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INTEREST ON RESERVES

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ABSTRACT

In a general equilibrium setting, we study versions of the proposal to pay interest on reserves at the market rate. We argue that the proposal makes the demand for total reserves indeterminate whether interest is paid on total reserves or on required reserves only. One consequence is that tax financing of the proposal gives rise to a continuum of equilibria, equilibria which differ in real returns and consumption allocations. Another consequence is that an attempt to finance the proposal through earnings on the central bank's portfolio either gives rise to an equilibrium with a zero nominal interest rate or fails to give rise to an equilibrium.

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At least since Friedman (1960) advocated payment of interest on reserves at the market rate, the proposal has been viewed with favor by many economists. Moreover, versions of it have recently been proposed as legislation in several countries. However, despite the attention the proposal has received, there has been little or no analysis of two related and problematical aspects of the proposal: its financing, and possible indeterminacy of equilibrium (existence of a continuum of equilibria).^{1/}

Indeterminacy of equilibrium is a possibility because the proposal eliminates the interest differential between the monetary base, at least the part held as reserves, and other assets. Since the proposal eliminates the foregone interest cost of holding reserves, it tends to produce an indeterminate demand for reserves and, hence, for the monetary base; that is, it makes the demand for the monetary base a correspondence rather than a function.

The version of the proposal most obviously conducive to indeterminacy in the demand for reserves is one that pays interest on total reserves. Under this version, financial intermediaries should be indifferent between holding excess reserves paying the market rate and holding other assets. This source of indeterminacy is widely recognized. However, there is also a "folk theorem" among economists who have studied interest on reserves that asserts that indeterminacy vanishes if interest is paid on required reserves only.

We doubt that this folk theorem is valid. With interest paid on required reserves only, the demand for required reserves

could itself be indeterminate even though financial institutions would face a foregone interest cost of holding excess reserves. Since financial institutions subject to reserve requirements would not be at a disadvantage relative to other financial institutions, all institutions could pay the market rate of interest on balances (deposits), while charging for services on a per unit basis--per check written, etc. In such circumstances, individuals should be indifferent between holding a relatively small fraction of their wealth in forms subject to reserves and holding much of it in such forms.^{2/} Moreover, as shown below, there exist portfolios of individuals and financial institutions consistent with required reserves being any fraction of net wealth. Thus, restricting payment of interest to required reserves does not eliminate indeterminacy in the demand for reserves.

However, an indeterminate demand for reserves and, hence, for base money is not sufficient to produce indeterminacy of equilibrium. Whether it produces a continuum of equilibria depends on how the proposal is financed.

In his 1960 discussion, Friedman noted that an interest on reserves plan could be financed either through taxes or through the earnings on the central bank's portfolio. Given an indeterminate demand for base money, we show that tax financing can give rise to indeterminacy of equilibrium. As for financing through earnings on the central bank's portfolio, we show that in a system with some outside money, either there is no equilibrium under such financing, or else the equilibrium is one in which real returns are equalized and interest is not paid (the nominal interest rate is zero).

The explicitly general equilibrium nature of our model accounts for the important role that our analysis assigns to the method of finance. Earlier discussions of interest on reserves by Patinkin (1961) and Fama (1983) did not mention financing. Patinkin conjectured that a necessary and sufficient condition for price level determinacy is the fixing of both (a) some nominal quantity and (b) some rate of return (Patinkin (1961, p. 116, 3rd paragraph)). Our analysis with interest payments financed by the government's earnings on its portfolio can be regarded as a counterexample to that conjecture, since we describe settings in which there is uniqueness among stationary and positive price level paths, even though there is a sense in which the government is not, under that scheme, fixing the rate of return on any asset. Consistent with Patinkin's conjecture, Fama (1983) claimed that if there is a positive and well behaved demand for real currency holdings when currency is dominated in rate of return and if the stock of nominal currency is fixed and no interest is paid on it, then price level determinacy obtains under schemes that pay interest on reserves at the market interest rate. While Fama's scheme may well deliver price level determinacy no matter how it is financed, it does not capture the spirit of interest-on-reserves proposals. The goal of such proposals is to achieve optimality by equalizing rates of return on all risk-free assets, including the monetary base, or as much of it as interest can be paid on. Consistent with this goal, we analyze payment of interest on the entire monetary base. Fama's scheme is equivalent to one without reserve requirements and without interest payments on any of the monetary base.

