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SEARCH FOR A THEORY OF MONEY

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ABSTRACT

The classical and early neoclassical economists knew that the essential function of money was its role as a medium of exchange. Recently, this idea has been formalized using search-theoretic noncooperative equilibrium models of the exchange process. The goal of this paper is to use a simple model of this class to analyze four substantive issues in monetary economics: the interaction between specialization and exchange, dual fiat currency regimes, the welfare improving role of money, and the susceptibility of monetary economies to extrinsic uncertainty

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I. Introduction

The classical and early neoclassical economists knew that the essential function of money was its role as a medium of exchange. Recently, this idea has been formalized using search-theoretic noncooperative equilibrium models of the exchange process. The goal of this paper is to use a simple model of this class to analyze four substantive issues in monetary economics: the interaction between specialization and exchange, dual fiat currency regimes, the welfare improving role of money, and the susceptibility of monetary economies to extrinsic uncertainty (like "animal spirits" or "sunspots").

The basic framework is similar to the model in Kiyotaki and Wright (1990), which generalizes the standard equilibrium search model to include a large number of differentiated commodities. This makes direct barter difficult, in that it involves Jevons' classic (1875) "double coincidence of wants" problem, and therefore makes for a natural setting in which to study the transactions role of fiat currency. We simplify our earlier model by assuming that an individual derives constant utility from an (exogenous) idiosyncratic set of commodity types. This makes the equilibrium trading strategies for commodities obvious – accept a good if and only if it is within the relevant set – and allows us to proceed almost immediately to the analysis of fiat money.[1]

In Section II we present the basic model, and characterize the set of equilibria from a certain class. When the stock of real money balances is given exogenously, there are exactly three possible nondegenerate outcomes consistent with individual maximization and rational expectations: a

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[1] We also assume in this paper that the production cost distribution is degenerate. These simplifying assumptions were suggested by Robert Lucas, in his discussion of our earlier model at the NBER's Economic Fluctuations Conference at Stanford in February 1989.

nonmonetary equilibrium where fiat currency is without value, a pure monetary equilibrium where fiat currency is the universally accepted medium of exchange, and a mixed equilibrium where fiat currency is only partially accepted.

In Section III we take up the issue of specialization, by facing agents with a trade-off between productivity and marketability. Greater output can be realized by increased specialization, but this reduces the potential market for the output. We find that the pure monetary equilibrium endogenously determines a degree of specialization that is increasing in the stock of real balances. In particular, monetary economies where fiat currency is universally accepted are more specialized and more productive than nonmonetary economies. Furthermore, as we reduce the search frictions in the exchange process to zero, production becomes completely specialized and the ratio of barter to monetary exchange approaches zero.

In Section IV we take up the issue of dual fiat currency regimes, by considering an economy with two types of money that are identical except for some intrinsically irrelevant characteristic, like color. For exogenously given stocks of the two monies, there exists a unique equilibrium (up to a relabeling) with the property that one is universally accepted while the other is only partially accepted. Although the two currencies circulate simultaneously, there is a clear sense in which the generally acceptable money is the superior medium of exchange. Nevertheless, the other is still valued, because it is still easier to trade using a partially acceptable currency than using direct barter in this equilibrium.

In Section V we take up the welfare improving role of fiat currency and the optimal quantity of money. We characterize the exogenous real money supply that maximizes steady state utility, and show how it depends on the diversity of tastes. We also discuss a simple decentralization procedure

for endogenously selecting the quantity of real balances (or the price level, for any given the stock of nominal balances). The procedure we discuss leads to a real quantity of money (or a price level) that does not maximize steady state utility; nevertheless, we show that this outcome is still Pareto optimal.

Finally, in Section VI we consider the possibility of equilibria that depend on some extrinsic random variable called a sunspot. We construct a class of equilibria in which agents accept money in one state of the world and do not accept money in another, even though these states have no effect on the fundamentals of the economy. Over time, the economy randomly cycles back and forth between monetary and nonmonetary exchange. This shows that our model is susceptible to the same type of extrinsic uncertainty that can impinge on some other popular models of fiat money, such as the overlapping generations model.

II. Monetary Equilibria

Consider an economy with a continuum of infinite lived agents having total population normalized to one, and also a continuum of differentiated commodities. Both consumers and commodities are uniformly distributed around a circle of circumference two. Individuals have idiosyncratic tastes: the agent indexed by point i most prefers the good indexed by i and receives utility u(z) from consuming one unit of good ω, where z is the distance between i and ω along the circle, and, in general, $u:[0,1]\rightarrow R$ is a decreasing function. For instance, one can think of position on the circle as representing a characteristic such as color, and utility as decreasing in the difference between a commodity's actual color and a consumer's favorite color.

The key simplifying assumption, compared to the model in Kiyotaki and Wright (1990), is that in this paper we adopt the following specification for preferences,

$$u(z) = \begin{cases} U \text{ for all } z \leq x \\ 0 \text{ for all } z > x, \end{cases} \tag{1}$$

where $U > 0$ and $x \in (0,1)$ are exogenous parameters. Thus, an agent derives the same utility from consuming any commodity within distance x of his ideal commodity, and no utility otherwise. As the distance between a randomly selected commodity and a given agent's ideal commodity is uniformly distributed on [0,1], the parameter x equals the probability that he can enjoy a good chosen at random. Alternatively, x represents the proportion of the population that can enjoy any particular good. Thus, x captures the extent to which either tastes or commodities are differentiated.

In addition to the consumption goods described above, there is an object that no individual ever consumes, because it always provides zero

utility, called *fiat money*. For example, it can be thought of as a collection of pieces of paper with zero intrinsic worth (see Wallace [1980] for a more detailed discussion of the definition of fiat money). Some individuals are initially endowed with this money, and the total supply is fixed at M, where $0 < M < 1$. At least until Section V, all objects including money are assumed to be indivisible and come in units of size one. All objects are costlessly storable, but only one unit at a time; therefore, individuals cannot store money and "real" commodities simultaneously. All objects are also freely disposable.

Agents with nothing in storage produce commodities according to a Poisson process with constant arrival rate $\alpha > 0$ (that is, $\alpha\Delta t$ is the probability of an arrival of a production opportunity in a short interval of time Δt). Each opportunity yields one unit of a good drawn at random from the set of commodities, at a cost we normalize to zero. It is natural to interpret α as labor productivity in the sense that it measures the rate of output per person in the production process. As is standard in the equilibrium search literature (see, e.g., Diamond [1982, 1984]), agents cannot consume their own output; therefore, after they produce, they proceed to an exchange process where they meet potential trading partners pair-wise, according to a Poisson process with constant arrival rate $\beta > 0$.[2] When two agents meet, if mutually agreeable, they swap inventories one-for-one. When

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[2] This is referred to as a *constant returns to scale* matching technology, since it is based on the assumption that the total number of meetings per unit time is proportional to the number of agents looking for a partner, which implies the arrival rate is constant for a representative individual (a similar model that allows for increasing returns to scale, and also non-steady state equilibria, is analyzed in Boldrin, Kiyotaki and Wright [1990], but only for nonmonetary economies). Note that when the set of agents in the exchange sector has measure zero, the arrival rate should be zero, which implies that there will always be a degenerate equilibrium with no one in exchange. We ignore the degenerate outcome for the rest of the analysis.

a real commodity is accepted in trade, however, there is a transaction cost ϵ in terms of utility that must be paid by the receiver, where $0 < \epsilon < U$. For simplicity, we assume a zero transaction cost to accepting fiat money.[3]

A strategy for agent i consists of two parts. First, there is a function $\tau_t^i(\omega,\omega')$ that determines the probability (we allow mixed strategies) i will trade object ω for object ω' at time t, if he has the chance, where ω and ω' are points in a set containing all real commodities and fiat money. Second, there is a function $\gamma_t^i(\omega)$ that determines the probabilities i will consume, dispose of, or continue to store object ω if he has it in stock at t, subject to not consuming his own produce or fiat money. It eases the presentation somewhat to simply assume individuals never dispose of real commodities, but this is actually not restrictive, because it turns out that they will never care to do so. However, it may be desirable in certain circumstances to freely dispose of fiat money, and so we analyze this decision explicitly.

