of those used by Christiano and Eichenbaum (1992). We choose the three parameters of the Markov

chain for government consumption to match three statistics of the postwar U.S. data: the average value of the ratio of government consumption to output, the variance of the detrended log of government consumption, and the serial autocorrelation of the detrended log of government consumption. We construct the Markov chain for the technology parameters by setting the mean of the technology shock equal to zero and use Prescott's (1986) statistics on the variance and serial correlation of the technology shock to determine the other two parameters.

For each setting of the parameter values, we simulate our economy starting from the steady state of the deterministic versions of our models. In Table 4 we report some properties of the fiscal variables for our baseline model. The table shows that tax on labor income fluctuates very little. For example, if the labor tax rate were approximately normally distributed, then 95 percent of the time the tax rate would fluctuate between 27.89 percent and 28.25 percent. The tax on capital income is zero. This is to be expected from the analytic results in Chari, Christiano, and Kehoe (1991a) since with $\psi = 0$ the utility function is separable between consumption and leisure and homothetic in consumption. For such preferences, this paper shows that the tax on capital is zero in all periods but the first.¹ In the baseline model, the tax on private assets has a large standard deviation. Intuitively, the tax on private assets acts as a shock absorber. The optimal tax rate on labor does not respond much to shocks to the economy. The government smooths labor tax rates by appropriately adjusting the tax on private assets in response to shocks. This variability of the tax on private assets does not distort capital accumulation since what matters for the capital accumulation decision is the ex ante tax rate on capital income. This can be seen by manipulating the first order condition for capital accumulation, (2.13).

In Table 4 we also report some properties of the fiscal policy variables for the high risk aversion model. Here, too, the tax rate on labor fluctuates very little. The tax rate on capital income has a mean of -0.11 percent, which is close to zero. We find this feature interesting

because it suggests that our analytical result approximately holds for the class of utility functions commonly used in the literature. This feature also suggests that Chamley's (1986) result on the undesirability of the taxation of capital income in a deterministic steady state approximately holds in stochastic steady states of stochastic models. As in the baseline model, we find here that the standard deviation of the tax rate on private assets is large.

To gain an appreciation of the magnitudes of some of the numbers for our model economies, we compute analogous numbers for the U.S. economy. In Table 4, we report these as well. For the labor tax rate, we use Barro and Sahasakul's (1983) estimate of the average marginal labor tax rate. The standard deviation of this rate is 2.39 percent, which is approximately 25 times the standard deviation in our baseline model. For the tax rate on capital income, we use Jorgenson and Sullivan's (1981) estimate of the effective corporate tax rate. This number probably underestimates the ex ante rate since it ignores the taxation of dividends and capital gains received by individuals. The mean effective rate in the data is 28.28 percent while our baseline model has an ex ante tax rate of zero. Finally, the standard deviation of the innovation in the tax on private assets in the baseline model is about six times that in the data.²

4. A Monetary Economy

In this section we study the properties of monetary policy using a version of Lucas and Stokey's (1983) cash-credit goods model. This economy is a version of the real economy of Section 1 with money but without capital. Here we focus on optimal monetary policy. For study of various exogenous policies in a related model, see Cooley and Hansen (1989, 1992). Here we study both the mean inflation rate and its cyclical properties. Friedman (1969) has argued that monetary policy should follow a rule: set nominal interest rates to zero. For a deterministic version of our economy, this would imply deflating at the rate of time preference. Phelps (1973) argues that

Friedman's rule is unlikely to be optimal in an economy with no lump-sum taxes. His argument is that optimal taxation generally requires using all available taxes, including the inflation tax. Thus, Phelps argues that the optimal inflation rate is higher than the Friedman rule implies.

In Section 3, we have shown how real state-contingent debt can serve a useful role as a shock absorber. Here we allow the government to issue only nominal state-noncontingent debt. We examine how the government should optimally use monetary policy to make this debt yield the appropriate real state-contingent returns.

Consider, then, a simple production economy with three goods. The goods are labor ℓ and two consumption goods: a cash good c_1 and a credit good c_2 . A stochastic constant returns-to-scale technology transforms labor into output according to

$$(4.1) \quad c_1(s^t) + c_2(s^t) + g(s^t) = z(s^t)\ell(s^t)$$

where $z(s^t)$ is a technology shock and, again, $g(s^t)$ is government consumption. The preferences of each consumer are given by

$$(4.2) \quad \sum_t \sum_{s^t} \beta^t \mu(s^t) U(c_1(s^t), c_2(s^t), \ell(s^t))$$

where U has the usual properties.

In period t , consumers trade money, assets, and goods in particular ways. At the start of period t , after observing the current state s_t , consumers trade money and assets in a centralized securities market. The assets are one-period state-noncontingent nominal claims. Let $M(s^t)$ and $B(s^t)$ denote the money and nominal bonds held at the end of the securities market trading. Let $R(s^t)$ denote the gross nominal return on these bonds payable in period $t + 1$ in all states s^{t+1} . After this trading, each consumer splits into a worker and a shopper. The shopper must use the money to purchase cash goods. To purchase credit goods, the shopper issues nominal claims which are settled in the securities market in the next period. The worker is paid in cash at the end of each period.

This environment leads to this constraint for the securities market:

$$(4.3) \quad M(s^t) + B(s^t) = R(s^{t-1})B(s^{t-1}) + M(s^{t-1}) - p(s^{t-1})c_1(s^{t-1}) - p(s^{t-1})c_2(s^{t-1}) \\ + p(s^{t-1})(1 - \tau(s^{t-1}))z(s^{t-1})\ell(s^{t-1}).$$

The left side of (4.4) is the nominal value of assets held at the end of securities market trading. The first term on the right side is the value of nominal debt bought in the preceding period. The next two terms are the shopper's unspent cash. The next is the payments for credit goods, and the last is the after-tax receipts from labor services. Besides this constraint, we will assume that the real holdings of debt, $B(s^t)/p(s^t)$, are bounded below by some arbitrarily large constant. Purchases of cash goods must satisfy a cash-in-advance constraint:

$$(4.4) \quad p(s^t)c_1(s^t) \leq M(s^t).$$

Money is introduced into and withdrawn from the economy through open market operations in the securities market. The constraint facing the government in this market is

$$(4.5) \quad M(s^t) - M(s^{t-1}) + B(s^t) = R(s^{t-1})B(s^{t-1}) + p(s^{t-1})g(s^{t-1}) - p(s^{t-1})\tau(s^{t-1})z(s^{t-1})\ell(s^{t-1}).$$

The terms on the left side of this equation are the assets sold by the government. The first term on the right is the payment on debt incurred in the preceding period, the second is the payment for government consumption, and the third is tax receipts. Notice that government consumption is bought on credit.

The consumer's problem is to maximize (4.2) subject to (4.3) and (4.4) and the bound on debt. Money earns a gross nominal return of one. If bonds earn a gross nominal return of less than one, then the consumer can make infinite profits by buying money and selling bonds. Thus, in any equilibrium, $R(s^t) \geq 1$. The consumer's first order conditions imply that $U_1(s^t)/U_2(s^t) = R(s^t)$; thus, in any equilibrium, this constraint must hold:

$$(4.6) \quad U_1(s^t) \geq U_2(s^t).$$

This feature of the competitive equilibrium constrains the set of Ramsey allocations.

