"Rational" Expectations, the Optimal Monetary Instrument and the Optimal Money Supply Rule

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This paper analyzes the effects of alternative ways of conducting monetary policy within the confines of an ad hoc macroeconomic model. By ad hoc we mean that the model is not derived from a consistent set of assumptions about individuals' and firms' objective functions and the information available to them. Despite this deplorable feature of the model, it closely resembles the macroeconometric models currently in use, which is our excuse for studying it. Following Poole [5], we compare two alternative strategies available to the monetary authority. One is to peg the interest rate period by period, letting the supply of money be whatever it must be to satisfy the demand for it. The other is to set the money supply period by period, accepting whatever interest rate equilibrates the system. We study the effects of such policies for two versions of the model: an autoregressive version in which the public's expectations are assumed formed via fixed autoregressive schemes on the variables being forecast, and a rational expectations version in which the public's expectations are assumed equal to objective (mathematical) expectations that depend upon, among other things, what is known about the rules governing monetary and fiscal policy (see Muth [3]).

The two versions have radically different policy implications. In the rational expectations version, (a) the probability distribution of output is independent of the deterministic money supply rule in effect, (b) if the loss function is the expected value of a discounted sum of a quadratic function of output and the price level, the optimal deterministic money supply rule is that which equates the expected value of next period's price level to the target value, and (c) a unique equilibrium price level does not exist if the monetary authority pegs
the interest rate period by period, regardless of how its value varies from period to period. None of these results emerges from the autoregressive version. It, instead, exhibits all the usual exploitable tradeoffs between output and inflation, implies that minimization of the above loss function is a well-defined nontrivial dynamic problem giving rise to a unique optimal deterministic feedback rule either for the money supply or the interest rate, and has a unique period by period equilibrium if the interest rate is pegged. Thus, in the autoregressive version of the model, which in principle is merely a variant of Poole's model, whether an interest rate feedback rule or a money supply feedback rule is superior depends, just as Poole asserted, on most of the parameters of the model including the covariance matrix of the disturbances.

1. The Ad Hoc Model

We assume a structure described by the following four equations:

(1) Aggregate Supply Schedule

\[ y_t = a_1 k_{t-1} + a_2 (p_t - p_{t-1}^*) + u_{1t} ; \quad a_1 > 0, a_2 > 0 \]

(2) Aggregate Demand Schedule or "IS" Curve:

\[ y_t = b_1 k_{t-1} + b_2 [r_t - (p_t - p_{t-1}^*)] + b_3 Z_t + u_{2t} ; \quad b_1 > 0, b_2 < 0 \]

(3) Portfolio Balance or "LM" Schedule:

\[ m_t = p_t + c_1 y_t + c_2 r_t + u_{3t} ; \quad c_1 > 0, c_2 < 0 \]

(4) Determination of Productive Capacity:

\[ k_t = d_1 k_{t-1} + d_2 [r_t - (p_t - p_{t-1}^*)] + d_3 Z_t + u_{4t} ; \quad d_2 < 0 \]
(5) Evolution of the Exogenous Variables:

\[ Z_t = \sum_{j=1}^{q} \rho_j Z_{t-j} + \xi_t \]

\[ u_{it} = \sum_{j=1}^{q} \rho_{ij} u_{i,t-j} + \xi_{i,t} \]

Here \( y_t, p_t, \) and \( m_t \) are the natural logarithms of output, the price level, and the money supply, respectively; \( r_t \) is the nominal rate of interest itself (not its logarithm) while \( Z_t \) is vector of exogenous variables. The variable \( p_{t+1}^* \) is the public's psychological expectation of the log of the price level to prevail at \( t+1 \), the expectation being held as of the end of period \( t-j \). The variable \( k_{t-1} \) is a measure of productive capacity, such as the logarithm of the stock of capital or labor or some linear combination of the logarithms of those stocks at the end of period \( t-1 \).