Individuals choose strategies to maximize their expected discounted utility of consumption net of transaction costs, taking as given the strategies of others and the probabilities of meeting agents with various objects at different points in time. Generally, an equilibrium is defined to be a set of strategies together with a meeting probability distribution, such that the strategy of each agent solves his maximization problem given the strategies of others and the meeting probabilities, and consistent with rational expectations. An equilibrium is called monetary if fiat money is

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[3] This is not essential. In Kiyotaki and Wright (1990), we demonstrate that there can exist equilibria with valued fiat currency even if the transaction cost on monetary exchange is larger than ϵ (this is one of the ways in which monetary equilibria are *robust* in these economies). That paper also generalizes the present framework to allow storage costs, random costs of production, and general utility functions.

accepted with positive probability, and nonmonetary otherwise. It is called a pure monetary equilibrium if money is accepted with probability one, which means that it is a universally acceptable medium of exchange, and a mixed monetary equilibrium if money is accepted with positive probability strictly less than one.

In this paper, we restrict attention to equilibria where each agent i uses a trading strategy of the form

$$(2) \qquad \tau_t^i(\omega,\omega') = \begin{cases} a_1(z) & \text{if } \omega \text{ and } \omega' = \text{real commodities} \\ a_m(z) & \text{if } \omega = \text{fiat money, } \omega' = \text{real commodity} \\ \pi & \text{if } \omega = \text{real commodity, } \omega' = \text{fiat money} \end{cases}$$

where z is the distance between ω' and i's ideal commodity type. These strategies are stationary (do not depend on time), satisfy the symmetry condition that only the distance z between i and ω' matters, and have the property that the decision to accept a trade may depend on whether the agent has fiat money or a real commodity, but not on *which* real commodity he has. We will show that if other agents use strategies with these properties, then a representative individual's best response is to do likewise.

To proceed, we formulate the representative agent's best response problem as a dynamic program, for a given expectation of the proportion of traders that have fiat money (as opposed to real commodities), denoted m. He also takes as given that a commodity trader selected at random is willing to accept fiat money with probability Π and that a commodity or money trader selected at random (*not* conditional on type) is willing to accept any real commodity ω with probability A_1 or A_m. In particular, he believes that A_1 and A_m are independent of ω, or that all real commodities are equally acceptable. Of course, all of these conjectures will have to be confirmed in a rational expectations equilibrium.

Let the value function of a producer, a trader with real commodity ω in stock (called a *commodity trader*), and a trader with fiat money in stock (called a *money trader*) be denoted V_0, $V_1(\omega)$, and V_m, respectively. Then Bellman's equations are

$$(3) \qquad rV_0 = \alpha E\left\{\max[0, V_1(\omega) - V_0]\right\}$$

$$(4) \qquad rV_1(\omega) = \beta(1-m)A_1 E\left[\max_{a_1} a_1\left\{\max[V_1(\omega'), u(\omega') + V_0] - \epsilon - V_1(\omega)\right\}\right]$$

$$+ \beta m A_m \max_{\pi} \pi[V_m - V_1(\omega)].$$

$$(5) \qquad rV_m = \max\left\{\beta(1-m)\Pi E\left[\max_{a_m} a_m\left\{\max[V_1(\omega'), u(\omega') + V_0] - \epsilon - V_m\right\}\right],\ rV_0\right\}$$

where $r > 0$ is the subjective rate of time preference. These expressions simply describe the events and decisions that arise in the life of our representative agent.

Equation (3) sets the return to search for a producer equal to the rate at which he locates production opportunities times the expected gain from becoming a commodity trader, if positive. Equation (4) sets the return to search for a trader with real commodity ω equal to the sum of two terms. The first is the expectation of meeting a commodity trader, $\beta(1-m)$, times the probability he is willing to trade, A_1, times the expected value of choosing the acceptance probability, a_1, conditional on the offer ω'. The gain from trading is the maximum value of either storing ω' or consuming it, minus the transaction cost ϵ and $V_1(\omega)$ (note that stationarity implies it is never useful to accept a good, store it for a while, and then consume it). The second term on the right hand side of (4) is the expectation of meeting a money trader, βm, times the probability he is willing to trade, A_m, times

value of choosing the probability of accepting fiat money, π. Finally, (5) sets the return to search for a money trader equal to the maximum of two terms. The first term is the expectation of meeting a commodity trader, $\beta(1-m)$, times the probability he is willing to accept money, Π, times the expected value of choosing the acceptance probability a_m conditional on ω'. The second term indicates V_m cannot fall below V_0, because of the free disposal option.

The following result indicates that V_1 is in fact independent of ω. This should not be too surprising, given that our agent only values ω for its use in exchange, and conjectures that all real commodities are equally likely to be accepted in a random meeting.

Lemma 1: $V_1(\omega) = V_1$ is independent of ω.

Proof: A fundamental theorem of dynamic programming is that value functions are the unique solutions to the functional equations (3)-(5) (see Lucas, Stokey and Prescott [1989], e.g.). This allows us to use a standard "guess and verify" technique: guess that the value functions have the property $V_1(\omega) = V_1$ and verify that this solves Bellman's equations. Now if $V_1(\omega)$ is in fact independent of ω, then so are the decision variables a_1 and π; but if a_1 and π are independent of ω then these functional equations indeed have a solution with V_1 is independent of ω. As the value functions uniquely solve Bellman's equations, we are done. ■

Lemma 1 implies we can ignore ω in equations (3)-(5), which simplifies the problem considerably. Now the value functions are obviously nonnegative and satisfy $V_1 \geq V_0$ and $V_m \geq V_0$ (because of free disposal). The next lemma verifies some other useful inequalities.

Lemma 2: The value functions satisfy

$$U - \epsilon + V_0 > V_1 \tag{6}$$

$$U - \epsilon + V_0 > V_m. \tag{7}$$

Proof: To show (6), suppose by way of contradiction that $U-\epsilon+V_0 \leq V_1$. Then the first term on the right hand side of (4) is necessarily 0. Case i. If $V_m \leq V_1$ then the second term on the right hand side of (4) is also 0, and therefore $V_1 = 0$, which contradicts our supposition $U-\epsilon+V_0 \leq V_1$. Case ii. If $V_m > V_1$, then our supposition implies $V_m > U-\epsilon+V_0$, but then (5) implies $V_m = 0$, which is also a contradiction. This establishes in either case that inequality (6) holds. Inequality (7) is similar. ■

The above inequalities together with (4) imply that a commodity trader will accept a real commodity if and only if it provides him with positive utility,

$$a_1(z) = \begin{cases} 1 \text{ if } z \leq x \\ 0 \text{ if } z > x \end{cases} \tag{8}$$

and that accepted commodities are consumed immediately. Also, at least in the relevant case where $m > 0$ (which requires $V_m \geq V_1$ if it is going to be consistent with equilibrium), these inequalities together with (5) imply that a money trader will accept a real commodity if and only if it provides him with positive utility,

$$a_m(z) = \begin{cases} 1 \text{ if } z \leq x \\ 0 \text{ if } z > x \end{cases} \tag{9}$$

and that accepted commodities are immediately consumed. When all agents use

these trading strategies, the probability that randomly selected commodity or money traders accept any good ω is $A_1 = A_m = \text{prob}(z \leq x) = x$, given the uniform distribution of types.[4]

Based on the above results, we can now write Bellman's equations in a much simplified form:

$$(10) \qquad rV_0 = \alpha(V_1 - V_0)$$

$$(11) \qquad rV_1 = \beta(1-m)x^2(U-\epsilon+V_0-V_1) + \beta mx \max_{\pi} \pi(V_m - V_1)$$

$$(12) \qquad rV_m = \max\left\{\beta(1-m)\Pi x(U-\epsilon+V_0-V_m),\ rV_0\right\}.$$

According to (11), commodity traders expect to meet other commodity traders at the rate $\beta(1-m)$, and consume if and only if each wants the other's good, which occurs probability x^2, while they expect to meet money traders at rate βm, who want their goods with probability x, whereupon they must choose a probability π of accepting money. According to (12), money traders expect to meet commodity traders at rate $\beta(1-m)$, who want their money with probability Π, whereupon they consume with probability x, unless they decide to take advantage of free disposal and immediately switch to production.