A Ramsey equilibrium for this economy is defined in the obvious way. As is well-known, if the initial stock of nominal assets held by consumers is positive, then welfare is maximized by increasing the initial price level to infinity. If the initial stock is negative, then welfare is maximized by setting the initial price level so low that the government raises all the revenue it needs without levying any distorting taxes. To make the problem interesting, we set the initial nominal assets of consumers to zero. Let $a(s_0)$ denote initial real claims that the government holds against private agents. The Ramsey allocation problem is

$$\max \sum_t \sum_{s^t} \beta^t \mu(s^t) U(c_1(s^t), c_2(s^t), \ell(s^t))$$

subject to (4.1), (4.6), and

$$(4.7) \quad \sum_t \sum_{s^t} \beta^t \mu(s^t) [U_1(s^t)c_1(s^t) + U_2(s^t)c_2(s^t) + U_3(s^t)\ell(s^t)] = U_2(s_0)a(s_0).$$

For convenience in studying the properties of the Ramsey allocation problem, let

$$(4.8) \quad W(c_1, c_2, \ell, \lambda) = U(c_1, c_2, \ell) + \lambda[U_1c_1 + U_2c_2 + U_3\ell]$$

where λ is the Lagrange multiplier on the implementability constraint (4.7). The Ramsey allocation problem is, then, to maximize

$$\sum_t \sum_{s^t} \beta^t \mu(s^t) W(c_1(s^t), c_2(s^t), \ell(s^t), \lambda)$$

subject to (4.2) and (4.6). Consider utility functions of the form

$$(4.9) \quad U(c_1, c_2, \ell) = h(c_1, c_2)v(\ell)$$

where h is homogenous of degree k and the utility function has the standard properties. We then have

PROPOSITION 3 (The Optimality of the Friedman Rule). For utility functions of the form (4.9), the Ramsey equilibrium has $R(s^t) = 1$ for all s^t .

Proof. Consider for a moment the Ramsey problem with constraint (4.5) dropped. A first order condition for this problem is

$$(4.10) \quad \frac{W_1(s^t)}{W_2(s^t)} = 1.$$

For utility functions of the form (4.9),

$$W = hv + \lambda[c_1h_1v + c_2h_2v + \ell hv'].$$

Since h is homogenous of degree k , $c_1h_1 + c_2h_2 = kh$. Thus, $W = h(c_1, c_2)Q(\ell, \lambda)$ for some function Q . Combining this feature with (4.10) gives

$$(4.11) \quad 1 = \frac{W_1}{W_2} = \frac{U_1}{U_2}.$$

Since the solution to this less-constrained problem satisfies (4.6), it is also a solution to the Ramsey problem. Then the consumer's first order condition $U_1(s^t)/U_2(s^t) = R(s^t)$ implies that $R(s^t) = 1$. \square

In Chari, Christiano, and Kehoe (1991b), we show that the Friedman rule is optimal for more general utility functions of the form

$$U(c_1, c_2, \ell) = V(h(c_1, c_2), \ell)$$

where h is homothetic. We also show that the Friedman rule is optimal for money-in-the-utility-function economies and transaction cost economies which satisfy a similar homotheticity condition.

The intuition for this result is as follows. In this economy, the tax on labor income implicitly taxes consumption of both goods at the same rate. A standard result in public finance is that if the utility function is separable in leisure and the subutility function over consumption goods is

homothetic, then the optimal policy is to tax all consumption goods at the same rate (Atkinson and Stiglitz 1972). If $R(s^t) > 1$, the cash good is effectively taxed at a higher rate than the credit good since cash goods must be paid for immediately but credit goods are paid for with a one-period lag. Thus, with such preferences, efficiency requires that $R(s^t) = 1$ and, therefore, that monetary policy follow the Friedman rule.

This intuition is not complete, however. As we mentioned earlier, the Friedman rule turns out to be optimal even in many models with money in the utility function or with money facilitating transactions. In such models, money and consumption goods are taxed at different rates. Specifically, money is not taxed at all while consumption goods are. Thus, the Phelps (1973) argument turns out to be more tenuous than it first appears. (For analyses of optimality of the Friedman rule in various deterministic models of money with distorting taxes, see Kimbrough 1986, Faig 1988, and Woodford 1990.)

We turn now to some numerical exercises which examine the cyclical properties of monetary policy in our model. In these exercises, we consider preferences of the form

$$U(c, \ell) = [c^{1-\gamma}(L-\ell)^\gamma]^\psi / \psi$$

where L is the endowment of labor and

$$c = [(1-\sigma)c_1^\nu + \sigma c_2^\nu]^{1/\nu}.$$

The technology shock z and government consumption both follow the same symmetric two-state Markov chains as in the model in Section 2.

For preferences, we set the discount factor $\beta = 0.97$, we set $\psi = 0$, which implies logarithmic preferences between the composite consumption good and leisure, and we set $\gamma = 0.8$. These values are the same as those in Christiano and Eichenbaum 1992. The parameters σ and ν are not available in the literature, so we estimate them using the consumer's first order conditions.

These conditions imply that $U_{1t}/U_{2t} = R_t$. For our specification of preferences, this condition can be manipulated to be

$$(4.12) \quad \frac{c_{2t}}{c_{1t}} = \left(\frac{\sigma}{1 - \sigma} \right)^{1/1-\nu} R_t^{1/1-\nu}.$$

With a binding cash-in-advance constraint, c_1 is real money balances and c_2 is aggregate consumption minus real money balances. We measure real money balances by the monetary base, R_t by the return on three-month Treasury bills, and consumption by consumption expenditures. Taking logs in (4.12) and running a regression using quarterly data for the period 1959–89 gives $\sigma = 0.57$ and $\nu = 0.83$.

Our regression turns out to be similar to those used in the money demand literature. To see this, note that (4.12) implies that

$$(4.13) \quad \frac{c_{1t}}{c_{1t} + c_{2t}} = \left[1 + \left(\frac{\sigma}{1 - \sigma} \right)^{1/1-\nu} R_t^{1/1-\nu} \right]^{-1}.$$

Taking logs in (4.13) and then taking a Taylor's expansion yields a money demand equation with consumption in the place of output and with the restriction that the coefficient of consumption is 1. Our estimates imply that the interest elasticity of money demand is 4.94. This estimate is somewhat smaller than estimates obtained when money balances are measured by M1 instead of the base.

Finally, we set the initial real claims on the government so that, in the resulting stationary equilibrium, the ratio of debt to output is 44 percent. This is approximately the ratio of U.S. federal government debt to GNP in 1989. For the second parameterization, we set $\psi = -8$, which implies a relatively high degree of risk aversion. For the third, we make both technology shocks and government consumption i.i.d.

In Table 5 we report the properties of the labor tax rate, the inflation rate, and the money growth rate for our monetary models. In all three, the labor tax rate has the same properties it did

in the real economy with capital: it fluctuates very little, and it inherits the persistence properties of the underlying shocks.

Consider next the inflation rate and the money growth rate. Recall that for these monetary models the nominal interest rate is identically zero. If government consumption and the technology shock were constant, then the price level and the money stock would fall at the rate of time preference, which is 3 percent. In a stochastic economy the inflation rate and the money growth rate vary with consumption. Therefore, the mean inflation rate depends not only on the rate of time preference, but also on the covariance of the inflation rate and the intertemporal marginal rate of substitution. This effect causes the inflation rate and the money growth rate to rise with an increase in the coefficient of risk aversion.

In the monetary models, the autocorrelations of the inflation rate are small or negative. Thus, they are far from a random walk. The correlations of inflation with government consumption and with the technology shock have the expected signs. Notice that these correlations have opposite signs, and in the baseline and high risk aversion models, this leads to inflation having essentially no correlation with output. The most striking feature of the inflation rates is their volatility. In the baseline model, for example, if the inflation rate were normally distributed, it would be higher than 20 percent or lower than -20 percent approximately a third of the time. The inflation rates for the high risk aversion model are even more volatile. The money growth rate has essentially the same properties as the inflation rate.

Note that our results are quite different from those of Mankiw (1987). Using a partial equilibrium model, he argues that optimal policy implies that inflation should follow a random walk. It might be worth investigating whether there are any general equilibrium settings which rationalize Mankiw's argument.