Equation (1) is an aggregate supply schedule relating output directly to productive capacity and the gap between the current price level and the public’s prior expectation of the current price level. Unexpected increases in the price level thus boost aggregate supply, the reason being that suppliers of labor and goods mistakenly interpret surprise increases in the aggregate price level as increases in the relative prices of the labor and goods they are supplying. This happens because suppliers receive information about the prices of their own goods faster than they receive information about the aggregate price level. This is the kind of aggregate supply schedule that Robert E. Lucas [2] has used to explain the inverse correlation between observed inflation and unemployment depicted by the Phillips curve.
Equation (2) is an aggregate demand or "IS" schedule showing
the dependence of aggregate demand on the real rate of interest and
capacity, a measure of wealth. The real rate of interest equals the
nominal rate \( r_t \) minus the rate of inflation between \( t \) and \( t+1 \) expected
by the public as of the end of period \( t-1 \), namely \( \pi^*_t = \pi^*_{t-1} - \pi_{t-1} \).
The rate \( r_t \) is assumed to be the yield to maturity on a one-period bond.
Aggregate demand also depends on a vector of exogenous variables \( Z_t \)
which includes government expenditures and tax rates.

Equation (3) summarizes the condition for portfolio balance. Owners of bonds and equities (which are assumed to be viewed as perfect
substitutes for one another) are satisfied with the division of their
portfolios between money, on the one hand, and bonds and equities, on
the other hand, when equation (3) is satisfied. Equation (3) posits
that the demand for real balances depends directly on real income and
inversely on the nominal rate of interest.

Equation (4) determines productive capacity for the next
period, while equation (5) describes autoregressive processes for the
exogenous variables. The \( \xi \)'s, which are sometimes called the "innovations" in the \( Z \) and \( u \) processes, are serially uncorrelated random
variables.

To complete the model, we need equations describing \( p^*_{t+1} \)
and \( p^*_t \). Adding those equations to (1) - (5) then results in a
system capable of determining the evolution over time of \( y_t \), \( p_t \), \( r_t \),
\( t+1p_{t-1}^* \), and \( t^*p_{t-1} \) and \( k_t \).

2. The Stabilization Policy Problem

The monetary authority's problem is to set \( r \) or \( m \) each period
in such a way as to minimize the quadratic loss function.
where \( K \) is diagonal with elements \( K_{ii} > 0, i = 1, 2 \). The operator \( E_{t-j} \) is the mathematical expectation operator, conditioned on data available as of the end of period \( t-j \). To minimize \( L \), the monetary authority compares two strategies. The first is to peg \( r_t \) via a deterministic linear feedback rule

\[
(6) \quad r_t = G \theta^*_{t-1}
\]

where \( \theta^*_t \) represents the set of current and past values of all of the endogenous and exogenous variables in the system as of the end of period \( t \); \( G \) is a vector of parameters conformable to \( \theta^*_{t-1} \). The monetary authority chooses the parameters in \( G \) to minimize \( L \). It must then compare the minimum loss associated with an interest rate rule having those \( G \)'s with the loss associated with the best money supply feedback rule of the form

\[
(7) \quad m_t = H \theta^*_{t-1}.
\]

Whichever rule delivers the lower loss is the one that should be used.

3. The Autoregressive Expectations Version

Here we assume that the psychological expectations \( p^*_t \) and \( p^*_{t+1} \) are governed by the distributed-lag or "adaptive" schemes

\[
(8) \quad p^*_{t+1} = \sum_{i=0}^{q} v_{1i} p_{t-i}
\]

\[
(9) \quad p^*_{t+2} = \sum_{i=0}^{q} v_{2i} p_{t-i}
\]
where the $v_{11}'s$ and $v_{21}'s$ are fixed numbers. Given that the money supply is used as the monetary instrument, the system formed by equations (1)-(5), (8) and (9) can be reduced to a difference equation of the form

\[ Y_{1t} = \sum_{i=0}^{q'} A_i^t Y_{1t-i} + \sum_{i=0}^{q'} B_i^t m_{t-i} + \phi_{1t} \]

where $Y_{1t} = (y_t, p_t, r_t, k_{t-1}, z_t)$ and $\phi_{1t}$ is a vector of serially uncorrelated random variables, the components of which are functions of the $\xi_t's$ in equations (5). The $A_i^t's$ are vectors conformable with $Y_{1t}$ and the $B_i^t's$ are scalars; both the $A_i^t's$ and $B_i^t's$ depend on the parameters of equations (1)-(5), (8) and (9). To find the best money-supply feedback rule, the monetary authority chooses the parameters $H$ of the rule (7) to minimize the loss $L$ subject to (10). Where loss is quadratic and the model is linear with known coefficients, rules of the linear form of (7) are known to be optimal.

