Understanding the Current View of Trends, Cycles, and the Persistence of Shocks.

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Abstract

The new classical view that macroeconomic fluctuations can be modeled as an equilibrium system perturbed by transitory monetary disturbances has been challenged in recent years by another equilibrium view of fluctuations, the so-called real business cycle theory. In this latter framework, shocks to the production function induce both intertemporal substitution of labor supply and permanent shifts in the stochastic trend of output. Monetary shocks, on the other hand, play only a minor role in this view of the cycle.

Much of the empirical support for the real business cycle view of fluctuations is based on a re-examination of traditional methods for detrending economic time series. The issues raised by the real business cycle theorists are not new; indeed, they go back at least to the NBER's first business cycle studies. However, the real business cycle theorists attach a radical economic interpretation to what, on the surface, appears to be a purely technical note on the proper method for detrending economic data.

This paper reviews the debate over stochastic trends, discusses the economic implications of the real business cycle interpretation of stochastic trend models, and weighs the time series evidence for some of the stronger claims made by real business cycle theorists. We conclude that, while this literature raises real and useful questions about the interpretation of observed fluctuations, the new classical view of the cycle is not ruled out by the data.

I Introduction

A fundamental concept in the study of business cycles is the distinction between trend growth and cyclical fluctuations. The long-run behavior of economic aggregates such as output and consumption is dominated by the trend component—the component supposed to be due to population growth, demographic shifts, technical progress, and the like. The cyclical component is just the remainder after the trend is subtracted. The up-and-down nature of the cyclical component presumably reflects the stream of transitory random shocks that affect the economy: policy actions, unusually good or bad harvests, labor and trade disputes, wars, etc.

Macroeconomic explanations of economic fluctuations focus on the cyclical component. The trend is considered a nuisance factor that obscures the events of greater interest. Thus methods for decomposing economic time series into trend and cycle are required so that the cyclical essence can be analyzed and the trend can be discarded.

The obvious problem is how to accurately distinguish between trend and cycle. It has been recognized almost since the beginning of business cycle research that estimates of central features of the business cycle, such as the lengths of cycles and the persistence of shocks, are sensitive to the methods used for removing the trend. (See Burns and Mitchell (1946, Chapters 3 and 7, esp. pp. 37–8) for an account of early studies of this problem.) Interest in this problem has revived in the 1980s and useful contributions have been made by Beveridge and Nelson (1981), Nelson and Kang (1981), Nelson and Plosser (1982), Watson (1986), and King, Plosser, Stock, and Watson (1987) among others.

On the face of it, the question of detrending economic time series would appear to be a dry, technical one, destined to catch the attention of only a few scholarly specialists. In its present incarnation however, this detrending literature is developing apace with and providing substantial support for a challenging new theory of economic fluctuations, the so-called real business cycle theory. (See King, Plosser, and Rebelo (1987) for a summary of the current state of this theory.) The real business cycle (RBC) theory is an equilibrium theory of the business cycle that models fluctations as being generated by a stochastic, neoclassical growth model. In particular, most of the variation in output is caused by *real shocks*, that is, random shocks to the rate of technical progress. One implication of the RBC theory is that most shocks to output are permanent; output has no tendency to return to a prior trend growth path.

The real business cycle theory is a direct challenge to the new classical theory of the cycle, another equilibrium theory that directly preceded the development of the RBC theory. In the new classical (NC) view of the cycle, fluctuations are generated by *nominal shocks*, that is, random increments to the money supply that temporarily obscure real relative prices. One implication of the NC theory is that most shocks to output are transitory; output tends to revert to its prior trend rate of growth. A principal focus of NC research is the analysis of

accelerator mechanisms, methods by which uncorrelated nominal shocks are translated into long-lasting deviations of economic aggregates from their trends.

This paper seeks to clarify the role of detrending in the debate over how best to model economic fluctuations. The detrending literature is scattered, the arguments are subtle and technical in nature, and the claims made about the implications of this work are sweeping and radically different from previous views of the business cycle. Our goals are to cut through some of the confusion about the new stochastic trend models used in RBC research, to weigh the evidence for and against these models of the trend, and, most important, to consider the interpretations of these models that have been advanced by their proponents. In particular, we wish to consider the extent to which stochastic trend models support the RBC view and challenge the NC view of economic fluctuations.

In the next section of the paper, we review the pitfalls that attend a mechanical approach to detrending economic time series. In the succeeding section, we examine more fully the stochastic trend models. In the next section, we discuss the interpretation of the stochastic trend models, and we consider the claims about business cycles that have been advanced based on the stochastic trend approach to modeling economic time series. In the final section, we summarize our findings and suggest directions for further research.

II The Detrending Problem

Secular trends of time series have been computed mainly by men who were concerned to get rid of them. Just as economic theorists have paid slight attention to the "other things" in their problems which they suppose to "remain the same," so the economic statisticians have paid slight attention to their trends beyond converting them into horizontal lines. Hence little is yet known about the trends themselves, their characteristics, similarities, and differences. Even their relations to cyclical fluctuations have been little considered. Here lies in obscurity a heap of problems, waiting for properly equipped investigators to exploit.

— Mitchell (1927)

II.1 The traditional trend/cycle decomposition

Figure 1 displays a plot of the log of quarterly real GNP from the first quarter of 1947 (1947:1) through the third quarter of 1987 (1987:3). Over this period, annual GNP has increased more than three-and-a-half times, from $1,067 billion in 1947 (1982 dollars) to $3,713 billion in 1986. Most of this increase is due to growth in population, which more than doubled over the same period. The unexplained portion of the in-

crease, about one-third of the total, is presumably the result of technical progress—more and better capital, a healthier and better-educated workforce, improvements in communications, and so on. The kinds of transitory economic disturbances studied by macroeconomists must play virtually no role in determining this long-run progress. Thus, if we wish to analyze these transitory disturbances, we must filter out the inexorable growth that is the central feature of Figure 1.

The simplest and most common method for estimating the secular trend in output is to regress the log of output against a constant and a time variable. In other words, it is assumed that the long-run equilibrium dynamics of output growth obey the relationship

$$y_t = \alpha + \beta t + c_t \tag{1}$$

where y_t is the log of real output, α is a constant, β is the trend rate of growth, t is an index of time, and c_t is the cyclical deviation from the secular trend. This is the so-called *deterministic trend* (DT) model.[1]

If we fit a DT model to the GNP data displayed in Figure 1, we obtain the estimate

$$\begin{aligned} \log \mathrm{GNP}_t &= \underset{(0.01)}{7.02} + \underset{(.00007)}{.0078\,t} \\ \hat{\sigma}_c^2 &= .044 \\ R^2 &= .99 \end{aligned}$$

This estimate implies that real GNP grows at a trend rate of 3.1 percent per year. Figure 2 redisplays the GNP data with the estimated trend line superimposed.

The residual from this regression is the deviation from trend, that is, the cyclical component of real GNP. Figure 3 displays a plot of this cyclical component over time. In the words of Wesley Mitchell quoted at the top of this section, Figure 3 converts the trend into a horizontal line. It is easy to locate familiar events in Figure 3: the remarkable economic performance in the 1960s, the oil shocks, the 1982 recession, and the recent sluggishness in real growth. [2]

In addition to conforming to our notions of the historical performance of the U.S. economy, the cyclical variations in Figure 3 also display features that conform well with traditional notions of macroeconomics. Note particularly that excursions from the secular trend are long-lived. In the eleven-and-a-half years from 1963:3 through 1974:4, for example, real GNP was consistently above its secular trend. As another

[1] Nelson and Plosser (1982) call this model the *trend stationary* (TS) model in contrast to their preferred *difference stationary* (DS) model. Much as we hate to introduce new jargon, we find the Nelson and Plosser terminology confusing. Both models assume that real output can be represented as the sum of a nonstationary trend component and a stationary cyclical component. The crucial difference is that the trend is stochastic in the model preferred by Nelson and Plosser. We emphasize this central distinction by using the terms *deterministic trend* (DT) and *stochastic trend* (ST) to qualify the models.

[2] Recall that a positive slope in the curve displayed in Figure 3 implies a higher than average growth rate of real GNP. In other words, economic conditions improve before GNP returns to its secular trend—before the cyclical residuals become positive. This arithmetic fact explains, for example, the negative residuals in the early 1960s.

example, real GNP has been below its secular trend for the last thirty quarters. This high serial correlation in output suggests that the economy is riddled with rigidities such as sticky wages and prices, long-term nominal contracts, and strong accelerator mechanisms.

We can sharpen the characterization of the cycle implicit in Figure 3. Following Blanchard (1981), we can fit an AR(2) model to the cyclical residuals.[3] The estimated model is

$$\begin{aligned} c_t &= \underset{(0.07)}{1.36}\, c_t - \underset{(.07)}{.40}\, c_{t-1} \qquad & \hat{\sigma}_\epsilon^2 = .01 \\ & & R^2 = .95 \end{aligned}$$

where ϵ_t is the random innovation in the AR(2) model.

The moving average representation of this model is displayed in Figure 4. This impulse response function displays the hump shape that is regarded as one of the stylized facts of the business cycle. The peak impact of a shock to real GNP occurs two quarters after the shock. The effects of the shock die off slowly; statistically significant effects occur more than ten years after the shock. According to these estimates, one-percent shock to real GNP today increases real GNP five years from today by nearly one-half of one percent.[4]

[3] A cursory examination of the autocorrelations of the cyclical residuals dispels the notion that these residuals can adequately be modeled by an AR(2) process. (We return to this point in the next subsection.) We adopt Blanchard's approach here because it is typical of other analyses based on the DT model and because it is a widely cited example of the supposed stylized facts about the modern business cycle.