5. Computational Issues

Our computational techniques use Propositions 1 and 2 to compute the equilibrium allocations and then use the constructive procedure outlined in the proof of Proposition 1 to compute the equilibrium policies. We illustrate our procedure by describing how we compute the Ramsey equilibrium for the real economy with capital accumulation. We assume that the transition probabilities for the state follow a Markov chain and that the exogenous state variables, s , take on S values, $s = 1, \dots, S$. Our computational procedure begins by fixing an initial value for the Lagrange multiplier λ on the implementability constraint (2.2). Given this value of λ , it is clear from the resource constraint (2.1) and from (3.4) and (3.5) that the consumption labor and capital allocations for $t \geq 1$ are stationary functions of the capital stock and the exogenous state. We denote these functions by $c(k,s;\lambda)$, $\ell(k,s;\lambda)$, and $k'(k,s;\lambda)$. There are a variety of standard methods for computing approximations to these functions (see Baxter 1988, Bizer and Judd 1989, Coleman 1989, and Judd 1991). We investigate two methods: the minimum weighted residual method described in Judd (1991) and the log-linearization method described in Christiano (1991). Both approaches yield approximations to the stationary allocation rules. We then use these, together with the appropriate resource constraint and Euler equations, to compute the date zero decisions. Finally, we adjust λ until the implementability constraint is satisfied. In what follows, we describe some details of these algorithms, and after that we compare results based on the minimum weighted residual and log-linearization methods.

We use (2.1) and (3.4) to solve for consumption and labor as functions of (k,k',s) and denote these functions as $\hat{c}(k,k',s)$ and $\hat{\ell}(k,k',s)$. (Here, and in what follows, we suppress dependence on λ .) We substitute these functions into (3.5) to obtain

$$(5.1) \quad H(k,k',s) = \sum_{s'} \beta \mu(s'|s) G(k',k'',s')$$

where $H(k, k', s) = W_c(\hat{c}(k, k', s), \hat{\ell}(k, k', s), s)$ and $G(k, k', s) = H(k, k', s)(1 + F_k(k, \hat{\ell}(k, k', s), s) - \delta)$, and k'' denotes the capital stock at the end of the next period. Substituting the stationary capital accumulation decision rule, $k'(k, s)$, into (5.1) gives

$$(5.2) \quad H(k, k'(k, s), s) - \sum \beta \mu(s' | s) G(k'(k, s), k'(k'(k, s), s'), s') = 0.$$

Then, for a given λ , our task is to find a function $k'(k, s)$ which solves (5.2). Given such a function the consumption and labor decision rules are simply, $c(k, s) = \hat{c}(k, k'(k, s), s)$ and $\ell(k, s) = \hat{\ell}(k, k'(k, s), s)$. Finding a function $k'(k, s)$ which solves (5.2) is complicated by the fact that k is a continuous variable. As a result, (5.2) is a continuum of equations in a continuum of unknowns, with equations and unknowns being indexed by all possible k, s pairs. Solving such a system is typically infeasible. The version of the minimum weighted residual approach that we adopt approximates $k'(k, s)$ by a finite parameter function based on polynomials, and requires evaluating (5.2) at only a finite number of points. The parameters of the function are adjusted until appropriately weighted averages of the left side of (5.2) are set equal to zero.

We consider decision rules of the form:

$$(5.3) \quad k'(k, s) = \exp \left[\sum_{i=0}^{M-1} \alpha_i(s) T_i(g(\log(k))) \right]$$

where $\alpha_i(s)$, $i = 0, \dots, M - 1$, $s = 1, \dots, S$ are the parameters of the decision rule and $T_i(\cdot)$ is a Chebyshev polynomial of order i which maps $(-1, 1)$ into $(-1, 1)$.³ The function g maps the range of the log of the capital stock that we consider, $(\log k_1, \log k_N)$, into the interval $(-1, 1)$, and is defined by

$$g(\log k) = (2 \log k - \log k_N - \log k_1) / (\log k_N - \log k_1).$$

Next, we construct a grid of capital stocks, $\{k_1, \dots, k_N\}$ such that $k_1 < k_2 < \dots < k_N$.⁴ Substituting (5.4) into (5.2) and evaluating the resulting expression at the $N \times S$ values of (k, s) gives $N \times S$ equations in $M \times S$ unknowns, namely:

$$(5.5) \quad R(k, s; \alpha) = 0, \quad k = k_1, \dots, k_N, \quad s = 1, \dots, S.$$

where $\alpha = (\alpha_i(s))$. We adopt the Galerkin version of the minimum weighted residual method, under which, $M < N$. Since there are more equations than unknowns, we consider $M \times S$ linearly independent combinations of the $N \times S$ equations. More specifically let $R(s; \alpha)$ be a vector of length N , namely $R(s; \alpha) = [R(k_1, s; \alpha), \dots, R(k_N, s; \alpha)]'$. Let A be a matrix of dimension $M \times N$, where $A = (a_{ij})$ with $a_{ij} = T_{i-1}(g(k_j))$. The parameters α are chosen to solve

$$(5.6) \quad AR(s, \alpha) = 0, \quad s = 1, \dots, S.$$

We also approximate the capital accumulation decision rule using a standard log-linearization procedure described in, for example, Christiano (1991). The procedure works with a modified version of (5.2) in which $H(\cdot)$ and $G(\cdot)$ are replaced by functions which are linear in the exogenous shocks and in the log of the capital stock. These functions are obtained by computing a suitable Taylor series expansion of $H(\cdot)$ and $G(\cdot)$ about the unconditional means of the exogenous shocks, and the log of the nonstochastic steady-state capital stock. It is then straightforward to solve the modified version of (5.2) exactly for a log-linear function, $k'(\cdot)$.⁵ We used this procedure to obtain starting parameter values for the minimum weighted residual algorithm and to set the bounds, k_1 and k_N , on the capital grid. These bounds are chosen to include the initial stock of capital and the ergodic set generated by the log-linear decision rule.

Either procedure described above gives us decision rules for a fixed value of λ . Next, we need to adjust λ until the implementability constraint (2.2) is satisfied. Our algorithm exploits the

stationarity and the recursive structure of the debt decision rule for $t \geq 1$. Recall from (2.28) that the debt decision rule satisfies

$$(5.7) \quad U_c(k,s)b(k,s) = \sum_{s'} \beta \mu(s'|s) [U_c(k',s')(c(k',s') + b(k',s') + k'(k,s)) + U_l(k',s')\ell(k',s')] - U_c(k,s)k'$$

where $k' = k'(k,s)$. Equation (5.7) defines a linear map from debt allocation rules into themselves. In practice we approximated the debt rule by a piecewise linear function of the capital stock for each value of the state s . We then used (5.7) to define a linear map from the space of the parameters of piecewise linear debt rules into itself. The fixed point of that map is a system of linear equations in the parameters of the debt rule, which is straightforward to solve. (See Chari, Christiano, and Kehoe 1991 for details.)

Next we substitute the resulting bond decision rules together with the other allocations into the consumer budget constraint (1.3) evaluated at date 0. Using the equality between the marginal rate of substitution between consumption and labor and the after-tax wage rate and setting $\theta(s_0)$ to its given value, we calculate a value for $R_b(s_0)b_{-1}$. We iterate on λ until the desired initial value for $R_b(s_0)b_{-1}$ is attained.

We found that the minimum weighted residual and the log-linearization methods gave very similar results for the allocations. For the policies, however, the minimum weighted residual method improved substantially the accuracy of the solutions. The reason is that the policies depend on ratios of the derivatives of the utility function and small errors in computing the allocations can lead to large errors in computing the policies. One way to compare the relative accuracy of the two methods is to check how close the residual function $R(k,s,\alpha)$ is to zero. Recall that ideally the intertemporal euler equation would hold exactly for all values of (k,s) and thus this function would be identically zero. In Figure 1 we plot this function against the capital stocks in the interval (k_1, k_N) for each of

the four states in our baseline model. These are ordered as follows: $s = 1$ corresponds to (g^t, z^t) ; $s = 2$: (g_t, z_h) ; $s = 3$: (g_h, z_t) ; $s = 4$: (g_h, z_h) . We plot the residual functions implied by the minimum weighted residual (MWR) method for $M = 2$ and 3, and by the log-linearization method. (For the MWR computations in Figure 1, $N = 20$.) Though the MWR ($M = 2$) and the log-linearization decision rules are both log linear, they still differ in two respects. First, the MRW rule allows both the constant and slope terms to differ for different values of s , while the log-linearized rule only allows the constant term to shift. Second, each method has a different criterion for choosing the coefficients of the decision rule, with the MRW method being focussed exclusively on the behavior of the residual function. These two differences explain why the MRW ($M = 2$) residual function looks closer to zero in Figure 1 than does the residual function corresponding to the log-linearized decision rule. Note also that increasing M from 2 to 3 moves the residual function closer to zero by a couple of orders of magnitude, so that MWR ($M = 3$) appears in Figure 1 as a straight line at zero. The maximum of the absolute value of the residual function implied by the log-linearized decision rule is 0.02, while it is 0.002 and 0.00002 for the MWR ($M = 2$) and MWR ($M = 3$) methods. All other calculations for the growth model with capital reported in this paper set $M = 10$ and $N = 41$, and the value of the resulting residual function is at most 10^{-10} in absolute value.

