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Abstract

We explore quantitatively the possibility of multiple equilibria in a model of sovereign debt crises. The source of multiplicity is the one identified by Calvo (1988). This type of multiplicity has been at the heart of the policy debate through the recent European sovereign debt crisis. Key for multiplicity in the model is a stochastic process for output featuring long periods of either high or low growth. We calibrate the output process in the model using data for the southern European countries that were exposed to the debt crisis. We find that expectations-driven sovereign debt crises are empirically plausible, but only in periods of stagnation. Multiplicity is state dependent: in periods of stagnation and for intermediate levels of debt, interest rates may be high for reasons unrelated to fundamentals.

Keywords: Self-fulfilling debt crises, sovereign default, multiplicity, good and bad times, stagnation.

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1 Introduction

How important are expectations in triggering sovereign debt crises? In this paper, we explore the quantitative implications of a model of sovereign debt crises that exhibits state-dependent multiplicity. The mechanism we consider to generate multiplicity is the one proposed by [Calvo \(1988\)](#) in which high interest rates induce high default probabilities that in turn justify the high rates. We show that the mechanism is quantitatively relevant. We build on [Lorenzoni and Werning \(2013\)](#) and [Ayes et al. \(2018\)](#), which argue that the mechanism in [Calvo \(1988\)](#) is of interest when the fundamental uncertainty is bimodal, with both good and bad times.

Our analysis of self-fulfilling equilibria in interest rate spreads is motivated by two particular episodes of sovereign debt crises. The first one is the recent European sovereign debt crisis that started in 2010, when most of the countries involved were facing a prolonged stagnation. The peak of the crisis was in the summer of 2012 and receded substantially after the policy announcements by the European Central Bank (ECB) in September of that year. The spreads on Italian and Spanish public debt, which were very close to zero since the introduction of the euro and until April 2009, were higher than 5 percent by the time the ECB announced the Outright Monetary Transactions (OMT) program. They were considerably higher in Portugal, Ireland, and Greece, particularly in the latter two cases. With the announcement of the OMT, according to which the central bank stands ready to purchase euro area sovereign debt in secondary markets, the spreads in most of those countries slid down to less than 2 percent, even though the ECB did not actually intervene. The potential self-fulfilling nature in the events leading to the high spreads in the summer of 2012 was explicitly used by the president of the ECB to justify the policy.¹

The second episode is the Argentine crisis of 1998–2002. Back in 1993, Argentina had regained access to international capital markets, but the average country risk spread on dollar-denominated bonds for the period 1993–1999, relative to the US bond, was 7 percent. The debt-to-GDP ratio was roughly 35 percent, very low by international standards, and the average yearly growth rate of GDP was around 5 percent. Still, the Argentine government defaulted in 2002, after four years of a long recession. Note that a 7 percent spread on a 35 percent debt to GDP ratio amounts to almost 2.5 percent of GDP on extra interest payments per year.² Accumulated over the 1993–1999 period, this represent 15 percent of GDP, or almost half of the debt-to-GDP ratio of Argentina in 1993. An obvious question arises: Had Argentina faced lower interest rates, would it have defaulted in 2002?

¹See [De Grauwe and Ji \(2013\)](#) on the poor correlation between spreads and fundamentals during the European sovereign debt crisis.

²This calculation unrealistically assumes one-period-maturity bonds only. Its purpose is just to illustrate the point in a simple way.

The main contribution of this paper is to show that the mechanism that generates multiplicity in [Calvo \(1988\)](#) is quantitatively plausible. Key for multiplicity is a bimodal output growth process, with persistent good and bad times. We modify an otherwise standard sovereign default model to incorporate an endowment growth rate process that is governed by a Markov chain with persistent high and low growth regimes. To calibrate the model, we estimate this output process for a set of countries that have recently been exposed to sovereign debt crises. We show that the calibrated model features self-fulfilling debt crises, which resemble the episodes just described.

In our calibrated model, there are equilibria in which interest rates can be high or low, depending on expectations. This happens only if fundamentals are weak. It is only in times of low and persistent growth that spreads can be high because of expectations. In the high-growth regime, the region of multiplicity is either empty or negligibly small. Thus, the multiplicity we compute is state dependent: expectations can trigger a crisis only when growth is low.

Jumps in interest rates may happen even absent exogenous changes in expectations. In this economy with a bimodal distribution for the growth process, the schedule of interest rates faced by the borrower can also exhibit large jumps due to fundamentals. It follows that jumps in interest rates are not necessarily a sign that a bad-expectations equilibrium is in the making. The discrete jumps, due to either fundamentals or expectations, induce policy responses by the borrower that can be interpreted as endogenous austerity. The borrower optimally refrains from increasing debt in order to avoid the costs associated with those jumps. This endogenous austerity is featured in some of the equilibrium simulations discussed in the paper.

Our model follows the quantitative sovereign debt crises literature that grew out of the work of [Eaton and Gersovitz \(1981\)](#) and was further developed by [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#).³ In these models, a single borrower faces a stochastic endowment and issues non-contingent debt to a large number of risk-neutral lenders. There is no commitment to repay. Timing and choice of actions are important: the assumptions in [Aguiar and Gopinath \(2006\)](#) or [Arellano \(2008\)](#) are that the borrower moves first and chooses the level of non-contingent debt at maturity. We make two main changes to the standard setup. First, we assume that the borrower chooses current debt rather than debt at maturity. This assumption is key to generating multiplicity. When the borrower chooses debt at maturity, it is implicitly choosing the default probability and therefore also the interest rate on the debt. Instead, when the borrower chooses current debt, default may be likely if interest rates are high or unlikely if interest rates are low.⁴

³Other related literature includes [Aguiar and Amador \(2018\)](#), [Aguiar et al. \(2014\)](#), [Bocola and Dovis \(2016\)](#), [Cole and Kehoe \(2000\)](#), [Conesa and Kehoe \(2017\)](#), [Corsetti and Dedola \(2014\)](#), [Lorenzoni and Werning \(2013\)](#), and [Roch and Uhlig \(2018\)](#), among others.

⁴[Ayres et al. \(2018\)](#) show that the timing of moves is also key. In particular, when lenders are first movers, there is multiplicity regardless of whether the borrower chooses current or future debt.

One fragility of the multiplicity mechanism in [Calvo \(1988\)](#) is that, for commonly used distributions of the endowment process, the high-rate schedule is downward sloping, meaning that the interest rates that the country faces decrease with the level of debt. That is not the case, as shown in [Lorenzoni and Werning \(2013\)](#) and [Ayres et al. \(2018\)](#), if the endowment is drawn from a bimodal distribution with good and bad times. Our second change to the standard setup, which is empirically founded, is that the exogenous endowment process is drawn from such a distribution. Specifically, we allow the endowment to follow a Markov-regime-switching process that alternates between persistent high and low growth. We view this distribution of the endowment process as reflecting the likelihood of relatively long periods of stagnation in a way that is consistent with the evidence in [Kahn and Rich \(2007\)](#). We emphasize the plausibility of long periods of stagnation as drivers of the multiplicity. To calibrate the bimodal distribution for the endowment process with periods of stagnation, we estimate a Markov-switching regime for the growth rate of output for Argentina, Brazil, Italy, Portugal, and Spain. The model is shown to be consistent with a sovereign debt crisis unraveling, triggered by coordinated expectations. This happens in periods of low growth when debt is neither too low nor too high.

The central results of this paper, that expectations-driven sovereign debt crises are empirically plausible, can contribute to the assessment of the role of policy in sovereign debt crises. It is when fundamentals are weak that a lender of last resort may be called in, not because fundamentals are weak but because the weak fundamentals create conditions for a role of expectations. Of course, the role of the lender of last resort in periods of stagnation will have effects on the economy beyond those periods in which interest rates could be high because of expectations.

The paper closest to ours in its motivation is [Lorenzoni and Werning \(2013\)](#). They show that bonds of long maturities are essential in generating the multiple equilibria they analyze. Our quantitative exploration is focused on the characteristics of the endowment process, as estimated using data of countries exposed to sovereign debt crises. In order to isolate this effect, we consider a model with one-period debt only.

The paper proceeds as follows. In [Section 2](#), we discuss a simple two-period model to show the key role played by the bimodal distribution in generating multiplicity. In this case, we can derive analytical expressions that highlight the importance of each of the few parameters in the model, and provide intuition for the results in the more complex quantitative model of [Section 3](#). There we also describe the calibration procedure, including the estimation of the endowment process. In [Section 4](#), we discuss the model results and summarize the robustness exercises. [Section 5](#) contains concluding remarks.

2 A two-period model

Here we illustrate the main mechanisms of the model in a simple two-period case. The economy is populated by a representative agent that draws utility from consumption in each period and by a continuum of risk-neutral foreign lenders. The initial wealth of the agent is denoted by ω . The endowment in the second period is distributed according to

$$y_2 = \begin{cases} y^l, & \text{with probability } p \\ y^h, & \text{with probability } (1 - p) \end{cases}$$

in which $y^l < y^h$.⁵

The representative agent preferences are given by $u(c_1) + \beta \mathbb{E}u(c_2)$, where u is strictly increasing, strictly concave, and satisfies standard Inada conditions. We assume that the initial wealth and the discount factor β are low enough so that the agent will want to borrow. In period one, the borrower moves first and issues a non-contingent debt level b . Lenders respond with an interest rate R . We denote by $R(b)$ the interest rate schedule faced by the borrower. In period two, after observing the endowment y_2 , the borrower decides whether to pay the debt or to default. In case of repayment, the borrower consumes the endowment net of debt repayment, $c_2 = y_2 - Rb$. In case of default, the borrower repays a fraction κ of the debt and consumes $c_2 = y^d - \kappa b$, $y^d < y^l$. The agent defaults if the cost of repayment is larger than the benefit:

$$\underbrace{(R - \kappa)b}_{\text{cost of repayment}} > \underbrace{y_2 - y^d}_{\text{benefit of repayment}}. \quad (1)$$

In the first period, given initial wealth ω and an interest rate schedule $R(b)$, the borrower solves the following problem:

$$\begin{aligned} V(\omega) &= \max_b \{u(c_1) + \beta \mathbb{E}u(c_2)\}, \\ \text{subject to } c_1 &= \omega + b, \\ c_2 &= \max \{y_2 - R(b)b, y^d - \kappa b\}, \end{aligned} \quad (2)$$

and is subject to a maximum debt level constraint, $b \leq \bar{B}$.

The assumption that the borrower moves first by choosing a level of debt and that lenders move next with an interest rate schedule is standard. We depart from the literature, as in [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#), in that we assume that the borrower chooses current debt b rather than debt at maturity, Rb . The risk-neutral lenders will be willing to lend to the agent as long as the expected return is the same as

⁵The discrete distribution will help make clear the main mechanisms for multiplicity. We owe this to an insightful discussion by Fernando Alvarez.

the risk-free rate R^* , that is,

$$R^* = h(R; b) \equiv [1 - \Pr(y_2 - y^d < (R - \kappa)b)] R + \Pr(y_2 - y^d < (R - \kappa)b) \kappa, \quad (3)$$

in which $h(R; b)$ is the expected return to the lender when the interest rate is R . Given a value for b , the expected return for lenders can be written as

$$h(R; b) = \begin{cases} R, & \text{if } R \leq \frac{y^l - y^d}{b} + \kappa \\ R(1 - p) + p\kappa, & \text{if } \frac{y^l - y^d}{b} + \kappa < R \leq \frac{y^h - y^d}{b} + \kappa \\ \kappa, & \text{if } R > \frac{y^h - y^d}{b} + \kappa. \end{cases} \quad (4)$$

In Figure 1, we plot the expected return as a function of the interest rate R , for three levels of debt, together with the risk-free rate R^* . Notice that for low levels of R , the expected return is equal to R since debt is repaid with probability one. In this region, as R increases, the expected return increases one to one. Eventually, R will be high enough that the borrower will default in the low output state, which happens with probability p . At this point, the expected return jumps down. As R increases, the expected return increases at a lower rate, $(1 - p)$, since repayment happens only in the high output state. Finally, for high enough R , default will happen with probability one, and the expected return will be the recovery rate κ . A higher level of debt decreases the expected return uniformly, shifting the curves downward.

For low levels of debt, there is only one solution to equation (3), with $R = R^*$. For intermediate levels of debt, there are two solutions: one solution has $R = R^*$ associated with a zero probability of default, and the other has $R = (R^* - p\kappa)/(1 - p)$ associated with a probability of default equal to p . For higher levels of debt, the only solution is the high rate $R = (R^* - p\kappa)/(1 - p)$. Finally, for even higher debt, there is no solution. There are multiple solutions only for intermediate levels of debt.

We can now define the following correspondence relating debt levels to interest rates:

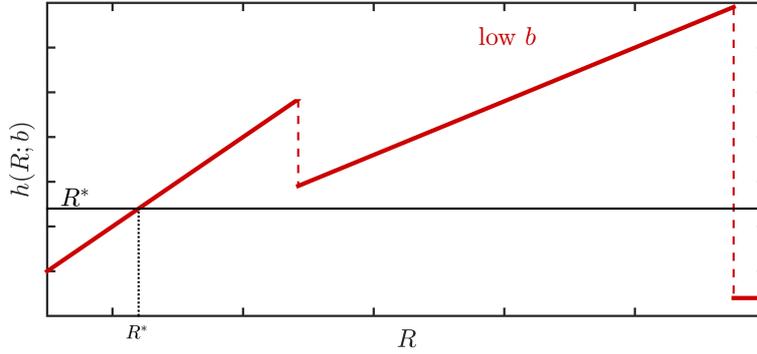
$$\mathcal{R}(b) = \begin{cases} R^*, & \text{if } b \leq \frac{y^l - y^d}{R^* - \kappa} \\ \frac{R^* - p\kappa}{1 - p}, & \text{if } (1 - p) \frac{y^l - y^d}{R^* - \kappa} < b \leq (1 - p) \frac{y^h - y^d}{R^* - \kappa} \\ \infty, & \text{if } b > (1 - p) \frac{y^h - y^d}{R^* - \kappa} \end{cases} \quad (5)$$

An *equilibrium* is an interest rate schedule $R(b)$ and a debt policy function $b(\omega)$ such that, given the schedule, the debt policy function solves the problem of the borrower in (2), and the schedule $R(b)$ is a selection of the correspondence $\mathcal{R}(b)$.