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[4] These results imply that real commodities are never accepted only to be traded again later, or that there is no commodity money, in the class of equilibria considered here. This is not to say that there could not be an equilibrium outside of this class, where some commodities are used as media of exchange. A closely related model with a finite number of types that lends itself to the study of commodity money was presented in Kiyotaki and Wright (1989), and has since been extended in Kehoe, Kiyotaki and Wright (1989), Aiyagari and Wallace (1989, 1990), and Marimon, McGratten and Sargent (1990).

This dynamic program describes an individual's choice of π (and the free disposal decision) as a best response to the economy-wide value of Π, conditional on his expectations concerning m. We call this the *conditional best response correspondence*, and write $\pi = \pi(\Pi,m)$. Its properties may be distilled by subtracting (11) from (12) to yield

$$V_m = \max\left\{V_0,\ V_1 + K(\Pi - x)\right\}, \tag{13}$$

where $K = \beta(1-m)x(U-\epsilon+V_0-V_1)/(r+\beta x\Pi) > 0$ for all $m < 1$. Clearly, (11) implies that a commodity trader will accept money if and only if V_m exceeds V_1; therefore, for all $m < 1$, (13) implies:

$$\pi(\Pi,m) = \begin{cases} \{1\} & \text{if } \Pi > x \\ [0,1] & \text{if } \Pi = x \\ \{0\} & \text{if } \Pi < x \end{cases} \tag{14}$$

As shown in Figure 1, the conditional best response correspondence always has three fixed points: $\Pi = 0$, x, and 1.

These fixed points are Nash equilibria conditional on the expectation of m being fixed. In case $\Pi = 0$, money is valueless and will be freely disposed of, which implies $m = 0$. Hence, there exists a unique nonmonetary rational expectations equilibrium. To consider the case with $\Pi > 0$ and $m > 0$, imagine that individuals have some conjecture about the fractions of the population who are producers, commodity traders, and money traders in the economy, say N_0, N_1, and N_m, which they use to compute $m = N_m/(N_1+N_m)$. For this conjectured distribution to equal the true distribution in steady state, it must satisfy the condition that the flow into the trading process equals the flow into the production process,

$$\alpha\cdot(1-N_1-N_m) = \phi(m,\Pi)\cdot(N_1+N_m), \tag{15}$$

where $\phi(m,\Pi) = \beta(1-m)[(1-m)x^2+m\Pi x]$ is consumption per trader per unit time.[5] Simple algebra implies that (15) is equivalent to

$$M = \frac{\alpha m}{\alpha+\phi(m,\Pi)} = \mu(m,\Pi). \tag{16}$$

Thus, expectations are rational if and only if $M = \mu(m,\Pi)$.[6]

Proposition 1: Given $0 < M < 1$, there exists a unique pure monetary rational expectations equilibrium and a unique mixed monetary equilibrium.

Proof: The function $\mu(\cdot)$ defined in (16) satisfies $\mu(0,\Pi) = 0$, $\mu(1,\Pi) = 1$, and $\partial\mu/\partial m > 0$. Therefore, for any $M\in(0,1)$ and $\Pi > 0$ there exists a unique value of $m > 0$ such that $M = \mu(m,\Pi)$. In particular, given M, there is a unique m such that $M = \mu(m,1)$, and therefore there is a unique m consistent with $\Pi = 1$ and rational expectations. Similarly, given M, there is a unique m such that $M = \mu(m,x)$, and therefore there is a unique m consistent with $\Pi = x$ and rational expectations. ■

The pure monetary equilibrium is depicted in Figure 2. Because money is universally acceptable, money traders consume whenever they meet someone

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[5] Note that ϕ is the rate at which a representative trader meets commodity traders, $\beta(1-m)$, times the probability that he has a real commodity and a deal is consummated, $(1-m)x^2$, plus the probability that he has money and a deal is consummated, $m\Pi x$.

[6] We can equivalently define a mapping from expected m to actual m and look for its fixed points, which may seem more direct, but turns out to involve slightly more complicated algebra. In either case, the distribution is completely characterized by one equation in one unknown, even though there are three states, because of the two identities $N_0 = 1-N_1-N_m$ and $N_m = M$.

who has a commodity they want, while commodity traders consume only if they meet someone who has a commodity they want and also wants the commodity they have. This is precisely Jevons (1875) "double coincidence of wants" problem with direct barter, which a universally acceptable medium of exchange avoids. Universal acceptability makes the expected utility of trading using money greater than the expected utility of trading using direct barter; but when the expected utility of trading using money exceeds the expected utility of barter any rational trader will always accept money. Universal acceptability is thereby self-fulfilling.

In the mixed monetary equilibrium, on the other hand, money is accepted with the same probability as a real commodity. This makes agents indifferent between using money and barter, and therefore we can set $\Pi = x$ for all agents, or equivalently, we can set $\Pi = 1$ for a fraction x of the agents and set $\Pi = 0$ for the rest (so that some agents always take money while others always insist upon real commodities). In either case, the use of money does not ameliorate the double coincidence problem. To the contrary, it is easy to see that the mixed monetary equilibrium is isomorphic to the nonmonetary equilibrium of another economy, where the arrival rate in the trading sector is reduced from β to $\beta(1-m)$ (a result that proves quite useful below). Hence, a currency that is only partially acceptable can make trade more difficult than no currency at all.

III. Specialization

An insight dating back at least to Adam Smith is that specialization is limited by the extent of market, and that the use of money encourages specialization by enlarging the extent of market.[7] In order to formalize one aspect of this idea, we introduce a trade-off between the arrival rate of production opportunities and the proportion of agents that can enjoy any particular good: $\alpha = \alpha(x)$, where $\alpha' < 0$. The idea is that specializing by choosing a small value of x reduces the proportion of the population that

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[7] "When the division of labour has been once thoroughly established, it is but a very small part of a man's wants which the produce of his own labour can supply. He supplies the far greater part of them by exchanging that surplus part of the produce of his own labour, which is over and above his own consumption, for such parts of the produce of other men's labour as he has occasion for. Every man thus lives by exchanging, or becomes in some measure a merchant, and the society itself grows to be what is properly a commercial society.

"But when the division of labour first began to take place, this power of exchanging must frequently have been very much clogged and embarrassed in its operations. One man, we shall suppose, has more of a certain commodity than he himself has occasion for, while another has less. The former consequently would be glad to dispose of, and the latter to purchase, a part of this superfluity. But if this latter should chance to have nothing that the former stands in need of, no exchange can be made between them. ... In order to avoid the inconveniency of such situations, every prudent man in every period of society, after the first establishment of the division of labour, must naturally have endeavoured to manage his affairs in such a manner, as to have at all times by him, besides the peculiar produce of his own industry, a certain quantity of some one commodity or other, such as he imagined few people would be likely to refuse in exchange for the produce of their industry." (Smith 1776, 22-23).