The remainder of this paper is organized as follows. Section 1 describes the model, an overlapping generations model which contains an indeterminate demand for base money under any scheme to pay interest on base money at the market rate of interest. Section 2 characterizes equilibria under tax financing, while Section 3 characterizes equilibria under financing by earnings on the central bank's portfolio. Section 4 briefly discusses the form that our results would take had we chosen to use various versions of an infinitely lived-agent model, rather than an overlapping generations model. Section 5 contains some concluding remarks.

1. The Model

We consider a pure-exchange (no production), one-good-per-date model of two-period lived overlapping generations which is defined over dates $t \geq 1$. We call the generation that is young at t and old at $t + 1$ generation t . We permit intra-generation diversity, but insist upon inter-generation homogeneity. Throughout, we assume competitive behavior and impose perfect foresight.

In the version we study, individuals hold the monetary base directly and all of it earns interest at the market rate.^{3/} In terms of the discussion of reserves given above, we are, in effect, consolidating the balance sheets of the public and those of financial institutions and we are omitting from the model the currency component of the base. The omission of currency is not important because if there is an indeterminate demand for one component of the base (the reserve component) and a determinant

demand for the remainder (the currency component), then there is an indeterminate demand for the total ^{4/}

Members of generation 0, the old at the first date, own among them H units of the monetary base, here assumed to be a government-issued fiat money. Each member of generation 0 acts competitively to maximize his or her consumption of time 1 good. It follows that the base, H, is supplied at any positive price in terms of the time 1 good.

For $t \geq 1$, member h in generation t has a twice-differentiable, strictly quasi-concave utility function whose arguments are $c_t^h(t)$ and $c_t^h(t+1)$, where $c_t^h(t+i)$ is h's consumption of time $t+i$ good. Member h in generation t has positive pre-tax endowments of good $t+i$, denoted $w_t^h(t+i)$, for $i = 0, 1$, and is liable to a tax payable at $t+1$ equal to $v_t(w_t^h(t)r_t + w_t^h(t+1))$, where v_t is the tax rate and r_t is the gross real rate of return at t (the price of time t good in units of time $t+1$ good) ^{5/} Individual h maximizes utility by choosing holdings of base money, m^h , loans, l^h , and consumption subject to

$$(1) \quad c_t^h(t) \leq w_t^h(t) - p_t(m^h + l^h)$$

$$(2) \quad c_t^h(t+1) \leq w_t^h(t+1) - v_t[w_t^h(t)r_t + w_t^h(t+1)] + r_t p_t(m^h + l^h)$$

where p_t is the price of base money at t in units of time t consumption (the inverse of the price level).

Notice that these constraints include the assumption that h earns the same return on base money and on loans. Moreover, since we assume that h can borrow or lend at r_t (l^h can be negative or positive), the constraint placed on consumption by (1)

and (2) is equivalent to the single constraint obtained by solving (1) for l^h and substituting the resulting inequality into (2); namely,

$$(3) \quad c_t^h(t) + c_t^h(t+1)/r_t \leq (1-v_t)[w_t^h(t)r_t + w_t^h(t+1)]/r_t$$

We express the result of h maximizing utility subject to (3) by a choice of $w_t^h(t) - c_t^h(t)$, of saving, which we denote by the function $s^h(r_t, v_t)$. The assumptions made about utility imply that s^h is a differentiable function that is defined for all $r_t > 0$ and $v_t \in [0, 1)$, is bounded above by $w_t^h(t)$ and, for fixed v_t , is such that $s^h(r_t, v_t) \geq 0$ as $r_t \geq \bar{r}^h$ for some \bar{r}^h .

We denote the sum of $s^h(r_t, v_t)$ over the members of generation t by $S(r_t, v_t)$ and denote the corresponding sum of $w_t^h(t)$ by W_1 and that of $w_t^h(t+1)$ by W_2 . It follows that S is differentiable, is bounded above by W_1 , and that there exists \bar{r} such that $S(\bar{r}, 0) = 0$ and $S(r, 0) > 0$ for all $r > \bar{r}$.