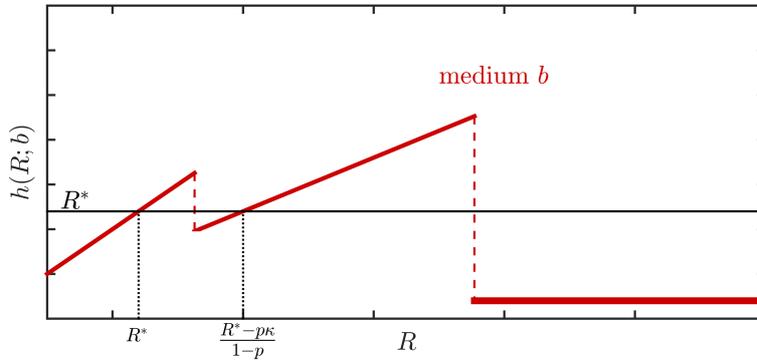
The correspondence $\mathcal{R}(b)$ is plotted (red line) in Figure 2. For all debt levels below $b_1 \equiv (1 - p) \frac{y^l - y^d}{R^* - \kappa}$, there is only one interest rate, the risk-free rate. For debt levels between b_1 and $b_2 \equiv (1 - p) \frac{y^h - y^d}{R^* - \kappa}$, there are two possible interest rates, the risk-free rate and a high rate.

Figure 1: Expected return function for different levels of debt

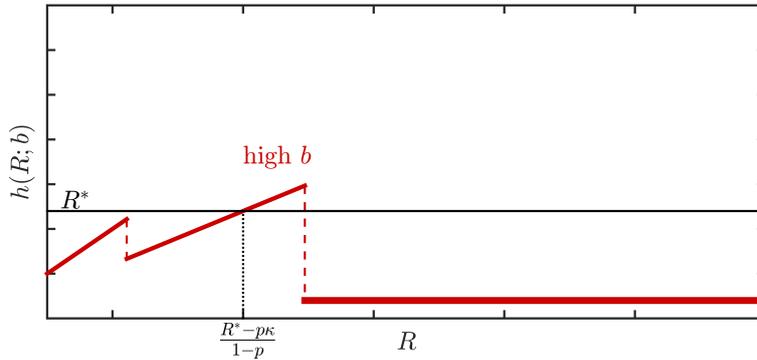
(a) low debt



(b) medium debt



(c) high debt



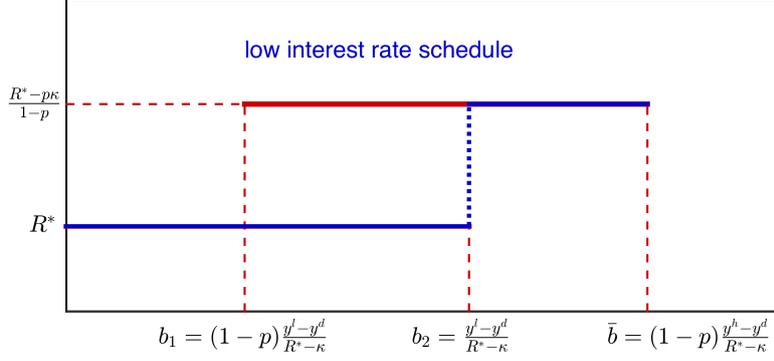
For debt levels between b_2 and $\bar{b} \equiv (1 - p) \frac{y^h - y^d}{R^* - \kappa}$, there is again only one interest rate, the high rate. There are multiple interest rate schedules that can be selected from this correspondence. We focus on two of those schedules: a low interest rate schedule, $R^{low}(b)$ in Figure 2a (blue line), and a high interest rate schedule, $R^{high}(b)$ in Figure 2b (blue line).

We think of b_1 as the debt level above which interest rates jump because of expectations, since alternative expectations could sustain low interest rates. We think of b_2

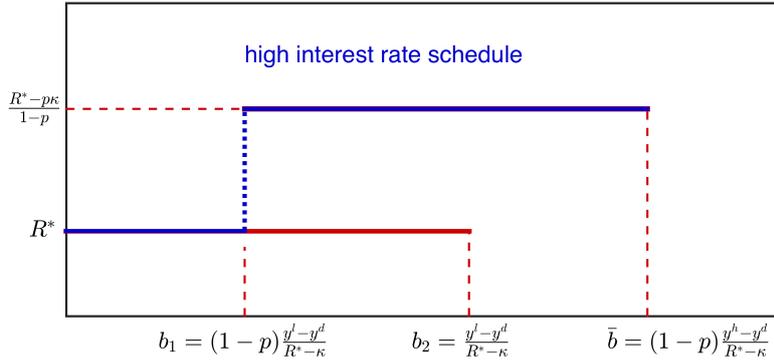
as the debt level above which interest rates jump because of fundamentals, since no expectations could sustain lower interest rates. We think of \bar{b} as an endogenous borrowing limit, since any debt issued above this level implies a default probability of one.

Figure 2: Interest rate schedules

(a) low interest rate schedule



(b) high interest rate schedule



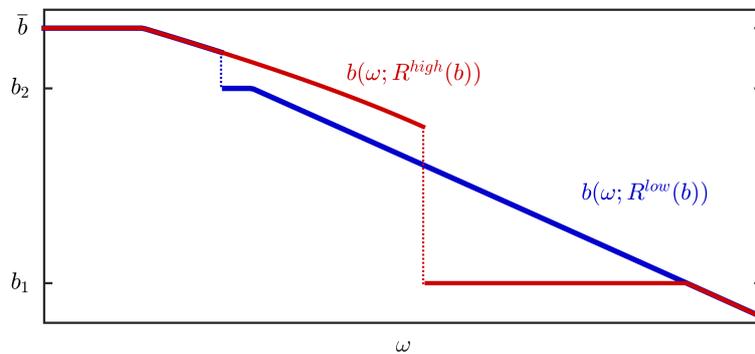
Whether spreads are low or high has implications for the level of debt that can be raised. The region of multiplicity happens for intermediate levels of debt, between b_1 and b_2 . If debt is sufficiently low, interest rates can only be low, whereas if debt is sufficiently high, rates can only be high. It is for intermediate levels of debt that interest rates can be either high or low depending on expectations. The size of the multiplicity region ($b_2 - b_1$) is $p \times b_2$, which depends on the probability of the low endowment and on the maximum amount of debt it can sustain at the low rate under favorable expectations.⁶

Figure 3 shows the optimal debt policy as a function of the initial wealth for the high and low interest rate schedules. For high levels of wealth, the optimal choice of debt is below b_1 , and thus the schedule does not matter. As wealth decreases, the schedule matters. For the high interest rate schedule, the borrower chooses to keep debt levels at b_1 in order to avoid the discrete jump in interest rates on the whole level of debt.

⁶The multiplicity region is $p \times b_2$ when $b_2 < \bar{b}$, which is the relevant one in the quantitative model of Section 3.

Eventually, for low enough wealth, the marginal utility of consumption in the first period is high enough that the borrower chooses to increase its debt level discretely. This discrete jump shows that the borrower has incentives to avoid at least part of the multiplicity region between b_1 and b_2 . As wealth decreases even more, debt levels keep increasing until they reach the endogenous borrowing limit \bar{b} . When the borrower faces the low interest rate schedule, borrowing keeps on increasing as wealth declines until it reaches the level b_2 . At this point, there is a choice to keep it constant for lower levels of wealth. Eventually, there is also a discrete jump, and debt levels continue to increase until they reach the endogenous borrowing limit.

Figure 3: Debt policy function



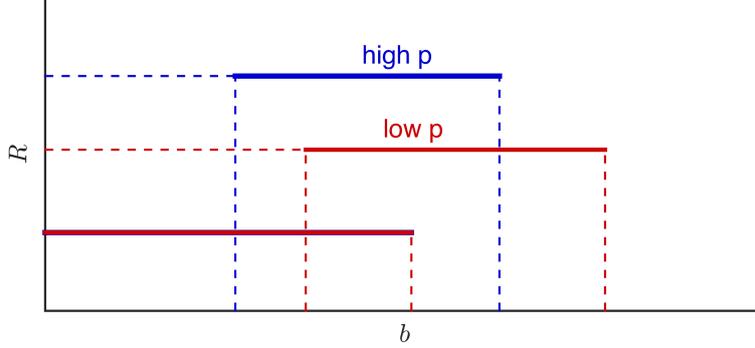
The choice of keeping debt levels constant as wealth decreases is a form of endogenous austerity. This happens in our model because of the discrete jumps in interest rates induced by either expectations or weak fundamentals. As we show, the quantitative model of Section 3 exhibits the same behavior.

The role of the bimodal distribution Here we analyze how the shape of the bimodal distribution affects the multiplicity of equilibria. We start by considering alternative probabilities of the low endowment state p . Figure 4 plots the interest rate correspondence $\mathcal{R}(b)$ for two values of p . The higher is p , the higher is the interest rate that the borrower faces if default happens in the low endowment state. The higher is the interest rate, the lower is the minimum debt level such that the borrower defaults in the low state. It follows that the higher p is associated with a higher interest rate and a larger region of multiplicity.

We relate the parameter p to the parameters in the full quantitative model in the next section. There, we have a two-state Markov process in the growth rates of output. The probability of switching to, or remaining at, the low growth regime is the analog of the value of p in this two-period model. In the estimations described in Section 3, we show that low output growth states are persistent, meaning higher p during stagnations. Consequently, in the quantitative model, stagnations come with larger regions of multiplicity

and higher interest rates.

Figure 4: Interest rate correspondence for different p 's



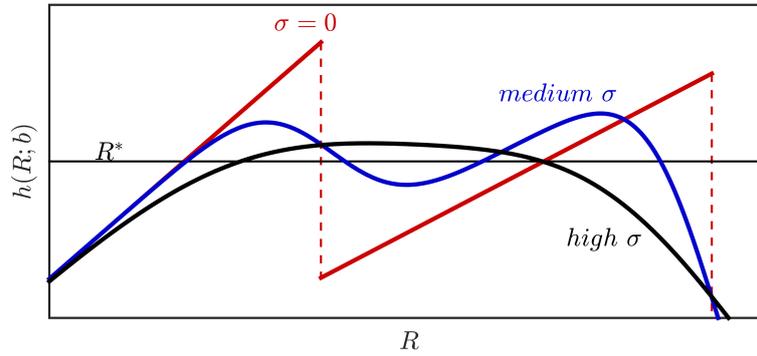
Next, we consider the case in which the endowment in the second period is drawn from a bimodal normal distribution, $y_2 \sim pN(y^l, \sigma^2) + (1 - p)N(y^h, \sigma^2)$. Figure 5 shows, for different values of σ , the expected return function $h(R; b)$ in Figure 5a and the implied interest rate correspondence $\mathcal{R}(b)$ in Figure 5b. The case with $\sigma = 0$ (red) is the one analyzed before, in which there are two solutions to equation (3). For strictly positive small levels of σ (blue), there are now four solutions to equation (3). However, the two solutions on the downward-sloping part of the expected return function are such that, if the interest rate decreases, the expected return increases. This implies segments of the interest rate schedule that decrease with debt b . These segments are clearly not very reasonable, and we rule them out for reasons discussed in Ayres et al. (2015). For higher values of σ (black), there are two solutions to equation (3), but one can be ruled out. Therefore, to have multiple admissible equilibria, we need to have relatively low levels of σ . We show this is the case in the estimations of Section 3.

The endowment levels y^l and y^h are also important for multiplicity, as is clear from Figure 2. As y^l approaches y^h , multiplicity disappears as the endowment distribution converges to the unimodal case. If instead y^l approaches y^d , multiplicity also disappears. For intermediate levels of y^l , when $b_2 < \bar{b}$, the multiplicity region $p \times b_2$ shrinks as y^l declines. The reason is that a lower y^l reduces the value of repayment in the low state, which in turn reduces the maximum amount of the debt that can be sustained at the low rate, b_2 . For a similar reasoning, higher y^h does not change the multiplicity region because it does not affect the amount of debt that can be sustained in the low endowment state. The multiplicity region is maximized for intermediate levels of y^l . A similar rationale will apply to the quantitative results of Section 3.