[4] Qualitatively similar results can be found in Barro (1977, 1978) and Barro and Rush (1980).

To summarize, the DT model specifies the trend as a fixed mean rate of growth. When real GNP is detrended by this method, the residuals—the cyclical component of GNP—displays the hump-shaped impulse response function that is familiar from textbook expositions of the multiplier-accelerator model of Samuelson (1944). In addition, shocks to GNP appear to have statistically and economically significant effects that persist for many years.

II.2 The pitfalls of deterministic trend models

Practitioners recognized the arbitrary nature of the DT model. The dogmatic assumption that the trend rate of growth (the parameter β in equation (1)) is constant is a convenient approximation at best. In addition, there had long been evidence that estimates of business cycle features such as the persistence of shocks is sensitive to the method used for detrending. (See Mitchell (1927) and Burns and Mitchell (1946) for references.) However the lack of clearly superior alternative detrending methods along with the conformity of the behavior of the cyclical component with accepted macroeconomic theories led economists to rely on the DT model.

The state of the art in detrending changed in the 1970s due largely to the publication of the first edition of the monograph by Box and Jenkins (1970). This book collected a body of specialist literature on time series models and combined this theory with tractable algorithms for calculating estimates of general linear models for

stationary and nonstationary time series. The new time series methods gained popularity rapidly, but the relative ease of the DT approach guaranteed its continued use by applied economists. More importantly, the fact that the cyclical residuals from DT models behave as macroeconomists expected suggested that the DT approach provided an adequate characterization of the data dynamics.

This belief, that the DT approach, while crude, does no serious harm, was challenged in papers by Chan, Hayya, and Ord (1977), Nelson and Kang (1981, 1984), and Nelson and Plosser (1982). These studies showed that the DT approach applied to a random walk (with or without drift) produces estimated models that appear reasonable but whose main features are almost entirely artifactual. The pitfalls of the DT method (when applied to a random walk) include the following[5]: (1) the sample autocorrelations of the residuals (the cyclical residuals in equation (1)) are purely artifactual and can be approximated as a function of the sample size alone; (2) the cyclical residuals appear to exhibit a long cycle that is spurious; (3) the estimated variance of the cyclical residuals is biased downward; (4) the R^2 statistic for estimates of equation (1) is biased upward; and (5) the standard error of the estimated coefficient on time (β in equation (1) is biased downward.

The DT model for real GNP reported in the previous subsection exhibits several of the symptoms described above. The estimated persistence of shocks in this model is very long, the R^2 is high, and the estimated standard error of $\hat{\beta}$ is very small. Figure 5 presents additional evidence that the DT model for real GNP is inadequate. This figure displays the first 20 autocorrelations and partial autocorrelations of the cyclical residuals, the c_t of equation (1). These residuals are assumed to be stationary, however the plots in Figure 5 clearly suggest that the c_t possess a unit root. If this is the case, then the AR(2) model cannot accurately capture the dynamics of cyclical GNP, and the impulse response function in Figure 4 is misleading.

Aside from the purely technical inadequacies of the DT model as a time series model, is it reasonable to suppose that the trend rate of real growth has been constant throughout the post-World War II years? A deterministic trend may provide a local linear summary of growth over a handful of years, but there is no *a priori* reason for believing that the average growth rate remained unchanged for forty years. Indeed there is an ample literature that claims to find a productivity slowdown in the U.S. in recent years. (Representative examples of this literature are Denison (1979) and Norsworthy, Harper, and Kunze (1979).) Absent a balancing increase in population growth, such a slowdown would decrease the trend rate of output growth.

The average growth rate of real GNP does decline over our sample period. The compound annual growth rate of GNP is 4.1 percent for the decade 1947–56, 3.8 percent for 1957–66, 2.6 percent for 1967–76, and 2.8 percent for 1977–86. Another way

[5]These points are taken from Nelson and Kang (1984)

of measuring this shift is to fit the regression

$$\log \mathrm{GNP}_t = \alpha + \beta t + \delta I_{<1970} + \gamma t I_{<1970} + u_t$$

where $I_{<1970}$ is an indicator variable that is equal to one if the observation is from a year before 1970 and equal to zero otherwise. This regression allows the trend line to shift both its slope and its intercept between the fourth quarter of 1969 and the first quarter of 1970. While this specification oversimplifies events, it does provide a measure of the statistical significance of the shift in the average growth rate. The ordinary least squares estimate of this equation is

$$\begin{aligned} \log \mathrm{GNP}_t \;=\; & \underset{(0.03)}{7.21} + \underset{(.0002)}{.0062\, t} \\ & - \underset{(.02)}{.24\, I_{<1970}} + \underset{(.0002)}{.0026\, t I_{<1970}} \\ & \hat{\sigma}_u^2 = .032 \\ & R^2 = .99 \end{aligned}$$

This regression indicates that the average growth rate of real output prior to 1970 was 3.6 percent. After 1970, the average growth rate was only 2.5 percent. The difference is highly significant; the F-statistic for the joint significance of both terms involving $I_{<1970}$ is $F_{2,159} = 75.52$.[6]

The evidence presented here against the DT model is necessarily abbreviated. In a far more ambitious study, Nelson and Plosser (1982) present a battery of formal and informal tests of the hypothesis that the DT model adequately characterizes a variety of macroeconomic time series including real and nominal GNP, consumption, and the money stock. They conclude that the DT model should be rejected for all of the nonstationary time series they examine.

III Stochastic trend models

The results of the previous section suggest that the deterministic trend (DT) model misrepresents the dynamic behavior of real output. We are left then with the problem of finding an acceptable model that can account for the nonstationarity of output. One possibility is to suppose that the log of output can be decomposed into the sum of two random variables: a nonstationary, but random, trend term, and a stationary cyclical term. We write this decomposition as

$$y_t = \overline{y}_t + c_t. \tag{2}$$

$\overline{y}_t$ is the stochastic trend, and c_t is used again to represent the stationary, cyclical

[6] We take no position here on the debate over the alleged productivity slowdown, neither do we suggest that this regression is an appropriate way of modeling productivity changes, nor do we claim that 1970 is the appropriate breakpoint in the data. (In fact, the evidence for a split is strongest when we break the sample between 1965 and 1966.) We use this regression only as a convenient summary of realized rates of measured growth in real GNP and to emphasize the *prima facie* untenability of the DT model for real GNP.

It is also important to note that we do not suggest that the DT model can be salvaged by converting a constant trend into a piecewise linear trend. The residuals from the piecewise linear trend model exhibit the same evidence of nonstationarity as the original cyclical residuals.

component of output.

Equation (2) is in the same form as the DT model of equation (1). The DT model also decomposes output into a nonstationary trend term and a stationary cyclical term. However in the DT model, the trend is given by the *deterministic* term $\alpha + \beta t$. This distinction between a deterministic and a random trend is the only formal difference between the DT and ST models. While this difference may appear slight—almost a technical footnote—in fact, it leads to a dramatic divergence in the abilities of the models to account for the data.

III.1 ARIMA models and stochastic trend models

The ARIMA model can usefully be thought of as the reduced form of time series analysis. The Wold decomposition theorem (1938) for stationary processes and the generalization by Cramér (1961) to nonstationary processes guarantee that any linear nonstationary process has an ARIMA representation. The assumption of invertibility identifies a unique ARIMA representation for each process. In other words, no matter how complicated the original (linear) structural model, there is a unique ARIMA representation for the resulting time series.

An immediate implication of this fact is that a stochastic trend model can be represented by an ARIMA model. Not as obvious though is the fact that any integrated process, that is, any ARIMA(p, d, q) with $d > 0$, is a stochastic trend model.[7] As an example, consider the simplest integrated process, the ARIMA$(0,1,0)$ or random walk with drift,

$$z_t = z_{t-1} + \delta + u_t. \tag{3}$$

In this model, δ is the drift parameter and u_t is a white noise process. By substituting repeatedly for z_{t-1}, we can rewrite equation (3) as

$$z_t = z_0 + \delta t + \sum_{j=1}^{t} u_j. \tag{4}$$

This equation has the same form as equation (2). The first two terms, $z_0 + \delta t$, comprise the stochastic trend while the last term, $\sum_{j=1}^{t} u_j$, is the cyclical component. Note that it is the intercept, z_0, not the slope, δt, that is random in the stochastic trend.

Since any linear nonstationary time series has an ARIMA representation, and since there is a correspondence between stochastic trend models and integrated processes, it is not surprising (at least in retrospect) that stochastic trend models provide a reasonable fit for most economic time series. (Nelson and Plosser (1982) examine the empirical performance of ST models.) DT models are a restricted form of ST models, hence the poor empirical performance of DT models is understandable, even though it may be regarded as unfortunate since DT models

[7]This point is implicit in much of the recent work on stochastic trends. Useful discussions can be found in Beveridge and Nelson (1981), Nelson and Plosser (1982), and Watson (1986). A remarkably prescient discussion of the role of integrated processes in explaining the stylized facts of economic aggregates can be found in Granger (1966) and the postscript in Granger and Newbold (1977).

are much simpler than ST models to apply and estimate.

