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SOME UNSOLVED PROBLEMS FOR MONETARY THEORY

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Some Unsolved Problems for Monetary Theory

In this paper, I discuss two policy questions which I regard as unresolved: should interest be paid on money and should currency provision be in the hands of the government? An affirmative answer to the first question stands as one of the few widely accepted general results of monetary theory. My discussion is intended to cast doubt on it. There is no widely accepted answer to the second question; some have asserted that currency provision is a public good while others have asserted that currency provision should be left to the market. My discussion of currency provision will not provide a resolution. Instead, I will discuss a way of formulating the question that seems to offer some hope for resolving it.

I. Payment of Interest on Money

The casual statement of the case for paying interest on money is familiar. Real balances are produced at zero social cost. In an equilibrium in which the real yield on other assets exceeds that on money--or, more generally, in which the marginal rate of substitution between future consumption and present consumption exceeds the real return on money--individuals face a positive alternative cost of holding money. Given the zero social cost, this positive alternative cost implies that too little money is being held. Payment of interest on money removes the positive alternative cost. The aspect of this casual statement on which I focus is how an equilibrium with a real return on money less than the relevant intertemporal marginal rate of substitution arises.

In order for a model to have such an equilibrium, it must contain something that prevents individuals from borrowing at the real return on money. Obviously, if individuals can freely borrow at the real return on money by issuing liabilities (forms of inside money) that are perfect substitutes for (outside) money, then such an equilibrium cannot come about. We are all familiar with the usual ways of ruling out such perfect substitution. Money-in-the-utility-function models rule it out through the assumption that the money in the utility function is outside money, not the sum of outside and inside money with individuals free to issue inside money. Cash-in-advance (Clower-constant) models rule it out by a similar identification of the objects that qualify as cash. It is ruled out in a more straightforward manner in a model like Bewley's (1983) in which it is simply assumed that there are no private credit instruments.

For appraising paying interest on money, it is desirable to be more explicit about what prevents individuals from borrowing at the real return on money. Explicit assumptions about what prevents individuals from borrowing at the real return on money are likely to have implications for the feasibility of tax-financed payment of interest on money and for the feasibility of other policies for dealing with the seeming nonoptimality that accompanies a real return on money lower than the relevant marginal rate of substitution. I will illustrate this by describing a formulation that is more explicit than the models just mentioned about what prevents individuals from borrowing at the real return

on outside money. In this formulation, the feature that prevents private borrowing makes feasibility of tax-financed interest on money questionable. And, in it, if tax-financed interest on money is feasible, then so are other policies that seem at least as desirable.

An Overlapping Generations Model Without Private Borrowing

This is a stationary, one good per date, pure exchange model of two-period lived overlapping generations. Each person cares only about his own lifetime pattern of consumption according to a twice differentiable strictly quasi concave utility function which implies that consumption at each date is a normal good. There is diversity in tastes and endowments within generations but no diversity across generations. Endowments are positive and the diversity within generations is such that there is someone for whom the marginal rate of substitution at the endowment is less than unity (h in Figure 1) and someone else for whom it exceeds

[Place Figure 1 approximately here]

unity (h' in Figure 1). Finally, suppose that the people who are in the second period of their lives at the initial date, the initial old, own in the aggregate one unit of fiat outside money.

Such settings are usually analyzed under the assumption that nothing prevents individuals when young from borrowing at the real return on outside money. I want to make a different assumption, namely that individuals cannot borrow at any return because

they cannot credibly commit themselves to repay when old. Although young people know they will receive an endowment of the good when they are old, I assume that they cannot commit this as collateral on a loan and that nothing prevents old people, whether or not they have borrowed, from consuming their endowment, and, hence, from reneging on any loan.

As I will now show, this model displays some features which seem to justify the prescription to pay interest on money. Thus, no nonintervention equilibrium is Pareto optimal. Also, there exists a nonintervention equilibrium with a positive and constant value of fiat money, one with a gross real return on money equal to unity. Finally, there exists an interest-on-money scheme financed by lump sum taxes payable when old which gives rise to a Pareto optimal equilibrium. The model, however, also displays features which should make us doubt the prescription. Lump sum taxation payable when old seems inconsistent with the inability of people to commit income when old as collateral on a loan. Although there exists an interest-on-money scheme financed by lump-sum taxes payable when young that supports an optimal equilibrium, such schemes require that the tax rate levied on an individual depend on the individual's saving propensity. Finally, the distributional consequences of any stationary interest on money equilibrium relative to the stationary monetary nonintervention equilibrium are such as to cast doubt on the desirability of such schemes.