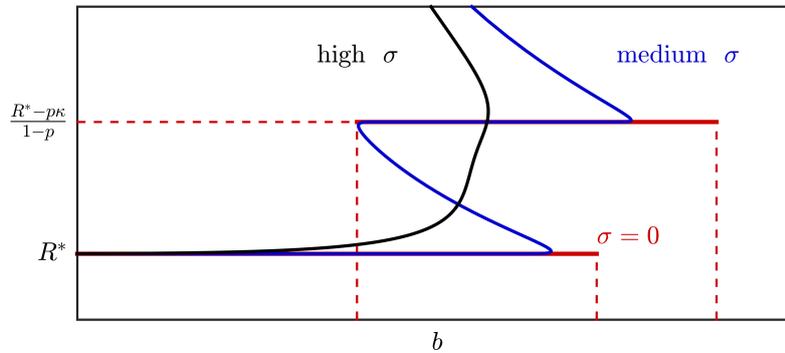
Varying the recovery rate κ A higher κ lowers the interest rate schedule (Figure 6) since it increases the return for lenders in case of default. The higher κ and the lower interest rate reduce the cost of repayment in the low endowment state, which in turn

Figure 5: Varying the standard deviation of endowment shock, σ

(a) expected return function

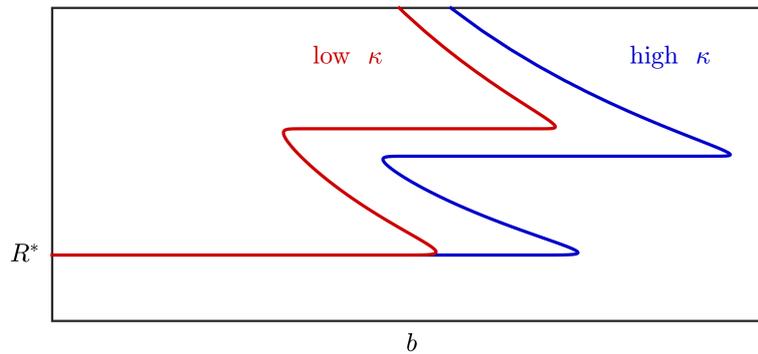


(b) interest rate correspondence



increases the maximum amount of the debt that can be sustained at the low rate, b_2 . Thus, higher κ increases the multiplicity region $p \times b_2$.

Figure 6: Interest rate correspondence for different κ 's



The comparative statics discussed above will help us to understand how changes in parameter values affect the equilibrium of our quantitative model, described in the next section.

3 A quantitative model of self-fulfilling debt crises

Next, we consider an infinite-horizon model that allows us to evaluate the quantitative role of multiplicity in triggering sovereign debt crises. We calibrate the endowment process in the model using data on GDP growth for a set of economies that were exposed to debt crises episodes. The calibrated model generates expectations-driven self-fulfilling debt crises with interest rate movements that resemble those observed in crises periods, including the recent European experience.

3.1 Model

We take the same structure of the model in the two-period case in Section 2 and extend it to the infinite horizon. We consider only one-period debt. This approach allows us to isolate the mechanism we want to evaluate from an alternative one studied by [Lorenzoni and Werning \(2013\)](#) that arises if debt is of longer maturity. For simplicity, we assume that, upon default, the borrower is permanently excluded from financial markets. Lenders recover a fraction of defaulted debt.

In the absence of default, the endowment grows over time as the result of a persistent and a transitory shock. In each period t , the endowment Y_t is given by

$$Y_t = \Gamma_t e^{\sigma \epsilon_t}, \quad (6)$$

$$\Gamma_t = e^{g_t} \Gamma_{t-1}, \quad (7)$$

where $\epsilon_t \sim \mathcal{N}(0, 1)$, and g_t follows a two-state Markov process with transition probability $p_g(g'|g)$. Thus, g_t is the current trend growth and Γ_t is the accumulated growth up to period t . We assume that g_t can be either high or low— $g_t \in \{g_H, g_L\}$ —representing (persistent) times of either fast growth or stagnation. The bimodal nature of g_t is empirically plausible and crucial for expectations playing a role in the model.

The preferences of the borrower are standard,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \quad u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}. \quad (8)$$

The period utility is consistent with balanced growth, which allows us to detrend the model as we show below.

The borrower can issue one-period non-contingent debt. We make the same assumptions on the timing of moves and actions of the borrower as in the two-period model. In particular, we assume that the borrower moves first and chooses current debt B' rather than debt at maturity. Lenders move next and offer an interest rate schedule $R(B', \Gamma_{t-1}, g_t, s_t)$, where s_t is a sunspot variable. The sunspot captures the role of expect-

tations in selecting the equilibrium interest rate schedule. The sunspot follows a two-state Markov process with transition probability given by $p_s(s'|s)$. We label the two states of s_t as good (low-rate schedule) or bad (high-rate schedule)— $s_t \in \{s_B, s_G\}$. The two increasing interest rate schedules are selected from the correspondence using the same approach as in the two-period case. The high-rate schedule corresponds to the highest interest rates for each level of debt, and the low-rate schedule corresponds to the lowest rate.

At the beginning of each period, after the endowment and sunspot are observed, the borrower decides whether or not to default. Default carries three consequences. First, the borrower is permanently excluded from financial markets. Second, the transitory component of the endowment becomes $\bar{\epsilon} < 0$, and trend growth is set to its unconditional mean $\mu_g = \mathbb{E}[g]$. Third, lenders recover a fraction κ of the face value of the debt. The recovery is implemented with a perpetual bond that, in each period, pays a coupon equal to x times the accumulated trend growth up to that period. That is, if the borrower defaults on an amount of debt B at time t , then x satisfies

$$\kappa B = \sum_{j=0}^{\infty} \frac{\tilde{\Gamma}_j x}{(R^*)^j}, \quad (9)$$

where $\tilde{\Gamma}_j = \mu_g^{j+1} \Gamma_-$ is the trend growth while in default, Γ_- is the accumulated trend growth up to $t-1$, and R^* is the risk-free rate. Equation (9) implicitly defines a function $x = x(B, \Gamma_-)$ such that the present discount value of coupon payments is equal to κB . Assuming recovery on defaulted debt is realistic and allows the model to predict rates in line with the observed ones.

Let $V^d(B, \Gamma_-)$ be the value to the borrower of defaulting on an amount of debt B with accumulated trend growth Γ_- . Since there is no re-entry to financial markets after default, this value is

$$V^d(B, \Gamma_-) = \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\gamma}}{1-\gamma}, \quad (10)$$

$$\text{where } C_{t+j} = \tilde{\Gamma}_j e^{\bar{\epsilon}} - \tilde{\Gamma}_j x,$$

with x defined in (9).

Let $V^{nd}(W, \Gamma_-, g, s)$ be the maximal attainable value to a borrower that never defaulted in the past and starts the period with wealth W , accumulated trend growth Γ_- , and current growth g when the value of the sunspot is s . The value of no default satisfies

the following Bellman equation:

$$\begin{aligned}
V^{nd}(W, \Gamma_-, g, s) &= \max_{C, B'} \left\{ \frac{C^{1-\gamma}}{1-\gamma} + \beta \mathbb{E} \left[\max \{ V^{nd}(W', \Gamma, g', s'), V^d(B', \Gamma) \} \middle| g, s \right] \right\}, \\
&\text{subject to} \\
C &\leq W + B', \\
W' &= \Gamma' e^{\sigma \epsilon'} - R(B', \Gamma_-, g, s) B', \\
B' &\leq \bar{b} \Gamma.
\end{aligned} \tag{11}$$

We use wealth W as a state variable (instead of current debt) because it reduces the dimensionality of the state space.⁷ The borrowing limit is important. Since the borrower receives a unit of consumption for every unit of debt issued, default could always be postponed by issuing more debt. This possibility is ruled out by imposing a maximum amount of debt. Making the borrowing limit proportional to Γ allows us to detrend the model.

The interest rate schedule $R(B', \Gamma_-, g, s)$ is a function of the amount of debt issued because default probabilities depend on it, and the interest rate reflects the likelihood of default. The rate is also a function of the fundamental state of the economy (Γ_-, g) since these variables contain information about future default probabilities. The interest rate schedule also depends on the sunspot, reflecting the fact that interest rates may fluctuate because of fundamentals or expectations.

The optimal default policy can be characterized by a *default set*. Let $\mathbb{D}(B', \Gamma_-, g, s)$ be the possible realizations next period that induce the sovereign to default. Formally,

$$\mathbb{D}(B', \Gamma_-, g, s) = \{(g', s', \epsilon') : V^d(B', \Gamma) > V^{nd}(W', \Gamma, g', s')\}, \tag{12}$$

in which $\Gamma = \Gamma_- e^g$ and W' is given as in (11). Given the set $\mathbb{D}(B', \Gamma_-, g, s)$, we can compute the next-period default probability as

$$\mathbb{P}(B', \Gamma_-, g, s) = \Pr \left((g', s', \epsilon') \in \mathbb{D}(B', \Gamma_-, g, s) \middle| g, s \right). \tag{13}$$

Interest rate schedule The lenders are willing to offer an interest rate schedule $R(B', \Gamma_-, g, s)$ as long as the following no-arbitrage condition is satisfied:

$$R^* = [1 - \mathbb{P}(B', \Gamma_-, g, s)] R(B', \Gamma_-, g, s) + \mathbb{P}(B', \Gamma_-, g, s) \kappa. \tag{14}$$

Equation (14) states that the expected return of lending to the borrower must be equal to the risk-free rate R^* . With probability $[1 - \mathbb{P}(B', \Gamma_-, g, s)]$, the borrower repays

⁷If we were to keep current debt B as a state, we would also need to know the previous period interest rate.

in full, and with the remaining probability the lender only recovers a fraction κ .

Definition 1 (Equilibrium). *An equilibrium is a set of value functions $\{V^d(B, \Gamma_-, g), V^{nd}(W, \Gamma_-, g, s)\}$, policy functions $\{C(W, \Gamma_-, g, s), B'(W, \Gamma_-, g, s)\}$, a default set $\mathbb{D}(B', \Gamma_-, g, s)$, and an interest rate schedule $R(B', \Gamma_-, g, s)$ such that: (i) the policies solve the borrower's problem in (11) and achieve value $V^{nd}(W, \Gamma_-, g, s)$; (ii) the default set is as in (12); and (iii) the interest rate schedule satisfies (14).*

Model normalization Since the endowment process has a trend, the state variables in the model are non-stationary. For computational purposes, we normalize all non-stationary variables by trend growth Γ_- . This requires showing homogeneity properties of the equilibrium functions. The following proposition states the required homogeneity properties and uses them to cast the model in stationary form. A more detail derivation can be found in Appendix A.

Lemma 1 (Model Normalization). *The model equilibrium admits a solution such that: (i) $V^d(B, \Gamma_-, g) = \Gamma_-^{1-\gamma} V^d(B, 1, g)$, (ii) $V^{nd}(W, \Gamma_-, g, s) = \Gamma_-^{1-\gamma} V^{nd}(W/\Gamma_-, 1, g, s)$, and (iii) $R(B', \Gamma_-, g, s) = R(B'/\Gamma_-, 1, g, s)$. Let $b = B/\Gamma_-$ and $\omega = W/\Gamma_-$ be the normalized states, and $v^d(b) = V^d(b, 1)$ and $v^{nd}(\omega, g, s) = V^{nd}(\omega, 1, g, s)$ be the normalized value functions. The normalized value functions satisfy*

$$v^{nd}(\omega, g, s) = \max_{c, b'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta g^{1-\gamma} \mathbb{E} \left[\max \{ v^{nd}(\omega', g', s'), v^d(b') \} \right] \right\}$$

subject to

$$c \leq \omega + gb'$$

$$\omega' = g' e^{\sigma \epsilon'} - r(b', g, s) b'$$

$$b' \leq \bar{b},$$

and the value of default is

$$v^d(b) = \frac{(e^{\bar{\epsilon}} - x(b))^{1-\gamma}}{1-\gamma} \frac{\mu_g^{1-\gamma}}{1 - \beta \mu_g^{1-\gamma}},$$

where $x(b) = \frac{\kappa}{\mu_g} \frac{R^* - \mu_g}{\mu_g} b$. Finally, the interest rate schedule satisfies

$$R^* = [1 - \mathbb{P}(b', g, s)] r(b', g, s) + \mathbb{P}(b', g, s) \kappa \tag{15}$$

with $\mathbb{P}(b', g, s) = \Pr \left((g', s', \epsilon') : v^d(b') > v^{nd}(\omega', g', s') \mid g, s \right)$.

Lemma 1 provides a stationary model suitable for computations. Intuitively, the lemma states that only debt (and wealth) relative to trend growth matters. In all that follows, we will use the normalization just described.

3.2 Calibration

A subset of our model parameters are standard, and we use common values in the sovereign debt literature. A period in our model is one year. We set $e^{\bar{\epsilon}} = 0.975$ so that 2.5 percent of output is lost upon default. This value for output loss lies between the ones considered in [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#), 2 and 3 percent, respectively.⁸ We set the annual risk-free rate to $R^* = 1.035$, consistent with the average (gross) annual return on US Treasury bills.⁹ The recovery rate is set to 75 percent ($\kappa = 0.75$), in line with the estimates in [Cruces and Trebesch \(2013\)](#).¹⁰ In order to make the borrower impatient enough to induce borrowing and default as possible equilibrium outcomes, we set the discount factor to $\beta = 0.75$. [Arellano \(2008\)](#) and [Chatterjee and Eyigungor \(2015\)](#) use quarterly discount factors of 0.95 and 0.94, respectively, which imply annual discount factors of 0.82 and 0.77, close to the value we chose.¹¹ We set the borrower’s risk aversion to $\gamma = 3$. This value is between the ones used by [Arellano \(2008\)](#), $\gamma = 2$, and [Bianchi, Hatchondo, and Martinez \(2018\)](#), $\gamma = 3.3$.¹² Lastly, the sunspot process is characterized by two transition probabilities: $p_B = p_s(s' = s_B | s = s_B)$ and $p_G = p_s(s' = s_G | s = s_G)$. We assume that the sunspot variable is *i.i.d.* and takes the bad realization with a probability of 5 percent—that is, $p_G = 1 - p_B = 0.95$.

In [Section 4](#) and [Appendix D](#) we perform sensitivity analysis for many of the parameters described above, and show that our conclusions are robust to alternative parameter values. Next, we turn to the second subset of parameters, which are related to the stochastic endowment process.

Estimation of the endowment process The endowment process in equations (6)-(7) is a regime-switching process characterized by five parameters $\{g_L, g_H, \sigma, p_L, p_H\}$, where $p_L = p_g(g' = g_L | g = g_L)$ and $p_H = p_g(g' = g_H | g = g_H)$. We estimate the process using the filter proposed in [Kim \(1994\)](#).¹³ We use annual GDP per capita data from The Conference Board Total Economy Database for years 1980–2017 and estimate the process separately for five countries: Argentina, Brazil, Italy, Portugal, and Spain.¹⁴ We assume bounded uniform priors for the five parameters and explore the posterior using

⁸Appendix D shows the results for $e^{\bar{\epsilon}} = 0.97$ and $e^{\bar{\epsilon}} = 0.98$.