To find the optimal interest rate rule, the system formed by equations (1)-(5), (8) and (9) is written as

\[ Y_{2t} = \sum_{i=0}^{q'} C_i^t Y_{2t-i} + \sum_{i=0}^{q'} D_i^t r_{t-i} + \phi_{2t} \]

where $Y_{2t} = (y_t, p_t, m_t, k_{t-1}, z_t)$. The optimal interest rate rule is the one with the $C_i^t's$ of (6) chosen so as to minimize loss $L$ subject to (11).

To show that (1)-(5), (8) and (9) yield versions of (10) and (11) that give rise to well defined, nontrivial dynamic problems, it is enough to examine the one-period reduced forms for $y_t$ and $p_t$. With the money supply as the monetary instrument we solve (1)-(3) for $y$, $r$, and $p$ and get as a reduced form for $p_t$.
where $X_t$ is a linear function (involving the parameters of (1)-(3)) of $k_{t-1}$, $Z_t$, and the $u_t$'s and where

$$J_0 = \frac{a_2(1+b_2c_2^{-1})+b_2}{a_2(1+b_2c_2^{-1})+b_2c_2^{-1}} < 1,$$

$$J_1 = \frac{1-J_0}{1-c_2^{-1}},$$

$$J_2 = -c_2^{-1}J_1.$$

Substitution of $p_t$ from equation (12) into equation (1) gives the one-period reduced form for $y_t$. Taking $E_{t-1}$ of $p_t$ and $y_t$ from these reduced forms and eliminating $m_t$ gives the set of pairs $(E_{t-1}y_t, E_{t-1}p_t)$ attainable by choice of $m_t$. The set is a line whose slope is neither infinity nor zero. Its position, obviously, depends on lagged values of $p$, via the $p^*$ variables, and on lagged values of other endogenous variables, the distributions of which depend on lagged values of $m$. In other words, the choice for the deterministic part of $m_t$ has effects in future periods, which is what we mean when we say that (10) gives rise to a nontrivial dynamic problem.

With the interest rate as the monetary instrument, equation (2) is the one-period reduced form for $y_t$ while that for $p_t$ is obtained by substituting the solution for $y_t$ into equation (1) and solving for $p_t$. The solution for $p_t$ is

$$a_2p_t = (a_2+b_2)[p^*_{t-1} - b_2(1+p^*_{t-1}) + b_2r_t + (b_1-a_1)k_{t-1} + b_3Z_t - u_{1t} + u_{2t}].$$

Again if we take $E_{t-1}$ of equation (2) and equation (13) and eliminate $r_t$ we find the set of pairs $(E_{t-1}y_t, E_{t-1}p_t)$ attainable by choice of $r_t$. 

(12) \[ p_t = J_0(p_{t-1}^*) + J_1(p_{t+1}^*) + J_2m_t + X_t \]
That set again depends on lagged values of $p$ which shows that (11) also gives rise to a nontrivial dynamic problem.

The monetary authority is supposed to solve each of the two dynamic problems, minimizing loss first under an $m$-rule, and then under an $r$-rule. Which policy is superior depends on which delivers the smaller loss, which in turn depends on all of the parameters of the model including the covariance matrix of the disturbances. Which rule is superior is therefore an empirical matter, an outcome which is completely consistent with Poole's analysis.

4. The Rational Expectations Version Under a Money Supply Rule

Here we impose the requirement that the public's expectations be rational by requiring that

$$\text{(14)} \quad p_{t+j}^* = E_{t-j} p_{t+i}$$

where $E_{t-j} p_{t+i}$ is the mathematical expectation of $p_{t+i}$ calculated using the model (i.e., the probability distribution of $p_{t+i}$) and all information assumed to be available as of the end of period $t-j$. The available information is assumed to consist of data on current and past values of all endogenous and exogenous variables observed as of the end of period $t-j$, i.e., $\theta_{t-j}$.

To begin, we again solve the system (1)-(3) for $y$, $r$, and $p$ given $m$. With expectations given by (14), what is now a pseudo reduced form equation for $p$ is

$$\text{(15)} \quad p_t = J_0 E_{t-1} p_t + J_1 E_{t-1} p_{t+1} + J_2 m_t + X_t$$

Computing $E_{t-1} p_t$ from (15) and subtracting the result from (15) we get
where the last equality follows from the assumption that a deterministic rule of the form (7) is being followed. But since \(X_t-E_{t-1}X_t\) is a linear combination of the innovations in the exogenous processes, it follows that \(P_t-E_{t-1}P_t\) is an exogenous process, unaffected by the rule chosen for determining the money supply.