III.2 Alternative specifications of trend and cycle

Equation (4) is *a* stochastic trend model; it is not *the* stochastic trend model. There are several ways to decompose a nonstationary time series into stochastic trend and cycle components. This point is important because RBC proponents base several of their arguments on alleged properties of empirical stochastic trend models. If those properties are sensitive to the particular specification chosen for the stochastic trend model, then the strength of these arguments is reduced.

To illustrate that there are different ways to specify the stochastic trend model and to show that these specifications have economic implications, let us augment equation (2) in a manner favored by Nelson and Plosser (1982). We assume that

$$(1-L)\overline{y}_t = \Theta(L)v_t \tag{5}$$

and

$$c_t = \Phi(L)u_t \tag{6}$$

where v_t and u_t are white noise processes, and $\Theta(L)$ and $\Phi(L)$ are polynomials (in the lag operator L) that satisfy the conditions for stationarity and invertibility.[8] Equation (5) specifies the trend as a random walk plus ARMA shocks. Equation (6) specifies the cyclical component as a general ARMA process. Substituting these two equations into equation (2) allows us to write the ARIMA$(p,1,q)$ model

$$(1-L)y_t = \Theta(L)v_t + \Phi(L)u_t \tag{7}$$

Note that differencing y_t does *not* remove the stochastic trend. This point is of the essence of the identification problem for stochastic trend models.

Because the trend, $\overline{y}_t$, follows a random walk, shocks to the trend are permanent; they do not dissipate over time. On the other hand, shocks to the cyclical component, c_t, die away by construction. Since most (though not all) macroeconomic theories imply that monetary disturbances have no permanent effect on real variables, Nelson and Plosser identify the innovation process u_t as the monetary or nominal disturbance. The v_t process is presumed to be a sequence of real shocks, that is, shocks to productivity or the rate of technical progress. Nelson and Plosser go on to argue, based on the observed autocorrelations of the log of real output, that $\sigma_v^2 > \sigma_u^2$, that is the variance of real shocks is greater than the variance of nominal shocks. They interpret this result to mean that most of the variation in output is due to real rather than nominal shocks. We will have more to say about this argument below, but this description serves to highlight the close ties between the specification of the stochastic trend model and the implied response of output to various economic events.

The model of equation (7) is not identified without further assumptions about the association between the real and nominal innovation series.[9] Beveridge and Nel-

[8] Following Nelson and Plosser, we ignore the drift parameter, δ, in order to simplify the exposition.

[9] Watson (1986) discusses the identification of

son (1981) provide a useful decomposition into trend and cycle that is always identified. Since several alternative decompositions (e.g., Watson (1986)) collapse to the Beveridge and Nelson form after sufficient identifying restrictions are incorporated, we examine this decomposition further.[10]

Define $w_t \equiv (1-L)y_t$. Consider forecasting y_{t+k} conditional on data through time period t. Denote this forecast by $\hat{y}_t(k)$. This can be written as

$$\hat{y}_t(k) = y_t + \hat{w}_t(1) + \ldots + \hat{w}_t(k). \qquad (8)$$

Write the moving average representation of w_t as

$$w_t = \delta + \epsilon_t + \psi_1 \epsilon_{t-1} + \ldots \qquad (9)$$

where δ is the unconditional mean of w_t, ϵ_t is a white noise process, and the ψ_j are the weights of the moving average representation. It is easy to show that (9) permits us to rewrite (8) as

$$\begin{aligned} \hat{y}_t(k) &= \delta k + y_t + \left(\sum_1^k \psi_j\right)\epsilon_t \\ &\quad + \left(\sum_2^{k+1} \psi_j\right)\epsilon_{t-1} + \ldots . \end{aligned} \qquad (10)$$

univariate stochastic trend models. King, Plosser, Stock, and Watson (1987) consider the identification of multivariate stochastic trend models. Peña and Box (1987) discuss the multivariate model from the perspective of dynamic factor models.

[10] An interesting question is whether all identifiable univariate stochastic trend models resolve to the Beveridge and Nelson form. Watson (1986) makes suggestive comments on this topic, but I am not aware of a complete answer to this question in the literature.

For long forecast horizons (large values of k), equation (10) approaches

$$\begin{aligned} \hat{y}_t(k) &\approx \delta k + y_t + \left(\sum_1^\infty \psi_j\right)\epsilon_t \\ &\quad + \left(\sum_2^\infty \psi_j\right)\epsilon_{t-1} + \ldots . \end{aligned} \qquad (11)$$

Define

$$\begin{aligned} \overline{y}_t &= y_t + \left(\sum_1^\infty \psi_j\right)\epsilon_t \\ &\quad + \left(\sum_2^\infty \psi_j\right)\epsilon_{t-1} + \ldots . \end{aligned} \qquad (12)$$

Substituting (12) into (11), we have

$$\hat{y}_t(k) \approx \overline{y}_t + \delta k. \qquad (13)$$

This last equation has the same form has equation (4): $\hat{y}_t(k)$ is a linear function of k with a random intercept $\overline{y}_t$. (There is no sum of innovations term in (13) because this is a forecast equation.) In other words, $\overline{y}_t$ can be regarded as the permanent or stochastic trend component of y_t.

Beveridge and Nelson show that $\overline{y}_t$ follows the random walk

$$(1-L)\overline{y}_t = \delta + \left(\sum_0^\infty \psi_j\right)\epsilon_t, \quad \psi_0 \equiv 0, \qquad (14)$$

thus

$$\sigma^2_{(1-L)\overline{y}_t} = \left(\sum_0^\infty \psi_j\right)^2 \sigma^2_\epsilon. \qquad (15)$$

This last equation relates the variance of the innovation in the stochastic trend to the variance of the innovation in y_t. The variance of the innovation in the trend may be

larger or smaller than the variance of the innovation in y_t.

From (12) it is immediate that c_t, the cyclical component of y_t, is

$$\begin{aligned} c_t &= \left(\sum_1^\infty \psi_j\right)\epsilon_t + \\ &\quad \left(\sum_2^\infty \psi_j\right)\epsilon_{t-1} + \dots \qquad (16) \\ &= \lim_{k\to\infty}\{[\hat{w}_t(1) + \dots + \hat{w}_t(k)] \\ &\qquad - \delta k\}. \qquad (17) \end{aligned}$$

The second equality comes from comparing equations (8), (12), and (13). Beveridge and Nelson interpret the cyclical component, c_t, to be the forecastable future changes in y_t, that is, its momentum.[11]

Equation (17) is used to calculate estimates of $\overline{y}_t$ and c_t. The algorithm is straightforward. First, estimate an ARIMA$(p,1,q)$ model for y_t. Use this estimated model to form estimates of the $\hat{w}_t(k)$. Let k grow until $\hat{w}_t(k) - \delta$ is negligible, then form c_t according to equation (17). The permanent component then is just $\overline{y}_t = y_t - c_t$.

IV Interpreting the stochastic trend models

The literature to date on stochastic trend (ST) models has two themes. The first theme is the superiority of the ST model to the deterministic trend (DT) model. Several aspects of that superiority have been analyzed in the previous two sections. The second theme is the alleged support given to the real business cycle (RBC) view of economic fluctuations by the ST model. In this section, we discuss some remaining questions about the relative merits of the ST and DT models, and we analyze the claims made for the RBC theory based on the ST models.

IV.1 How different are the ST and DT models?

The evidence presented and cited in previous sections leaves no doubt that the DT model misrepresents central features of the business cycle. In this regard, the ST model is clearly superior, and, in this sense, the two models are very different. It is also claimed that the two models generate very different views of the long-run behavior of economic time series. In particular, equation (1) specifies that the variance of the error in forecasting y_{t+k} is a constant, that is, it does not grow as k, the forecast horizon grows. Equation (4), on the other hand, specifies that the variance of the forecast error is proportional to k. As the forecast horizon approaches infinity, so does the uncertainty surrounding the forecast.[12]

Figure 6 shows the three different long-run forecast profiles for three different mod-

[11] We noted above that the Beveridge and Nelson decomposition has the advantages of being both identified and computationally feasible. Note however that there is only one series of fundamental innovations in this decomposition, that is, the innovations to the cyclical component are perfectly correlated with the innovations to the trend component.

[12] It is interesting to note that *estimates* of theoretically equivalent stochastic trend models can have very different long-run implications. Watson (1986) presents an illuminating example.

els of the log of real GNP: the DT model estimated in section II, an ARIMA(0, 1, 1) model, and an ARIMA(2, 1, 2) model.[13] Figure 6a displays the forecasts for the period 1960–1990[14], Figure 6b displays the forecasts for 1970–1990, and Figure 6c displays the forecasts for 1980–1990.

In Figure 6a, the longest of the forecast horizons, the DT model and the ARIMA(0, 1, 1) models give essentially the same forecast, while the ARIMA(2, 1, 2) model forecasts lower GNP in each period. All three forecast profiles are essentially straight lines over this period. Figure 6b gives a very different picture than Figure 6a. For this twenty-year forecast, the DT model and the ARIMA(2, 1, 2) give essentially the same forecast, while the ARIMA(0, 1, 1) model forecasts higher GNP in each period. The ARIMA(0, 1, 1) model again gives a straight line forecast profile, but the other two models exhibit some fluctuations during the earlier portion of the forecast profile. In Figure 6c, all three models exhibit roughly the same, straight line forecast profile.

The DT forecasts change little across the three figures. [15] Thus the fact that the ARIMA(0, 1, 1) model 'agrees' with the DT model for the 1960–1990 forecast while the ARIMA(2, 1, 2) model 'agrees' with the DT model for the 1970–1990 forecast indicates that the ARIMA forecasts are highly sensitive to the choice of forecast origin. In other words, one's view of the future course of real GNP is just as sensitive to *which* ARIMA model is used as it is to the choice between a DT or ST model.