⁹[Bianchi, Hatchondo, and Martinez \(2018\)](#) and [Aguiar et al. \(2016\)](#), for example, use $R^* = 1.04$. Appendix D shows the results for $R^* = 1.03$ and $R^* = 1.04$.

¹⁰The haircut estimates in [Cruces and Trebesch \(2013\)](#) vary from 16 percent to 40 percent, corresponding to values of κ between 0.84 and 0.60. We perform sensitivity analysis for different values of κ in [Section 4](#).

¹¹[Aguiar and Gopinath \(2006\)](#) use a quarterly discount factor of 0.8, which corresponds to 0.41 annually. On the other hand, [Aguiar et al. \(2016\)](#) use annual discount factors between 0.84 and 0.89. Appendix D shows the interest rate correspondence for different values of β .

¹²Appendix D shows the results for $\gamma = 2.5$ and $\gamma = 3.5$.

¹³We do not use the filter in [Hamilton \(1989\)](#) directly because output growth has a moving average component. We use the filter in [Kim \(1994\)](#) instead.

¹⁴We start in 1980 to avoid the high growth rates of the period of convergence in the 1960s and 1970s.

Table 1: Prior and posterior distributions

	g_L	g_H	p_L	p_H	σ
Prior distribution					
	$U[-0.1, 0.1]$	$U[-0.1, 0.1]$	$U[0.1, 1.0]$	$U[0.1, 1.0]$	$U[10^{-3}, 0.5]$
Posterior distribution (mean, and 5th to 95th percentile intervals)					
Countries					
Italy	-0.017 [-0.037,-0.008]	0.022 [0.018,0.028]	0.646 [0.050,0.990]	0.843 [0.627,0.990]	0.016 [0.012,0.023]
Portugal	-0.002 [-0.011,0.003]	0.048 [0.041,0.057]	0.805 [0.516,0.990]	0.720 [0.454,0.939]	0.019 [0.014,0.025]
Spain	-0.018 [-0.026,-0.010]	0.033 [0.026,0.039]	0.629 [0.308,0.990]	0.838 [0.653,0.990]	0.017 [0.013,0.025]
Argentina	-0.040 [-0.049,-0.022]	0.060 [0.051,0.078]	0.620 [0.346,0.877]	0.581 [0.358,0.781]	0.033 [0.025,0.044]
Brazil	-0.033 [-0.071,-0.022]	0.029 [0.025,0.032]	0.589 [0.103,0.860]	0.793 [0.627,0.923]	0.019 [0.014,0.025]

Note: For each country, we estimate an output process as: $\Delta \ln y_t = g_t + \sigma(\epsilon_t - \epsilon_{t-1})$, in which $\epsilon_t \sim N(0, 1)$ and $g_t \in \{g_L, g_H\}$, with $\Pr(g_{t+1} = g_L | g_t = g_L) = p_L$ and $\Pr(g_{t+1} = g_H | g_t = g_H) = p_H$. The table reports the mean and the interval between the 5th and 95th percentiles of the posterior distributions of each of the parameters for each country. The table also reports the prior distributions we used, which were chosen to be the same across countries. For each country, we use data on GDP per capita in 2016 US\$ (converted to 2016 price level with updated 2011 PPPs) between 1980 and 2017 from The Conference Board Total Economy Database as the measure of y_t . See Appendix B for a description of the estimation.

a Metropolis-Hastings Markov Chain Monte Carlo (MCMC) algorithm. Table 1 shows the estimates for all countries, and Appendix B contains more details on the estimation and data sources. As in the model, the estimation assumes the same standard deviation of shocks, σ , across the two growth states. In Appendix B, we also estimate the process allowing for state-dependent σ and find they are similar across states.

Two things are worth mentioning regarding the estimates in Table 1. First, the estimates show clear evidence of a bimodal distribution for output growth. The average across countries of the difference between g_L and g_H is 6 percentage points, more than three times the standard deviation of the transitory shock. The difference between g_L and g_H is a key ingredient for expectations to play a role. Second, both the low- and high-growth states are persistent (between 60 and 80 percent persistence). As we show below, this relatively high persistence for the low-growth state generates high but plausible interest rates in the bad-sunspot state. As a benchmark calibration, we use $p_L = 0.6$, $p_H = 0.8$, $\sigma = 0.015$, $g_L = -0.01$, and $g_H = 0.03$.¹⁵ Table 2 contains all the model

¹⁵We are being conservative in considering a difference in growth rates of 4 percentage points, the

Table 2: Benchmark calibration

Description	Parameter	Value
discount factor	β	0.75
risk aversion	γ	3.0
risk-free rate	R^*	1.035
low-growth rate	g_L	-0.01
high-growth rate	g_H	0.03
standard deviation of transitory shock	σ	0.015
fraction of debt recovered after default	κ	0.750
default cost	$e^{\bar{\epsilon}}$	0.975
probability of remaining in low growth	p_L	0.60
probability of remaining in high growth	p_H	0.80
probability of remaining in bad sunspot	p_B	0.05
probability of remaining in good sunspot	p_G	0.95

parameter values.

3.3 Model computation

Multiple interest rate schedules satisfy equation (15). Some of those interest rate schedules will be decreasing in the level of debt. We use the same approach as in the two-period model to select only upward-sloping schedules.¹⁶ Again, we focus on two polar cases, with either low or high rates. The infinite-horizon problem introduces an additional complexity because the selection affects the value function, which in turn affects the selection.

The standard algorithm used to compute the recursive equilibrium in quantitative sovereign debt models, as in Arellano (2008) and others, is not suitable for our analysis. They typically start with a guess for price/interest rate functions, but our case has multitude of those functions. To overcome that, we develop a new algorithm that iterates only on the value function and that, in each iteration, computes the respective interest rate correspondences and selects the high and low interest rate schedules. In Appendix C, we explain in detail the algorithm used to build the equilibrium.

4 Model evaluation

We now describe the computation exercises and discuss the results. In Figure 7 we plot the solutions of equation (15) for debt issuance b' , for the low (dotted red) and high

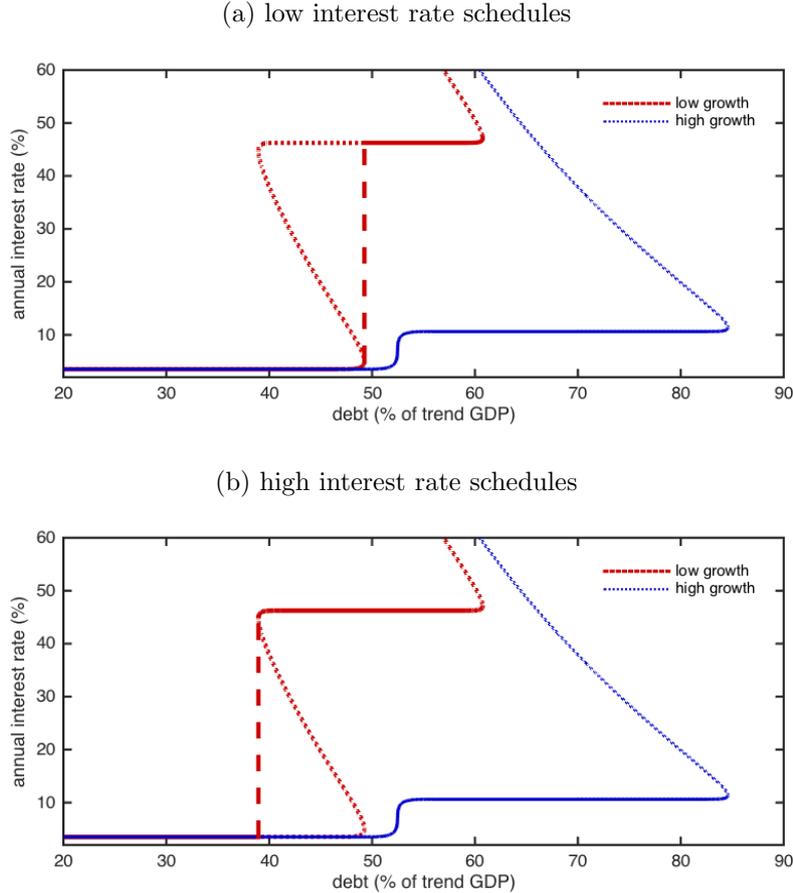
figure for Italy, rather than the country average of 6.

¹⁶See Ayres et al. (2015) for a discussion on this.

(dotted blue) growth states. The solid lines depict the selected schedules for the good sunspot (Figure 7a) and the bad sunspot (Figure 7b).

While in the high-growth state, the schedules associated with the good and bad sunspots are the same, but that is not the case for the low growth state. In the low-growth state, for intermediate levels of debt, interest rates can be either low or high depending on the sunspot.

Figure 7: Interest rate schedules under benchmark calibration



In this model, expectations play a role when fundamentals are weak. The reason for this state-dependent role of expectations is the following. In periods of low growth, the probability of observing low growth in the future is high. For this reason, if the borrower is expected to default in the low growth state, the interest rate must be high. This high interest rate will, in turn, induce the borrower to default in the low-growth state, confirming the expectations. This will happen even for low debt levels. If the borrower is not expected to default in the low-growth state, however, the interest rates will be relatively low and the borrower will be able to issue a larger amount of debt without risking default next period. This translates into a large region of multiplicity, in which for intermediate levels of debt, interest rates can be either high or low depending on expectations.

In periods of high growth, the probability of switching to low growth is low. Thus, even if the borrower is expected to default in the low state, the interest rate consistent with these expectations will be low. Expectations will be confirmed, meaning that the borrower will default in the low-growth state, but only when debt levels are relatively large. If the borrower is not expected to default in the low-growth state, however, the interest rates are only marginally lower, and the debt levels such that the borrower will not default are not much higher. This translates into a small region of multiplicity, if any.

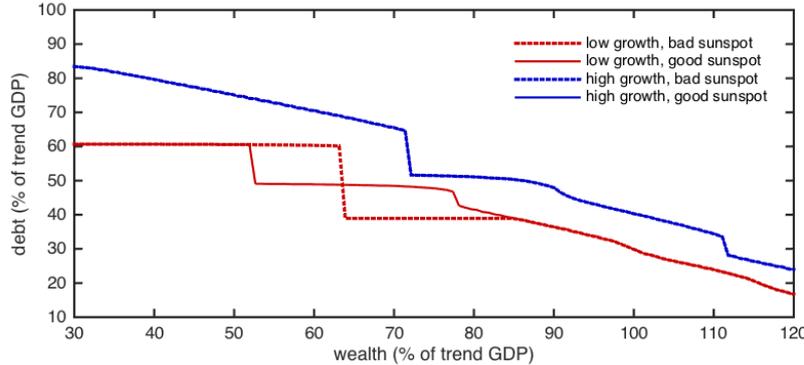
The probability of switching to the low-growth state in the infinite-horizon model is the analog of the probability of the low endowment in the two-period model of Section 2. When that probability is low, the region of multiplicity is small, whereas when that probability is high, the region of multiplicity is large. In the infinite-horizon model, the probabilities are functions of the state. In low-growth states, the probability of future low growth is high, and the region of multiplicity is large. In high-growth states, that probability is low, and the region of multiplicity is small.

The spread at the high interest rate equilibrium that the model delivers is higher than the ones observed in countries such as Italy, Spain (around 5 percent), and even those of Portugal or Ireland (around 15 percent). We do not think of this finding an issue for several reasons. First, as we show below, the spread is very sensitive to small changes in both the persistence of the low-growth state and the recovery rate, and there is substantial uncertainty about the exact value for those parameters (see Table 1). Second, and most important to us, the spreads observed in Europe at the time were most surely affected by the chances that the European Central Bank or some other European institution would intervene, as the ECB finally did. The relevant question to us is what would have been the spreads in Spain, Portugal, and Italy by December 2012 if they had not been part of the euro area? On the other hand, the spread generated by the model is closer to the one observed for Argentina in 2001. A main difference, of course, is that Argentina did default in 2002, whereas Italy, Portugal, and Spain did not. As we show in the discussion of the model dynamics below, both paths are consistent with the model.

Figure 8 shows the optimal debt policies as functions of wealth, for the different growth and sunspot states. In the high-growth state (blue), the policies under the two sunspots coincide. Debt increases as wealth goes down, except for a region where debt remains constant before jumping to a higher level. The reason why debt remains constant is to avoid the discrete jump in the interest rate that higher debt levels would induce, as seen in Figure 7. The interest rate schedules in the high-growth state are the same for the two sunspots. This means that the discrete jump in interest rates is due to fundamentals and not to expectations. There is endogenous austerity due to fundamentals.

Debt levels are lower in the low-growth state (red). In this case, the debt policy functions are different across sunspot states, reflecting the different schedules. For the

Figure 8: Debt policy functions under benchmark calibration



good sunspot (solid red), the debt policy is similar to the one in the high-growth state, including endogenous austerity due to fundamentals. For the bad sunspot (dotted red), instead, there is also endogenous austerity and a discrete jump in debt levels, but now this happens for lower levels of debt and is due to the discrete jump in interest rates induced by expectations. The model features endogenous austerity due to both fundamentals and expectations.

When facing the high interest rate schedule, the borrower chooses to either keep debt below the multiplicity region or discretely raise debt above it. Thus, high interest rates due to expectations induce debt behavior that is either austerity-like, or gambling-for-redemption-like as in [Conesa and Kehoe \(2017\)](#).

4.1 Dynamics

In this section, we show how the model can generate an expectations-driven crisis that resembles the dynamics observed during the European debt crises.

