On Simplifying the Theory of Fiat Money—Abstract
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This paper argues that versions of Samuelson/Cass-Yaari overlapping-generations consumption-loans models ought to be taken seriously as models of fiat money. The case is made by summarizing and interpreting what these models have to say about fiat money and by arguing that these properties are robust in the sense that they can be expected to hold in any model of fiat money.

Two of the properties establish the connection between, on the one hand, the existence of equilibria in which value is attached to a fixed stock of fiat money and, on the other hand, the optimality of such equilibria and the nonoptimality of nonfiat-money equilibria. Other properties describe aspects of the tenuousness of monetary equilibria in such models: The nonuniqueness of such equilibria in the sense that there always exists a nonfiat-money equilibrium and the dependence of the existence of the monetary equilibrium on the physical characteristics of other potential assets and on other institutional features like the tax-transfer scheme in effect. Rather than being defects of these models, it is argued that this tenuousness is helpful in interpreting various monetary systems and, in any case, is unavoidable; it will turn up in any good model of fiat money. Still other properties summarize what these models imply about the connection—or, better, lack of such—between fiat money and private borrowing and lending (financial intermediation) and what they imply about country-specific monies.

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Most monetary theorists consider themselves as facing two problems: (1) explaining the pattern of exchange; and (2) explaining valued fiat money. As regards the pattern of exchange, the task is to explain why a few objects appear on one side of most transactions. More generally, if for each object we define a transaction velocity—the ratio of the number of units exchanged per unit time to the number of units in existence, a pure number per unit time—the task is to explain the ranking of transaction velocities of the different objects. Put differently, why do a few objects tend to have very high transaction velocities? As regards valued fiat money, the task is to explain how it can be that value is attached to objects that play no direct role in production and are not arguments in anyone's utility function.

There is fairly wide agreement that a satisfactory solution to the pattern-of-exchange problem is to be obtained by adding technological constraints to the standard Walrasian general equilibrium model, constraints that in some way or other make the process of exchange a resource- or utility-using activity. There is also, I think, wide agreement that no good solution has so far been found. But I will not

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in this paper be offering a new solution to the pattern-of-exchange problem. My goal in this paper is to try to convince you to take seriously as models of fiat money various versions of the Samuelson/Cass-Yaari overlapping-generations consumption-loans model.

It is, of course, widely recognized that overlapping-generations models do indeed explain why fiat money is valued. My task is two fold: (1) to summarize and interpret what those models have to say about valued fiat money; and (2) to argue that the implications of those models are robust in the sense that they are likely to hold in models that in a more interesting way confront the pattern-of-exchange problem.

What are these implications? Subject to an important qualification concerning taxation through fiat money issue as part of an optimal tax structure, the main implication is that the government's only role is to make available and maintain a fixed stock or number of pieces of fiat money. There is nothing in those models to suggest that the government ought to maintain the value of fiat money, or that it should regulate financial intermediaries.

This paper is directed primarily to those who would argue that the valued-fiat-money problem cannot fruitfully be taken up without simultaneously taking up and solving in a more interesting way the pattern-of-exchange problem. After all, so this argument would go, a solution to the pattern-of-exchange problem will have implications about the kinds of objects that end up having high transaction velocities, and presumably therefore implications about whether fiat money can be one of those objects. Since the Samuelson/Cass-Yaari models do not contain the kinds of constraints that give rise to an interesting pattern-of-
exchange problem—information constraints, primarily—any conclusions drawn from them should be viewed with great suspicion. 1/

But the overlapping-generations models do confront the pattern-of-exchange problem, although perhaps in a rudimentary and special way. The burden of my argument is that although these models are very specific in the way they confront the pattern-of-exchange problem, their implications about fiat money are general. We do not need to go beyond such models in order to be able to say a great deal about fiat money and the government's role in the financial system.