To prove that no optimal nonintervention equilibrium exists one simply notes that optimality requires equality of marginal rates of substitution for all members of generation t . Under our assumption that individuals cannot borrow, this requires a real gross return on money no smaller than the maximum marginal rate of substitution at the endowment, denoted r^* . Since r^* is assumed to exceed unity, this requires a return that exceeds and is bounded away from the unity for all t , or, equivalently, a minimum positive rate of appreciation of money. Since the real value of money is bounded by the sum of first period endowments for a generation, a constant in this model, such appreciation cannot occur as an equilibrium.

That a positive and constant value monetary equilibrium exists follows from the assumption that the marginal rate of substitution of some person is less than unity at the endowment (e.g., person h in Figure 1). The relevant equilibrium condition can be expressed as follows. First, for each h in generation t , let $s^h(r, v)$ be the utility maximizing choice of $w_1^h - c_1^h$ (saving) subject to

$$(1) \quad c_1^h + c_2^h/r \leq (1-v)(w_1^h + w_2^h/r)$$

$$(2) \quad c_1^h - w_1^h \leq 0$$

where (c_1^h, c_2^h) , the vector of arguments of h 's utility function, is h 's consumption when young and when old respectively, (w_1^h, w_2^h) is the corresponding vector of pretax endowments of h , v is a tax rate levied on wealth, and r is the gross real

return faced by h . Note that constraint (2) is the version of the no-borrowing restriction implied by the assumption that any taxes are payable when old. The positive and constant equilibrium value of money is simply $\sum s^h(1,0)$, where the summation (and all those below) is over the members of a single generation. Our assumption about diversity guarantees that the sum is positive.

In order to prove that there exists a v that supports an optimal equilibrium, first note that the normal goods assumption implies that with positive taxation payable when old, the marginal rate of substitution at the after-tax endowment is smaller than at the pre-tax endowment. Thus, at any return $r > r^*$ and positive tax rate, everyone wants to save so that constraint (2) is not binding. Thus, we need only show that there exists a triplet (r,v,p) that satisfies $r > r^*$, $v \in (0,1)$, $p > 0$ and

$$(3) \quad \sum s^h(r,v) = p$$

$$(4) \quad (r-1)p = v(rW_1 + W_2)$$

where W_1 is the sum of endowments when young in a generation and W_2 is the corresponding sum when old. Equation (4) is the condition that interest payments (the left-side) be equal to tax revenue (the right side). One way to prove that such a triplet exists is to show that for any $r > r^*$, there exists a $v \in (0,1)$ that satisfies

$$(5) \quad (r-1) \sum s^h(r,v) = v(rW_1 + W_2)$$

Existence of such a v is implied by three facts. First, with $r > 1$, the left side of (5) exceeds the right side at $v = 0$. Second, there exists a v large enough but less than unity for which the right side exceeds the left side (essentially because $\sum s^h(r, v) < W_1$). Third, both sides of (5) are continuous functions of v . That any such $(r, v(r))$ implies a positive p from (3) follows because $r > r^*$ implies $s^h(r, v(r)) > 0$ for all h . This, in turn, implies equality of marginal rates of substitution and Pareto optimality because $r^* > 1$.

We now discuss features which call into doubt the prescription to pay interest on money in this model.

I have shown that with lump-sum taxes payable when old, it is easy to support an optimal equilibrium. However, such taxing seems to contradict the assumption that people cannot commit themselves to repay debts. If the government can enforce taxes payable when old, why can't it enforce debt repayment?

Taxes payable when young would not seem to suffer from the same enforcement problem, but as I now indicate, such tax schemes are subject to other difficulties. Above, I showed that an optimal equilibrium exists by showing that there are high enough returns and corresponding supporting taxes so that constraint (2) is not binding for any person. With taxes payable when young, each person faces constraint (1), but a more stringent version of (2), namely

$$(2') \quad c_1^h - w_1^h \leq -v(w_1^h + w_2^h/r)$$

which is equivalent to $c_2^h > w_2^h$ if (1) holds with equality. In terms of Figure 2, for a given

[Place Figure 2 approximately here]

return and tax rate, constraint (2') requires that each person choose to be northwest of point s' , while constraint (2) only requires the person to choose to be northwest of point s . It is easy to produce innocent examples for which there does not exist a steady state with a uniform tax rate, with taxes payable when young, and with (2') nonbinding.^{1/} Although there exist vectors of tax rates which support a steady state with (2') not binding, the tax rates must vary across people and must be such that relatively high saving people pay relatively high taxes, while still regarding those taxes as lump-sum. Obviously, the informational requirements for such taxing are substantial.