Although our main result is an existence result which depends on no more than the above assumptions, we do present one result that uses more restrictive assumptions. Since s^h is the utility maximizing choice of $w_t^h(t) - c_t^h(t)$, the partial derivatives of s^h --denoted s_1^h and s_2^h --are minus the partial derivatives of the utility maximizing choice of $c_t^h(t)$. And since (3) has the form of the usual budget set with endowments $(1-v_t)w_t^h(t)$ and $(1-v_t)w_t^h(t+1)$ and relative price r_t , the usual gross substitutes assumption implies $s_1^h > 0$ and, hence, $S_1 > 0$. As for s_2^h , we have $s_2^h = n^h[w_t^h(t)r_t + w_t^h(t+1)]/r_t$, where n^h is the partial derivative of the demand for $c_t^h(t)$ with respect to the R.H.S. of

(3)--i.e., with respect to wealth. Thus, if first and second period consumption are normal goods, then $\eta^h \in (0,1)$. It would then follow that $S_2 = n(W_1r_t + W_2)/r_t$ with $n \in (0,1)$, because $n = \sum \theta^h \eta^h$, where $\theta^h = [w_t^h(t)r_t + w_t^h(t+1)]/(W_1r_t + W_2)$, h's share of the wealth of generation t.

2. Equilibrium Under Tax Financing

Here we suppose that v_t is set to raise enough revenue to pay for interest payments on the monetary base. The total tax collected at date $t + 1$ is $v_t(W_1r_t + W_2)$. Since the time $t + 1$ interest payments total $p_t H(r_t - p_{t+1}/p_t)$, one condition for equilibrium is

$$(4) \quad v_t(W_1r_t + W_2) = p_t H(r_t - p_{t+1}/p_t)$$

The other equilibrium condition is that generation t's saving be equal to generation $t - 1$'s dissaving, or that

$$(5) \quad S(r_t, v_t) = p_t H$$

An equilibrium, therefore, consists of sequences for r_t , v_t and p_t that satisfy (4) and (5) for all $t \geq 1$.

We will prove that there is a continuum of constant sequences that are equilibria. To do that, it is convenient to substitute from (5) into (4) and to note that constant values of r_t and v_t --denoted r and v , respectively--that are equilibria must satisfy

$$(6) \quad v = S(r, v)(r-1)/(W_1r + W_2) \equiv g(r, v)$$

Conversely, if a pair (r, v) satisfies (6) and is such that $S(r, v) > 0$, then $(r_t, v_t, p_t) = (r, v, S(r, v)/H)$ for all $t \geq 1$ is an equilibrium, i.e., satisfies (4) and (5).

The following proposition describes a continuum of equilibria.

Proposition 1. Let $r^* = \max(1, \bar{r})$, where, recall, \bar{r} is such that $S(r, 0) > 0$ for $r > \bar{r}$. For each $r > r^*$, there exists $(r, v(r), p(r))$ with $v(r) \in (0, 1)$ and $p(r) > 0$ such that $(r_t, v_t, p_t) = (r, v(r), p(r))$ for all $t \geq 1$ is an equilibrium.

Proof. Given the remarks made above about how to associate equilibria with solutions to (6), we have to show only that for any $r > r^*$, there exists a $v(r) \in (0, 1)$ satisfying (6) and implying that $S(r, v(r)) > 0$.

Since $S(r, 0) > 0$ for any such r , $v < g(r, v)$ at $v = 0$. Since $S(r, v) < W_1$, $g(r, v) < W_1(r-1)/(W_1r+W_2) < 1$. Thus $v > g(r, v)$ for $v \geq W_1(r-1)/(W_1r+W_2)$. These facts and continuity of $g(r, v)$ in v imply that there exists a $v(r) \in (0, 1)$ satisfying (6). Finally, since $v(r) > 0$ and $r > 1$, (6) implies that $S(r, v(r)) > 0$. Δ

Since (6) is one equation in two unknowns, it comes as no surprise that there is a continuum of solutions and a corresponding continuum of equilibria.^{6/} Moreover, as is typical of continua of equilibria, each equilibrium in the continuum can be interpreted as a neutral equilibrium, rather than as a stable or unstable equilibrium. To see this, suppose that starting in an equilibrium, some saver switches marginally from private loans to base money, which can be interpreted as a switch from intermediary

accounts which do not require reserves to ones which do. This creates excess demands for current consumption and for base money, presumably driving upward both the interest rate and the price of base money. These price changes do not, however, seem to force the system back to the initial position. Instead, they can be regarded as giving rise to a higher tax which equilibrates the system at a position different from the initial one.