Using (14) and (16) we can write equation (1) as

\[
y_t = a_1 a_{k_t} + a_2 [X_t - E_{t-1}X_t] u_{lt}
\]

If we substitute the RHS for \(y_t\) in equation (2), we can obtain the real interest rate as a function of \(k_{t-1}\) and exogenous processes. Substituting that function into equation (4), we get a difference equation in \(k\) driven by exogenous processes. This proves that \(k\) is an exogenous process, which by (17) implies that \(y\) is an exogenous process, i.e., has a distribution independent of the deterministic rule for the money supply. So we have proved assertion (a) above: the distribution of output does not depend on the parameters of the feedback rule for the money supply.

To prove assertion (b) we write the \(t^{th}\) term of the loss function \(L\) as

\[
L_t = E_t\{E_t E_{t-1}\left[K_0 P_t + K_2 P_t^2 + K_1 y_t + K_1^2 y_t^2\right]\}
\]

where the insertion of \(E_{t-1}\) is valid for \(t > 0\). Using \(E(x^2) = E[(x-Ex)^2]\) and \((E)^2\) we have

\[
L_t = E_t\{K_{0t} + K_2 E_{t-1}P_t + K_{22}(E_{t-1}P_t)^2\}
\]

where

\[
K_{0t} = E_t\{K_2 (P_t - E_{t-1}P_t)^2 + K_1 y_t + K_1^2 y_t^2\}
\]

and where, given the exogeneity of \(y_t\) and \(P_t - E_{t-1}P_t\) proved above, \(K_0\) is
an exogenous process. Moreover, it is possible, as we shall show below, to find a rule for \( m \) that implies choosing \( E_{t-1}P_t \) to minimize

\[
K_0 + 2E_{t-1}P_t + K_2(E_{t-1}P_t)^2.
\]

And because \( K_0 \) is unaffected by settings for the money supply at any time, a rule which minimizes \( L_t \) also minimizes \( L \).

To show that there exists such a rule for \( m \) we must solve the model. Again, we take \( E_{t-1}P_t \) in (15), and write the result as

\[
(18) \quad (1-J_0)E_{t-1}P_t = J_1E_{t-1}P_{t+1} + J_2E_{t-1}m_t + E_{t-1}X_t
\]

Since this holds for all \( t \) it follows that

\[
(19) \quad (1-J_0)E_{t-1}P_{t+j} = J_1E_{t-1}P_{t+j+1} + J_2E_{t-1}m_{t+j} + E_{t-1}X_{t+j}
\]

By repeated substitution from (19) into (18) we obtain

\[
(20) \quad (1-J_0)E_{t-1}P_t = \sum_{j=0}^{n} [J_1/(1-J_0)]^j (E_{t-1}X_{t+j} + J_2E_{t-1}m_{t+j})
\]

\[
+ [J_1/(1-J_0)]^{n+1}E_tX_{t+n+1}
\]

where

\[
0 < J_1/(1-J_0) = 1/(1-c_2^{-1}) < 1
\]

We assume that the limit as \( n \to \infty \) of the second term on the RHS of (20) is zero, which is a terminal condition that has the effect of ruling out "speculative bubbles." Then from (20),

\[
(21) \quad (1-J_0)E_{t-1}P_t = \sum_{j=0}^{\infty} [J_1/(1-J_0)]^j E_{t-1}(X_{t+j} + J_2m_{t+j})
\]

Since this holds for all \( t \), we may replace \( t \) by \( t+1 \) and compute \( E_{t-1} \) of the result to get
For an arbitrary money supply rule of the form (7), substituting (21) and (22) into (15) gives the solution for $p_t$; substituting (21) and (22) into (2) gives the solution for $r_t$. This assumes that the rule is not such as to imply too explosive a process for $X_{t+j} + J_m t+j$. To find the optimal money supply rule, multiply (22) by $J/(1-J_o)$ and subtract the result from (21) to get:

\begin{equation}
(1-J_o)E_{t-1}p_{t+1} = \sum_{j=0}^{\infty} \frac{J^j}{(1-J_o)^j} E_{t-1}(X_{t+j+1} + J_m t+j+1)
\end{equation}

The value of $E_{t-1}p_t$ that minimizes $L_t$ for all $t$ is:

\begin{equation}
E_{t-1}p_t = -K_2/2K_{22}
\end{equation}

so that

\begin{equation}
E_{t-1}p_{t+1} = -K_2/2K_{22}
\end{equation}

The optimal rule for the money supply is obtained by substituting (24) and (25) into (23). The resulting expression for $m_t$ is a feedback rule of the form (7).