Figure 9 condenses the dynamic behavior of the model. For each wealth level, growth state, and sunspot state this period, it provides the distribution of wealth for each growth states next period.¹⁷ In blue we show the distribution conditional on the high-growth state next period, and the low-growth state is shown in red. Shaded areas correspond to the 90 percent distribution, and the solid lines are the means. The dashed blue and red lines correspond to the wealth default thresholds for the high- and low-growth realizations in the next period, respectively. The solid black line corresponds to the 45-degree line. Figure 9, together with the debt policy functions (Figure 8) and interest rate schedules (Figure 7), completely describes the dynamics of the quantitative model for our benchmark calibration.

Next-period wealth tends to decrease, as the next-period distributions are located below the 45-degree line. In the high-growth and good/bad sunspot state (Figure 9a),

¹⁷We do not separate the good from the bad sunspot case in the high-growth state because they are the same. This follows from the fact that the interest rate schedule is unique in the high-growth state.

there is a fixed point at which wealth converges, around 12.1 percent of trend GDP, corresponding to the point at which the solid black line crosses the solid blue line. At that wealth level, the borrower chooses debt equal to 84.4 percent of trend GDP, associated with an annual interest rate of 10.9 percent. Notice that, in this case, if growth switches to the low state, default occurs regardless of the sunspot realization in the next period. This means that the probability of default is around 20 percent, which explains the interest rate of 10.9 percent.

There is no such fixed point for the low-growth state, regardless of the sunspot realization. The solid black line crosses the solid red line in Figures 9b and 9c, but at levels below the wealth-default thresholds for the low-growth state. That means that the economy will already be in default in these cases. Therefore, if a long enough sequence of low-growth realizations realizes, the borrower will default.

Using Figures 7, 8, and 9, we can simulate a sequence of shocks where the sunspot plays a crucial role in determining interest rates and default probabilities. We compare two scenarios that differ only in the sunspot realization: scenario 1, where the sunspot is always good, and scenario 2, where the sunspot takes a bad realization.

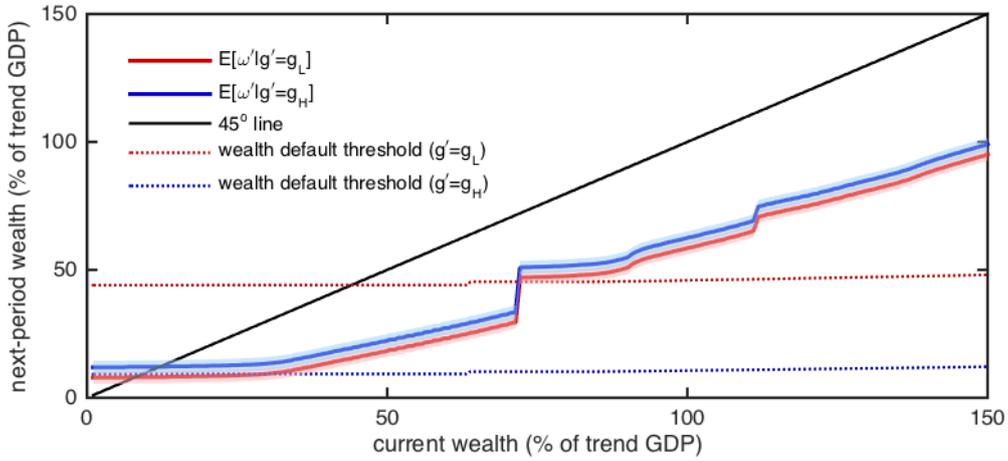
Figure 10 summarizes the dynamics of an economy that starts with wealth equal to 130 percent of trend GDP and is in the high-growth state in $t = 0$. In scenario 1, the sunspot realization is always good. In scenario 2, the sunspot realization is bad in $t = 2$ and then returns to the good state. The realizations of the growth shock are the same in both scenarios. The economy starts in the high-growth state, switches to low growth in $t = 1$ and $t = 2$, and returns to high growth in $t = 3, 4, 5$. The figure reports the optimal choices of debt, b' , in each period together with the interest rates and next-period probabilities of default.

The sunspot realization in $t = 2$ determines whether the economy faces a high or low probability of default. If the sunspot realization is good (scenario 1), the economy chooses debt equal to 49.0 percent of trend GDP, with a low interest rate, 4.0 percent, and a low probability of default in the following period, 1.7 percent. On the other hand, if the sunspot realization is bad (scenario 2), the economy chooses debt equal to 60.4 percent of trend GDP, with a high interest rate, 46.4 percent, and a high probability of default in the following period, 60.1 percent. Therefore, the high interest rate in $t = 2$ for scenario 2 is only driven by bad expectations, representing a self-fulfilling debt crisis.

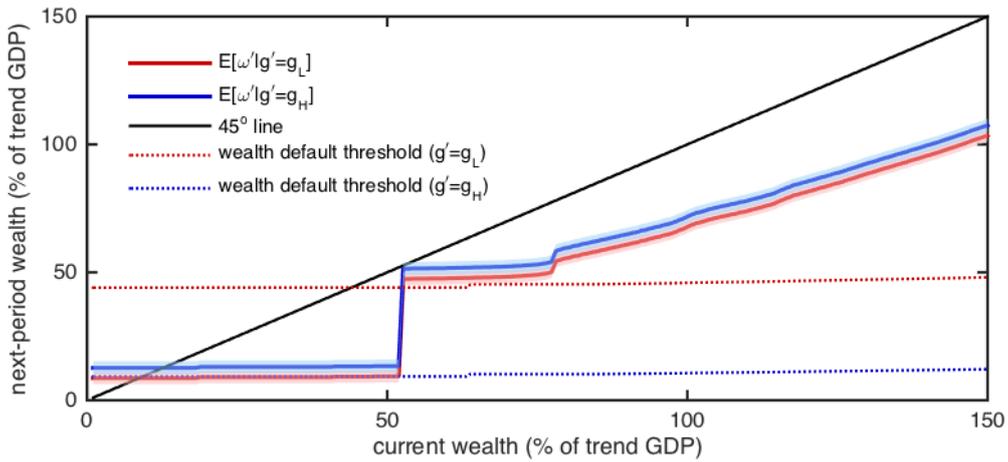
We can use the simulation in Figure 10 to interpret the European debt crisis. The economies in southern Europe switched from high to low growth in the 2000s. The low-growth state made these economies vulnerable to a self-fulfilling debt crisis, which eventually happened in 2010. However, the intervention by the ECB was able to rule out the bad (high interest rate) equilibrium, and the economies switched to scenario 1, with low interest rates and low probabilities of default. These types of interventions by a lender of last resort are fully justified when view through the lenses of this model.

Figure 9: Model dynamics

(a) high growth, good or bad sunspot



(b) low growth, good sunspot



(c) low growth, bad sunspot

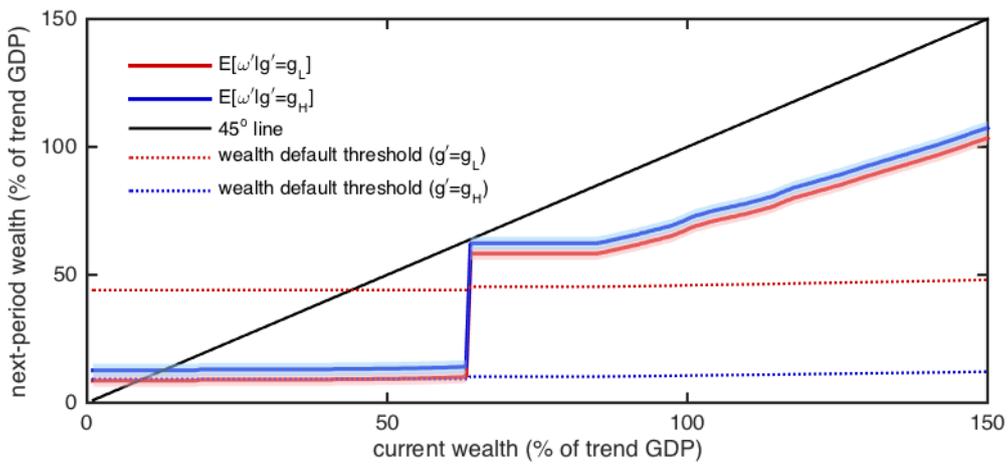
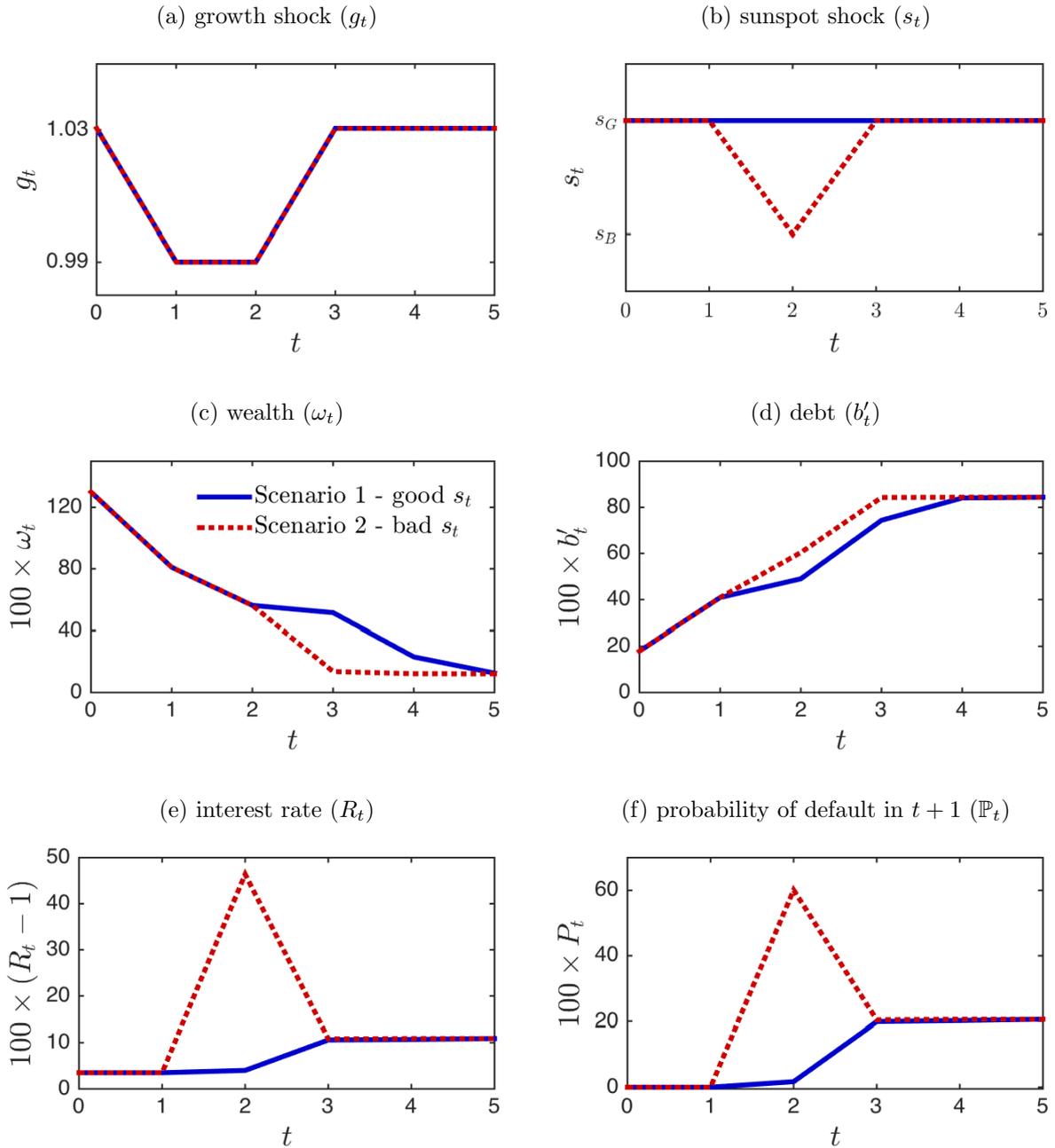


Figure 10: Scenario Simulation



4.2 Robustness

We finish this section by showing that our results are robust to reasonable perturbations in the value of the key model parameters, p_L , κ , σ , g_H , and g_L . We show that our results for the multiplicity region and interest rate schedules are robust to different parameter values.¹⁸ We argue that the effect of each parameter on multiplicity of equilibria

¹⁸As Figure 7 shows, there is no multiplicity in the high-growth state for our benchmark calibration, so here we focus only on the interest rate correspondence in the low-growth state. The correspondences for the high-growth state as well as the robustness exercises with the remaining parameters are presented in Appendix D.

is very similar to the one discussed in the two-period model of Section 2 for the analog parameters, p , κ , y^l , and y^h .

Varying p_L Figure 11a shows the interest rate correspondence for different values of p_L , the probability of remaining in the low-growth state. As can be seen, multiplicity of equilibria is robust to perturbations in the value of p_L . This parameter is analogous to p in the two-period model, the probability of observing the low endowment. As in the two-period model, a higher p_L results in higher interest rates and a larger multiplicity region. The higher is p_L , the higher is the interest rate because default is more likely in the low-growth state. At the same time, the higher is the interest rate, the lower is the amount of debt on which the borrower defaults in the low-growth state. This is the same rationale as the one discussed in the two-period model, which leads to higher interest rates and a larger multiplicity region the higher that p_L is.