So far we have shown that there is a continuum of equilibria when there is complete indifference on the part of individuals between base money and other assets bearing the same return. The logic behind this result suggests that it would carry over to situations in which there is indifference only beyond some minimum holding of base money. We establish this only under the gross substitutes and normal goods assumptions.

We now suppose that in addition to (4) and (5), an equilibrium must satisfy the inequality $S(r_t, v_t) > S_0$, where S_0 is to be interpreted as some positive minimum holding of real base money. Under the restrictive assumptions on S , we show that if there is an equilibrium with base money equal to the minimum, then there is a continuum of equilibria.

Proposition 2. If first and second period consumption are normal goods and gross substitutes and if there exists (r_0, v_0) such that $v_0 > 0$, $S(r_0, v_0) = S_0 > 0$ and $v_0 = g(r_0, v_0)$, then for any $r > r_0$, (i) there exists a $v(r)$ such that $v(r) = g(r, v(r))$ and (ii) $S(r, v(r)) > S_0$.

Proof. The hypotheses imply that $r_0 > 1$. Differentiating the function g , we find that for any $r > 1$, the gross substitutes assumption implies $g_1 > 0$ and the normal goods assumption implies $g_2 \in (0,1)$. The restriction on g_1 implies that $g(r, v_0) > g(r_0, v_0) = v_0$ for any $r > r_0$. This and the upper bound on g imply existence--that is, conclusion (i)--exactly as in the proof of Proposition 1. The restriction on g_2 implies that $v(r)$, the solution to (6), is a differentiable function, differentiable because S is differentiable. Then differentiating (6), we find that $v'(r) = g_1/(1-g_2) > 0$. It follows that $dS(r, v(r))/dr = S_1 + S_2 v'(r) > 0$ which establishes (ii). Δ

It may seem that the continua of propositions 1 and 2 overstate the degree of indeterminacy that would prevail if interest is paid on required reserves only. After all, these continua include equilibria in which $s^h(r, v(r)) > 0$ for all h , equilibria in which everyone is a net saver. That being so, one may wonder whether such equilibria can be consistent with $S(r, v(r))$ being equal to required reserves, where required reserves are some given fraction of the liabilities of a class of financial intermediaries. We show that they can be. The demonstration involves constructing gross portfolios of individuals and of intermediaries consistent with $S(r, v(r))$ being equal to required reserves.

Given $s^h(r, v(r))$ for each h and $p(r) > 0$, the requirement that the base consist entirely of required reserves is $\sum s^h(r, v(r)) = kp(r)L_1 = kp(r)\sum l_1^h$ where $k \in (0,1)$ is the reserve requirement, L_1 is total intermediary liabilities subject to reserves, and l_1^h is h 's loans to (deposits in) such intermedi-

aries. Letting $l_2^h < 0$ be the gross debt of h to such intermediaries, we have only to find $l_1^h \geq 0$ and $l_2^h \leq 0$ for each h such that $l_1^h + l_2^h = s^h(r, v(r))/p(r)$ and $k \Sigma l_1^h = \Sigma s^h(r, v(r))/p(r)$. This can always be done.

This kind of construction suggests why a scheme of limiting payment of interest to required reserves cannot be expected to produce determinacy. So long as required reserves are defined in terms of the portfolios of a class of financial institutions whose size is endogenous, individuals can engage in borrowing and lending with such institutions so as to make the fraction of their net saving held in the form of required reserves as large as they want.

3. Equilibrium Under Financing by Earnings on the Central Bank's Portfolio

If financing interest payments by taxation gives rise to too many equilibria, financing them by earnings on the central bank's portfolio gives rise to too few equilibria. When interest payments on reserves are financed entirely by earnings on the central bank's portfolio, the equilibria are identical with those for a system with an unchanging stock of outside base money on which interest is not paid and with rate of return equality between base money and other assets. Moreover, there may be no equilibrium with valued outside base money, in which case we will say that there is no equilibrium with interest payments on base money being financed by interest on the central bank's holding of securities.