5. The Rational Expectations Version Under an Interest Rate Rule

Above we showed that a certain terminal condition implied the existence of a unique equilibrium price level for the rational expectations version under a money supply rule that is not too explosive. That analysis took as a starting point the difference equation (18). With the interest rate determined by the feedback rule (6), a seemingly analogous difference equation is obtained by imposing rationality, equation (14), in (13) and taking $E_{t-1}$ of the result.
If we solve (26) by recursion, proceeding as we did in deriving (20) from (18), we find:

\[
E_{t-1}P_t = -\sum_{j=0}^{n} E_{t-1}[r_{t+j} + (b_1-a_1)/b_2]k_{t+j-1} + (b_3/b_2)(Z_{t+j} - u_{t+j}u_{t+j})]
\]

To obtain a particular solution for \(E_{t-1}P_t\) from (27) requires imposing a terminal condition in the form of taking as exogenous a value of \(E_{t-1}P_{t+j}\) for some \(j \geq 0\). This is obviously a very much stronger terminal condition than we had to impose on (20), a consequence of (26) being a nonconvergent difference equation. Thus, when the interest rate is pegged, the model cannot determine a path of expected prices \(E_{t-1}P_{t+j}\), \(j=0, 1, \ldots\), and by implication cannot determine the price level \(p_t\). Neither can it determine the money supply.

The economics behind the underdetermined expected price level is pretty obvious. Under the interest rate rule (6), the public correctly expects that the monetary authority will accommodate whatever quantity of money is demanded at the pegged interest rate. The public therefore expects that, ceteris paribus, any increase in \(p_t\) will be met by an equal increase in \(m_t\). But that means that one \(E_{t-1}P_t\) is as good as any other from the point of view of being rational. There is nothing to anchor the expected price level. And this is not simply a matter of choosing the "wrong" level or rule for the interest rate. There is no interest rate rule that is associated with a determinate price level.

At least since the time of Wicksell, it has been known that in the context of a static analysis of a full employment model with wages
and prices that are flexible instantaneously, it can happen that the price level is indeterminate if the monetary authority pegs the interest rate.\(^5\) In such a static analysis, the indeterminacy of the price level depends critically on output and employment being exogenous with respect to shocks to aggregate demand or portfolio balance, i.e., the Phillips curve must be vertical. In our model, however, the Phillips curve is not vertical, but Wicksell's indeterminacy still arises.

6. An Information Advantage for the Monetary Authority

Here we shall examine some consequences of the monetary authority having more information than the public. We shall first show that if the monetary authority follows the money supply rule that is optimal if there is no information discrepancy, then the loss attained is the same as attained when there is no information discrepancy. Then we consider whether that rule is optimal given an information discrepancy.

We shall write \(E_{t-1}\) for the expectation conditional on what the monetary authority knows at the end of period \(t-1\) and \(E_{\theta,t-1}\) for the expectation conditional on what the public knows at the end of period \(t-1\), where \(\theta\) is a subset of what the monetary authority knows. Then in place of (14), we impose

\[
\text{(28)} \quad p^*_t = E_{\theta,t-1} p_{t+1}
\]

so that in place of (15), we have

\[
\text{(29)} \quad p_t = J_0 E_{\theta,t-1} p_{t-1} + J_1 E_{\theta,t-1} p_{t+1} + J_2 m + X
\]

Then taking \(E_{\theta,t-1}\) of \(p_t\) and subtracting from \(p_t\), we have
The rule that we found to be optimal without an information discrepancy is

\[ J_2 m_t = -(K_2 / 2K_{22})(1-J_o - J_1) - E_{t-1}X_t \]

From this it follows that

\[ J_2 (m_t - E_{t-1}t_m_t) = -E_{t-1}X_t + E_{t-1}X_t \]