Figures 2 through 4 illustrate that the differences in the accuracy of the two methods can have significant effects on the accuracy of the computed policies. In Figure 2 we plot the autocorrelation of the labor tax rate against the autocorrelation of the shocks for two values of the risk-aversion parameter, ψ . Figure 2b shows that when $\psi = 0$, the two methods give quite different values for the autocorrelation of the labor tax rate when government consumption is close to i.i.d. In Figure 3, we plot the standard deviation of the labor tax rate against the risk-aversion parameter for the two methods. We find that the two methods can give quite different answers, especially when the risk-

aversion parameter is around -4 . In Figure 4, we plot the mean and standard deviation of the ex ante capital tax rate and the private assets tax rate against the risk-aversion parameter. Figure 4a shows that the log-linear method performs worse in terms of the mean of the ex ante capital tax rate for high values of risk aversion. Figure 4b illustrates that the log linear approach performs worse in terms of the standard deviation of the ex ante capital tax rate for low values of risk aversion. The case $\psi = 0$ is of particular interest, since in this case we know theoretically that the mean and standard deviation of the ex ante tax rate on capital must be zero. Figure 4 indicates that when $\psi = 0$, then the minimum weighted residual gets these statistics right, while the log-linear method gets them wrong. Figure 5a illustrates that the log linear approach works worse in terms of the mean of the private assets tax rate for high values of risk aversion. Taken together, these figures suggest that the two methods give quite different answers for policies, and that the minimum weighted residual method is preferable for the issues we consider.

6. Conclusions

In this paper, we have analyzed the quantitative properties of policies in business cycle models. We have abstracted from a host of issues. We have considered a representative agent model. It would be interesting to explore the quantitative implications of policies in dynamic models with heterogeneous agents. For example, Auerbach and Kotlikoff (1987) have analyzed the quantitative implications of fiscal policies in overlapping generations models. (See also Escolano 1992 for an analysis of optimal policy in such models.) In our model growth is exogenous. For some interesting work on optimal policies in endogenous growth models see Lucas (1990) and Jones, Manuelli, and Rossi (1991).

Our work and the related work mentioned above takes as given that the government has available only proportional taxes to finance its spending. A different approach stemming from

Mirrlees (1971) derives optimal policies given the restrictions imposed by incentive constraints due to private information. A fruitful area for research is to derive the quantitative implications for optimal fiscal policies in dynamic economies with private information.

Finally, we have assumed there is a technology for commitment by the government. It should be clear that the Ramsey policies in these economies are not time consistent. Perhaps the most promising area for research is to consider economies in which policy decisions are made sequentially and to analyze the quantitative properties of equilibrium policy and allocations. For recent theoretical discussions see Chari and Kehoe (1990, 1992) and Stokey (1990).

Footnotes

Separability between consumption and leisure and homotheticity in consumption are the well-known conditions under which the optimal policy is uniform consumption taxes in all periods except the first. (See Atkinson and Stiglitz 1972 for an analysis in a partial equilibrium setting.) In our model, uniform consumption taxes are equivalent to zero capital income taxes; thus, with $\psi = 0$, the result that capital income taxes are zero in a stochastic steady state is not surprising. More interesting is the result that, even for the high risk aversion model, which is not separable between consumption and leisure, the mean of the capital income tax is close to zero in a stochastic steady state.

We compute the tax on private assets by first constructing a value for total debt. Following Jorgenson and Sullivan (1981), we note that the present value of depreciation allowances is a claim on the government similar to conventional debt. We thus define total debt to be the sum of the market value of federal debt and the value of depreciation allowances. For further details, see Chari, Christiano, and Kehoe (1991b).

These polynomials are recursively defined by $T_0(x) = 1$, $T_1(x) = x$, and $T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x)$ for $i \geq 2$. See Press, Flannery, Teukolsky, and Vetterling (1988) for details.

This grid is computed as follows. Let z_i denote the i^{th} zero of $T_N(z)$, $i = 1, \dots, N$. Then, $\log k_i = g^{-1}(z_i)$, $i = 1, \dots, N$.

There are actually two $k'(\cdot)$ that solve the log-linearized version of (5.2). We chose the solution which implies that for fixed s and any initial k_0 , $\beta^{1/2}k_i \rightarrow 0$, where $k_i = k'(k_{i-1}, s)$, $i \geq 1$.

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Table 1

Parameter Values for the Exogenous Policy Analysis in Example 4

Models	Parameters and Values		
Preferences	$\gamma = .75$		$\beta = .98$
Technology	$\alpha = .34$	$\delta = .08$	$\rho = .016$
Initial Conditions	$b_{-1} = .2$	$k_{-1} = 1$	
Government Consumption	$g = .07$		
Markov Chains for			
Technology Shock	$z_l = -.04$	$z_h = .04$	$\pi = .91$
Baseline Policy	$\tau = .24$	$\theta = .27$	

Table 2

Countercyclical Policy Exercise in Example 4

	Naive	Budget Balance
Taxes		
τ_H	.24	.27
τ_L	.18	.21
Welfare relative to baseline policy	.65	-.1
Standard deviation of output relative to baseline policy	.31	.27
Standard deviation of employment relative to baseline policy	1.53	1.64

Notes: The welfare measure is the percentage of increase in baseline consumption levels which gives the same utility as the policy exercise.

Table 3

Parameter Values for the Real Models

Models	Parameters and Values			
<u>Baseline Model</u>				
Preferences	$\gamma = .80$	$\psi = 0$	$\beta = .97$	$L = 5,475$
Technology	$\alpha = .34$	$\delta = .08$	$\rho = .016$	
Markov Chains for				
Government Consumption	$g_t = 350$	$g_h = 402$	$\phi = 0.95$	
Technology Shock	$z_t = -0.04$	$z_h = 0.04$	$\pi = 0.91$	
<u>High Risk Aversion Model</u>				
Preferences	$\psi = -8$			

Source: Chari, Christiano, and Kehoe 1990b

Table 4

Properties of the Real Models and the U.S. Economy

Tax Rates	Models		U.S. Economy
	Baseline	High Risk Aversion	
<u>Labor</u>			
Mean	28.77	25.55	24.76
Standard Deviation	.10	.09	2.39
Autocorrelation	.83	.90	.77
<u>Capital</u>			
Mean	.00	-.11	28.28
Standard Deviation	.00	2.64	8.75
Autocorrelation	—	.83	.74
<u>Private Assets</u>			
Mean	1.46	-1.27	.00
Standard Deviation	84.94	70.29	.73
Autocorrelation	-.01	.00	-.32

Notes: All statistics are based on 400 simulated observations. The means and standard deviations are in percentage terms. For the U.S. economy, the labor tax rate is measured by the average marginal tax rate of Barro and Sahasakul (1983), the capital tax rate is measured by the effective corporate tax rate of Jorgenson and Sullivan (1981), and the tax on private assets is constructed as described by Chari, Christiano, and Kehoe (1990b). For the baseline model, the capital tax rate is zero; thus, its autocorrelation is not defined.