Figures 7a, 7b, and 7c display the 95% confidence bands for the forecast profiles shown in Figure 6. The DT model, of course, produces the narrowest confidence bands. However, our purely informal, subjective evaluation of these figures is that the differences between the confidence bands of the DT and ARIMA models are not that important, even at a thirty year horizon. In addition, the DT confidence band appear to perform reasonably well in bounding the movements of real GNP.

In summary, Figures 6 and 7 suggest that the differences between DT and ARIMA models in forecasting real output have been overemphasized. In addition, there are quite large differences in performance between different ARIMA specifications. However, the DT model can lead to spurious inferences about the stylized behavior of the cyclical residuals.

IV.2 Are real shocks more important than nominal shocks?

Beveridge and Nelson (1981) suggest that the variance of the innovations to the trend is larger than the variance of the innovation to the cyclical component for most economic time series. Nelson and Plosser (1982) claim to prove this sug-

[13] The ARIMA(0, 1, 1) is suggested by Beveridge and Nelson (1981), Nelson and Plosser (1982), and Watson (1986). The ARIMA(2, 1, 2) model is suggested by Campbell and Mankiw (1986, 1987).

[14] The quarters 1960:1 through 1989:4 are forecast using 1959:4 as the origin of the forecast. The other two forecast profiles are defined analogously.

[15] The only source of deviations from the trend line in the DT model are the initial conditions in the AR(2) model for the cyclical residuals.

gestion. King, Plosser, Stock, and Watson (1987) assert that 2/3 of the variance of the error in forecasting output is due to innovations in the stochastic trend.[16] These authors all maintain that their findings are evidence in favor of the RBC view that real shocks are more important than nominal shocks in explaining the behavior of aggregate output.

Are these claims justified? The answer to this question is long and complicated. Let us begin constructing our answer by pointing out one obvious fact. Real factors are clearly more important than nominal factors in explaining the gross behavior of real output. Nominal shocks do not account for the more than 350 percent increase in real GNP over the last forty years. In this trivial sense, the trend is always more important than the cyclical component, even in a DT model.

This point is obvious, but it is not a side issue. ARIMA models, the reduced form for all stochastic trend models, confound trend and cycle components. (See equation (7.) Some of the forecasting power of the ARIMA models comes from their inclusion of the innovations to the trend.

This consideration leads us to the central issue: what does it mean to say that real shocks are more important than nominal shocks? In the context of the DT model, there is a simple answer to this question. The goal in the DT framework is to explain the variation of output *around its trend*. Assuming that real and nominal factors can somehow be identified, their contribution to explaining this variation can be measured by classical regression or analysis of variance methods.

In the ST model, the trend varies at the same time as the cyclical component (and the original series) do. We would like to measure how much variation is 'left over' to be explained by the cyclical components after the ups and downs of the stochastic trend have been removed. This step poses no problems, but then how is this contribution to be compared to the contribution of the stochastic trend? Both the original series and the stochastic trend are nonstationary; their variances are infinite.

One approach is to partition the forecast error variance into a portion due to trend shocks and a portion due to cycle shocks. If the trend shocks account for a greater percentage of the error variance than do the cycle shocks, then we say that the real shocks are more important than the nominal shocks. This accounting is a well-defined ANOVA decomposition if the trend and cycle innovations are uncorrelated. In a more general specification, the calculation and interpretation of 'sums of squares' is more complicated. Nonetheless, we favor this approach over competing ones.[17]

[16] Watson (1986) finds some contrary evidence. In his estimates of an unobserved components model for the log of real GNP, the variance of the innovation to the cyclical component is 1 1/3 times as great as the variance of the innovations to the trend.

[17] Nelson and Plosser (1982) directly compare the variances of the innovations to the trend and to the cycle. This approach is not identical in general to the comparison of contributions to the forecast error variance, although it is clearly related. The Nelson and Plosser comparison does have the advantage of picking a clear 'winner' even if the innovation series are correlated.

One way to provide intuition for the approach of decomposing the forecast error variance is to imagine the following counterfactual choice. Imagine that a machine is devised which will reveal the entire sequence—past, present, and future—of one, but only one, of the innovation series. A forecaster operating this machine would choose to observe the innovation series that is of the most help in reducing forecast errors. In this sense, it is possible to think of one of the series of shocks as being more important than the other series.

The difficulty with this proposal is the interpretation of the innovations to the trend as real shocks and the innovations to the cyclical component as nominal shocks. In the always identified decomposition of Beveridge and Nelson (1981), there are no separate fundamental real and nominal shock series. The only distinctions between the 'real' and 'nominal' innovations are the coefficient vectors applied to the past and present fundamental shocks. Other models posit separate fundamental shock series, but these are not identified in general, and when they are identified, they often can be written in a form that is equivalent to the Beveridge and Nelson decomposition. The problem of attaching an economic interpretation to fundamental disturbances is not unique to the RBC model. Indeed one of the virtues of the RBC model is that it highlights this conundrum that is more easily masked in other models.

Because of its persuasiveness in this literature, it is worthwhile to temporarily set aside our objections to the RBC interpretation of shocks and to review the Nelson and Plosser (1982) 'proof' that the variance of real shocks is greater than the variance of nominal shocks. The syllogism goes as follows: (1) the cyclical component of real output is stationary, (2) the first difference of the log of real output has a single positive autocorrelation at lag one and zero (insignificant) autocorrelations at all other lags, therefore (3) the variance of real shocks is greater than the variance of nominal shocks. We will not reproduce the proof of this syllogism here. The interested reader can consult Nelson and Plosser (1982, pp. 155–56). Instead we will consider whether the premises are true.

The first premise is purely definitional. Thus the syllogism stands or falls on the second premise, that the log of real output possesses a single positive autocorrelation, that is, that the log of real output can be modeled as an ARIMA(0, 1, 1) process. If some other process is the correct model for real output, then the syllogism fails (although the conclusion may, of course, still be true).

The first thing to point out is that the identification of ARIMA models is problematic. Useful identifications cannot be made by an objective, mechanical procedure; the subjective opinion of a skilled practitioner is required. Indeed, Granger and Newbold have said that 'identification is a technique that should not be attempted for the first time' (Granger and Newbold, 1977). And, in this case, different practitioners have identified different ARIMA specifications for real output. Watson (1986) selects an ARIMA(1, 1, 0) model for quarterly real GNP (although he says that an ARIMA(0, 1, 1) model is also ac-

ceptable). Campbell and Mankiw (1986) present an exhaustive evaluation of ARIMA models for quarterly real output and select an ARIMA(2, 1, 2) model.

Figure 8 displays the autocorrelations and partial autocorrelations of the first difference of the log of real output for both quarterly (1947:1–1987:3) and annual (1947–1986) data. Bands of plus-or-minus two standard errors of the estimates are drawn around the horizontal axes. Recall that Nelson and Plosser's identification requires that *only* the first autocorrelation is significant and that it is positive.[18] For the quarterly data, the first two autocorrelations and the first partial autocorrelation are positive and significant. For the annual data, none of the estimated autocorrelations and partial autocorrelations are significant. These patterns of autocorrelations and partial autocorrelations do not strongly suggest that one and only one identification is possible for these data.

One reason for the differences between the estimated autocorrelations in Nelson and Plosser and the ones shown in Figure 8 may be the long sample used by Nelson and Plosser (1909-1970). Romer (1986) has shown that measured GNP for the pre-World War II era may not be comparable to (may not follow the same process as) the more recent data. A defense of Nelson and Plosser's argument is the fact that they estimate the autocorrelations of numerous series and base their identification on a common tendency for these series to exhibit a single significant and positive autocorrelation at lag one.

[18] Nelson and Plosser report the following estimates for the first six autocorrelations (annual data, 1909-1970): 0.34, 0.04, -0.18, -0.23, -0.19, and 0.01. The large sample standard error under the null hypothesis of no autocorrelation is 0.13.

IV.3 Why is consumption so smooth?

If most shocks to output are permanent—are embodied in the stochastic trend—then most variations in output represent variations in permanent income. Indeed, many estimates indicate that a one-percent increase in output today leads to a rational forecast of a greater than one-percent increase in permanent income. If this is so, then why is consumption so smooth? Why doesn't consumption vary at least as much as output does?

This question is a challenge to all schools of thought on the cycle, since the permanent income hypothesis is almost universally accepted in theoretical macroeconomics. Deaton (1986) has analyzed this question, but no definitive answer has been obtained so far.

IV.4 What is left of the accelerator?

In the heyday of the Lucas theory of the business cycle, a key question was the role of various accelerator mechanisms in propagating white noise shocks. This line of research has faded somewhat in the furor over stochastic trend models. There is no logical basis for this lack of attention. McCallum (1977) points out (albeit in a stationary context) that the ψ_j weights of equation (9)

are, in principle, largely determined by accelerator mechanisms.

V Conclusion

This paper has examined the stochastic trend models that underlie much of the recent literature on real business cycles. There is no doubt that stochastic trend models provide a superior framework for analyzing the dynamic behavior of economic time series. It is not clear, though, that stochastic trend models will bear the interpretations placed on them by real business cycle theorists. Indeed the stochastic trend models appear rather to highlight the difficulty of attaching substantive economic interpretations to fundamental innovations. The debate over the relative merits of real business cycle and monetary models of economic fluctuations will have to be resolved, if it is resolved at all, by appealing to some other evidence than the estimates of stochastic trend models.