Substituting into (30) we have

\[ p_t - E_{\theta, t-1}p_t = J_2 (m_t - E_{\theta, t-1}m_t) + (X_t - E_{\theta, t-1}X_t) \]

Upon substituting from (33) into equation (1) of the structure we get equation (17). And if we substitute for \( y_t \) in equation (2) the RHS of (17) we obtain \([r_t - E_{\theta, t-1}(p_{t+1} - p_t)]\) as a function of \( k_{t-1} \) and the exogenous processes, the same function that we previously found for \( r_t - E_{t-1}(p_{t+1} - p_t) \). Then substituting this function into equation (5) of the structure we get the same first order difference equation in \( k \) as we had without an information discrepancy. This proves that under the rule given by (31), the distribution of \( k \) does not depend on the information discrepancy. It follows then from equation (17) that the same is true for \( y \).

To find the distribution of \( p_t \), we proceed to solve the difference equation,

\[ (1-J_o)E_{\theta, t-1}p_t = J_1E_{\theta, t-1}p_{t+1} + J_2E_{\theta, t-1}m_t + E_{\theta, t-1}X_t \]

which is obtained by taking \( E_{\theta, t-1} \) of (29). Then proceeding as
we did for (18), we obtain expressions exactly like (21) and (22) except
that in place of $E_{t-1}$ on the left and right we have $E_{\theta,t-1}$.

But from (31),

$$X_{t+j} + J_2 m_{t+j} = -(K_2/2K_2)(1-J_0-J_1) + X_{t+j} - E_{t-1} X_{t+j}$$

so for $j \geq 0$

$$E_{\theta,t-1}(X_{t+j} + J_2 m_{t+j}) = -(K_2/2K_2)(1-J_0-J_1) = E_{t-1}(X_{t+j} + J_2 m_{t+j})$$

Thus use of the rule given by (31) implies $E_{\theta,t-1} p_{t+j} = E_{t-1} p_{t+j}$,

$j=0,1$, which by (29) implies that under the rule given by (31) the
distribution of $p$ does not depend on $\theta$; i.e., does not depend on the
information discrepancy. It follows that the loss attained under the
rule given by (31) does not depend on the information discrepancy.

This shows that the monetary authority can do as well given an
information discrepancy as it can if there is none. But can it do
better? Can it, as it were, take advantage of the presence of an infor­
mation discrepancy? We are not sure. But within our structure the
answer seems to be that it can take advantage of a discrepancy, although
necessarily in a limited and rather subtle way.

To indicate why, let us focus first on how the distribution of
$y$ depends on the rule for $m$. Under present assumptions, equation (1) of
the structure is:

$$(35) \quad y_t = a_1 k_{t-1} + a_2 (p_{t-1} - E_{\theta,t-1} p_{t-1}) + u_{1t}$$

It follows that as of the end of $t-1$, $E_{\theta,t-1} y_t$ is unaffected by the
choice of $m_{t}$, since
To find the variance of $y_t$, we subtract (36) from (35) and obtain

$$
\hat{y}_t = a_1 \hat{k}_{t-1} + a_2 \hat{u}_t + \hat{u}_t
$$

where $\hat{x}_t = x_t - E_{\theta}x_{t-1}$. The variance of $y_t$ around $E_{\theta}x_{t-1}$ is, therefore,

$$
E_{\theta}, t-1 (y_t^2) = E_{\theta}, t-1 [(a_1 \hat{k}_{t-1} + \hat{u}_t)^2] + E_{\theta}, t-1 (p_t^2)
$$

+ other terms

where $\hat{x}_t = E_{t-1}x_t - E_{\theta}, t-1 x_t$ and where the omitted terms are unaffected by the setting for the deterministic part of $m_t$. Thus, setting $m_t$ according to (31) (i.e., setting $p_t = 0$) minimizes $E_{\theta}, t-1 (y_t^2)$ only if the first term on the RHS of (37) cannot be made negative. That term can be made negative by a rule different from (31) if $a_1 \hat{k}_{t-1} + \hat{u}_t \neq 0$, that is, if the monetary authority knows more about either the $k_{t-1}$ or the $u_t$ process than does the public. Of course, to take advantage of this information discrepancy, the monetary authority must know precisely how the public's information differs from its own.