Table 5

Properties of the Monetary Models

	Models		
	Baseline	High Risk Aversion	I.I.D.
<u>Labor Tax</u>			
Mean	20.05	20.18	20.05
Standard Deviation	.11	.06	.11
Autocorrelation	.89	.89	.00
Correlation With			
Government Consumption	.93	-.93	.93
Technology Shock	-.36	.35	-.36
Output	.03	-.06	.02
<u>Inflation</u>			
Mean	-.44	4.78	-2.39
Standard Deviation	19.93	60.37	9.83
Autocorrelation	.02	.06	-.41
Correlation With			
Government Consumption	.37	.26	.43
Technology Shock	-.21	-.21	-.70
Output	-.05	-.08	-.48
<u>Money Growth</u>			
Mean	-.70	4.03	-2.78
Standard Deviation	18.00	54.43	3.74
Autocorrelation	.04	.07	.00
Correlation With			
Government Consumption	.40	.28	.92
Technology Shock	-.17	-.20	-.36
Output	.00	-.07	.02

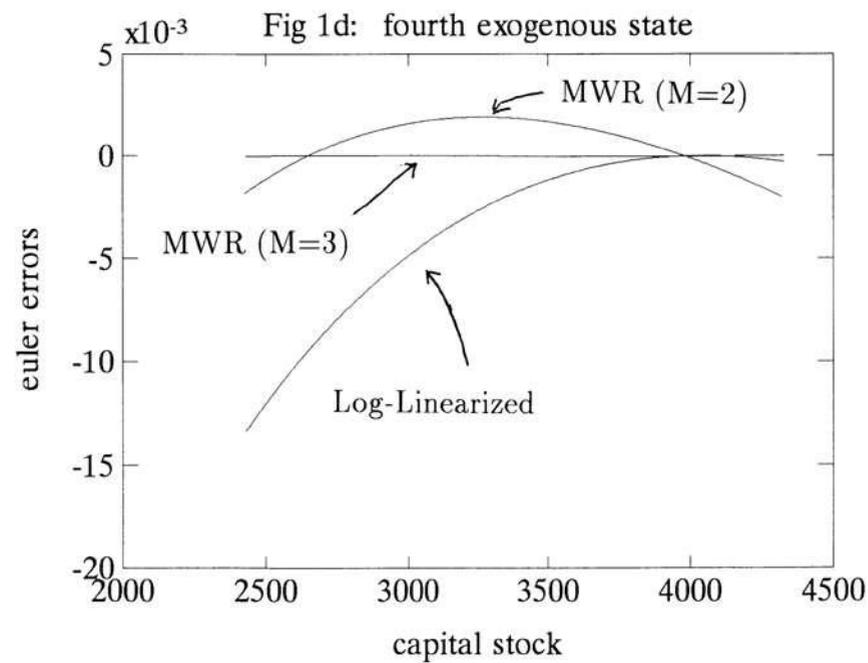
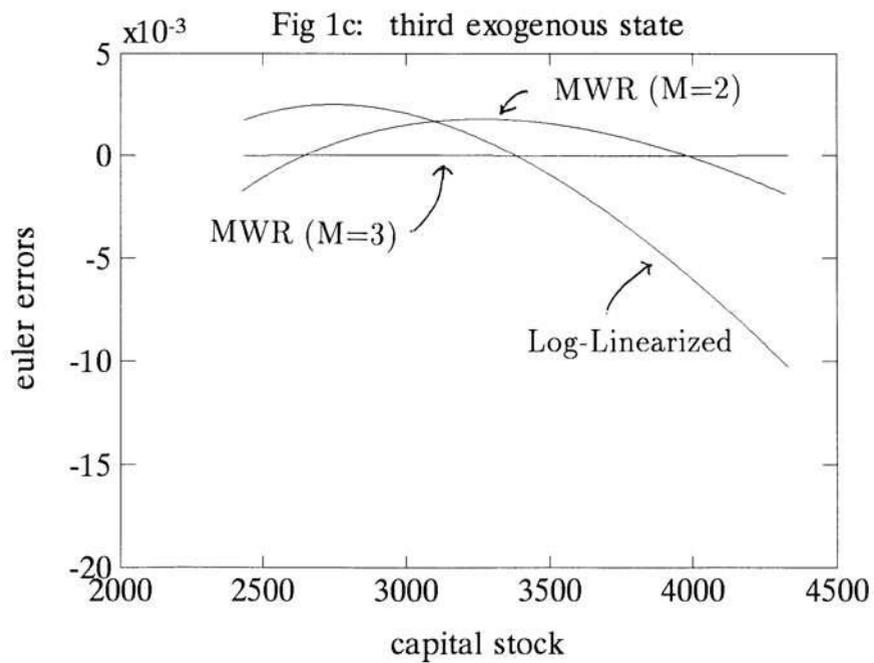
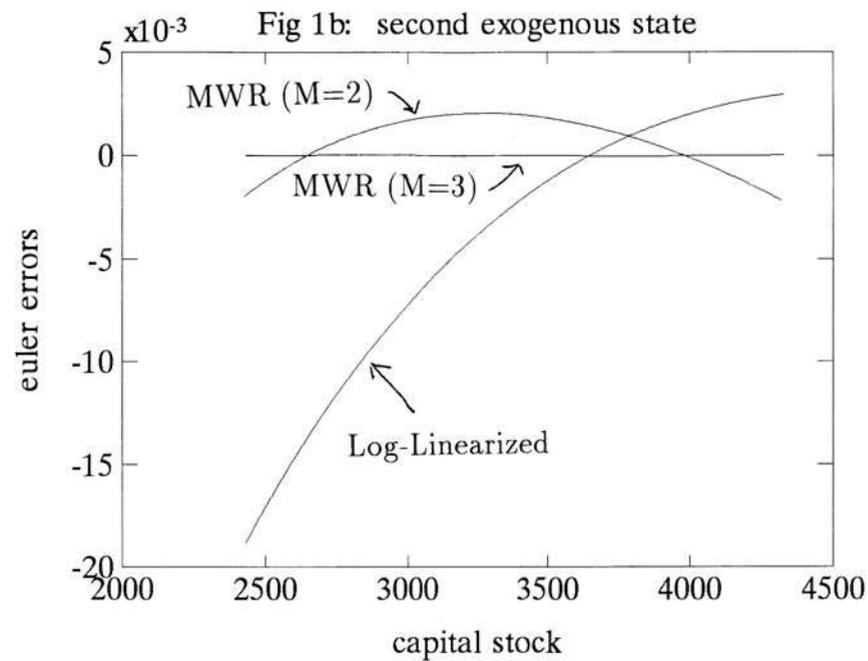
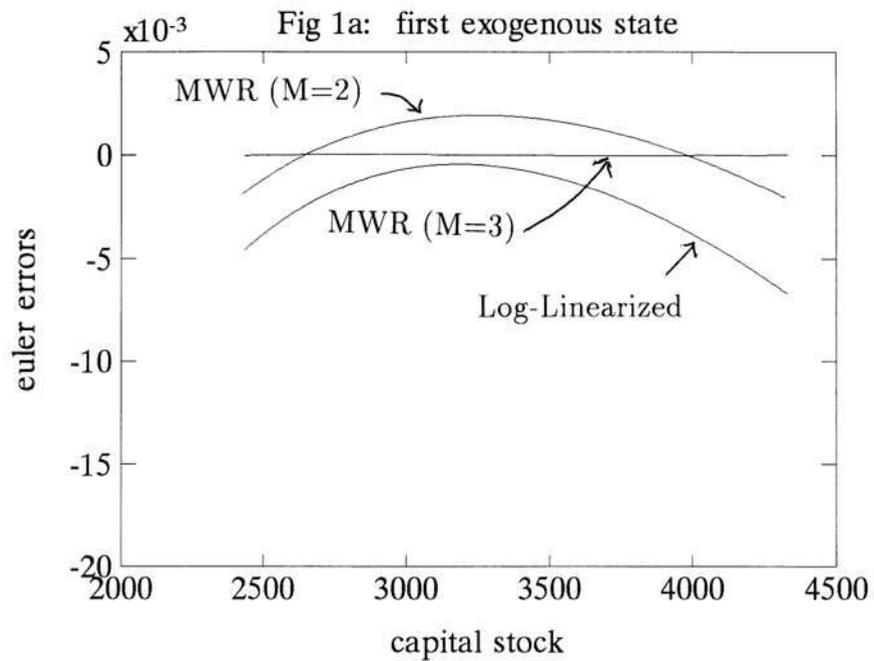