Smith goes on to discuss the properties of metals that lead them to be chosen almost universally as media of exchange, focusing mainly on their divisibility. A much more comprehensive early discussion of the various desirable intrinsic properties of money is provided by Menger (1892). In this paper, we abstract from all issues relating to these intrinsic properties and focus exclusively on the *endogenous* property of marketability (what Menger called "saleability"). We also note that Smith suggested individuals should special in production but generalize in consumption. Our model is easily reinterpreted as one in which an agent always produces the same output, but over time consumes all goods, if we assume his ideal good is given by a random draw from the set of commodities after production.

may be willing to consume your output and hence reduces its marketability, but increases productivity in the sense of output per of unit time in the production process.[8]

Assume that, before entering the production process, an individual makes a specialization decision by choosing x, and therefore α, taking as given the decisions of others as reflected in the economy-wide choice X. We restrict attention for the moment to pure monetary equilibria, by looking for equilibria with $\Pi = 1$. By arguments analogous to those in the previous section, Bellman's equations are now

$$(17) \qquad rV_0 = \max_x \left\{\alpha(x)[V_1(x)-V_0]\right\}$$

$$(18) \qquad rV_1(x) = \beta(1-m)Xx[U-\epsilon+V_0-V_1(x))]+ \beta mx[V_m-V_1(x)]$$

$$(19) \qquad rV_m = \beta(1-m)\dot{X}(U-\epsilon+V_0-V_m).$$

Notice that the value function for a commodity trader $V_1(x)$ depends on the specialization decision carried over as a state variable from the production process. In particular, (18) yields

$$(20) \qquad V_1'(x) = \frac{\beta(1-m)X}{r+\beta(1-m)xX+\beta mx}\,[U-\epsilon+V_0-V_1(x)] > 0.$$

The first order condition for the individual's maximizing choice of x given the economy-wide value X is found by differentiating (17),

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[8] An alternative approach would be to assume that specialized output can be sold at a higher price. Our approach (modeling greater specialization as leading to greater productivity) captures the essence of the idea and still allows us to maintain the tractability of the one-for-one exchange framework. King and Plosser (1986) and Cole and Stockman (1987) are other recent attempts to model the interaction between money and specialization.

$$\alpha'(x)[V_1(x)-V_0] + \alpha(x)V_1'(x) = 0. \tag{21}$$

The first term reflects the marginal cost of increasing x, which lowers the arrival rate α, while the second term is the marginal benefit of greater marketability. Substituting (20) into (21) and then setting X = x yields the symmetric equilibrium condition, given the fixed expectation of m. For the sake of illustration, consider the example

$$\alpha = \alpha(x) = x^{-\eta}, \tag{22}$$

where $\eta > 0$. In this case the symmetric equilibrium condition reduces to

$$\eta = \frac{r+x^{-\eta}}{r+\beta(1-m)x^2+\beta mx} = \psi(m,x). \tag{23}$$

If $1+r < \eta(r+\beta)$, there exists a unique $x\in(0,1)$ satisfying $\eta = \psi(m,x)$ for any $m\in(0,1)$; otherwise, x = 1 and every good that is produced is 100 percent marketable, although productivity is low.

In case $1+r < \eta(r+\beta)$, the locus of points in (x,m) space satisfying (23) is downward sloping, as shown in Figure 3. Further, the rational expectations condition can now be written

$$M = \alpha m/(\alpha+\phi) = \tilde{\mu}(m,x) \tag{24}$$

where $\phi = \beta(1-m)[(1-m)x^2+mx]$, as in the previous section. The locus of points satisfying (24) is upward sloping, as shown. Given M, there is a unique point $(m,x)\in(0,1)\times(0,1)$ where the curves cross, and therefore a unique pure monetary rational expectations equilibrium with endogenous specialization. An increase in M shifts the $M = \tilde{\mu}(m,x)$ curve upward. Hence, an increase in the supply of real balances reduces x and increases α.

We conclude that more extensive use of money, by lowering the marginal gain to marketability, leads to an increase in specialization and productivity.

An increase in the arrival rate β shifts both curves in Figure 3 to the left, again increasing specialization and productivity. In fact, as β becomes large, x approaches zero, and specialization becomes complete. As this happens, the ratio of the volume of barter to monetary transactions, $\beta(1-m)^2x^2/[\beta m(1-m)x] = (1-m)x/m$, approaches zero, since the probability of a double coincidence decreases in proportion to x^2 while the probability of a monetary exchange decreases only in proportion to x. Hence, Clower's (1965) observation that, at least in advanced economies, there is apparently very little direct barter – "money buys goods and goods buy money; but goods do not buy goods" – can be modeled endogenously, without imposing the constraint that agents *must* use cash. The useful part about modeling this endogenously is that our model predicts barter will resurface if some change in the economy significantly reduces the desirability of fiat money, such as a tax.

Finally, we recall from Proposition 1 that for any given M the economy actually has three equilibria. The nonmonetary equilibrium is equivalent to the monetary equilibrium when $M \to 0$, and therefore, we can conclude that the degree of specialization in the economy is always greater in pure monetary equilibrium than in its nonmonetary equilibrium. Also, the mixed monetary equilibrium is equivalent to a nonmonetary equilibrium in another economy with a reduced value of β, as discussed at the end of the previous section. Therefore, the degree of specialization is even lower in the mixed monetary equilibrium than in the nonmonetary equilibrium.

We summarize the results of this section in the following statement.

Proposition 2: There exists a unique pure monetary rational expectations equilibrium with endogenous specialization, with $x < 1$ if and only if $1+r < \eta(r+\beta)$. An increase in M increases specialization. An increase in β increases specialization, and the ratio of barter to monetary exchange vanishes as β become large. Given M and β, the nonmonetary equilibrium has less specialization than the pure monetary equilibrium, but more than the mixed monetary equilibrium.

IV. Dual Fiat Currency Regimes

In some economies, we seem to observe more than one type of fiat money in simultaneous circulation; for example, in certain countries, there can be a generally acceptable domestic currency as well as a foreign currency used as media of exchange, although the latter may only be partially acceptable.[9] In this section we investigate the issue of dual fiat currency equilibria, and provide an example where one type of fiat money is generally acceptable while a second is partially acceptable.[10] To simplify the presentation, we only consider the case where α and x are fixed exogenously. The argument is based on the observation that a mixed monetary equilibrium is isomorphic to a nonmonetary equilibrium in another economy with a reduced arrival rate, and there is always room for a generally acceptable medium of exchange in such an economy.

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[9] For instance, it is common for Canadian dollars to circulate just over the U.S. border, and vice-versa, although the foreign currency is certainly not universally acceptable.

[10] The famous bimetalism debates were concerned with whether it is desirable to have one or two monies, and in particular, whether we should use gold, silver, or both. As Kindleberger (1984) describes things, it was thought that gold and silver coins were required to facilitate different size purchases, but if both circulated, stability in their relative prices was necessary for the "efficient discharge of the unit-of-account function" (p. 55). Establishment of official mint prices for the two metals was supposed to stabilize their market prices; more often, however, fluctuations in market prices "destabilized the mint price through the workings of Gresham's law" (p. 56).