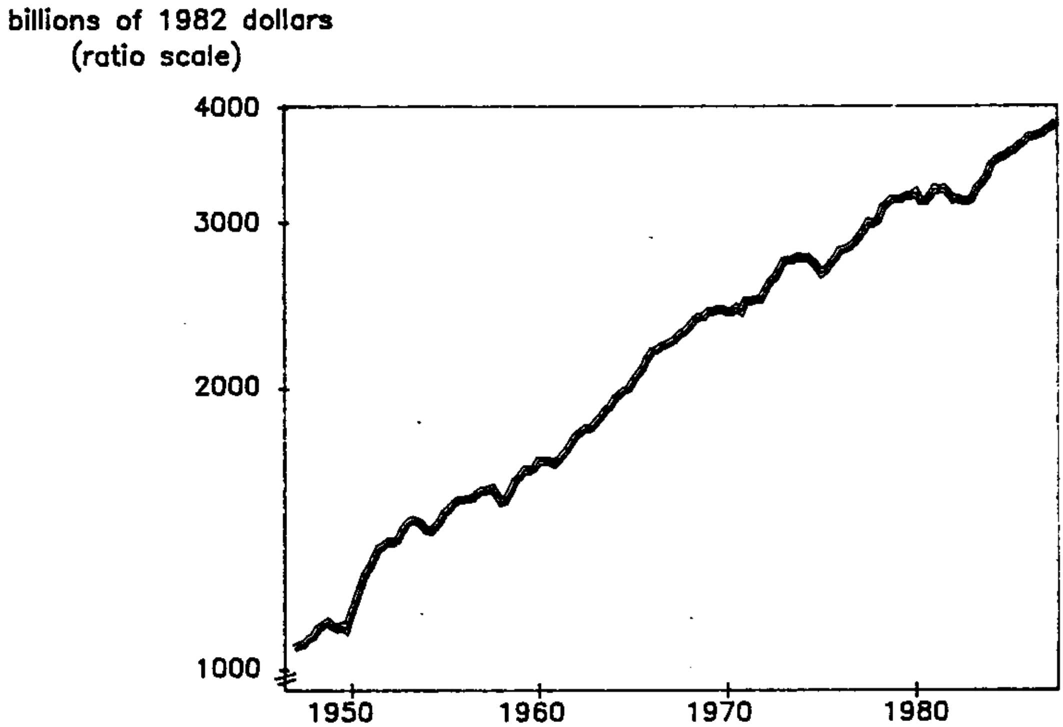


Figure 1: The Log of Real GNP (1947:1–1987:3)

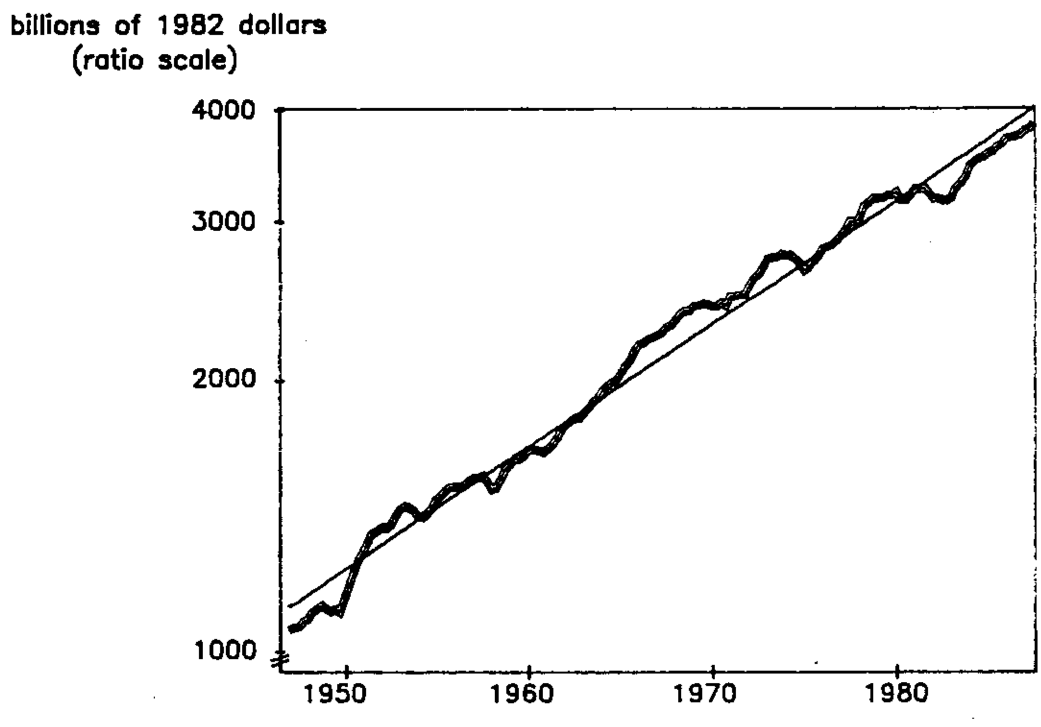


Figure 2: Estimated Trend of Real GNP (1947:1–1987:3)

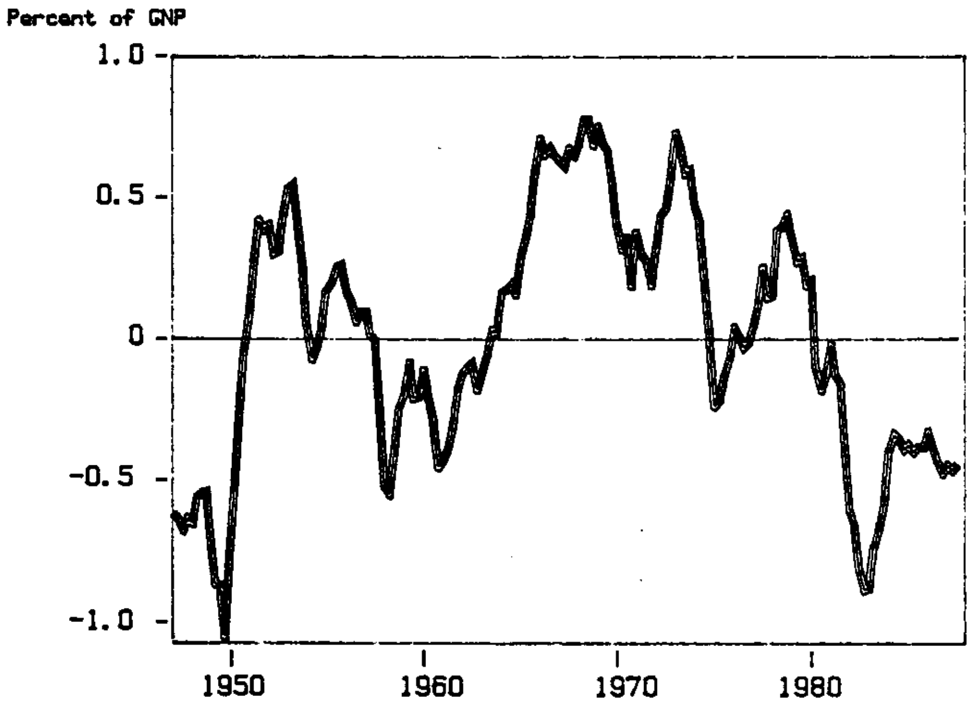


Figure 3: Cyclical Component of Real GNP (1947:1–1987:3)