Secondarily, this paper is addressed to those who think that the two problems I cited at the outset are not problems at all, that monetary wisdom lies in the Employment Act of 1946 and that the remaining problems are largely empirical (e.g., finding the measure of the "money supply" that displays the highest correlation with GNP). Such readers will find in this paper a model of the monetary system that allows them simultaneously to answer questions about regulation of financial intermediaries and international monetary relations, and one that need not be abandoned everytime the private sector comes up with a new form of private debt, whether it be CDs, NOW accounts, or electronic funds transfer.

The paper is organized as follows. After briefly describing two alternative versions of the basic overlapping-generations model—versions that provide some focus for the subsequent discussion—Section I sets out some conjectures about the existence and optimality of equilibria that are characterized by positive prices for fiat money.

1/ Clower, Hahn, and Samuelson (1968) are among those who seem not to take these models seriously as models of fiat money.
Sections II through IV deal with other properties of fiat money equilibria. Section II takes up several aspects of the tenuousness of valued-fiat-money equilibria in the overlapping-generations models. I argue that this tenuousness is not a defect but is both unavoidable and rich in implications for interpreting various monetary systems. Section III takes up private borrowing and lending and, by implication, financial intermediation, while Section IV contains a brief discussion of country-specific fiat monies. The paper concludes with some remarks about currently available alternative ways of approaching problems in monetary theory.

I. Existence and Optimality of Fiat-Money Equilibria

Before setting out some properties that I conjecture hold quite generally, it will help fix ideas to have before us several specific Samuelson/Cass-Yaari structures and some definitions.

As structures, consider the following discrete time economies. At time $t$, $N(t)$ two-period lived members of generation $t$ (age 1 at time $t$) appear. Each is endowed with some amount of nonstorable period $t$ labor and each maximizes the expected value of $u[c^h_1(t), c^h_2(t)] - c^h_j(t)$ is age $j$ consumption of member $h$ of generation $t$—where $u$ is a smooth, well-behaved utility function. As possible technologies for production of the single consumption good we have:

Model A—period $t$ labor equals output so that each age 1 person is endowed with output. There is a common linear technology: $k$ units of period $t-1$ output can be transformed into $x(t)k$ units of period $t$ output, where $x(t) > 0$ is in general a random variable. $N(t)/N(t-1) = n > 0.$
Model B—period t labor and land, a fixed and unchanging quantity of which is available, are inputs in the production of non-storable output via a smooth and in general stochastic production function. N(t) = N.

As an initial condition, the current old are endowed with all the "assets." In all versions, one of these assets is some "stuff" whose quantity, for the moment, we take to be fixed for all time. This "stuff" is distinct from anything else in the models and, hence, by implication satisfies the properties we assign to fiat money: It plays no direct role in production and is not an argument in anyone's utility function. In Model A we also endow the current old with some output stored from the previous period while in Model B we endow them with all the land. In Model B, the old get both the rental of land and the proceeds from selling it to the young.

I define a perfect foresight competitive equilibrium (henceforth, simply equilibrium) to be sequences beginning at the arbitrary current date—one sequence for each of the endogenous variables—consistent with optimizing price-taking behavior, period-by-period market clearing, and equality between subjective and objective distributions. A nonfiat-money equilibrium is one in which at every date the "stuff" has no value in terms of the consumption good at that date, while a fiat-money equilibrium is any other equilibrium. I will assume in the discussion that follows that all the models under consideration have structures

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2/ If the model is stochastic, there is one sequence for each of the endogenous variables for each realization, a realization being one possible infinite sequence of drawings.

3/ If the "stuff" has value at any date, then it has value at every date.
consistent with the existence of a unique nonfiat-money equilibrium.

I next want to define an optimal allocation. An allocation describes lifetime consumption for the current young and future generations and second-period consumption of the current old. Allocation A is Pareto superior to allocation B if A is strictly preferred to B by someone—someone currently alive or some member of some future generation—and is judged inferior to B by no one. An allocation is optimal if there does not exist an allocation that is technologically feasible and Pareto superior to it.  

I can now state two conjectures.

Conjecture I: There exist fixed-supply fiat-money equilibria if and only if the nonfiat-money equilibrium is nonoptimal.