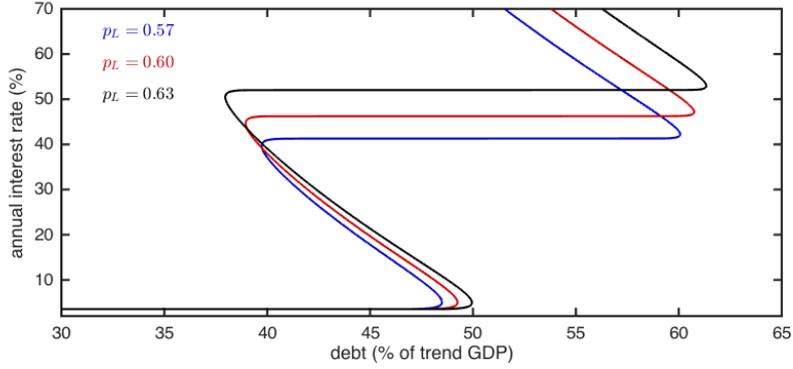
In addition, two dynamic considerations emerge in the quantitative model. First, a higher p_L means a more persistent low-growth state, a lower probability of switching to high growth, and thus a lower value of being in the low-growth state. As a result, the amount of debt that can be sustained in the low-growth state decreases. This is different from the two-period model, where the value of low endowment does not depend on p . Second, a higher p_L reduces the value of default by reducing the average growth rate of the economy, μ_g . This makes default less attractive, and thus more debt can be sustained. This is different from the two-period model, where the value of default is independent of p . Thus, the first and second effects work in opposite directions. Overall, interest rates and the multiplicity region increase with p_L .

Varying κ A higher κ lowers the interest rate schedule and increases the region of multiplicity, as Figure 11b shows. The intuition is the same as in the two-period model. A higher κ decreases the value of default and also increases the expected return to lenders, both of which reduce interest rates. In turn, the amount of borrowing that can be sustained in equilibrium increases.

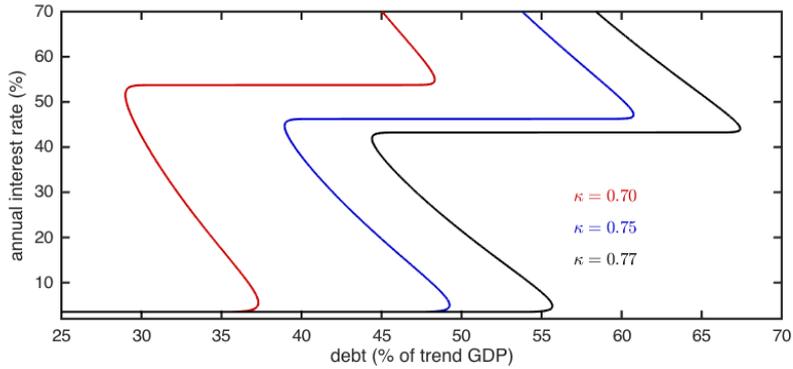
Varying σ A lower σ is associated with a larger region of multiplicity, as Figure 12a shows. This has the same effect as decreasing σ in the two-period model, but now there is also a dynamic consideration. While σ does not affect the value of default by assumption, it decreases the value of repayment because the borrower is risk-averse. This makes default more attractive, reducing the levels of debt that can be sustained in equilibrium. However, our estimation results are such that reasonable perturbations in σ do not substantially alter the multiplicity of equilibria.

Figure 11: Robustness exercises: p_L and κ

(a) varying the persistence of the low-growth state p_L



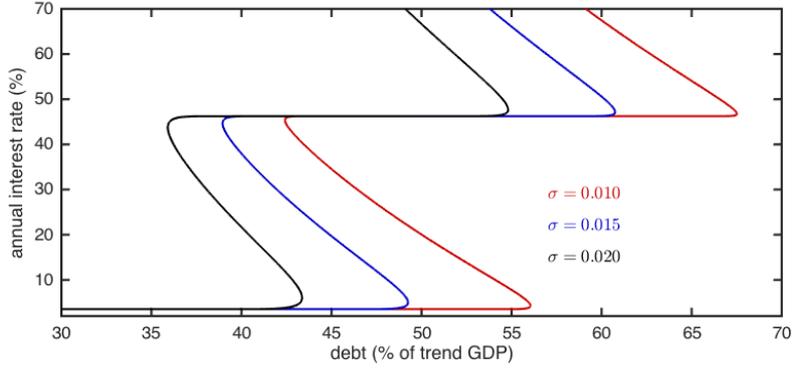
(b) varying the recovery rate κ



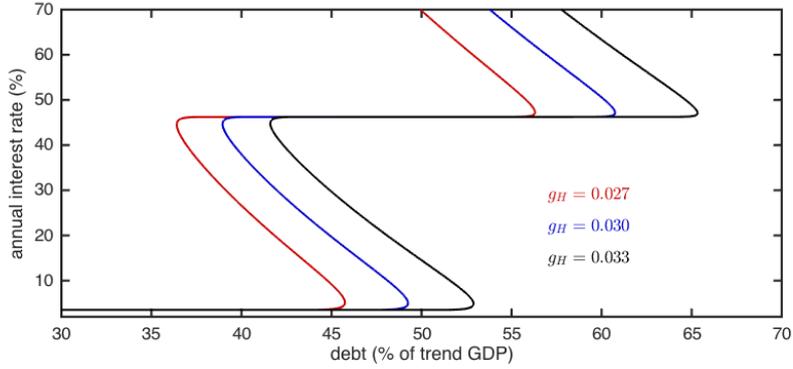
Varying g_L and g_H Changes in g_L and g_H do not affect interest rate levels, which are largely determined by p_L , but do affect the levels of debt that can be sustained in equilibrium, as Figures 12b and 12c show. The higher is g_H , the higher is the value of repayment, since the economy will eventually switch to the high-growth state, and thus the larger are the debt levels that can be sustained in equilibrium. However, differences in g_L and g_H also affect the value of default—unlike in the two-period model where the value of default is given by y^d . The higher are g_L and g_H , the higher is the value of default, and the smaller is the amount of debt that can be sustained. Because in equilibrium the borrower only defaults in low-growth states, the effect of g_H on the default value is irrelevant, but a higher g_L makes default more attractive in the low-growth state. This is why sustainable debt levels are smaller for higher g_L levels. In any case, as with σ , the empirical plausible estimates we obtain for g_L and g_H are such that small perturbations do not substantially alter our results.

Figure 12: Robustness exercises: σ , g_H , and g_L

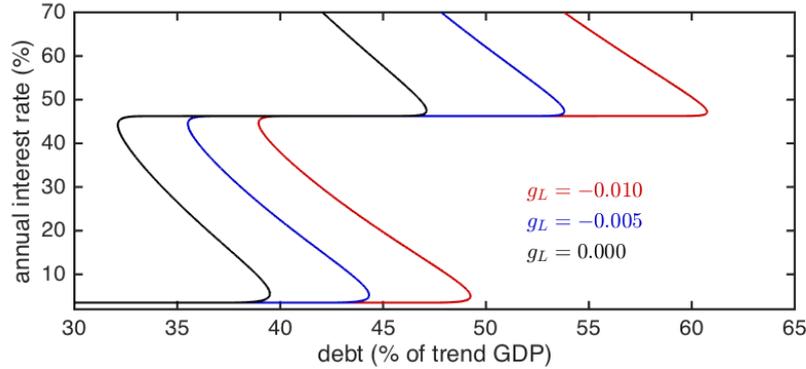
(a) varying the standard deviation of transitory shocks σ



(b) varying the high-growth rate g_H



(c) varying the low-growth rate g_L



5 Concluding remarks

In the model of sovereign debt crises of Calvo (1988), there are multiple interest rate schedules because expectations of high probabilities of default are self-confirming. In particular, if expectations of default are high, interest rates must be high, and high interest rates increase the burden of debt, inducing the borrower to default. While this mechanism is highly intuitive, the multiplicity in Calvo (1988) is fragile. In particular, for commonly used distributions of the output process, the high-rate schedules are downward

sloping. This means that the higher is the debt, the lower is the interest rate that the borrower faces. The reason is that, when interest rates are high, the negative effect of increasing the interest rate on the probability of default that dominates the direct positive effect. [Ayres et al. \(2018\)](#) analyze a simple two-period model to show that when the distribution of the output process is bimodal, with good and bad times, there are multiple upward-sloping interest rate schedules. The question that remains is whether that source of multiplicity is quantitatively relevant. In this paper, we show that it is indeed the case, but only in times of persistent low growth.

We estimate the output process for Argentina, Brazil, Italy, Portugal, and Spain, and find that the distribution of growth rates is bimodal and has high persistence. We calibrate an infinite-horizon Calvo-type model with the estimated process and obtain that expectations do play a role in periods of stagnation. In those periods, growth is low and expected to remain low. If a country is expected to default in bad times, then the probability of that happening is high, so interest rates must be high. If interest rates are high, the country will indeed default in bad times, confirming the expectations. With those expectations, the level of debt that the country will be able to issue at low rates is low. Instead, if the country is not expected to default in bad times, interest rates will be low, and the country will not be induced to default, confirming the low rates. The level of debt that can be issued at low rates is high. Thus, for intermediate levels of debt, the country will be facing either high or low rates depending on expectations. Instead, when growth rates are high and are expected to remain high, if a country is expected to default in bad times, that will happen with low probability. Interest rates will be low, and the borrower will not be induced to default. There is no role for expectations in good times. This state-dependent role of expectations is one of the central results of this paper.

The calibrated policy functions also feature behavior that can be interpreted as endogenous fiscal austerity. The bimodal distribution gives rise to large discrete jumps in interest rates due to expectations, but also due to fundamentals. In order to avoid the costs associated with these discrete jumps in interest rates, the borrower sacrifices consumption in order to refrain from increasing debt.

The main policy implication of our analysis is that the role of a lender of last resort is effective in times when fundamentals are weak, since it is when fundamentals are weak that expectations play a role.

References

- Aguiar, M. and M. Amador (2018). Self-fulfilling debt dilution: Maturity and multiplicity in debt models. Staff Report 565, Federal Reserve Bank of Minneapolis.
- Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2014). Coordination and crisis in

- monetary unions. Working Paper 20277, National Bureau of Economic Research.
- Aguiar, M., S. Chatterjee, H. Cole, and Z. Stangebye (2016). Quantitative models of sovereign debt crises. In B. T. Taylor and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Volume 2B, pp. 1697–1755. Elsevier.
- Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics* 69(1), 64–83.
- Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review* 98(3), 690–712.
- Ayres, J., G. Navarro, J. P. Nicolini, and P. Teles (2015). Sovereign default: The role of expectations. Working Paper 723, Federal Reserve Bank of Minneapolis.
- Ayres, J., G. Navarro, J. P. Nicolini, and P. Teles (2018). Sovereign default: The role of expectations. *Journal of Economic Theory* 175, 803–812.
- Bianchi, J., J. C. Hatchondo, and L. Martinez (2018). International reserves and rollover risk. *American Economic Review* 108(9), 2629–2670.
- Bocola, L. and A. Dovis (2016). Self-fulfilling debt crises: A quantitative analysis. Working Paper 22694, National Bureau of Economic Research.
- Calvo, G. A. (1988). Servicing the public debt: The role of expectations. *American Economic Review* 78(4), 647–661.
- Chatterjee, S. and B. Eyigungor (2015). A seniority arrangement for sovereign debt. *American Economic Review* 105(12), 3740–3765.
- Cole, H. L. and T. J. Kehoe (2000). Self-fulfilling debt crises. *Review of Economic Studies* 67(1), 91–116.
- Conesa, J. C. and T. J. Kehoe (2017). Gambling for redemption and self-fulfilling debt crises. *Economic Theory* 64(4), 707–740.
- Corsetti, G. and L. Dedola (2014). The “mystery of the printing press” monetary policy and self-fulfilling debt crises. Discussion Papers 1424, Centre for Macroeconomics (CFM).
- Cruces, J. J. and C. Trebesch (2013). Sovereign defaults: The price of haircuts. *American Economic Journal: Macroeconomics* 5(3), 85–117.
- De Grauwe, P. and Y. Ji (2013). Self-fulfilling crises in the eurozone: An empirical test. *Journal of International Money and Finance* 34, 15–36.

- Eaton, J. and M. Gersovitz (1981). Debt with potential repudiation: Theoretical and empirical analysis. *Review of Economic Studies* 48(2), 289–309.
- Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* 57(2), 357–384.
- Kahn, J. A. and R. W. Rich (2007). Tracking the new economy: Using growth theory to detect changes in trend productivity. *Journal of Monetary Economics* 54(6), 1670–1701.
- Kim, C.-J. (1994). Dynamic linear models with markov-switching. *Journal of Econometrics* 60(1), 1–22.
- Lorenzoni, G. and I. Werning (2013). Slow moving debt crises. Working Paper 19228, National Bureau of Economic Research.
- Roch, F. and H. Uhlig (2018). The dynamics of sovereign debt crises and bailouts. *Journal of International Economics* 114, 1–13.

A Model Normalization

In this appendix, we provide more details on the normalization step. First, we show that the value functions and the interest rate schedule satisfy certain homogeneity properties. Second, we use these homogeneity properties to derive the stationary system in Lemma 1.

We proceed by guessing that equilibrium functions satisfy certain homogeneity properties and verify that the guess is consistent with the equilibrium definition. Thus, we do not claim that all possible equilibria satisfy these properties, but rather that the equilibrium definition admits a solution with these properties.