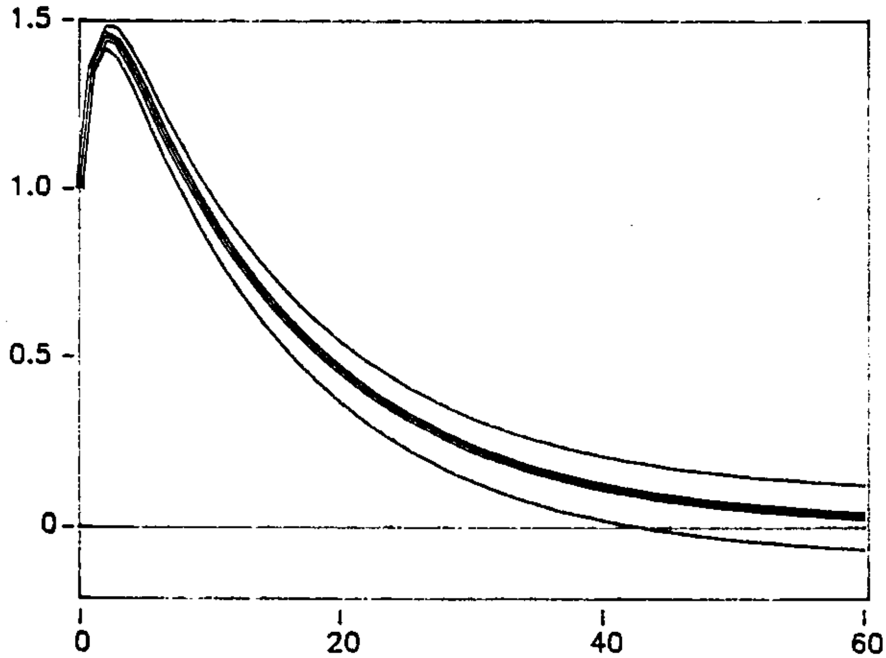


Figure 4: Impulse Response Function of the Cyclical Residuals

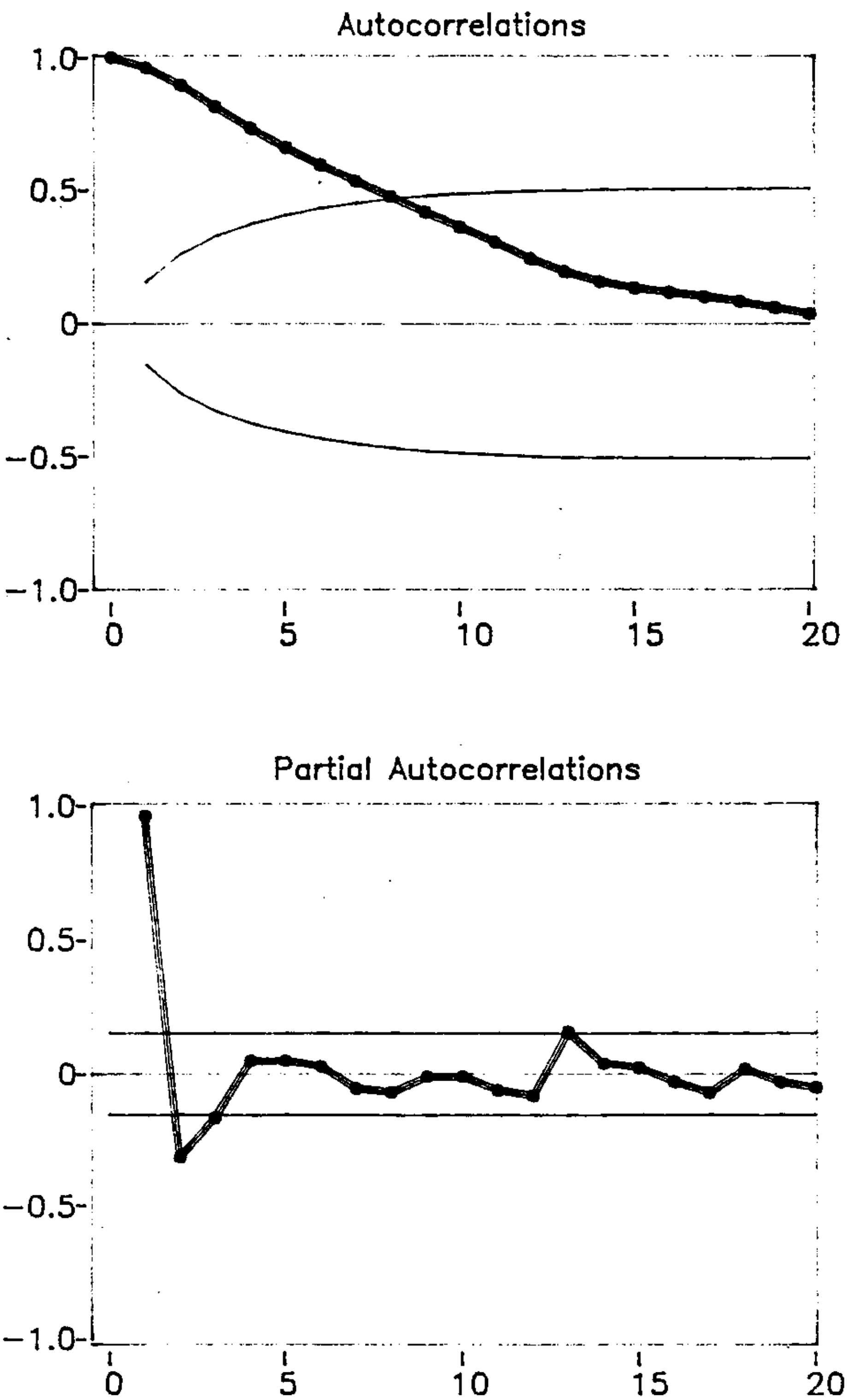
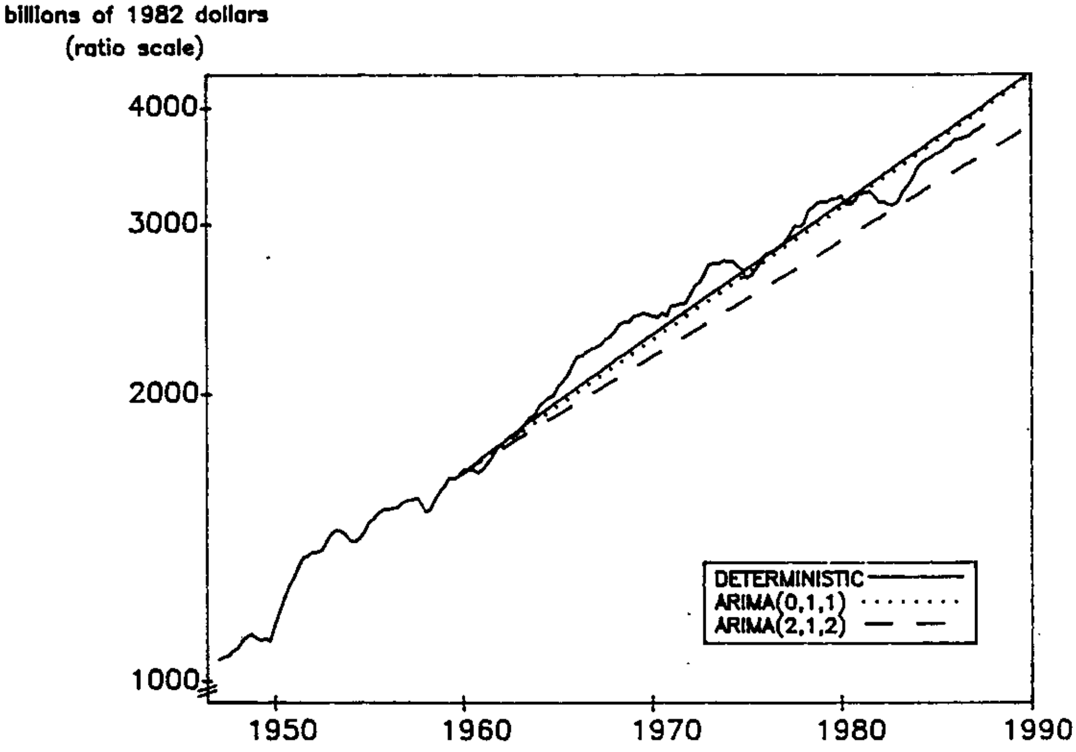
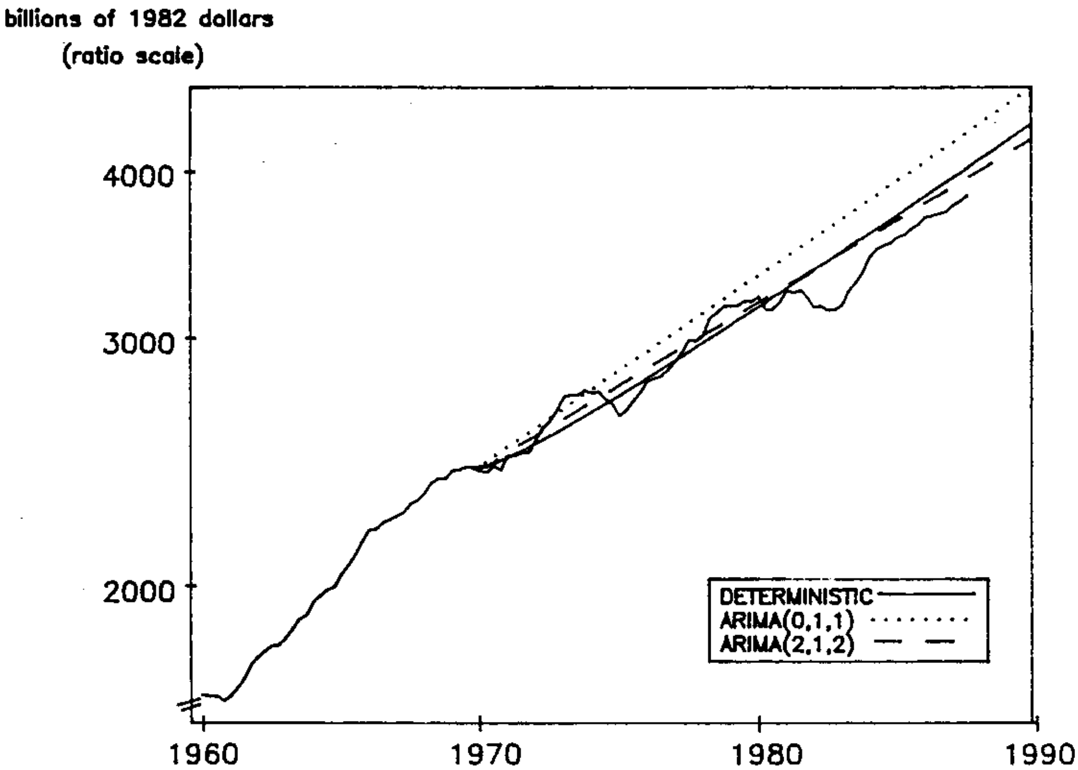


Figure 5: Autocorrelations and Partial Autocorrelations of the Cyclical Residuals