Regarding distributional consequences, at least some people in each generation other than generation 0 are worse off under any stationary optimal interest-on-money equilibrium than under the nonintervention equilibrium. Indeed, in a sense, the average member of each such generation is worse off. This follows from the fact that the interest-on-money equilibrium satisfies feasibility with equality,

$$(6) \quad \sum [c_1^h + c_2^h - (w_1^h + w_2^h)] = 0$$

and is such that everyone is saving. It follows that if someone in generation t ends up outside the budget set faced in the non-

intervention equilibrium, then someone else ends up inside it and, hence, is worse off. The sense in which the "average" person in generation t is worse off under the interest payment scheme is depicted in Figure 2. It shows the situation of someone who ends up on the boundary of the non-intervention budget set, or, equivalently, of someone who ends up paying taxes equal to the interest on his money holdings. This worse off person is the "average" person because (6) holds. It is, of course, the initial old who benefit from the interest on money scheme.

For several reasons, then, the above model makes tax-financed interest on money a questionable policy. A tax scheme with taxes payable when old seems to contradict the assumption that private debt agreements cannot be enforced. A tax scheme with taxes payable when young must have tax rates depend on individual characteristics. And, finally, any such tax schemes seem to have strange distributional consequences.

Of course, the above is simply one particular model--and, perhaps a bizarre one. However, there are few, if any, other models which both account for the inability of individuals to borrow at the real return on outside money and imply that tax supported interest on money is a feasible and good policy. The one model I know of which comes closest to displaying both properties is Townsend's "Turnpike model of exchange" (1980, pages 267-275). But even in his set-up, as he notes (1980, page 297), the feature that rules out private borrowing, an extreme form of spatial and informational separation, makes one uneasy about the feasibility of tax schemes for supporting interest on money.

II. Currency Provision

Here, I will discuss an approach to studying currency provision. The goal of such study is ultimately to answer questions of the following sort. Should currency provision be a government activity? And, if it should be, how should it be managed? Armed with a theory that address such questions, we might hope to appraise the United States coinage act of 1792 which, among other things, created a government mint, set up a denominational structure, directed the mint to issue coins to those depositing gold and silver bullion at the mint with no expense to the depositor, and yet made no provision for replacing worn coins with new coins. We also might hope to appraise the current system of currency provision under which, for example, ten \$1 bills exchange for one \$10 bill despite the obvious production cost discrepancies, and under which worn units of currency are exchanged for new units despite the cost of making the replacement. (The latter policy is currently coming under pressure because of the large demand for new or almost new units of currency arising from the use of automatic teller machines.) We might also be able to better interpret the fairly general view that some economies have suffered from an inadequate currency. It is claimed, for example, that during the Colonial period and during periods in the 19th century, the United States suffered a shortage of small denomination currency (see Hanson 1979 and Timberlake 1978, Chapter 9). It is also claimed that England during the 19th Century suffered from a currency that was badly worn (see Jevons 1918).

As these comments suggest, issues about currency provision arise in both what we may loosely call fiat monetary systems and in commodity monetary systems. That being so, it may be helpful to interpret the following discussion in the context of commodity money systems and, in so doing, avoid the additional complications that must be faced in studying fiat monetary systems.

I want to approach the study of currency provision in commodity money systems by using suggestions, made repeatedly, about the properties of objects that render them suitable as commodity monies. Among the suggested properties are: divisibility, durability, portability, and recognizability. However, I want to use this kind of list in what may be a new way. I want to treat the listed properties both as conjectures about properties that permit an object to play a prominent role in exchange and as conjectures about properties that a decentralized system, a laissez-faire system, will in some sense misproduce--underproduce or overproduce. Naturally, to even discuss such possibilities the environments or models we use better not have these properties be too freely available. Indeed, even aside from the welfare economics objective, these suggestions about properties of commodity monies make no sense in environments in which all objects share these properties to an unlimited degree--are perfectly divisible, durable, portable, and recognizable. Thus, for example, if we want to study divisibility, then we had better start out with a model in which not everything is divisible to start with and in

which making things more divisible is costly. Also, in order to give the conjectures a chance, we want to have an environment in which there is conceivably some nontrivial role for media of exchange. Obviously, study of Robinson Crusoe alone on his island will not do. Before I discuss some features of a candidate model, I want to express in a loose way why one might be willing to entertain the possibility that a decentralized system misproduces some of the listed properties.