Proposition 1 (Homogeneity of equilibrium functions). *For any $\lambda > 0$, the equilibrium functions admit a solution that satisfies the following homogeneity properties: (i) $x(\lambda B, \lambda \Gamma_-) = x(B, \Gamma_-)$; (ii) $V^d(\lambda B, \lambda \Gamma_-) = \lambda^{1-\gamma} V^d(B, \Gamma_-)$; (iii) $V^{nd}(\lambda W, \lambda \Gamma_-, g, s) = \lambda^{1-\gamma} V^{nd}(W, \Gamma_-, g, s)$, (iv) $R(\lambda B', \lambda \Gamma_-, g, s) = R(B', \Gamma_-, g, s)$.*

Proof. Start with (i). From equation (9), provided that $\mu_g < R^*$, we can solve for $x(\cdot)$ as $x(B, \Gamma_-) = \frac{\kappa}{\mu_g} \frac{R^* - \mu_g}{R^*} \frac{B}{\Gamma_-}$. Thus, $x(\lambda B, \lambda \Gamma_-) = x(B, \Gamma_-)$. To show (ii), we can solve for $V^d(\cdot)$ in equation (10) as $V^d(B, \Gamma_-) = \Gamma_-^{1-\gamma} \frac{(e^{\bar{c}} - x(B, \Gamma_-))^{1-\gamma}}{1-\gamma} \frac{\mu_g^{1-\gamma}}{1-\beta\mu_g^{1-\gamma}}$. Since $x(\lambda B, \lambda \Gamma_-) = x(B, \Gamma_-)$, it follows that $V^d(\lambda B, \lambda \Gamma_-) = \lambda^{1-\gamma} V^d(B, \Gamma_-)$.

Finally, we guess and verify (iii) and (iv) jointly. Using equation (11), we have

$$\begin{aligned}
V^{nd}(\lambda W, \lambda \Gamma_-, g, s) &= \max_{B'} \left\{ \frac{(\lambda W + B')^{1-\gamma}}{1-\gamma} + \right. & (16) \\
&+ \mathbb{E} \left[\max \left\{ \frac{V^{nd} \left((\lambda \Gamma_-) g g' e^{\epsilon'} - R(B', \lambda \Gamma_-, g, s) B', (\lambda \Gamma_-) g, g', s' \right)}{V^d(B', (\lambda \Gamma_-) g)}, \right\} |g, s \right] \Big\} \\
&= \max_{\tilde{B}'} \left\{ \frac{(\lambda W + \lambda \tilde{B}')^{1-\gamma}}{1-\gamma} + \right. \\
&+ \mathbb{E} \left[\max \left\{ \frac{V^{nd} \left((\lambda \Gamma_-) g g' e^{\epsilon'} - R(\lambda \tilde{B}', \lambda \Gamma_-, g, s) \lambda \tilde{B}', (\lambda \Gamma_-) g, g', s' \right)}{V^d(\lambda \tilde{B}', (\lambda \Gamma_-) g)}, \right\} |g, s \right] \Big\} \\
&= \max_{\tilde{B}'} \left\{ \frac{(\lambda W + \lambda \tilde{B}')^{1-\gamma}}{1-\gamma} + \right. \\
&+ \mathbb{E} \left[\max \left\{ \frac{\lambda^{1-\gamma} V^{nd} \left(\Gamma_- g g' e^{\epsilon'} - R(\tilde{B}', \Gamma_-, g, s) \tilde{B}', \Gamma_- g, g', s' \right)}{\lambda^{1-\gamma} V^d(\tilde{B}', \Gamma_- g)}, \right\} |g, s \right] \Big\} \\
&= \lambda^{1-\gamma} \max_{\tilde{B}'} \left\{ \frac{(W + \tilde{B}')^{1-\gamma}}{1-\gamma} + \right. \\
&+ \mathbb{E} \left[\max \left\{ \frac{V^{nd} \left(\Gamma_- g g' e^{\epsilon'} - R(\tilde{B}', \Gamma_-, g, s) \tilde{B}', \Gamma_- g, g', s' \right)}{V^d(\tilde{B}', \Gamma_- g)}, \right\} |g, s \right] \Big\} \\
&= \lambda^{1-\gamma} V^{nd}(W, \Gamma_-, g, s) ,
\end{aligned}$$

where the second equality of (16) uses a change of variable $B' = \lambda\tilde{B}'$; the third equality uses property (ii) for $V^d(\cdot)$ and the guess of properties (iii) and (iv) for $V^{nd}(\cdot)$ and $R(\cdot)$. We still need to show that $R(\lambda B', \lambda\Gamma_-, g, s) = R(B', \Gamma_-, g, s)$ is consistent with equation (14), since we used this property in the derivation of (16). For this, it's enough to show that the default probability under (B', Γ_-) is the same as under $(\lambda B', \lambda\Gamma_-)$. Notice that

$$\begin{aligned}
& \mathbb{P}(\lambda B', \lambda\Gamma_-, g, s) \\
&= \Pr\left((g', s', \epsilon') : V^d(\lambda B', (\lambda\Gamma_-)g) > V^{nd}\left((\lambda\Gamma_-)gg'e^{\epsilon'} - R(\lambda B', \lambda\Gamma_-, g, s)\lambda B', (\lambda\Gamma_-)g, g', s'\right) \mid g, s\right) \\
&= \Pr\left((g', s', \epsilon') : \lambda^{1-\gamma}V^d(B', \Gamma_-g) > \lambda^{1-\gamma}V^{nd}\left(\Gamma_-gg'e^{\epsilon'} - R(B', \Gamma_-, g, s)B', \Gamma_-g, g', s'\right) \mid g, s\right) \\
&= \Pr\left((g', s', \epsilon') : V^d(B', \Gamma_-g) > V^{nd}\left(\Gamma_-gg'e^{\epsilon'} - R(B', \Gamma_-, g, s)B', \Gamma_-g, g', s'\right) \mid g, s\right) \\
&= \mathbb{P}(B', \Gamma_-, g, s) \quad .
\end{aligned}$$

□

With these properties, the stationary system in Lemma 1 can be obtained by substituting $V^{nd}(W, \Gamma_-, g, s) = \Gamma_-^{1-\gamma}v^{nd}(W/\Gamma_-, g, s)$ and $V^d(B, \Gamma_-) = \Gamma_-^{1-\gamma}v^d(B/\Gamma_-, 1)$, where $v^{nd}(\omega, g, s) = V^{nd}(\omega, 1, g, s)$ and $v^d(b) = V^d(b, 1)$.

B Endowment Process Estimation

In this appendix, we provide more details on the estimation of the GDP process used in the quantitative evaluation of the model. We use the filter in [Kim \(1994\)](#) to obtain the likelihood function and explore the posterior using a Metropolis-Hastings MCMC algorithm with a random walk proposal density.

Let Y_t denote the country's GDP during year t , and let $\Delta y_t = \log(Y_t) - \log(Y_{t-1})$ denote GDP growth. The process in equations (6)-(7) implies

$$\Delta y_t = g_t + \sigma (\epsilon_t - \epsilon_{t-1}) \quad (17)$$

where $\epsilon_t \sim \mathcal{N}(0, 1)$, and g_t follows a two-state Markov process with transition probabilities $p_g(g'|g)$. Denote g_L and g_H to the possible values of g : $g_t \in \{g_L, g_H\}$. The transition probability is fully summarized by the two parameters p_L and p_H where $p_L = p_g(g_{t+1} = g_L | g_t = g_L)$ and $p_H = p_g(g_{t+1} = g_H | g_t = g_H)$.

Let $\theta = \{g_L, g_H, \sigma, p_L, p_H\}$ collect all parameters determining the process in equation (17). Denote $\mathcal{Y}_t = \{\Delta y_0, \Delta y_1, \dots, \Delta y_t\}$ to be all observations up to period t , and $\mathcal{L}(\theta | \mathcal{Y}_T)$ to be the likelihood of parameters θ where T is the total number of observations. We construct the likelihood $\mathcal{L}(\theta | \mathcal{Y}_T)$ using the filter in [Kim \(1994\)](#).

We assume uniform priors on parameters, which bounds the possible space for θ (see [Table 1](#)). Additionally, we include the normalization of $g_L \leq g_H$ as part of our priors. Let $p(\theta)$ denote the prior selection. The posterior of parameters θ is then given as

$$\mathcal{P}(\theta | \mathcal{Y}_T) = \mathcal{L}(\theta | \mathcal{Y}_T) p(\theta) \quad (18)$$

We explore the posterior $\mathcal{P}(\theta | \mathcal{Y}_T)$ using a Metropolis-Hastings MCMC algorithm with a random walk as a proposal density. In particular, the proposal density is a normal $\mathcal{N}(\theta_{n-1}, \bar{\sigma} \Sigma_\theta)$, where θ_{n-1} is the last draw of the chain, Σ_θ has $\theta^* = \arg \max_\theta \mathcal{P}(\theta | \mathcal{Y}_T)$ in its diagonal and zeros otherwise, and $\bar{\sigma}$ is selected so that the rejection rate in the chain is between 60 percent and 70 percent. We simulate 10 chains of length 125,000 each and compute posteriors by pooling 1 out of every 10 draws from the last 100,000 observations in each chain. [Table 1](#) contains all posterior estimates.

Data are from The Conference Board Total Economy Database, and we used GDP per capita in 2016 US\$ (converted to 2016 price level with updated 2011 PPPs) as our measure of output.

Different standard deviations across states We also estimate the endowment process for the case in which the standard deviation of the temporary shock depends on whether the economy is at the low-growth state, $g_t = g_L$, or high-growth state, $g_t = g_H$. We denote the standard deviations in the low- and high-growth states by σ_L and σ_H ,

Table 3: Prior and posterior distributions: different σ 's across growth states

	g_L	g_H	p_L	p_H	σ_L	σ_H
	Prior distribution					
	$U[-0.1, 0.1]$	$U[-0.1, 0.1]$	$U[0.1, 1.0]$	$U[0.1, 1.0]$	$U[10^{-3}, 0.5]$	$U[10^{-3}, 0.5]$
	Posterior distribution (mean, and 5th to 95th percentile intervals)					
Countries						
Italy	-0.014 [-0.022,-0.008]	0.026 [0.019,0.030]	0.654 [0.356,0.974]	0.766 [0.523,0.990]	0.020 [0.013,0.031]	0.011 [0.006,0.020]
Portugal	-0.002 [-0.011,0.003]	0.048 [0.040,0.058]	0.790 [0.503,0.976]	0.703 [0.403,0.990]	0.020 [0.012,0.029]	0.018 [0.011,0.029]
Spain	-0.017 [-0.025,-0.009]	0.035 [0.027,0.040]	0.608 [0.325,0.866]	0.810 [0.618,0.978]	0.019 [0.009,0.031]	0.015 [0.009,0.025]
Argentina	-0.027 [-0.043,-0.023]	0.072 [0.057,0.076]	0.776 [0.562,0.916]	0.570 [0.344,0.783]	0.046 [0.034,0.075]	0.013 [0.008,0.024]
Brazil	-0.032 [-0.071,-0.019]	0.029 [0.026,0.032]	0.605 [0.059,0.874]	0.790 [0.619,0.926]	0.022 [0.011,0.063]	0.019 [0.014,0.026]

Note: For each country, we estimate an output process as: $\Delta \ln y_t = g_t + \sigma_t \epsilon_t - \sigma_{t-1} \epsilon_{t-1}$, in which $\epsilon_t \sim N(0, 1)$ and $g_t \in \{g_L, g_H\}$, with $\Pr(g_{t+1} = g_L | g_t = g_L) = p_L$ and $\Pr(g_{t+1} = g_H | g_t = g_H) = p_H$. If $g_t = g_L$, then $\sigma_t = \sigma_L$, otherwise $\sigma_t = \sigma_H$. The table reports the mean and the interval between the 5th and 95th percentiles of the posterior distributions of each of the parameters for each country. The table also reports the prior distributions we used, which were chosen to be the same across countries. For each country, we use data on GDP per capita in 2016 US\$ (converted to 2016 price level with updated 2011 PPPs) between 1980 and 2017 from The Conference Board Total Economy Database as the measure of y_t .

respectively. Table 3 shows the estimates for all countries. First, note that the mean of the posterior distributions of the standard deviations is not very different across growth states, except for the case of Argentina, and that the mean of σ_L is larger than the mean of σ_H in all cases. In the case of Italy, the difference between σ 's is somewhat larger, but the values are still within the interval analyzed in Figure 12a, which was shown to not have significant implications for our results. Finally, Table 3 also shows that the posterior distributions of the remaining parameters are very similar to their counterparts in Table 1.

C Algorithm

Since we are dealing with multiple interest rate schedules, the standard algorithm to compute the recursive equilibrium in quantitative sovereign debt models (e.g., [Arellano, 2008](#)), which starts with a guess for the price/interest rate functions, is not suitable. We develop a new algorithm that iterates only on the value function. In each iteration, we construct the interest rate correspondence and select the high and low interest rate schedules associated with the respective value function. The algorithm consists of the following steps:

Step 1: Guess $v^{nd,n}(\omega, g, s)$, increasing function in all arguments. We use a grid for wealth, ω , consisting of 200 points evenly spaced between 0.01 and 1.5.

Step 2: For each growth state g , we construct the interest rate correspondence, that is, the values of debt b' and interest rates r such that lenders have an expected return equal to R^* . While we know that for a given b' , we might have multiple R 's satisfying this condition, we explore the fact that for a given $R > R^*$, there is a unique level of debt such that the expected return is equal to R^* . We construct a grid with 500 interest rates evenly spaced between $R^* = 1.2$ and 4. For each r in the grid, we compute b' such that

$$r = \frac{R^* - \kappa}{1 - \Pr(v^{nd,n}(g'e^{\sigma\epsilon'} - rb', g', s') < v^d(b'))} + \kappa.$$

We use the bisection method, with extreme values of b' as starting points. We know that for very high levels of debt, interest rates should tend to infinity and that, for very low levels of debt, interest rates should be equal to the risk-free rate.

Remember that ϵ follows a standard normal distribution. For each b' , it is useful to compute, for each one of the four combinations of (g', s') , the thresholds $\bar{\epsilon}(b', g', s')$ such that

$$v^{nd,n}(g'e^{\sigma\bar{\epsilon}(b', g', s')} - rb', g', s') = v^d(b').$$

In this case, we just need to compute b' that satisfies

$$\frac{R - \kappa}{R^* - \kappa} = \frac{1}{1 - \sum_{s'} \sum_{g'} p_g(g'|g) p_s(s'|s) \Phi(\bar{\epsilon}(b', g', s'))},$$

where Φ is the standard normal cumulative distribution.