Conjecture II: Among fixed-supply fiat-money equilibria, that which attaches the highest value to the "stuff" is optimal.

Model A may but need not have fiat-money equilibria. If x is constant, then there exists a fiat-money equilibria if and only if x ≤ n. In Model B there does not exist a fiat-money equilibria. The land of Model B dominates the "stuff" even if its rental and price are stochastic and even if its equilibrium rate-of-return distribution ranges over negative outcomes.

If the model is stochastic, an allocation describes what each person gets for each realization. But how should an allocation be judged? In particular, what should condition the expectation in computing expected utility. For example, in Model B if the young (the workers) appear after the random draw that determines the technology for the period, then any risk sharing between the old (the landowners) and the young is ruled out. In such circumstances, if allocations are judged by an expectation of utility that is not conditioned on this random draw, then any market scheme is nonoptimal. If the expectation is conditional on this random draw, then one must proceed by calling one allocation Pareto superior to another if the former is weakly preferred for each possible value of the conditioning information.

For a proof of this last assertion, see Wallace.
Conjectures I and II are consistent with multiple fiat-money equilibria. While perhaps in some sense not likely, multiple fiat-money equilibria can occur. As is well known, Model A with \( x = n \) possesses a continuum of stationary equilibria: the fraction \( \alpha \), say, of a fixed amount of savings that is in the form of storage of the consumption good can be anything in the interval \([0,1]\). All equilibria with \( \alpha < 1 \) are fiat-money equilibria and all equilibria with \( \alpha > 0 \) are nonoptimal. In addition, in many of these models it seems difficult to rule out fiat-money equilibria in which the value of fiat money converges to zero.

II. The Tenuousness of Fiat-Money Equilibria

The overlapping-generations models seem to imply that (absent restrictions of various kinds) the demand for fiat money is extremely tenuous. Is this a defect? I think not. This tenuousness is not only unavoidable but is suggestive in the sense that it is helpful in interpreting observations. Three aspects of this tenuousness will be discussed.

1. Nonuniqueness

One aspect of the tenuousness of valued fiat money in the overlapping-generations models is that there always exists a nonfiat-money equilibrium. Should we not have as a goal a model with a unique equilibrium, where that equilibrium is monetary in the sense of the last section, i.e., attributes value to fiat money? Absent governmental restrictions, such uniqueness would not seem to be achievable. If we maintain as properties of fiat money that it not appear as an argument of utility functions or as an input into production in the ordinary sense, then any well-specified model must have an equilibrium in which fiat money has no value because a person's utility does not depend on the amount of valueless fiat money held.
This nonfiat-money equilibrium should not, though, be thought of as being a "barter" equilibrium. In a model with technological constraints that make exchange costly, a nonfiat-money equilibrium could well display a transaction velocity pattern among different objects in which one, or a small number of objects is playing a medium-of-exchange role. Similarly, one can have overlapping-generations models in which many markets operate in the nonfiat-money equilibrium. Only in very simple versions of these models is the nonfiat-money equilibrium autarkic.

2. Fiat Money and the Technology

The particular overlapping-generations models set out above make existence of a fiat-money equilibrium depend on the physical characteristics of other potential assets. Thus, in Model A if the storage technology is too productive, there will not exist a fixed-supply fiat-money equilibrium. And in Model B there does not exist a fixed-supply monetary equilibrium. Should we not therefore look beyond these models to other models of fiat money, models that would allow fixed-supply fiat-money to coexist along with, say, the land of Model B or productive storage in Model A? Two comments are in order.

The first is in the nature of a conjectured theorem: The search for such models will be fruitless. In other words, one will not get valued fiat money to coexist along with physical assets that are as attractive as consumption goods that are constantly and costlessly appreciating or the land of Model B.

The second comment is in the nature of an observation. The challenge posed by this question seems to be in a sense empty. Most of monetary experience is inconsistent with the existence of physical assets with these properties. The land of Model B is Henry George land.
It is homogeneous and in exogenous supply. For some purposes it may be a useful abstraction to assume that there exists a homogeneous nonreproducible form of capital. But such an abstraction is disastrous for explaining why Henry George's tax proposal has not been implemented, or for explaining why titles to land have not played a prominent commodity money role; i.e., titles to land have not historically been among the objects with the highest transaction velocity.