1960–1990



1970–1990

Figures 6a & 6b: Forecasts of the Log of Real GNP

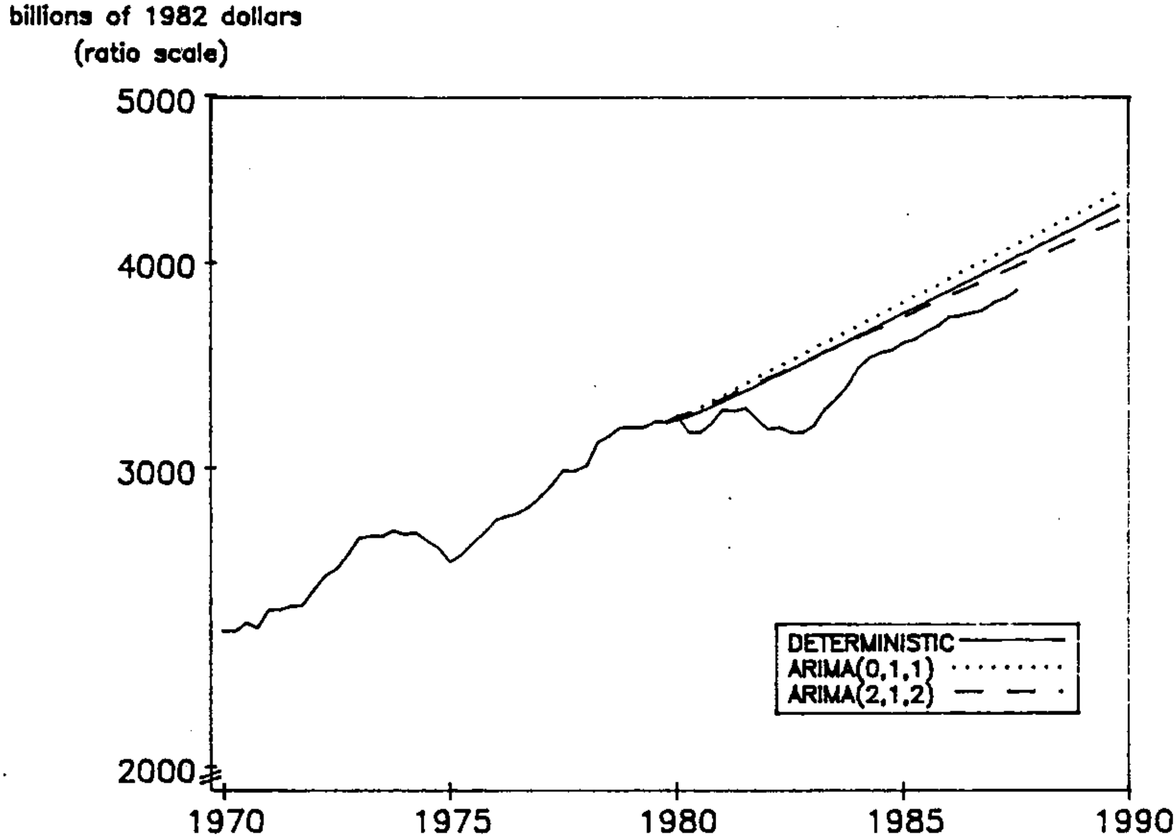
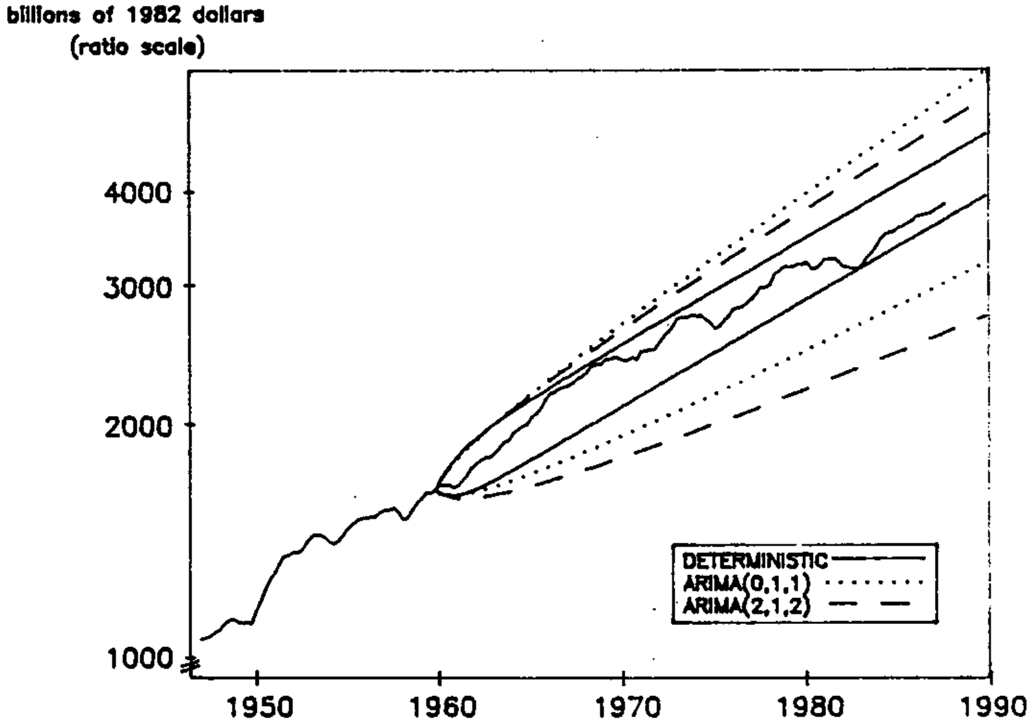
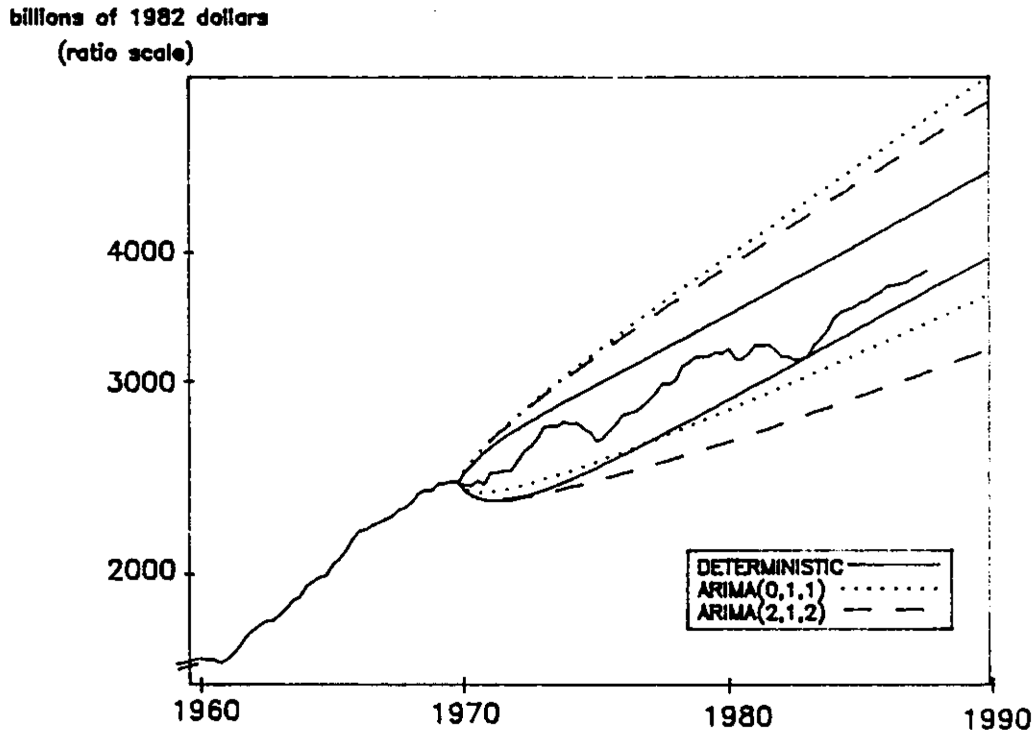