The procedure above allows us to compute the correspondence, that is, the pairs (r, b') that, given $v^{nd,n}(\omega, g, s)$, imply an expected return equal to R^* .

Step 3: For each g , we use the respective interest rate correspondence to compute

the high and low interest rate schedules associated with the bad and good sunspots, respectively. We only consider the increasing schedules for reasons explained in the text.

Step 4: For each growth and sunspot states, (g, s) , and for each level of wealth in the grid, we compute the optimal choice of debt b' given the interest rate schedule associated with (g, s) . We use a golden search method. We split the problem into two, optimal choice of debt, $b' \geq 0$, and optimal choice of savings, $-\omega \leq b' \leq 0$, and select the one that achieves higher utility. The reason is that for $b' \leq 0$, the interest rate is constant and equal to R^* , which simplifies the problem. To compute the optimal choice of debt, $b' \geq 0$, we further split the problem according to the number of continuous segments in the respective interest rate schedule. Since there are discontinuities in the schedule, we compute the optimal level of debt in each of the continuous segments and then compute the optimal choice of debt among the selected candidates.

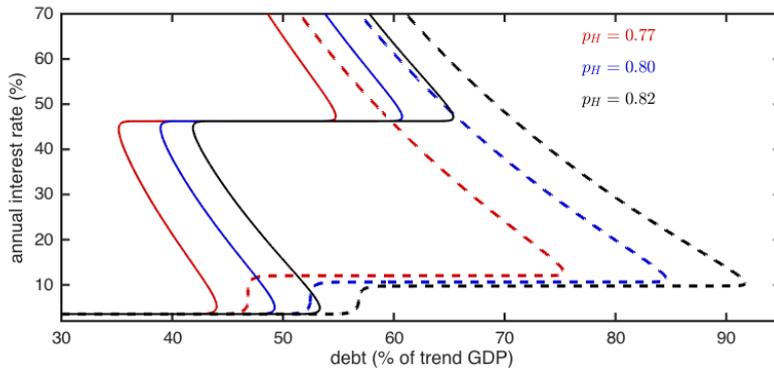
Step 5: Update $v^{nd, n+1}(\omega, g, s)$ and return to Step 1. Keep iterating until convergence is achieved. We used $1.0e^{-6}$ as our convergence criterion.

D Robustness

In this appendix, we complete the analysis of the robustness exercises in Section 3. There, we analyzed how the interest rate correspondences in the low-growth state varied for different values of p_L , σ , g_H , g_L , and κ . Here, we analyze how the interest rate correspondences vary for changes in the remaining parameters p_H , R^* , $\bar{\epsilon}$, γ , and β , and the transition probabilities of the sunspot state, and show the interest rate correspondences in the high-growth state for the cases analyzed in Section 3.

Figure 13: Robustness exercises: p_H and R^*

(a) varying the persistence of the high-growth state p_H



(b) varying the risk-free rate R^*

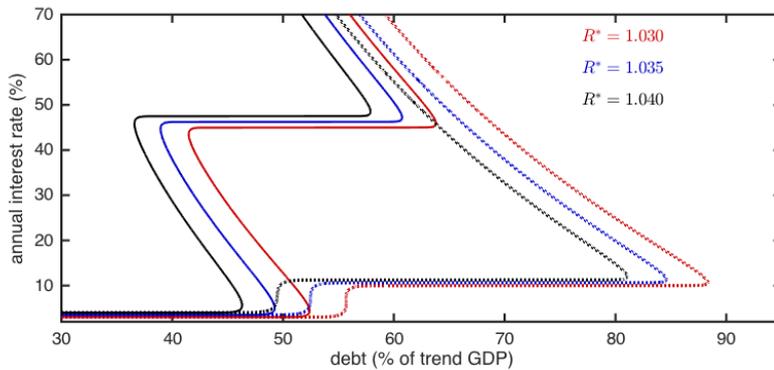


Figure 13a shows the interest rate correspondence in the low-growth (solid) and high-growth (dotted) states for three different values of the persistence of the high-growth state p_H . Similar to the argument used for p_L , illustrated in Figure 11a, the change in p_H only affects the higher interest rate of the high-growth schedules. The interest rate schedules are unique in the high-growth state, that is, they do not depend on the sunspot realization.¹⁹ The probability of default is based only on fundamentals. This implies that, in the high-growth state, the interest rates associated with levels of debt in which

¹⁹In our quantitative model, it is possible to observe multiple interest rate schedules in the high-growth state. But given our estimation of the endowment process, with relatively high values of p_H , the high-growth schedule happens to be unique.

the borrower chooses to default in the low-growth state increase as we decrease p_H . This explains why the high-growth schedules shift upward and to the left as we decrease p_H . The higher persistence of the high-growth state also affects the values of default and non-default in non-trivial ways. While lower persistence of the high-growth state reduces the value of default, it also reduces the value of non-default. In this exercise, the second effect dominates and we observe a shift to the left in both correspondences, with the borrower being able to issue more debt as p_H increases.

Figure 13b shows the interest rate correspondences for different values of the risk-free rate R^* . Lower risk-free rates imply lower costs of servicing the debt in any given state, which implies that probabilities of default are lower, and the borrower is able to sustain higher levels of debt. That explains why both low-growth (solid) and high-growth (dotted) interest rate correspondences shift to the right as we reduce R^* . In addition, the size of the multiplicity region of the interest rate correspondence in the low-growth state is virtually the same for the different values of the risk-free rate.

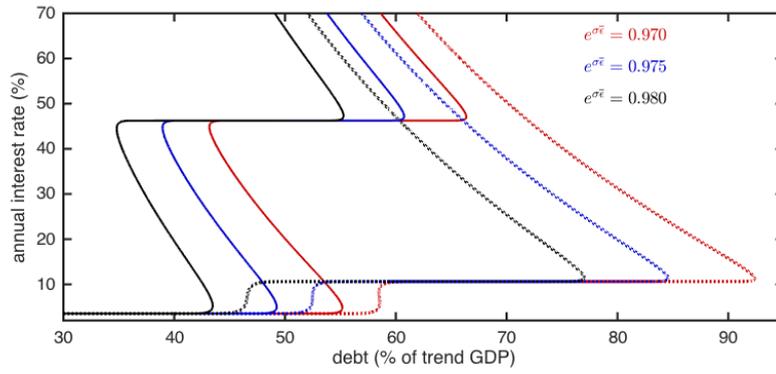
Figures 14a, 14b, and 14c show the interest rate correspondences for different values of the default cost $\bar{\epsilon}$, coefficient of risk aversion γ , and discount factor β , respectively. The figures show that the variations in those parameters only shift the correspondences to the left and right, without changing the high interest rates and with very little (if any) effect on the size of the multiplicity region. These shifts to the left or right reflect the effect of different values of the parameters on the values of default and non-default. When the value of default becomes relatively more attractive, the maximum amount of debt that the borrower can issue decreases and the schedules shift to the left. The opposite happens when the value of default becomes relatively less attractive.

Of course, default becomes less attractive for lower values of $\bar{\epsilon}$ in Figure 14a, and we observe that both correspondences shift to the right. Default also becomes relatively less attractive when the borrower becomes less risk averse, with lower values of γ in Figure 14b, and when the borrower becomes more patient, with higher values of β in Figure 14c.

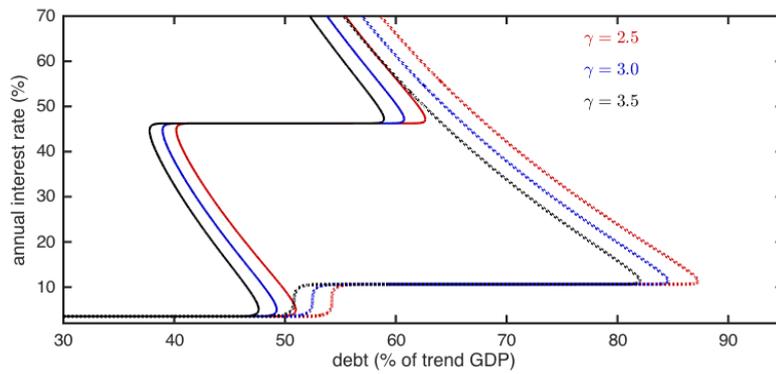
Finally, Figures 15 and 16 show the interest rate correspondences in the low-growth state for the parameters analyzed in Section 3.

Figure 14: Robustness exercises: $\bar{\tau}$ and γ

(a) varying the cost of default $\bar{\tau}$



(b) varying the risk aversion γ



(c) varying the discount factor β

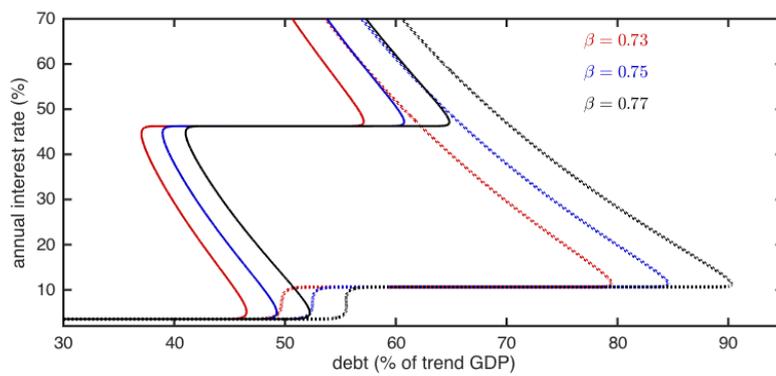
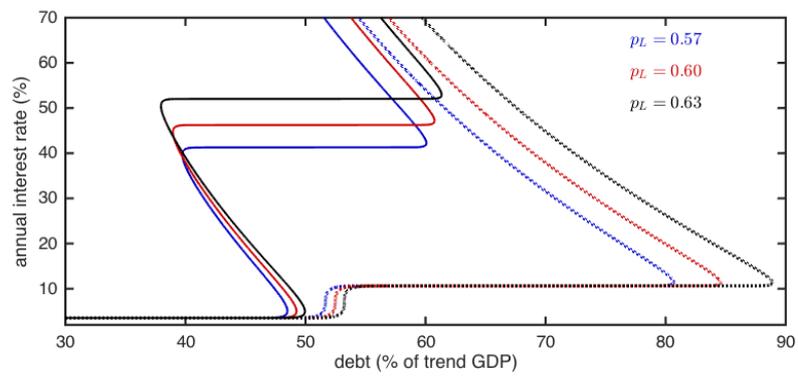


Figure 15: Robustness exercises: p_L and σ

(a) varying the persistence of the low-growth state p_L



(b) varying the standard deviation of transitory shocks σ

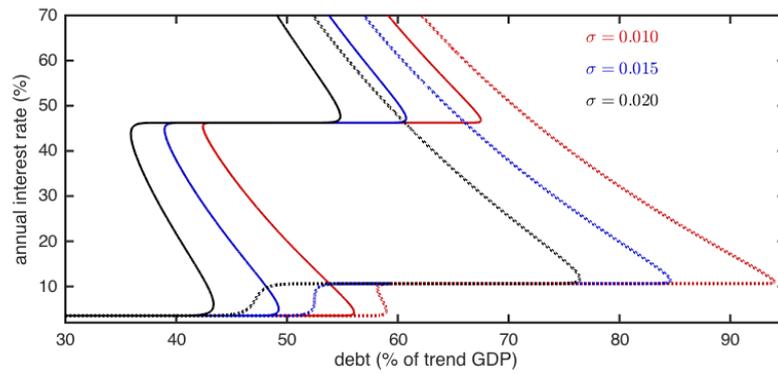
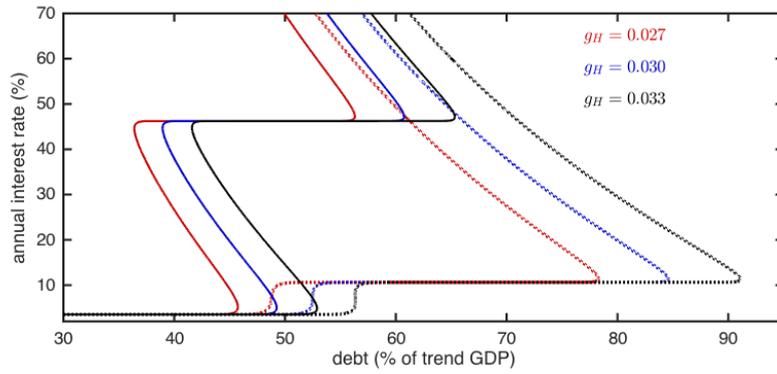
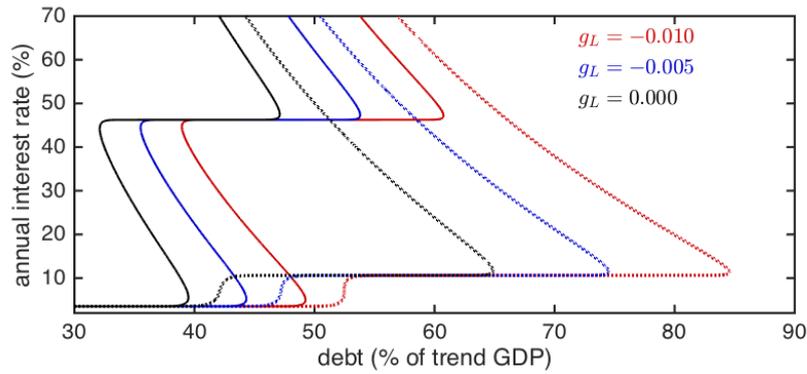


Figure 16: Robustness exercises: g_H , g_L , and κ

(a) varying the high-growth rate g_H



(b) varying the low-growth rate g_L



(c) varying the recovery rate κ