Indeed, I am tempted to go further and argue that there is no stuff found in nature with the asset properties of either the land of Model B or the "stuff" of any of those models. In my view, those objects that have played the commodity-money role have been objects that have to some extent homogeneity and a fixed supply. But it seems likely that a well-managed fiat money could approximate the fixed-supply property better than could any natural object.

3. Fiat Money and Alternative Institutions

A third sense in which valued fiat money is tenuous in the overlapping generations models is that other institutions may substitute for it. As Samuelson emphasized, a tax-transfer scheme can substitute for it. Posing a seemingly more serious challenge to my interpretation, Earl Thompson has argued that there are private incentives to create a store of value and that a "social contrivance," to use Samuelson's phrase, is not needed. But this I think is misleading.

Consider, say, Model A, with \( x < n \) for all \( t \), so that the nonfiat-money equilibrium is nonoptimal. Every person alive at \( t \) would like to be the sole issuer of a store of value. There are, as it were, as many fiat-money equilibria as there are possible issuers. In fact, there are many more than that. Suppose that one person issues some
number of pieces of red paper and that another person issues some pieces of green paper. Then in any equilibrium, each red piece must exchange for the same number of green pieces in every period. But for how many green pieces must it exchange? Name any nonnegative number and there is a corresponding equilibrium. How do we settle on one of these equilibria? It may be begging the question to say that there must be a social decision, but it is certainly the case that the usual appeal to free entry and private incentives does not seem to determine an equilibrium. This is important because if Thompson were right, then there would be no reason to interpret the "stuff" of the overlapping-generations model as fiat money.\footnote{Friedman argues that the only competitive solution is commodity money--fiat money that is worth no more than the paper on which it is printed. Klein's notion of competitively produced money is flawed by his failure to carefully distinguish between fiat and commodity money. Commodity or convertible money can be produced competitively (and need not be regulated). See the discussion of financial intermediaries below.}

The fact that a tax-transfer scheme can substitute for fiat money should not, I think, disturb us. In general, unconstrained tax-transfer schemes can achieve any feasible allocation and there is every reason to think that this result would hold in virtually any model.

III. Private Borrowing and Lending and Financial Intermediation

In the overlapping-generations models described in Section I, there is no trade among members of the same generation. This is because all members of the same generation have the same tastes and endowments. But it is easy enough to formulate models in which differences among members of the same generation give rise to trade among them, trade that can resemble dealings in securities markets. Thus, for example, consider...
a version of model A with no storage (that is, \(x=0\)), but in which some of the young are endowed with first-period consumption and others are endowed with second-period consumption.

If the aggregate endowment is tilted sufficiently toward first-period consumption, then this model will have a fixed-supply fiat-money equilibrium. Valued fiat money will coexist along with private borrowing and lending. Moreover, if the second-period endowment is stochastic, then individual portfolios will be diversified in a determinate way. But there is no reason to adjust the rule for managing fiat money because there is private borrowing and lending. There is every reason to believe that conjectures I and II hold for such models. In this sense, private borrowing and lending is not a substitute for the existence of fiat money.

But in other senses internal debt and fiat money are substitutes. Thus, the fixed-supply fiat-money equilibria for closed economies that differ only in the degree to which endowments are tilted toward first-period consumption display an inverse relationship between the amount of private debt and the value of fiat money. Also, the fiat-money equilibria of alternative closed economies that differ only in the way fiat money is managed display an inverse relationship between the value of fiat money and its rate of return. The differences in the equilibria result from differences in the rates at which fiat money is being created via a lump-sum transfer scheme. And, finally, in any such model various restrictions on private borrowing and lending—for example, an outright prohibition on private borrowing and lending or a tax on it—enhance the demand for fiat money. But so what. The fact that private debt substitutes for fiat money in these senses does not justify government interference with private debt creation.
And what does all this have to do with financial intermediation? In my view, everything. Why not regard various forms of intermediation as ways of carrying out private borrowing and lending? According to such a view, proper management of fiat money does not call for regulation of the financial intermediation industry.