Such debates revolve almost exclusively around money's role as a unit of account. Kindleberger reflects the conventional wisdom when he argues that "The medium-of-exchange function of money can tolerate more than one money without too much trouble; the unit-of-account function cannot" (p. 55). Although the equilibrium analyzed in this section has nothing to say about exchange rate instability, it does provide an example where having two monies that are treated asymmetrically has important implications for money's role as a medium of exchange.

Assume that there are now two colors of fiat money, red and blue, whose supplies are fixed at M_R and M_B with $M_R+M_B < 1$. We continue to assume that money is indivisible and the agent can carry only one unit of either red money, blue money, or real commodities at a time. Let m_R and m_B denote agents expectations concerning the proportions of red money traders and blue money traders in the economy, and $m_C = 1-m_R-m_B$ the expected proportion of commodity traders. Let the economy-wide probability of commodity traders accepting red money and blue money be Π_R and Π_B. A representative agent's dynamic programming problem can then be described by the following versions of Bellman's equations:

$$(25) \qquad rV_0 = \alpha(V_1-V_0)$$

$$(26) \qquad rV_1 = \beta m_C x^2(U-\epsilon+V_0-V_1) + \beta m_R x \max_{\pi_R} \pi_R(V_R-V_1) + \beta m_B x \max_{\pi_B} \pi(V_B-V_1)$$

$$(27) \qquad rV_R = \beta m_C x \Pi_R(U-\epsilon+V_0-V_R)$$

$$(28) \qquad rV_B = \beta m_C x \Pi_B(U-\epsilon+V_0-V_B).$$

Note that we are assuming in (27) and (28) that neither money will be disposed of; it is obvious that there will be equilibria where one color of money is not accepted at all, in which case we are back to a one currency regime and the previous results apply. Similarly, it is obvious that there will be equilibria where agents ignore color and treat the two monies perfectly symmetrically, which also reduces to the previous model. We wish to concentrate on *genuine* dual currency regimes, in which both monies circulate but with different acceptabilities. Without loss in generality, assume $\Pi_R > \Pi_B > 0$. Note that this implies red money traders will never accept blue money, and so there is no trade between money holders, as is implicit in (27) and (28). Furthermore, it is clear that if Π_B and Π_R are

both strictly less than one, then $V_R = V_B = V_1$, which can only be true if red money, blue money, and real commodities are all equally acceptable, which means $\Pi_B = \Pi_R = x$. Therefore, the only possible genuine two currency regime is one in which $1 = \Pi_R > \Pi_B > 0$.

One can show for any positive m_R and m_B, when $1 = \Pi_R > \Pi_B > 0$, the conditional best response is to set $\pi_R = 1$ and to set $\pi_B \in (0,1)$ if and only if $V_B = V_1$. Now $V_B = V_1$ if and only if

$$\Pi_B = \left[\frac{r+\beta x m_C+\beta m_R}{r+\beta x m_C+\beta x m_R}\right] x. \tag{28}$$

Notice that $1 > \Pi_B > x$, so that blue money is less acceptable than red money but more acceptable than real commodities.[11] Thus, $1 = \Pi_R > \Pi_B > 0$ is a Nash equilibrium conditional on expectations m_R and m_B.

For these expectations to be rational, they must satisfy the steady state condition

$$\alpha\cdot(1-N_1-M_R-M_B) = \tilde{\phi}(m_R,m_B,\Pi_R,\Pi_B)\cdot(N_1+M_R+M_B) \tag{30}$$

where in this case $\tilde{\phi}(m_R,m_B,\Pi_R,\Pi_B) = \beta m_C x(m_C x+m_R\Pi_R+m_B\Pi_B)$ (which still has the interpretation as the rate of consumption per trader). An equivalent condition to (30) is

$$M_R = \frac{\alpha m_R}{\alpha+\phi(m_R,m_R M_B/M_R,\Pi_R,\Pi_B)} = \bar{\mu}(m_R,M_R/M_B), \tag{31}$$

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[11] This is because a commodity trader always has a chance to trade up to red money, while a blue money trader does not, and so the acceptance probability of blue money has to exceed x in order to equalize V_1 and V_B.

which is one equation in the single unknown m_R.[12] Thus, expectations are rational if and only if $M_R = \bar{\mu}(m_R, M_R/M_B)$.

As in Section II, it can easily be shown that for any M_R and M_B such that $M_R+M_B < 1$, there exists a unique m_R solving (31). This m_R, along with $m_B = m_R M_B/M_R$, constitutes the unique rational expectations equilibrium where red currency is universally accepted while blue currency is accepted by everyone with some probability less than 1 (or equivalently, always accepted by some agents and never accepted by others). As the details are similar to the proof of Proposition 1, we simply state the result.

Proposition 3: For any M_R and M_B such that $M_R+M_B < 1$, there exists a unique dual currency rational expectations equilibrium with $1 = \Pi_R > \Pi_B > x$.

To close this section, suppose that the two monies pay flow yields (in utility per period) described by rates of return R_R and R_B. Then the perturbation argument in Kiyotaki and Wright (1990, Section IV) suggests that it is still possible to construct equilibria with $\Pi_R > \Pi_B$, whether R_R is greater than, equal to, or less than R_B. The case where $R_R < R_B$, but red money is still generally acceptable while blue money is only partially acceptable, is a form of rate of return dominance stronger than that displayed by any model of which we are aware. In this case, agents use one object as a medium of exchange even though it bears a lower rate of return than another object, simply because the former is more readily acceptable. This would seem to answer Hicks' (1935) long standing challenge to explain a phenomena he called the "central issue in the pure theory of money."

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[12] Although there is now an additional state, the distribution can still be characterized by one equation due to the additional identity $m_B = m_R M_B/M_R$.

V. Welfare and the Optimal Quantity of Money

In this section we take up the issue of welfare, and the optimal quantity of money. We begin by defining what seems to be a natural social welfare function,

$$W = N_0V_0 + N_1V_1 + N_mV_m \tag{32}$$

(we also consider the Pareto criterion below). This can be interpreted as average utility across agents at a point in time, or expected utility over a long horizon for a representative agent, not conditional on his current state. It should be clear from our earlier discussions that mixed monetary equilibria are inefficient, and that the inefficiency gets worse when M becomes larger.[13] Thus, we concentrate on pure monetary equilibria. We also fix α and x exogenously, and consider only single currency economies.[14]

With $\Pi = \pi = 1$, equations (10)-(12) can be solved explicitly for the reduced form value functions,

$$rV_0 = \Delta(U-\epsilon)\alpha[rx(1-m)+\phi] \tag{33}$$

$$rV_1 = \Delta(U-\epsilon)(\alpha+r)[rx(1-m)+\phi] \tag{34}$$

$$rV_m = \Delta(U-\epsilon)(\alpha+r)[r(1-m)+\phi] \tag{35}$$

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[13] That is, W is globally decreasing in M in the mixed monetary equilibrium. Since $\Pi = x$ in this case, money does nothing to speed up trade, and simply crowds out real commodities.

[14] In the dual currency regime, the partially acceptable medium of exchange is also inefficient – that is, W is globally decreasing in the stock of blue money – even though blue money is accepted with probability strictly greater than x.

where $\Delta = \beta x/\{(\alpha+r)(r+\beta x)+\beta x[rx(1-m)+\phi]\} > 0$, and $\phi = \beta(1-m)x[(1-m)x+m] > 0$. Substitution of these into (32) implies

$$rW = (U-\epsilon)\alpha\phi/(\alpha+\phi) \tag{36}$$

Notice that $\alpha\phi/(\alpha+\phi) = (N_1+N_M)\phi$ equals aggregate consumption, consistent with W measuring steady state utility. Furthermore, W is a monotonically increasing function of ϕ, the per trader consumption rate.