Before demonstrating these results, it may help to describe them as follows. Given some initial outstanding stock of (outside) base money and some prospective rates of return on its holdings of interest-bearing securities, suppose that a central bank undertakes an open market strategy--a strategy of lending to or borrowing from the public--with the goal of earning enough on its portfolio to pay interest at the market rate on all newly issued liabilities and on the initially outstanding stock. This goal can be achieved only if the real rate on loans is driven down to the real rate on base money absent any interest payments. Otherwise, the surplus needed to cover interest on the initial stock of base money is never achieved. Therefore, the effort to implement this open-market strategy will either lead to a zero nominal interest rate or else fail to lead to an equilibrium.

To demonstrate the results, we begin by setting out two conditions, the analogues of (4) and (5), that any equilibrium under this scheme must satisfy. The relevant analogue of (4) is

$$(7) \quad r_t p_t L_t^G - p_{t+1} L_{t+1}^G + (H_{t+1} - H_t) p_{t+1} = p_t H_t (r_t - p_{t+1}/p_t)$$

where L_t^G is nominal one-period government loans to the private sector at t and H_t is base money held by the private sector from t to $t + 1$. The R.H.S. of (7) is the same as that of (4), except that (7) takes into account that the nominal stock on which interest is to be paid can change. The L.H.S. says that receipts at $t + 1$ consist of interest on loans granted earlier less new loans granted plus additions to the monetary base.^{7/} The relevant analogue of (5) is

$$(8) \quad S(r_t, 0) = p_t H_t - p_t L_t^G$$

We also require that $S(r_1, 0) > 0$, which is to require that the scheme be consistent with a positive value for the initial stock of outside base money.

We now derive a consequence of (7) and (8). Note that (7) can be rewritten as $p_{t+1} H_{t+1} - p_{t+1} L_{t+1}^G = r_t (p_t H_t - p_t L_t^G)$. Then, using (8) for t and $t + 1$, we have

$$(9) \quad S(r_{t+1}, 0) = r_t S(r_t, 0); \quad t > 1$$

Thus, any equilibrium r_t sequence under the present scheme must satisfy (9) and $S(r_1, 0) > 0$.

From (9), it is easy to see that an equilibrium may not exist. For example, if $S_1 > 0$ and $\bar{r} > 1$, then no equilibrium exists, because, then, no positive and bounded sequence for $S(r_t, 0)$ can satisfy (9). Also, any equilibrium in which r_t converges to a constant, r , is necessarily one in which either $r = 1$ or $S(r, 0) = 0$. More generally, any equilibrium under this scheme has the same real return sequence and consumption allocation as an equilibrium under a seemingly different scheme, one in which there is a fixed stock of outside base money on which interest is not paid and in which the return on private loans is equal to the return on base money. In our notation, these conditions are

$$(10) \quad S(r_t, 0) = p_t H$$

$$(11) \quad r_t = p_{t+1}/p_t$$

By substituting from (10) for t and $t + 1$ into (11), we find that any r_t sequence satisfying (10) and (11) with $S(r_1, 0) > 0$ satisfies (9), and vice versa.

This equivalence verifies our casual discussion which suggested that if financing of interest on reserves by earnings on the central bank's portfolio is accomplished, then it is accomplished because the real return on loans is driven down to the real return on base money so that interest does not have to be paid. We doubt that Friedman or other advocates of interest on reserves intend that this be the outcome of their proposal.

4. Results in Alternative Models

Now we briefly discuss the implications of paying interest on reserves under the alternative financing schemes in several versions of the infinitely-lived agent model.

In a cash-in-advance model of the kind used by Lucas (1982), paying interest on base money at the risk-free market rate of interest makes the cash-in-advance constraint nonbinding and makes the demand for base money indeterminate. It can be shown that with interest on base money financed by lump sum taxes, there exists a continuum of equilibria with distinct price level and real balance paths. However, each of these equilibria is associated with the same consumption allocation. With interest payments on base money financed by interest earnings on the government's portfolio, it can be shown that no equilibrium exists. These results are proved in Sargent (1984). Like the results in the present paper, these results with cash-in-advance models highlight the importance of the method of finance.