Figure 2a: persistence of labor tax

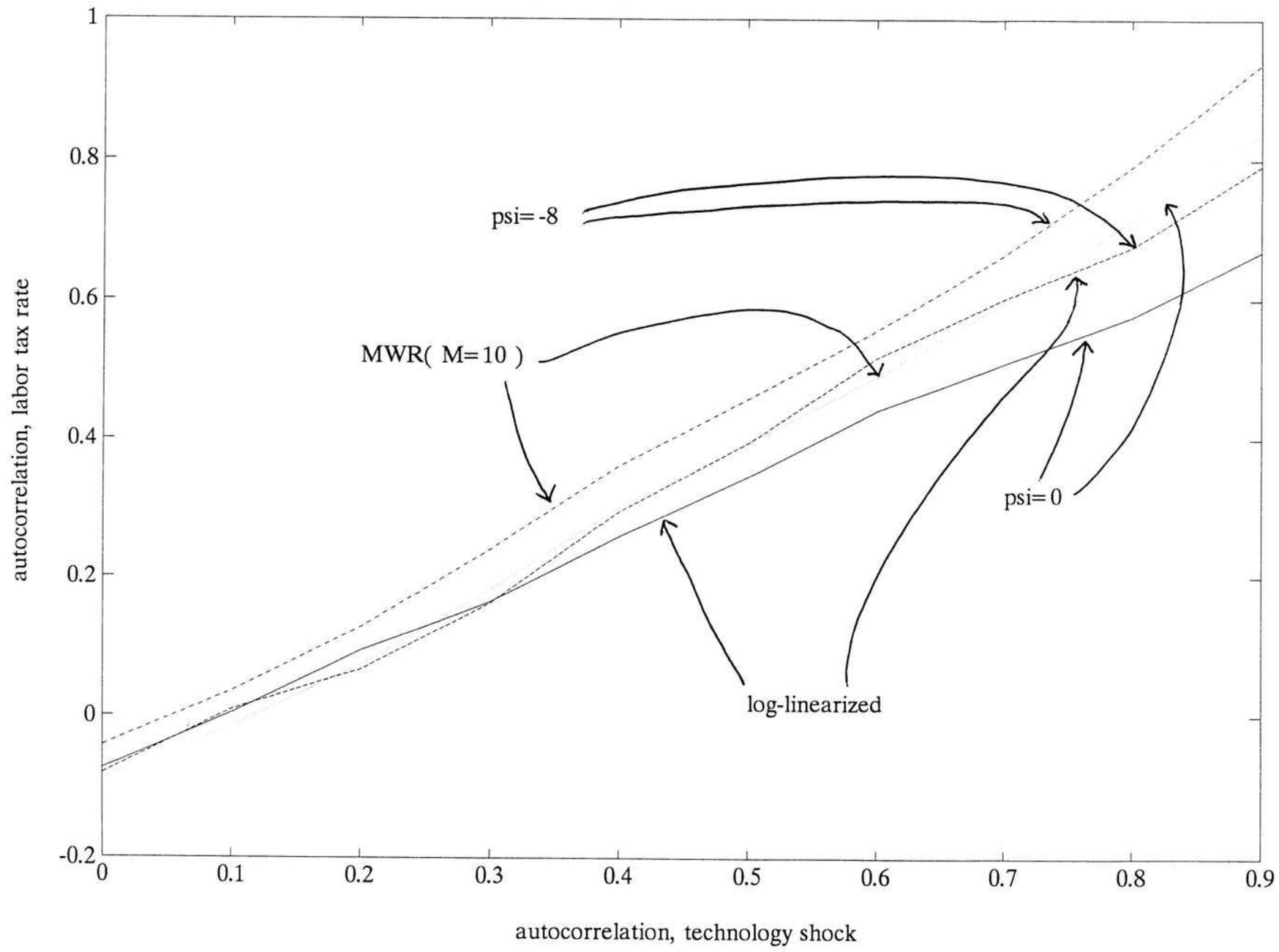


Figure 2b: persistence of labor tax

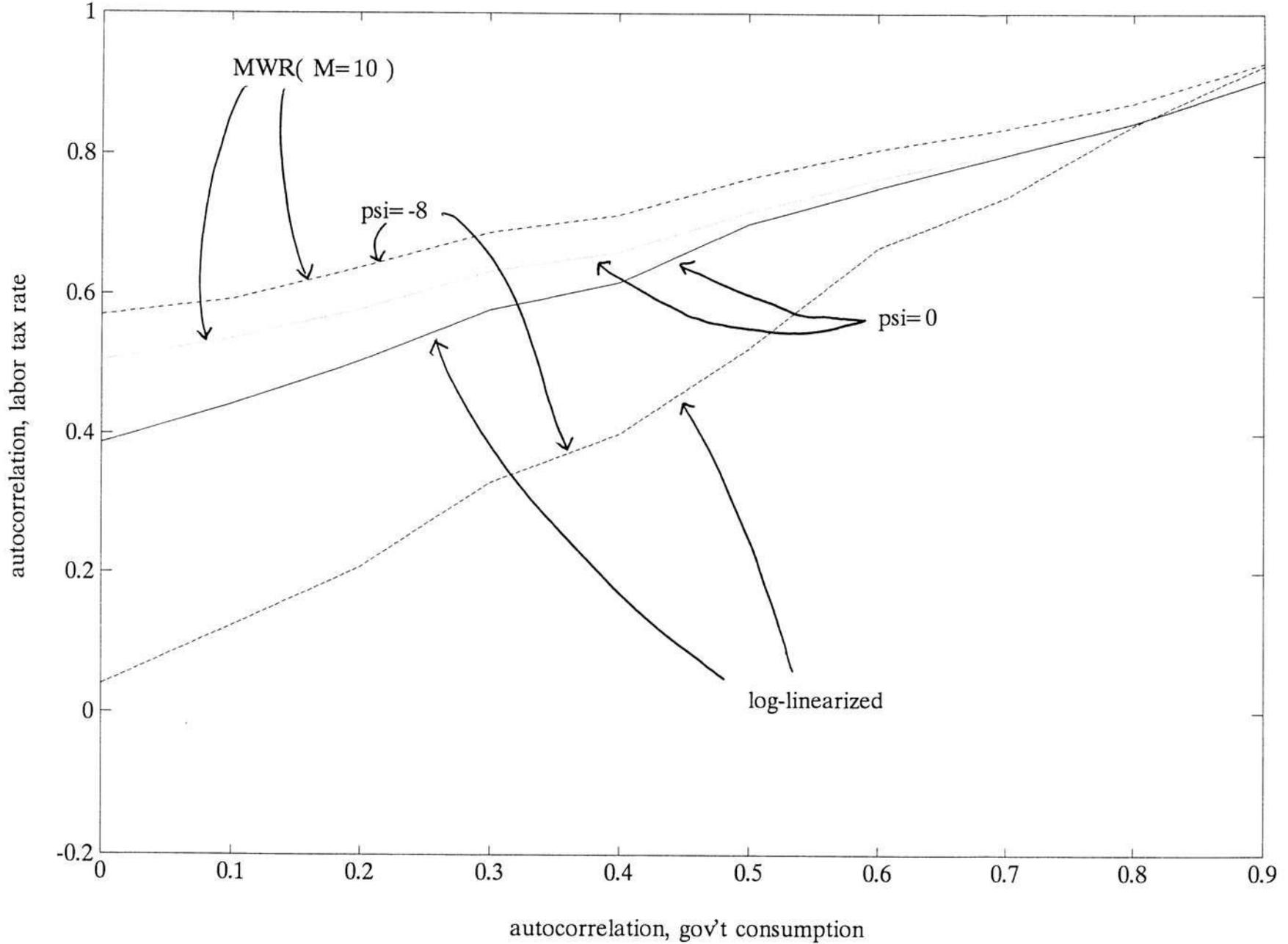