Proposition 4: Let M^o be the value of the exogenous fiat money supply that maximizes W in pure monetary equilibrium; then $x \geq 1/2$ implies $M^o = 0$, while $x < 1/2$ implies $0 < M^o < 1$.

Proof: We first find the value m^o that maximizes W with respect to m, and then determine M^o from the equilibrium condition $M = \mu(m,\Pi) = \alpha m/[\alpha+\phi(m,\Pi)]$. Recall that $\partial\mu/\partial m > 0$, $\mu(0,1) = 0$, and $\mu(1,1) = 1$; hence, $0 < M^o < 1$ if and only if $0 < m^o < 1$. Now W is maximized when ϕ is maximized, and ϕ is strictly concave in m. Thus, from the derivative

$$\partial\phi/\partial m = \beta x[1-2x-2m(1-x)], \tag{37}$$

we conclude that $x \geq 1/2$ implies $m^o = 0$ and therefore $M^o = 0$, while $x < 1/2$ implies $m^o = (1-2x)/(2-2x)$ and therefore $0 < M^o < 1$. ■

In this model welfare eventually must decrease with M, because at $M = 1$ there are only money traders and no commodity traders in the economy (an extreme case of "too much money chasing too few goods"). This result stems from our assumption that agents cannot store money and real commodities

simultaneously. The less obvious and more interesting part of Proposition 4 is that, in spite of this assumption, which seems to be unfavorable to the use of fiat money, it is still possible for the introduction of money to improve welfare as long as tastes vary enough across people. Exactly how much tastes have to differ is described by the surprisingly simple result that x must be less than 1/2. Furthermore, it is easy to show that a smaller value of x, which makes the double coincidence problem more difficult, necessarily entails a greater value of M^o.

So far we have been assuming that fiat money is indivisible, and therefore the exchange of money for a real commodity is always one-for-one. In order to pursue the welfare issues in more detail, we now modify this assumption so that fiat money is perfectly divisible (while real commodities remain indivisible). We look for equilibria where each money trader carries P units of cash, all P units are required to purchase one real commodity (so that P is the aggregate price level), and real balances are $M = S/P$, given any stock of nominal currency, S. The above discussion has concentrated on characterizing outcomes with M exogenous; but if we alternatively assume that S is exogenous, we need to impose an additional equilibrium condition to pin down P and M. One can then study the welfare properties of any particular method of selecting the nominal price level.

Consider a method used by Diamond (1984) in his cash-in-advance model, which is to impose as an equilibrium condition that the gains from trade for a money trader and a commodity trader are equalized whenever an exchange is is made between them,

$$V_m - V_1 = U - \epsilon + V_0 - V_m. \tag{38}$$

We call this a split-the-surplus equilibrium, since it equally divides the total surplus generated when a money trader meets a commodity trader with a

good the former has need of.[15] If we substitute (33)-(35) into (38), the unique value of m that satisfies this condition is

$$m^* = (1-2x)/(2-2x) - r/2\beta x(1-x), \tag{39}$$

which is positive if and only if $r < \beta x(1-2x)$, a restriction we impose for the remainder of the discussion. This implies $M^* = \mu(m^*,1)$, and therefore, $P^* = S/M^*$ for any exogenous supply of nominal balances S.

Recall that the value of m that maximizes W is $m^o = (1-2x)/(2-2x)$, and therefore, $m^* < m^o$. This means the split-the-surplus equilibrium always yields a lower value of m – thus, a lower value of M, and a higher value of P for any given S – than that which maximizes welfare. However, consider the individual value functions V_m and V_1 (V_0 is proportional to V_1, and need not be considered independently). It is not difficult to check that both are concave, and that V_1 is increasing but V_m is decreasing in m at m^*, as shown in Figure 4. Hence, m^* is optimal according to the Pareto criteria, which only requires that the outcome cannot be revised so as to make some agents better off without making others worse off. In fact, if we let $\underline{m}$ and $\bar{m}$ denote the values of m that maximize V_m and V_1, then $\underline{m} < m^* < \bar{m} < m^o$, as shown in Figure 4, and an equilibrium is Pareto optimal if and only if $m \in [\underline{m}, \bar{m}]$.[16]

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[15] This is meant to suggest that, when commodity and money traders meet, they are faced with a bargaining problem that is resolved according to the Nash solution, although Diamond was not explicit about the value of having more or less than P units of fiat money, or the value of having real goods and money simultaneously.

[16] It may be shown that the nonmonetary equilibrium is Pareto dominated by a monetary equilibrium (i.e., $\underline{m} > 0$) if and only if

$$[\beta x(1-2x)-r]\ (r+\alpha)(r+\beta x) > r\beta^2 x^2 (1-x)^2;$$

It should not be too surprising that our split-the-surplus solution does not maximize W, even though it is Pareto optimal. It is true that if agents could get together at some date before the start of the economy, they would agree that they are all better off at m^o than any other m, in an ex ante expected utility sense. Nevertheless, as soon as they are assigned to initial conditions as either producers, commodity traders, or money traders, they revise their opinion. As shown in Figure 4, at m^o, all agents prefer a different value of m.[17] This type of dynamic inconsistency has been analyzed previously in similar models (see Wright [1986] for an extended discussion in the context of redistributive taxation), and is also related to the distinction between various optimality criteria in stochastic overlapping generations models (see Peled [1985]).

Agents in the model are not interested in average expected utility, as measured by W, but only in their own utility, as measured by the V_j. A decentralized mechanism has little hope of maximizing a welfare criterion that individuals are not interested in. What is a more interesting is whether it fails to achieve an objective that they would all agree is desirable – contemporaneously, and not *ex ante*. An example of a genuine coordination failure of the type celebrated in Cooper and John (1989) is when the economy fails to settle on a medium of exchange at all, or even worse, in our model, when it settles on a partially acceptable money as in a mixed monetary equilibrium. The failure to maximize W certainly seems less

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this condition is certainly true for small r, given that $r < \beta x(1-2x)$ is required for a split-the-surplus monetary equilibrium to exist, although it is not true in general.

[17] In fact, at m^o, all agents actually prefer a lower value of m. This may seem paradoxical, given m^o maximizes W, which is a weighted average of their expected utilities; but it can easily be understood by recalling that the weights N_0, N_1, and N_m themselves change with m.

troublesome, given it is Pareto optimal. In any case, we summarize the properties of the split-the-surplus equilibrium as follows.

Proposition 5: There exists a unique positive equilibrium level of real balances, and therefore a unique finite price level for any exogenous stock of nominal balances, consistent with the split-the-surplus rule as long as $r < \beta x(1-2x)$. It is Pareto optimal, although it does not maximize W.

VI. Stationary Sunspot Equilibria

In this section we demonstrate that our economy is susceptible to the same type of extrinsic uncertainty that can impinge on other models of fiat money (including overlapping generations models). Let there be two discernible states of extrinsic uncertainty, called *sunspot activity*: $s = s_0$ and s_1. Given the system is in state s, it switches to the other state randomly over time according to a Poisson process with parameter λ^s; typically, $\lambda^0 \neq \lambda^1$. By the definition of extrinsic uncertainty, the fundamental structure of the model does not dependent on s. However, in equilibrium, agents' strategies potentially might be influenced by this seemingly irrelevant consideration, and if this is the case, it is referred to as a *sunspot equilibrium*. In such an equilibrium, extrinsic randomness can cause the economy to fluctuate simply by virtue of the fact that agents believe that it will.