This is not to say, though, that technological payments mechanism developments have no effect. In particular, they are very likely to affect the value of a fixed stock of fiat money. But what follows from that? Such changes would seem to be analytically equivalent to a trend in the degree to which aggregate endowments are tilted toward second-period consumption. In either case, a fixed-supply fiat-money equilibrium could display a falling value of fiat money, something we have come to call inflation. And in either case, nothing need be done about it. We can, though, understand why a monetary authority charged with maintaining the value of fiat money would seek to impose various restrictions on financial intermediation. What is not easy to understand is why maintenance of the value of fiat money should be a goal.

IV. Country-Specific Fiat Monies

Here I will briefly indicate what the overlapping-generations models have to say about country-specific fiat monies.\(^7\)

\(^7\)/This is not to say that we do not need a theory of the firm. In fact, there would seem to be a close connection between a deep theory of the structure of exchange and a deep theory of the firm, one that follows Coase’s suggestion for a theory that allocates interactions between market interactions and nonmarket interactions. For a model of an intermediary, see Townsend.

\(^8\)/The details of such an analysis are spelled out in a forthcoming paper by John Kareken and me entitled, "Samuelson’s Consumption Loan Model with Country-Specific Fiat Monies."
Roughly speaking, the key implication is that one fiat money is enough. Separate demands for country-specific monies—that is, demands consistent with the different monies being imperfect substitutes—are entirely the result of government rules, capital controls and other rules that make it difficult for residents of one country to hold and use fiat money issued by another country. Absent such restrictions, the indeterminacy in the relative values of two or more fiat monies referred to above in the discussion of free entry into fiat money issue holds in the case of several national fiat monies. Put differently, the demand for a particular fiat money in a world of several such monies is extremely tenuous.

Is this implication to be taken seriously? The way to avoid this implication is to get a model that generates separate demands for country-specific fiat monies. But can that be done? Could a model built from postulates about preferences and the technology produce separate demands for country-specific fiat monies? By now, you know my answer.

V. Concluding Remarks

Most of my message has been in the nature of predictions about the implications of still-to-be-developed models of money. I have argued that you ought to take seriously as models of fiat money Samuelson/Cass-Yaari overlapping-generations models because their implications are likely to carry over to other models of money. I now want to take a slightly different tack and argue that the Samuelson/Cass-Yaari models are the only currently available models of money.

What are the currently available alternatives? There seem to be two: One can begin either by postulating demand functions for money
or by postulating that money is an argument of utility or production functions. The inherent weaknesses of either approach are revealed by posing some hypothetical (or, perhaps, not so hypothetical) questions. What would be the demand, say, for a new money issued by a newly independent province of Quebec? Does that demand depend on whether the money is convertible on demand into a commodity or another currency? Does it depend on the system of exchange controls that is imposed? The Samuelson/Cass-Yaari overlapping-generations models can address these questions. The currently available alternative approaches cannot.\(^9\)

I would like to close by emphasizing an important qualification to virtually everything I have said. Throughout this paper I have ignored the role of fiat money issue as part of an optimal tax structure. If we are to come to grips in a positive sense with the different monetary policies followed by different countries, then that role of fiat money cannot be ignored. In future work, I hope to show that versions of Samuelson/Cass-Yaari overlapping-generations models can confront questions of optimal tax structure in a setting that allows fiat money issue, interest bearing debt issue, and ordinary forms of taxation to be alternative possible ways of financing government expenditures. If we assume that any form of taxation causes distortions, then to no one's surprise we will find that some rate of fiat money issue is a feature of an optimal tax structure.

\(^9\)/More generally, are the postulated demand functions invariant over the range of policy regimes most analyses attempt to study? And what of the way money is assumed to enter utility and/or production functions.
References