Figure 6c: Forecasts of the Log of Real GNP 1980–1990



1960–1990



1970–1990

Figures 7a & 7b: 95% Confidence Bands

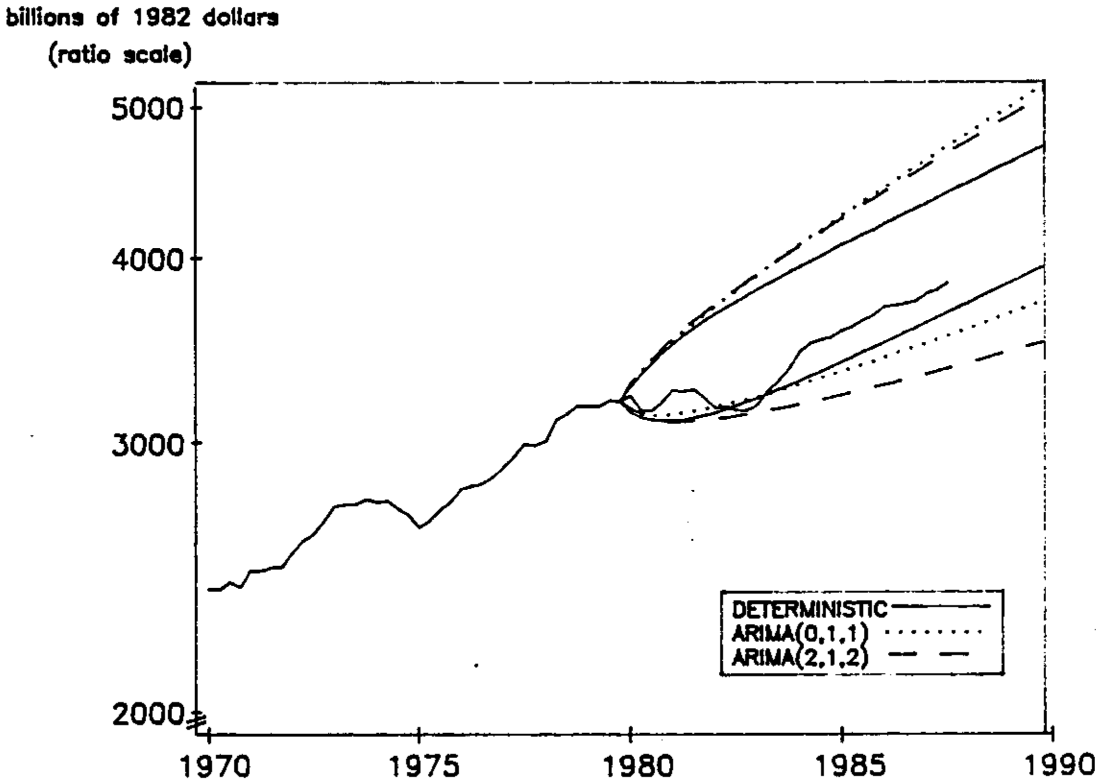


Figure 7c: 95% Confidence Bands 1980–1990

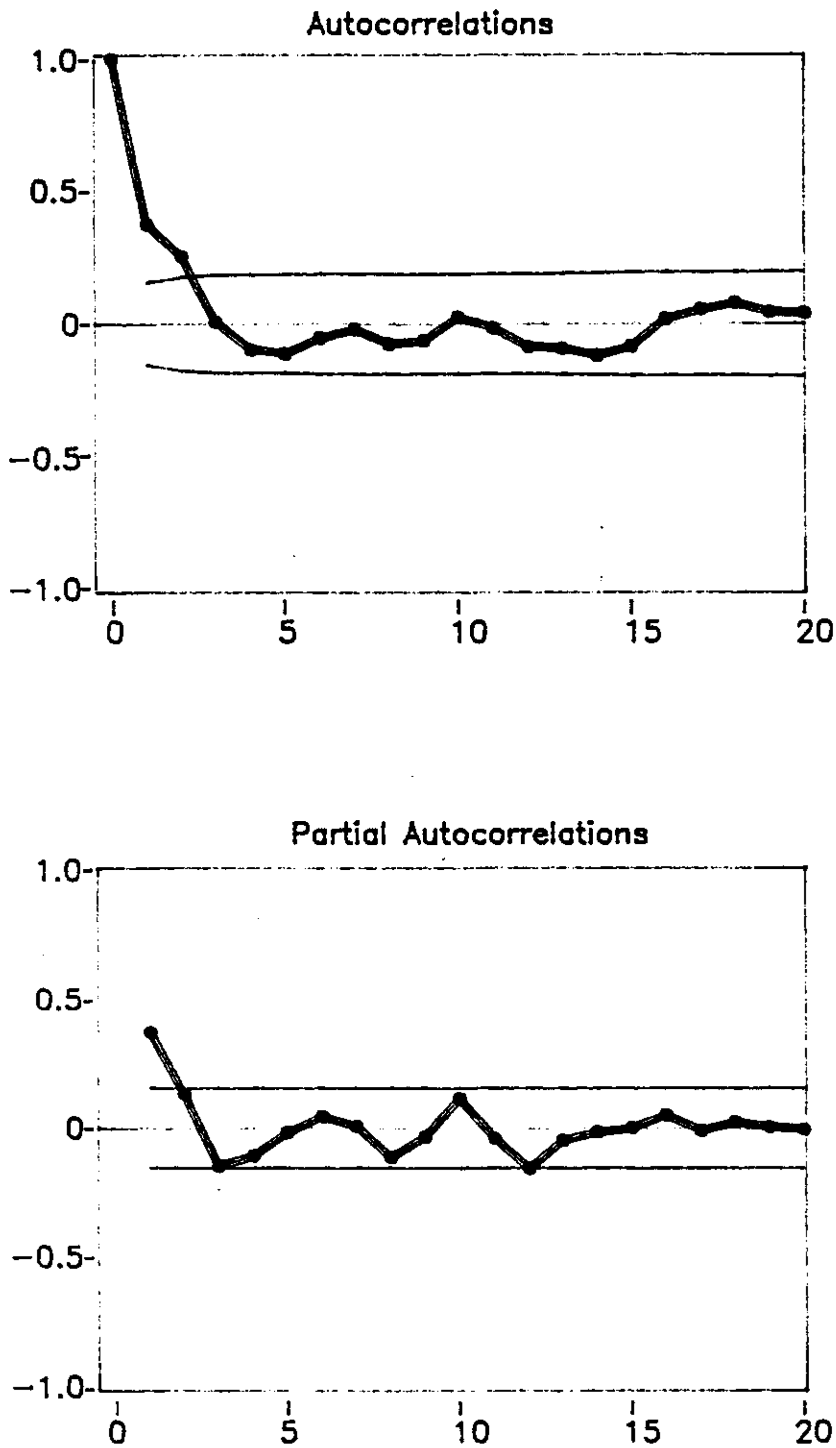


Figure 8a: Autocorrelations and Partial Autocorrelations of $\triangle$ Log Real GNP, Quarterly (1947:1–1987:3)

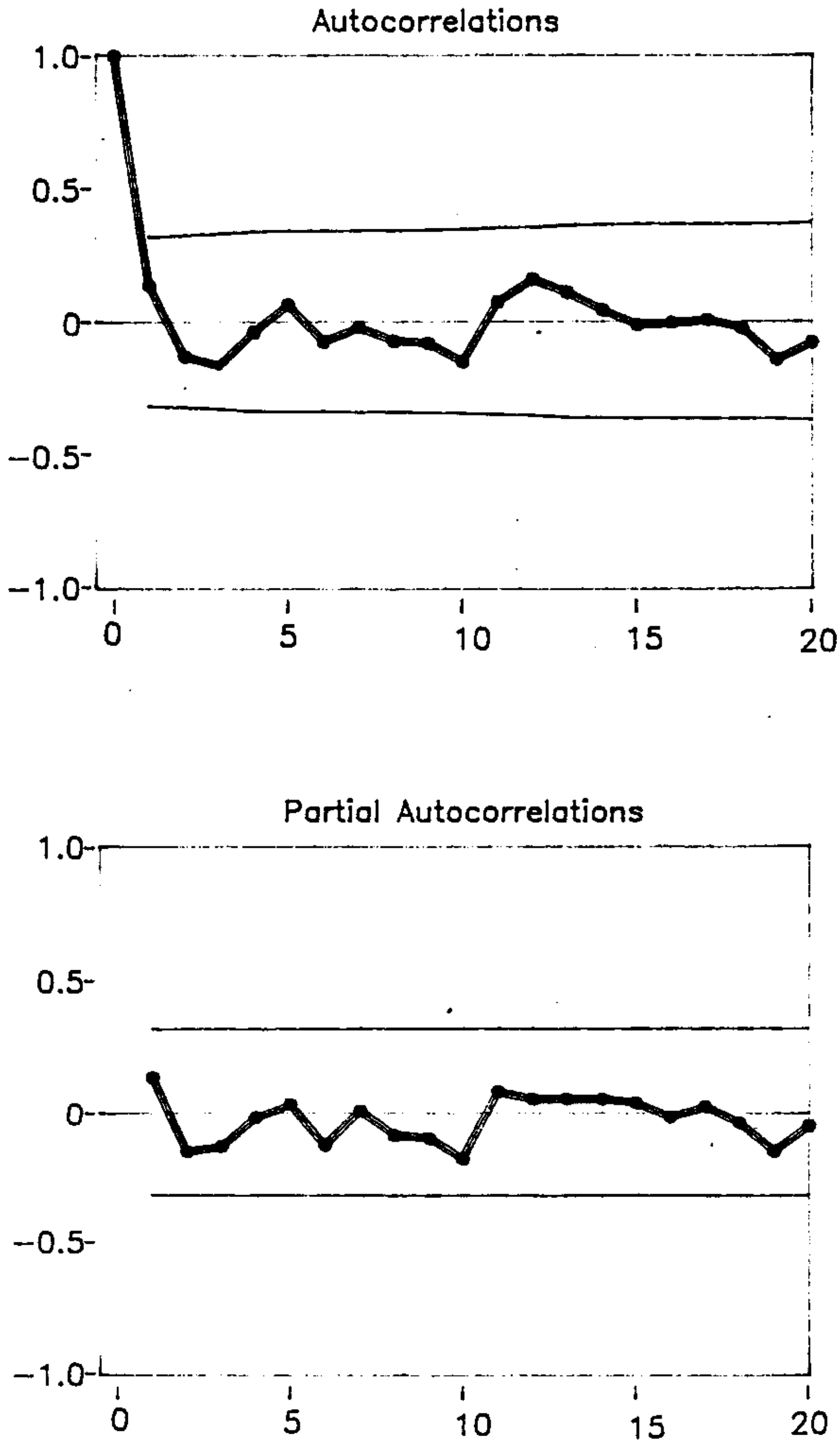


Figure 8b: Autocorrelations and Partial Autocorrelations of $\triangle$ Log Real GNP, Annually (1947–1986)

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