Figure 3: standard deviation of labor tax

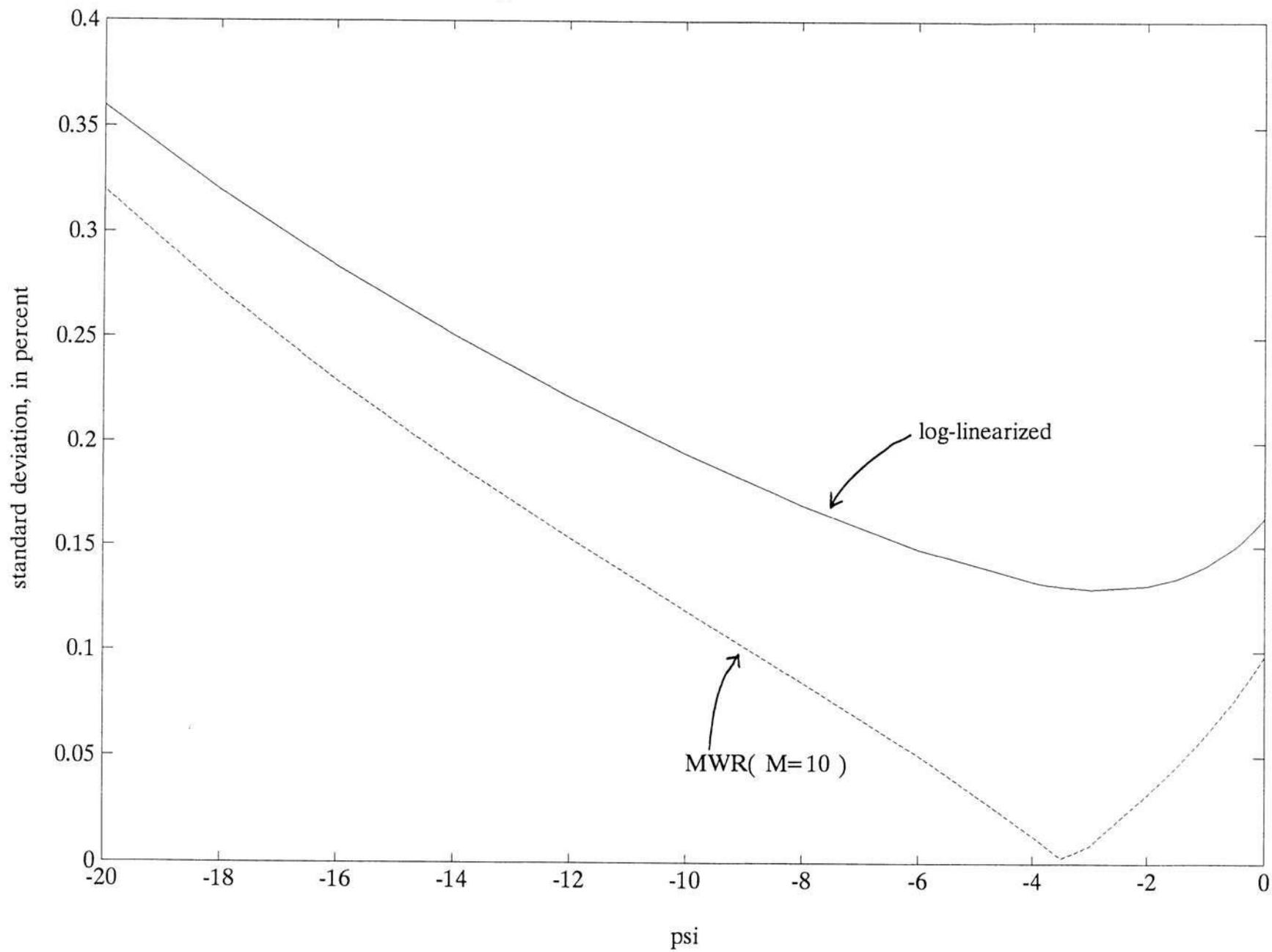
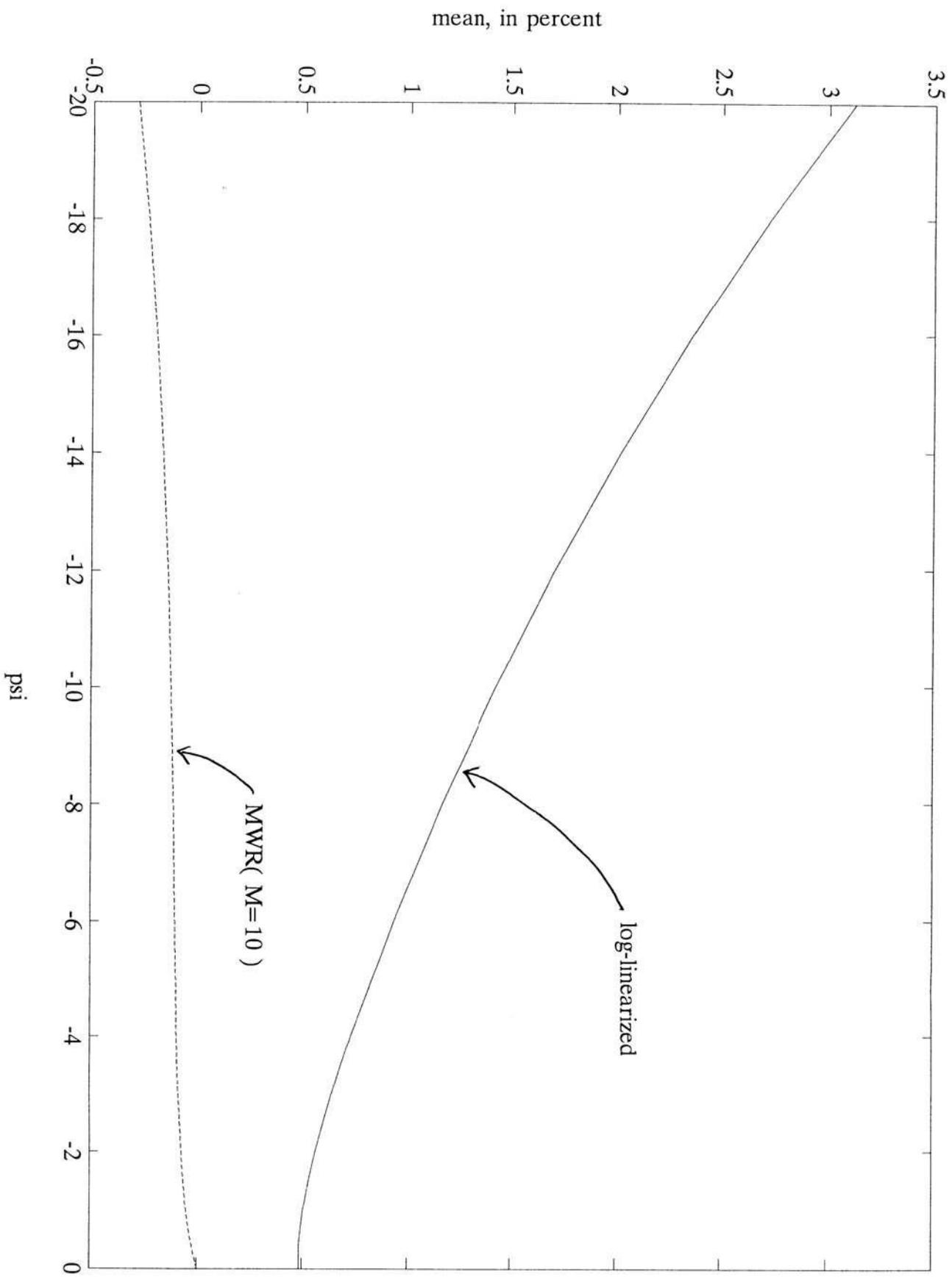