Similar conclusions hold for the distribution of $k_t$. The expectation $E_{\theta}, t-1 k_t$ is unaffected by the setting for $m_t$, but, in general, the variance, $E_{\theta}, t-1 (\hat{k}_t^2)$ depends on it and is not minimized by use of the rule given by (31). And since the setting for $m_t$ affects the distribution of $(y_{t+j}, p_{t+j})$ for $j > 0$ by way of its effect on the distribution of $k_t$, this means that given an information discrepancy, our structure gives rise to a nontrivial dynamic problem.
But this should not be taken to mean that we are back in the setting produced by the assumption that expectations are formed on the basis of fixed autoregressive schemes. The information discrepancy assumption does not produce any simple trade-off between the means of output and the price level. The fact that $E_{\theta, t-1} y_t$ and $E_{\theta, t-1} k_t$ are unaffected by $m_t$ is very limiting if $\theta$ contains, say, as little as $(1, p_{t-1}, y_{t-1})$. Secondly, to exploit the information discrepancy the monetary authority must know what it is. To assume that it does seems far fetched. Indeed, we suspect that estimating the discrepancy is a very subtle and perhaps intractable econometric problem.

For these reasons, we think some comfort can be taken from the first result established in this section. Use of the rule given by (31) is optimal if the public is as well informed as the monetary authority. The loss attained under that rule does not depend on how well informed the public is, and implementation of the rule does not require knowledge of how well-informed the public is.

This does not, of course, deny that there is a gain from learning more about the exogenous processes. Settings for the money supply under the rule given by (31) depend on what the monetary authority knows. Operating under that rule, loss is smaller, the more the monetary authority knows about the exogenous processes.

7. Concluding Remarks

Given that conclusions (a) - (c) are derived from an ad hoc model, should they be taken seriously? In one sense, they should not be. Because of their ad hoc nature, neither the structure set out in section 1 nor the loss function of section 2 should be accepted as
providing a suitable context within which to study macroeconomic policy. Nevertheless, some aspects of our model cannot be dismissed so easily. First, the hypothesis that expectations are rational must be taken seriously, if only because its alternatives, e.g., various fixed weight autoregressive models, are subject to so many objections. Second, the aggregate supply hypothesis is one that has some microeconomic foundations; and it has proved difficult to dispose of empirically. It is precisely these two aspects of our model—rational expectations in conjunction with Lucas's aggregate supply hypothesis—that account for most of our results. We believe that the results concerning systematic countercyclical macroeconomic policy are fairly robust to alterations of other features of the model, such as the aggregate demand schedule and the portfolio balance condition. In particular, the dramatically different implications associated with assuming rational expectations, on the one hand, or fixed autoregressive expectations, on the other hand, will survive such alterations.
Footnote

1/ The structure closely resembles the model used by Sargent [7].

2/ See Gregory Chow [1].

3/ Gregory Chow [1] describes how optimal rules of the form (6) or (7) are found for a system like (10 or (11).

4/ A workable "reduced form" for \( p_t \) can be obtained by substituting (20) into (15), and then by using (5) and (7) to replace \( E_{t-1} m_{t+j} \) and \( E_{t-1} X_{t+j} \) with the linear functions of past variables that they equal. These linear functions are easily calculated from the feedback rule for \( m_t \) and the autoregressions governing components of \( X_t \). While the resulting "reduced form" for \( p_t \) formally resembles the corresponding equation in the system with "adaptive" expectations, there is a crucial difference. For now changes in the parameters of the feedback rule for \( m_t \) produce changes in the parameters of the reduced form for \( p_t \). This feature of the system is what renders Poole's results inapplicable. For an explicit illustration of the dependence of the reduced form parameters on the form of the policy rule, see Sargent and Wallace [6, p. 332-333].

5/ See Olivera [4]. In both our model and the standard static model, the aggregate demand schedule must exclude any components of real wealth that vary with the price level if Wicksell's indeterminacy is to arise. For example, if the anticipated rate of capital gains on real (outside) money balances is included in the aggregate demand schedule, the price level is determinate with a pegged interest rate. However, such a system has peculiar stability characteristics, since stability hinges on the sign of the expected rate of inflation.

6/ The reader may verify this by finding \( k_t \) as a function of \( k_{t-1} \) and \( p_t - E_{t-1} p_t \) using (35) and equations (2) and (5) of the structure.

7/ For example, see Lucas [2].

8/ Tests of the aggregate supply hypothesis are reported by Lucas [2] and Sargent [7].
REFERENCES