In a money-in-the-utility-function model of the sort used by Brock (1974, 1976) and LeRoy (1984), the implications of paying interest on base money depend importantly on how the utility function is specified, and, in particular, on whether satiation in real balances can occur. If satiation is ruled out, as in Brock (1974, 1976), then no equilibrium exists with base money bearing the market rate of return because the demand for it would be unbounded. Infinite real balances supported by infinite lump sums taxes is not an admissible equilibrium. If the utility function is specified so that satiation in real base money occurs at some finite level, then the demand for base money is indeterminate when base money bears the market rate of return. Under this specification, as one would suspect, the results are the same as in the cash-in-advance specification.

In the model of Bewley (1980, 1983), the demand for real base money is insatiable. Base money is the only asset available, all state-contingent futures markets being ruled out. By accumulating and decumulating base money, consumers can smooth their consumption across time and across states of nature. However, so long as base money pays less than some critical interest rate, consumers bear some risk in equilibrium. As the rate of return on base money is driven toward that critical rate from below, consumers' demand for real balances grows without bound. As in the no-satiation version of the money-in-the-utility-function model, such unboundedness of the demand for real balances prevents an equilibrium from existing with interest paid at that critical rate.

The main difference between the implications of these models (when they have equilibria under payment of interest on base money at the market rate) and our model concerns the extent of the indeterminacy. In these models, the indeterminacy applies only to real balances and the level of taxes, while in our model it extends to real returns, consumption allocations, and utility levels. This difference is less significant than it may at first appear; it would not survive substituting distorting taxation for lump-sum taxation in both models.

5. Concluding Remarks

Paying interest on reserves was Friedman's way of circumventing defects in the "Chicago Plan of Banking Reform" of Henry Simons and Lloyd Mints. The original Chicago plan called for a government monopoly on issuing currency, imposition of 100 percent reserves on deposit liabilities of banks, and stabilization of the quantity of high-powered money.^{8/} Friedman argued that the Simons-Mints plan suffered from inferior "economic results" and difficulties of avoidance (Friedman, 1960, p. 66). Our recent analysis of the Simons-Mints plan, which we called a "quantity theory" plan, arrived at similar conclusions (see Sargent-Wallace (1982)). We found that the plan leads to bad economic results in the form of different intertemporal marginal rates of substitutions for different people, and difficulties of avoidance in the form of incentives on the part of private borrowers to issue claims that compete with low-yielding outside money.^{9/} We did not, however, analyze Friedman's suggestion for paying inter-

est on reserves. In part, the present paper was undertaken to correct that oversight.

The equality of rates of return that Friedman wanted to achieve is built into our model at the outset, in the budget constraints (see (1) and (2)). As Friedman suggested it would, such equality eliminates both problems: discrepancies in intertemporal marginal rates of substitution and enforcement problems. However, some new problems arise when financing requirements are explicitly considered. Under tax financing, the interest rate, tax rate, price level, and consumption allocation all become indeterminate. Under financing through earnings on the central bank's portfolio, an equilibrium may fail to exist; and when an equilibrium does exist, it undoes the effects on the price level and interest rate of the Simons-Mints restrictions giving the government a monopoly of currency issue, requiring 100 percent reserves, and stabilizing the quantity of outside base money.

Without doubt, current proposals to pay interest on reserves are proposals that involve tax financing. There has even been some discussion in the U.S. concerning the budgetary impact of the proposal. That discussion, however, presumes that the current level of reserves--roughly \$40 billion, or about 5 percent of federal debt in the hands of the public--is indicative of the level of reserves under the proposal. We have offered grounds for doubting that presumption. With interest on reserves, the principal incentive for minimizing holdings of reserves disappears. That opens the possibility that the level of reserves in real terms or relative to GNP could be quite different under the pro-

posal from what it is now. If, as we suggest, the demand for reserves under the proposal is indeterminate, then the tax financing version of the proposal is problematic. It is ill-defined because it leaves indeterminate real interest rates, real taxes, and real consumption allocations.

Footnotes

1/Possibilities of indeterminacy of equilibrium under schemes equivalent to paying interest on reserves were discussed by S. C. Tsiang (1969) using a framework different from ours.

2/In the U.S., interest payments are taxed, but transactions services (free check-writing) are not. This creates an incentive not to price services directly and to finance services by paying less than the market rate on balances. Such pricing gives customers an incentive to economize holdings of high service balances. We ignore this effect in our analysis.

3/Although our results do not depend on having a description of what happens if interest is not paid, we have in mind that there would then be rate-of-return discrepancies among riskless assets. Such discrepancies appear as a result of binding legal restrictions on private intermediation in Sargent-Wallace (1982). They appear for other reasons in the settings described below in Section 4.