Figure 4a: ex ante capital tax



standard deviation, in percent

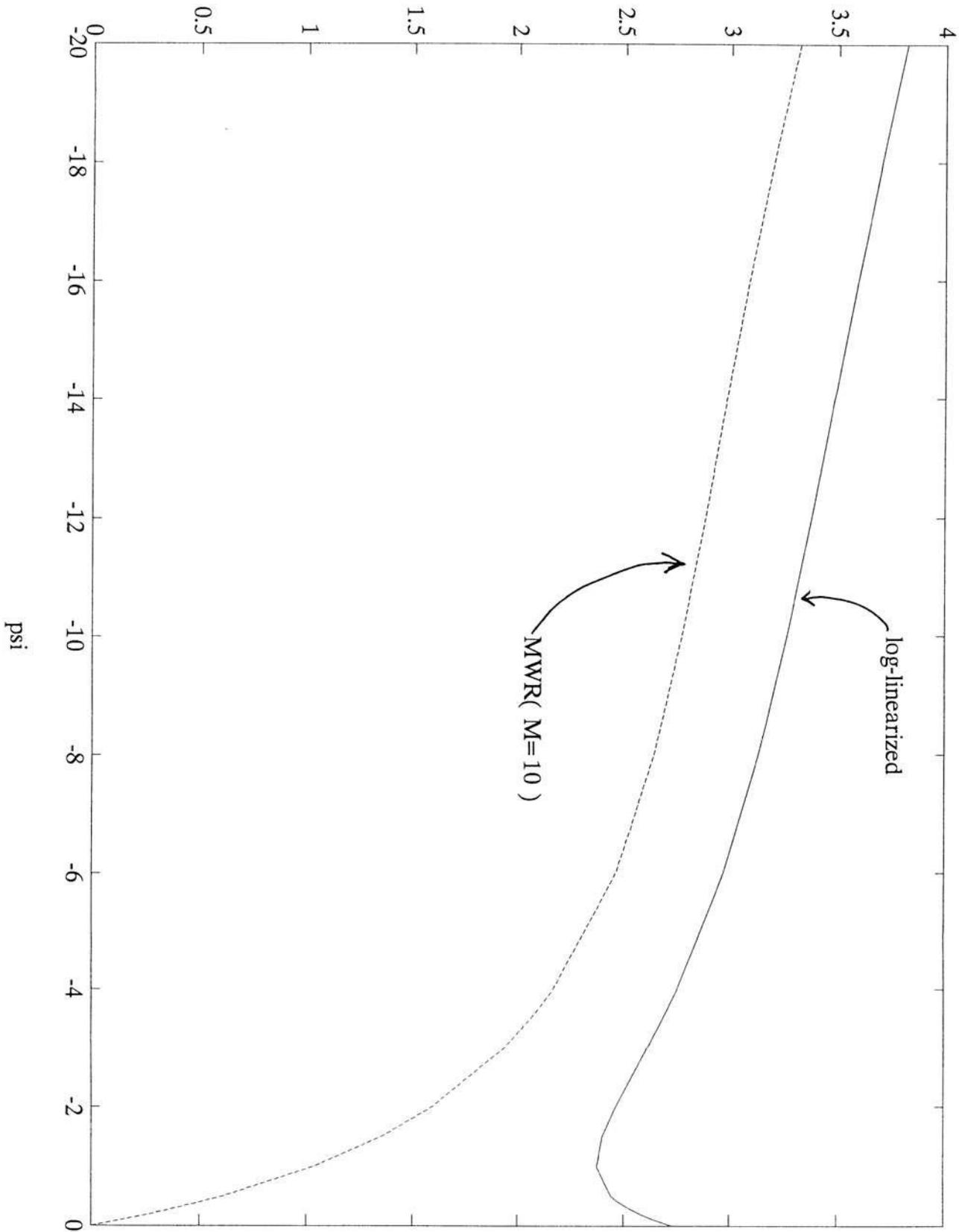


Figure 4b: ex ante capital tax

Figure 5a: private assets tax rate

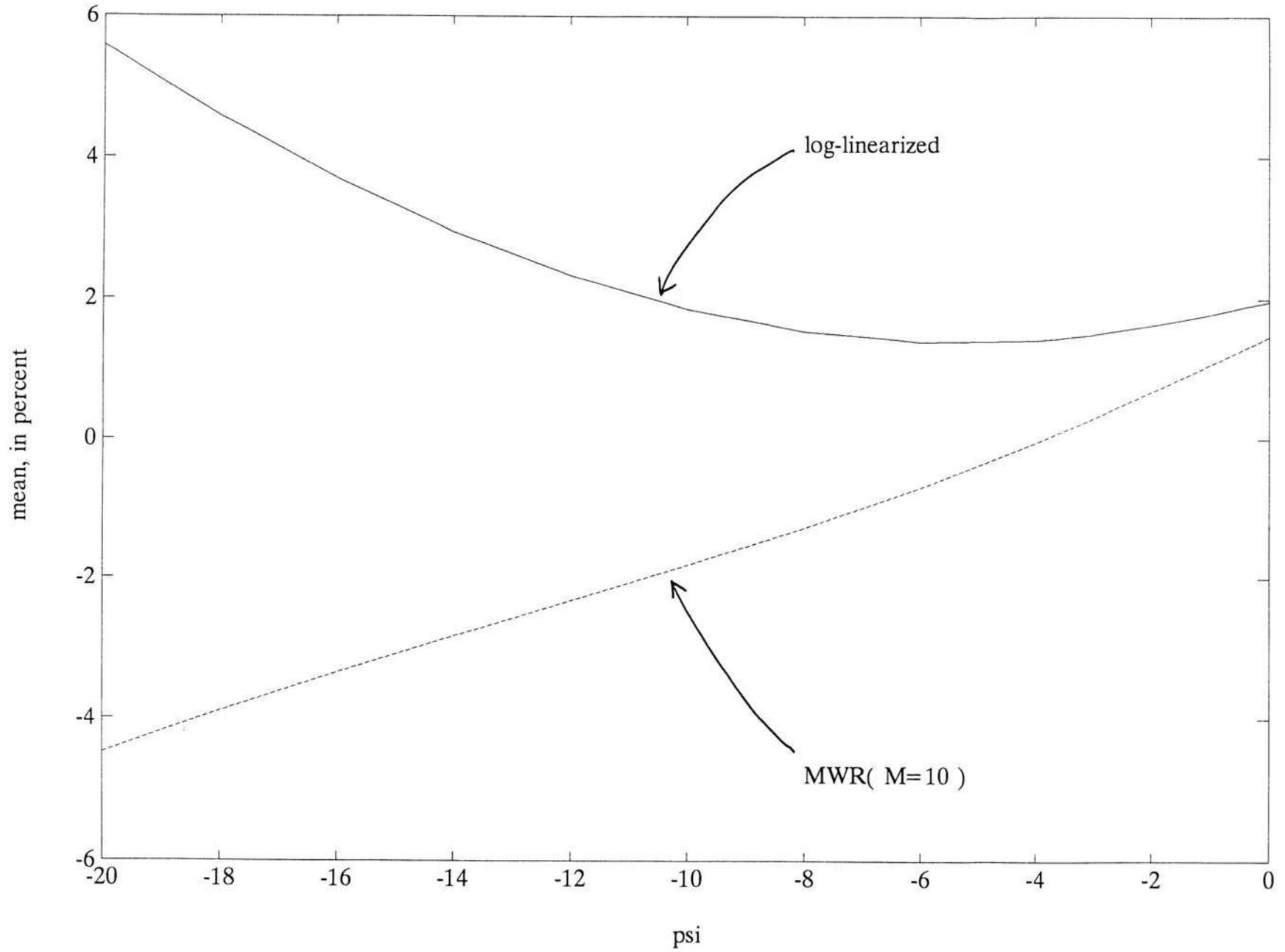


Figure 5b: private assets tax rate