For simplicity, we restrict attention to pure strategies. This means that a sunspot equilibrium will involve agents accepting money in one state, say s_1, and not in the other. We are particularly interested in *stationary* sunspot equilibria, where the effect of the extrinsic uncertainty lasts forever. This means that in the nonmonetary state s_0, agents hold onto money and wait for things to turn around (rather than disposing of it). For the sake of illustration, we also restrict attention to the case where production is instantaneous (i.e., the limiting case where $\alpha \to \infty$). This means that $m = M$. It also means that, if money is not accepted in state s_0 and yet money traders do not dispose of it, then agents must be indifferent between bartering and holding onto their currency until the state switches

back to s_1.[18]

Let V_j^k denote the value function of a representative agent, where $j = 1$ or m indicates whether he has a real commodity or fiat money, and $k = 0$ or 1 indicates that $s = s_k$. The best response problem, given that other agents accept money if and only if $s = s_1$, is described by the following version of Bellman's equations:

$$rV_1^1 = \beta(1-M)x^2(U-\epsilon) + \beta Mx \max(V_m^1-V_1^1,0) + \lambda^1(V_1^0-V_1^1) \tag{40}$$

$$rV_m^1 = \beta(1-M)x(U-\epsilon+V_1^1-V_m^1) + \lambda^1(V_m^0-V_m^1) \tag{41}$$

$$rV_1^0 = \beta(1-M)x^2(U-\epsilon) + \beta Mx \max(V_m^0-V_1^0,0) + \lambda^0(V_1^1-V_1^0) \tag{42}$$

$$rV_m^0 = \lambda^0(V_m^1-V_m^0). \tag{43}$$

Note that the way (41) and (43) are written assumes the agent will not dispose of fiat money in either state; it will be shown that this is true as we proceed.

The goal is to verify, under certain restrictions on the probabilities λ^0 and λ^1, that $V_m^1 > V_1^1$ and $V_1^0 = V_m^0 > 0$. This means we have an equilibrium where commodity traders accept money in state s_1 and not in state s_0, and

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[18] If production took time, or otherwise involved some cost, then it would be possible for agents to *strictly* prefer barter to holding money and waiting for the state to switch, and also *strictly* prefer holding money and waiting for the state to switch to disposing of money and producing. In the case where there is a cost $c > 0$ to production, for instance, we have constructed sunspot equilibria with

$$V_0^0 < V_m^0 < V_1^0,$$

which means that commodity traders reject currency but money traders prefer to hold on to it rather than pay the cost of production. With $c = 0$, as in the example in the text, these inequalities must be replaced by equalities.

that money is never disposed of. Assuming that such an equilibrium exists leads to the following condition on λ^0 and λ^1:

$$\lambda^0 = \frac{x}{1-x}(r+\beta x+\lambda^1) \tag{44}$$

Proposition 6: For any λ^1 and λ^0 such that (44) holds, there is a stationary sunspot equilibrium where money is always accepted in state s_1, money is never accepted in state s_0, agents never dispose of money, and the value function satisfy $V_m^0 = V_1^0$ and $V_1^1 < V_m^1$.

Proof: Given λ^1 and λ^0 satisfying the required condition, it is a routine matter to check that these strategies satisfy Bellman's equations and imply the asserted inequalities. ■

We have constructed a class of equilibria with the property that in state s_1 money is universally acceptable, while in state s_0 no one accepts it and money traders simply wait for the system to switch back to state s_1. The rate at which the economy switches from the monetary to the nonmonetary state, λ^1, can be arbitrarily large, as long as the rate at which it switches back, λ^0, is great enough to entice money traders to hold onto the stuff. They do so because as soon as money regains its value they gain the advantage afforded by a generally acceptable medium of exchange, and they are willing to forgo the opportunity of production in anticipation of this event. At the same time, λ^0 has to be sufficiently low so that commodity traders do not want to accept money in state s^0. In the simple version of the model presented here, with free and instantaneous production, these conditions give us the tight relation between λ^0 and λ^1 described by (44). However, in the case where there is a strictly positive production cost, for

a given λ^1 there is a *range* of λ^0 such that equilibrium exists where money traders strictly prefer to wait for the money to regain its value while commodity traders strictly prefer to reject money in favor of barter in the nonmonetary state.

VII. Conclusion

We have presented a simple model of exchange in which Jevons' double coincidence of wants problem leads to a transactions role for fiat currency. Even with the many simplifying assumptions, the model seems useful for addressing several substantive issues. It does seem to describe an important way in which specialization and the exchange process are interconnected. It allows one to explicitly construct rational expectations equilibria where two currencies circulate simultaneously and yet one is unambiguously preferred, a phenomena that is difficult to capture with other approaches, such as overlapping generations models. It makes sharp predictions concerning the welfare improving role of generally and partially acceptable media of exchange. Finally, it can be used to illustrate the way in which extrinsic uncertainty can impinge on monetary economies. The framework presented here is a long way from being applicable to matters of daily policy relevance, but we think there is still something to learn from it.

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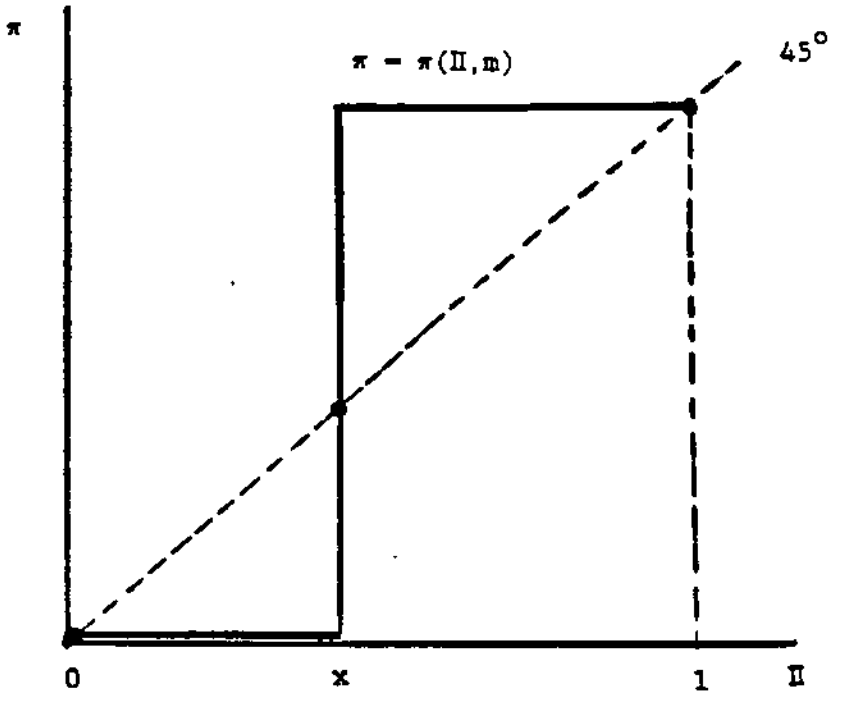