Consider divisibility and imagine a world in which the production of anything that is durable--think, of a coin--costs more in resources than one slice of bread, bread being nondurable. It would seem, then, that if there is only private coinage production, one would never observe a coin that exchanges for as little as one slice of bread. Could this also be a world in which it would be convenient to have coins that exchange for as little as one slice of bread, convenient in that people would in some sense be willing to be taxed to provide a subsidy to be used to support the production of a stock of such coins? If it could be, then we would have an instance of underproduction of small denomination currency. Notice that in this loose kind of story, coins differ from other durable goods because it is posited that the services people get from coins depend upon what they exchange for. One modeling challenge is to determine whether something like this story can be made to emerge as an implication of a coherent environment or model.

My own thinking about candidate models takes off from Milton Harris' model in his 1979 paper, "Expectations and Money in a Dynamic Exchange Model." Harris' model is one of pure exchange in which the resources or endowments consist of stocks of non-depreciating durable goods that yield services in proportion to the amounts held. Time is discrete and there are a large number of infinitely lived people, each of whom maximizes the expected value of discounted instantaneous utility, where instantaneous utility at date t depends upon the vector of services from the vector of durable goods carried over from date $t - 1$ to date t . The special feature of the model--and the one that attracts me to it--is that people meet pairwise and at random at each date. This pairwise random meeting pattern is treated by Harris as ruling out all trades except spot trades in durables between people meeting at a date. Treated that way, Harris' model is as conducive to a role for media of exchange as any setup I can imagine. However, to use the Harris model for the purposes I have in mind, it has to be amended.^{2/}

All of Harris' durable goods are endowed to an infinite degree with the properties I listed above as those that make objects suitable as commodity monies. In particular, they are all perfectly divisible, durable, portable, and recognizable. Thus, for example, if we want to study divisibility, we at least have to have a version in which some or all durable goods are initially indivisible to a degree and in which there are costly technologies available for making things more divisible, say by splitting them up in some way.

Unfortunately, I have not gotten beyond this very loose way of formulating questions about currency provision. Nevertheless, even this very preliminary discussion is suggestive both about directions to pursue and about how far we have to go. Moreover, I think there is considerable hope for progress along these lines. The conjecture that properties like divisibility, durability, and recognizability are particularly important for objects that are traded frequently still seems to be a good one. And now our modelling capabilities are such that we can actually study the role of such properties.

III. Concluding Remarks

The two policy problems I have discussed--payment of interest on money and government currency provision--seem very different. I chose to discuss the first because it is often taken to be a solved problem. I chose to discuss the second because it seems so fundamental. An answer to it will not only determine whether some governmental role is desirable--the alternative being some sort of competitive money system--but will identify what that role ought to be. Certainly, the second problem subsumes the first in that any model that allows us to analyze the government's role in the financial system will also have implications for whether there should be an interest subsidy on some or all of the assets in the model.

Footnotes

1/ One example has a common utility function given by $c_1^h c_2^h$ so that the choice of c_2^h subject only to (1) is $c_2^h/w_2^h = [1-v(r)]r(w_1^h/w_2^h)/2 + [1-v(r)]/2 \leq [1-v(r)]r(w_1^h/w_2^h)/2 + 1/2$. In this case, the solution for $v(r)$ from (5) is $v(r) = [(r-1)/(r+1)](rW_1-W_2)/(rW_1+W_2)$, a function of r and aggregate endowments only. It follows that $[1-v(r)]r$ is bounded above for all r by $2(W_1+W_2)/W_1$. Therefore, if for some h , $w_1^h/w_2^h < W_1/2(W_1+W_2)$, then for that h , $c_2^h/w_2^h < 1$, which violates (2').

2/ One should also impose some version of rational expectations which Harris failed to do. To accomplish this, subjective views about future trading possibilities must be endogenous--not exogenous as in Harris' formulation--and consistent with what agents know about other agents.

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Figure 1

Figure 2