4/The claim that omitting currency is not important presumes that the nominal supply of the total base, any currency plus reserves, is being fixed exogenously, not the currency component only, as under the Fama scheme described above.

5/The assumption that taxes are payable when old permits a steady state to be an equilibrium (path) under schemes initiated at $t = 1$. It is easily shown that associated with each stationary equilibrium under that tax payment assumption is an equilibrium (path) under the assumption that tax payments are distributed over the lifetime, an equilibrium (path) which is identical for $t \geq 2$.

6/The proposition 1 multiplicity of equilibria is closely related to the nonneutrality of alternative (negative) growth rates of outside base money when these are financed by lump-sum taxation (see, for example, Tobin 1965 and Wallace 1980). Indeed, the proposition 1 equilibria are identical in real terms to those of a closely related model under such a scheme. The closely related model has the same S function and the following equilibrium conditions: (a) $v_t(W_1 r_t + W_2) = (H_t - H_{t+1})p_{t+1}$ with $H_1 = H$, an initial condition; (b) $S(r_t, v_t) = p_t H_t$; and (c) $r_t = p_{t+1}/p_t$. The following proposition can be verified directly: equations (4) and (5) are satisfied by sequences (r'_t, v'_t, p'_t) if and only if (a)-(c) are satisfied by (r'_t, v'_t, p''_t) with $p''_1 = p'_1$.

7/One could conceivably constrain the L.H.S. of (7) further by requiring that additions to base money come about only in connection with net additions to loans granted. Our results also hold for such schemes.

8/For a description of the Simons-Mints plan, see Friedman (1960, p. 65) and the references listed in footnote 8 of Friedman (1960, p. 108).

9/In Sargent-Wallace (1982), we assumed that these incentives are completely thwarted by a perfectly and costlessly enforced legal restriction on the minimum denomination of privately issued securities.

By the way, we made an error in describing the implications for individual budget sets of this restriction. We mistakenly assumed that net borrowers never hold base money. Luckily for us, we chose parameters so that all the equilibria we described are correct; they are such that the omitted opportunity is not, in fact, chosen.

References

- Bewley, Truman, 1980, The Optimum Quantity of Money, Models of Monetary Economies ed. by John Kareken and Neil Wallace, (Federal Reserve Bank of Minneapolis, Minneapolis) 169-210.
- _____, 1983, A Difficulty with the Optimum Quantity of Money, Econometrica, 51, September, 1485-1504.
- Brock, William A., 1975, A Simple Perfect Foresight Monetary Model, Journal of Monetary Economics, 1, April, 133-50.
- _____, 1974, Money and Growth: The Case of Long-Run Perfect Foresight, International Economic Review, 15, October, 750-77.
- Fama, Eugene, 1983, Financial Intermediation and Price Level Control, Journal of Monetary Economics, 12, July, 7-28.
- Friedman, Milton, 1960, A Program for Monetary Stability, (Fordham University Press, New York).
- LeRoy, Stephen F., 1984, Nominal Prices and Interest Rates in General Equilibrium: Money Shocks, Journal of Business, 57, April, 177-195.
- Lucas, Robert E., Jr., 1982, Interest Rates and Currency Prices in a Two-Country World, Journal of Monetary Economics, 10, November, 335-360.
- Patinkin, Don, 1961, Financial Intermediaries and the Logical Structure of Monetary Theory, American Economic Review, 51, March, 95-116.
- Sargent, Thomas J. and Neil Wallace, 1982, The Real Bills Doctrine vs. the Quantity Theory: A Reconsideration, Journal of Political Economy, 90, December, 1212-36.

Sargent, Thomas J., 1984, Dynamic Macroeconomic Theory, Ch. III, manuscript.

Tobin, James, 1965, Money and Economic Growth, Econometrica, 33, October, 671-84.

Tsiang, S. C., 1969, A Critical Note on the Optimum Supply of Money, Journal of Money, Credit, and Banking, 1, May, 266-80.

Wallace, Neil, 1980, The Overlapping Generations Model of Fiat Money, Models of Monetary Economies, ed. by John H. Kareken and Neil Wallace, (Federal Reserve Bank of Minneapolis, Minneapolis), 48-82.