FIGURE 1

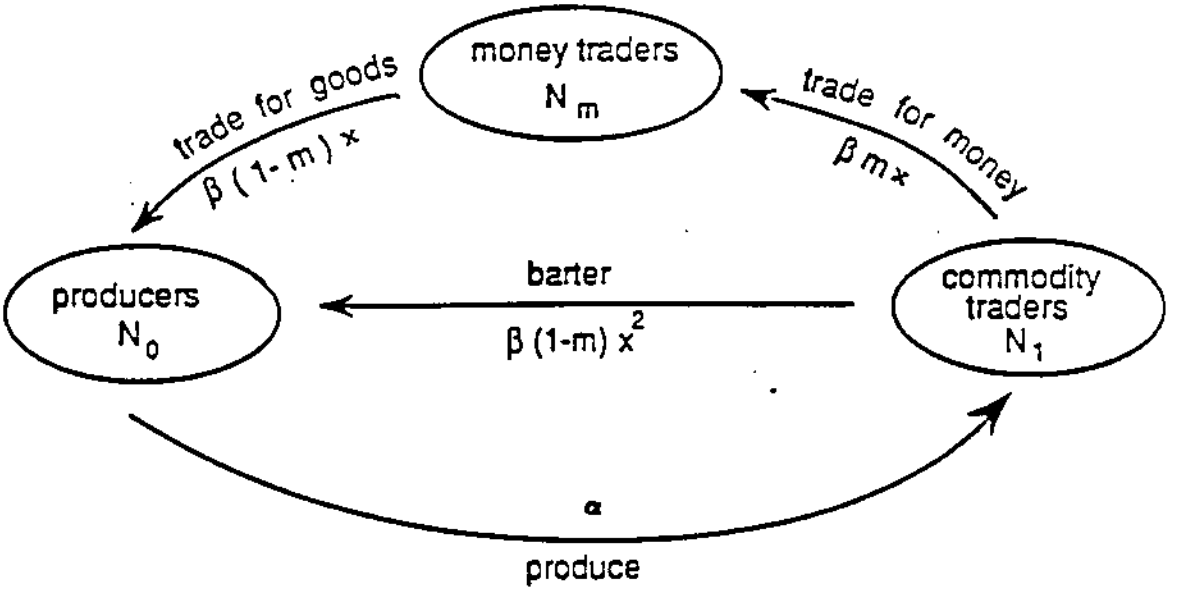

FIGURE 2

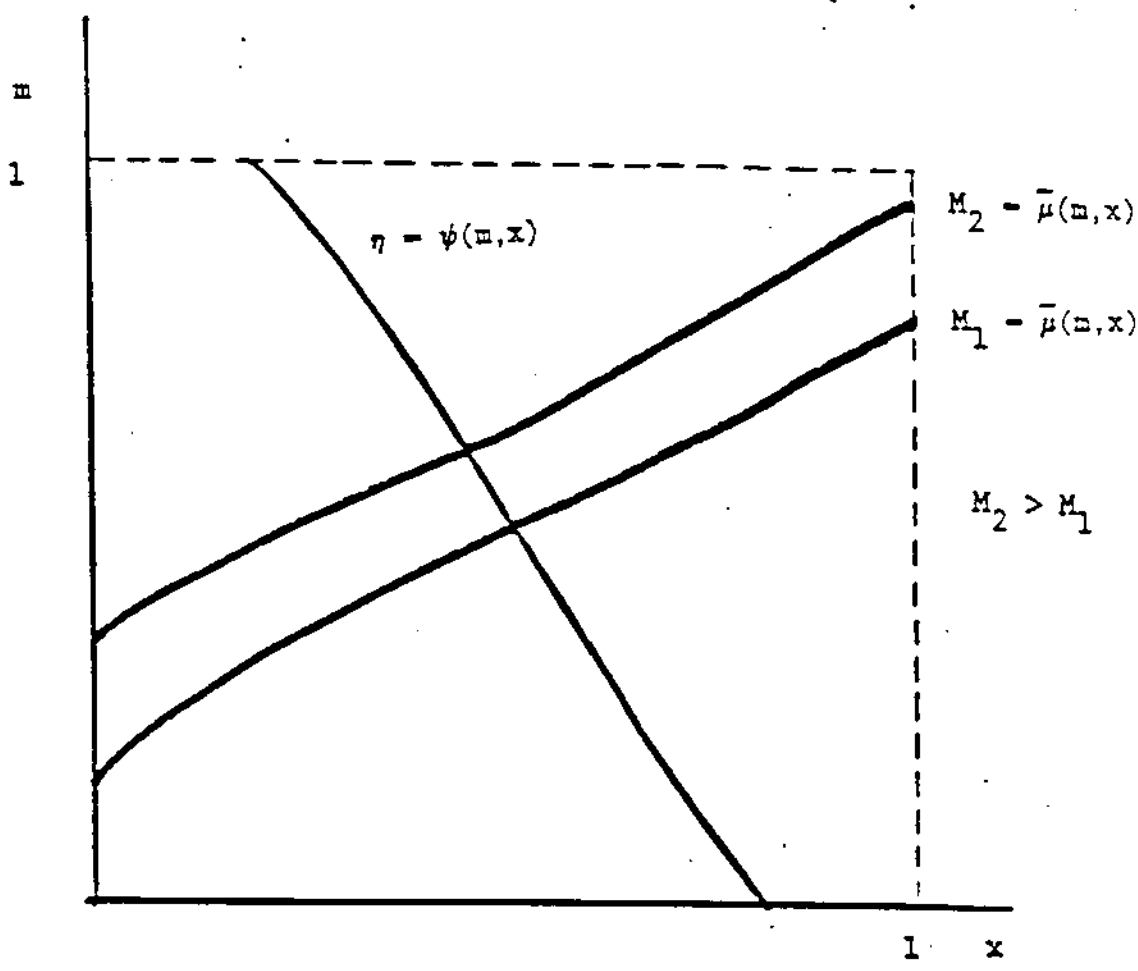

FIGURE 3

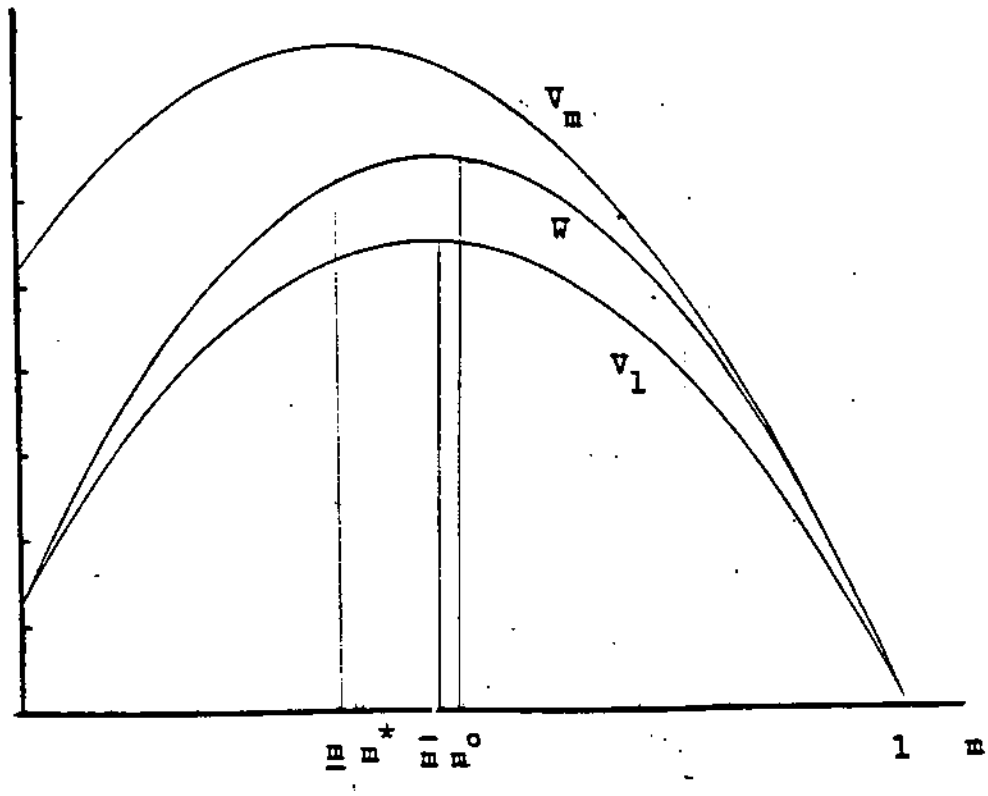

FIGURE 4